Recent development in hydrodynamic description of small colliding systems

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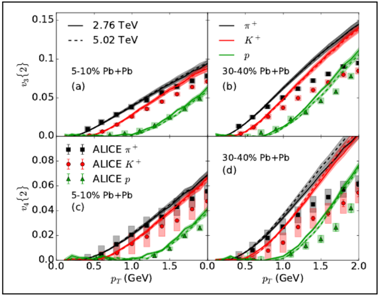
Hydro is remarkably successful in AA

Magnitude of v_n



[McDonald, Gale, Jeon, Shen, Shenke, Denicol, Ollitrault, Pal, Luzum, Niemi, Heinz,...]

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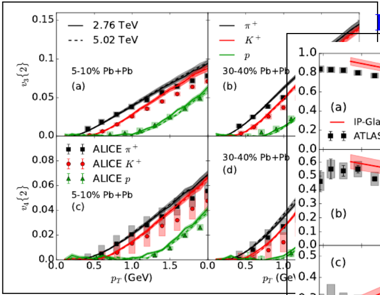

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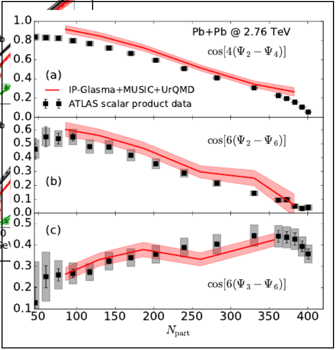
Flow correlations



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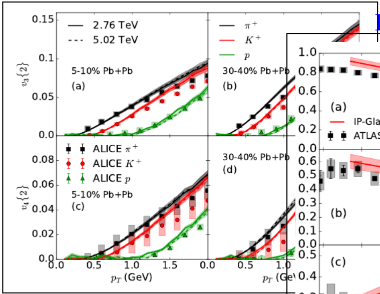

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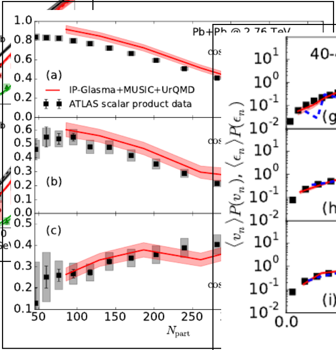
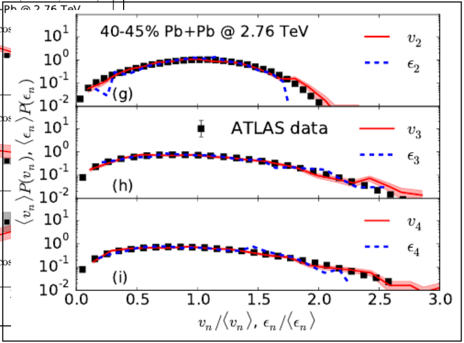


Flow fluctuations



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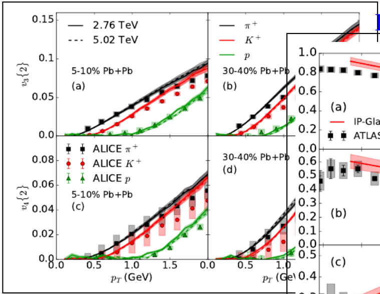

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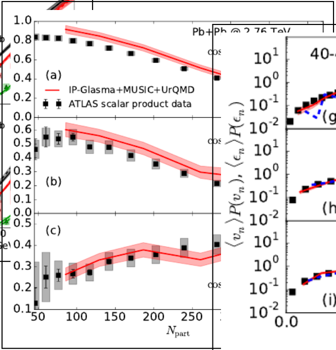
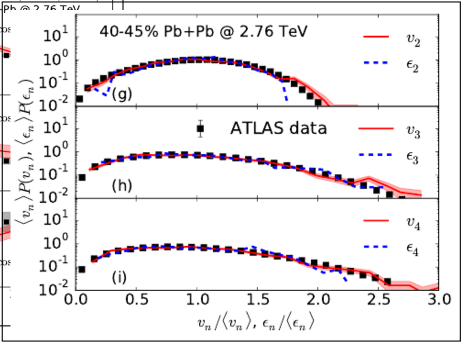


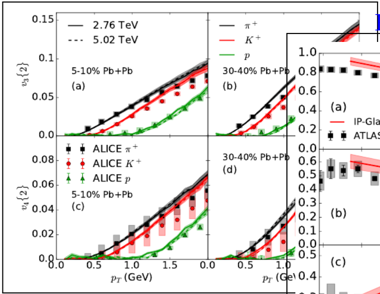
Flow fluctuations



- Flow fluctuations and correlations are more differential measurables.

Hydro is remarkably successful in AA

Magnitude of v_n



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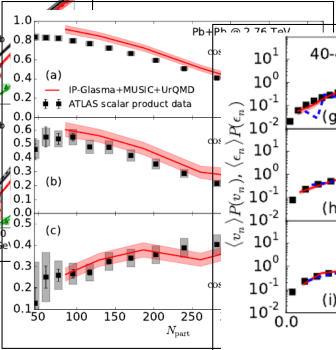
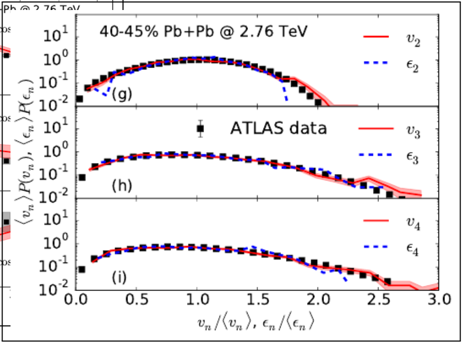
Flow correlations



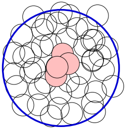
Flow fluctuations

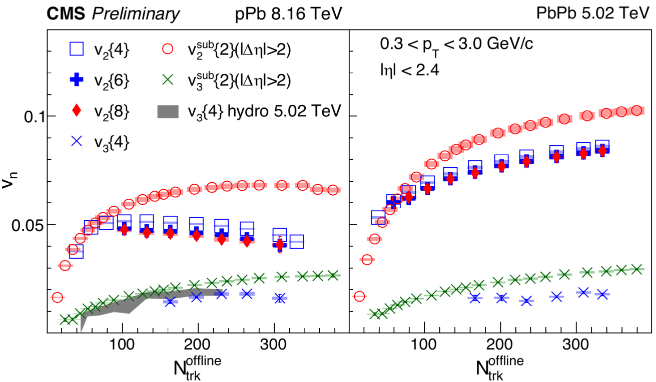


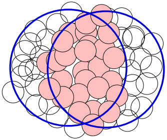
- Flow fluctuations and correlations are more differential measurables.
- Hydro paradigm: $v_n \Leftrightarrow$ QGP hydrodynamic response to IC !

i.e., $V_n = V_n(\mathcal{E}_n, \text{medium resp.})$ $\begin{cases} \mathcal{E}_n : \text{initial state geometry} \\ \text{medium resp.: fluid dynamics } \eta\,,\ \zeta \end{cases}$

Fluctuations of flow in small systems: flow cumulants !





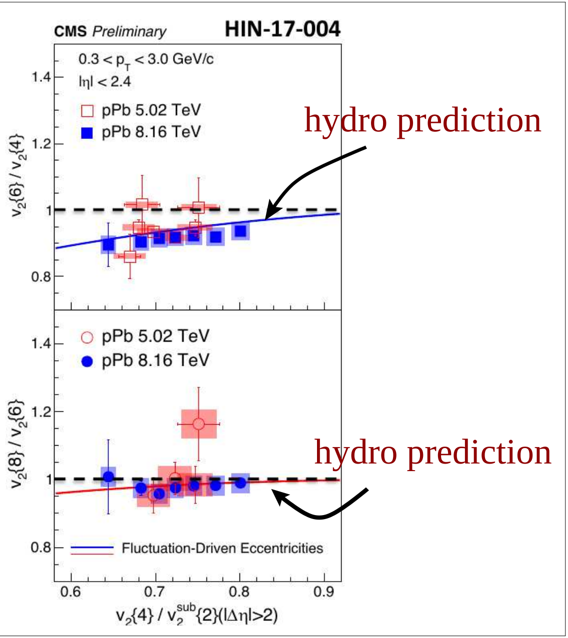


- Cumulants involve multi-particle correlations ⇒ collective phenomenon.
- Quantitative description of flow fluctuations further confirms fluidity.
- Hydro predicts universal non-Gaussianity in pA from IC (and for v_n):

  [LY, Ollitrault, Poskanzer]

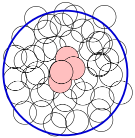
$$v_n\{2\} > v_n\{4\} \gtrsim v_n\{6\} \gtrsim v_n\{8\}, \qquad \text{NOT JUST ‘}\approx\text{’}$$

- If one takes ratios:



- Quantitatively agreement for very fine structures in flow fluctuations.

- No intrinsic shape ?



- Hydro is remarkably successful in pA!

- Why hydro is remarkably successful in pA?

Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- $T^{\mu\nu}$ is expanded in gradients *perturbatively*,

$$T^{\mu\nu} = \underbrace{eu^\mu u^\nu - P\Delta^{\mu\nu}}_{\text{ideal hydro}} + \underbrace{\eta\langle\nabla^\mu u^\nu\rangle + \zeta\Delta^{\mu\nu}\nabla\cdot u}_{\text{Navier-Stokes hydro}} + \underbrace{O(\nabla^2)}_{\text{2nd order hydro}} + O(\nabla^3) + \ldots$$

- Applicability condition of hydro: close to thermal equilibrium

$$|\nabla| \sim \mathrm{Kn} \sim \frac{\lambda_{\mathrm{mfp}}}{L} \ll 1 \quad \begin{cases} \lambda_{\mathrm{mfp}}: \text{ mean free path} \\ L: \text{ system size} \end{cases}$$

- Truncation in practical simulations: 2nd order viscous hydro.

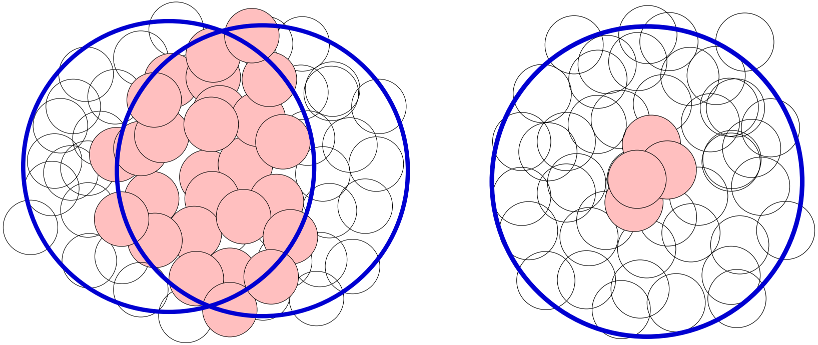
higher order (≥ 3) contributions are ignored!

- Number of gradient structures grows as n!

[Grad, Heller, Spanlinski, Denicol, Noronha, LY, Blaizot, ...]

$\Rightarrow$ Perturbative expansion in hydro is not *convergent*!

Challenges of hydro in pA: system is out-of-equilibrium!



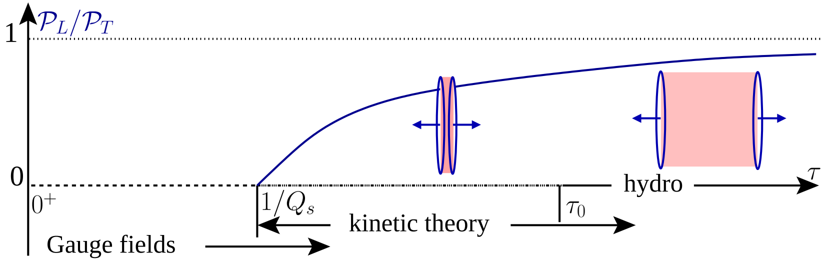
- From AA to pA system transverse size reduces,

$$L_{pA} \sim \frac{L_{AA}}{10} \sim O(1)\ \text{fm} \quad \Rightarrow \quad \text{Kn increases}$$

such that higher order gradients CANNOT be simply ignored.

A similar case of out-of-equilibrium quarks and gluons

- Pre-equilibrium evolution and thermalization in AA:



right before the onset of hydro, system size along the beam axis:

$$L_{long} \sim \tau_0 \sim O(1) \text{ fm}$$

- Thermalization (or hydrodynamization) is a long-standing question ...

We study out-of-equilibrium 1D Bjorken expansion.

Solution: out-of-equilibrium hydro from kinetic theory

[see also Heller, Svensson, Romatschke, Strickland,...]

- Transport equation with relaxation time approximation and Bjorken exp:

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p},\tau) = -\frac{f(\mathbf{p},\tau) - f_{\rm eq}(p/T)}{\tau_R}, \qquad \tau_R = \tau_R(T) \sim \frac{\eta}{s}$$

Define $\mathcal{L}_n = \int_{\mathbf{p}} p^2 P_{2n}(p_z/p)$, [LY, Blaizot]

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = \underbrace{-\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right]}_{\text{free streaming}} - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}), \quad n = 0, 1, ...$$

- $\mathcal{L}_0 = e$, $\mathcal{L}_1 = P_L - P_T$
- a_n, b_n and c_n are constant coefficients.

$$a_0 = \frac{4}{3}, \quad a_1 = \frac{38}{21}, \quad \ldots$$

- Kn$=\tau_R/\tau$ defines Knudsen number.

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}) \quad n = 0, 1, ...$$

- Truncate at n-th order: ignore all $\mathcal{L}$-moments higher than n-th order

- at $n = 0$

$$\frac{\partial e}{\partial \tau} + \frac{4}{3}\frac{e}{\tau} = 0 \quad \rightarrow \quad e \sim \tau^{-4/3} \qquad \text{ideal hydro}$$

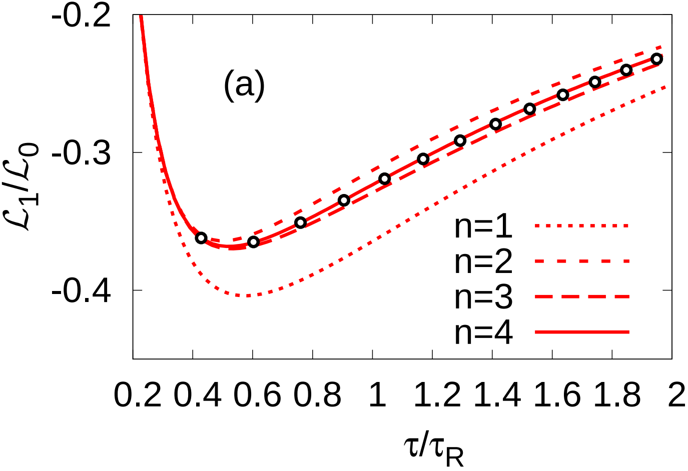
- at $n = 1$

$$\frac{\partial \mathcal{L}_0}{\partial \tau} = -\frac{1}{\tau}\left[a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right]$$
$$\frac{\partial \mathcal{L}_1}{\partial \tau} = -\frac{1}{\tau}\left[a_1\mathcal{L}_1 + b_1\mathcal{L}_0\right] - \frac{\mathcal{L}_1}{\tau_R} \qquad \supset \text{2nd order viscous hydro.}$$

- at higher orders $\Rightarrow$ higher order viscous hydro.

$\mathcal{L}_n$: nth order viscous correction term $\sim 1/\tau^n \Rightarrow$ determine hydro fixed point

A test of truncation



Truncation works well, converges to the exact solution.

The free-streaming fixed points: $\tau/\tau_R \to 0$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right]$$

For infinite n:

- $\mathcal{L}_0 = \mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \ldots$

$$\Rightarrow \mathcal{L}_n = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right)^2 \qquad \to g_n \equiv \tau\partial_\tau \ln \mathcal{L}_n = -2$$

- $\mathcal{L}_n(\tau) = P_{2n}(0)\mathcal{L}_0(\tau)$,

$$\Rightarrow \mathcal{L}_n(\tau) = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right) \qquad \to g_n \equiv \tau\partial_\tau \ln \mathcal{L}_n = -1$$

For finite n,

$$\begin{pmatrix} a_0 & c_0 & 0 & 0 & \ldots \\ b_1 & a_1 & c_1 & 0 & \ldots \\ 0 & b_2 & a_2 & c_2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} \Rightarrow \qquad \approx -2(\text{unstable}) \text{ and } \approx -1(\text{stable})$$

The hydro fixed points: $\tau/\tau_R \to \infty$

Ansatz form of gradient expansion

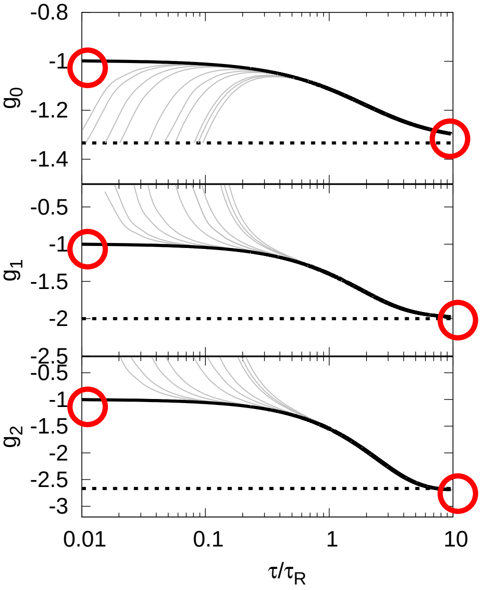
$$\mathcal{L}_n = \sum_{m=0} \frac{\alpha_m^{(n)}}{\tau^n}$$

asymptotic decay rate determined by the leading term: $\mathcal{L}_n \sim \alpha_n^{(n)}/\tau$

$$\Rightarrow g_n \equiv \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+2n}{3} \quad (\tau_R \propto 1/T)$$
$$\Rightarrow g_n \equiv \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+3n}{3} \quad (\tau_R \text{ constant})$$

These are stable fixed points in the hydro regime.

Attractor solutions: transition between fixed points



- Attractor solution for every g_n. (Infinite number of attractors).
- Attractor solution exists, with or without conformal symmetry, beyond Bjorken symmetry.

  P. Romatschke, M. Martinez, M. Strickland, G. Denicol, ...
- Attractor corresponds to Borel-summation of hydro gradient expansion.

  M.Heller, M. Spalinski, R. Janik, P. Witaszczky, G. Basar, G. Dunne, ...

Renormalization of η/s

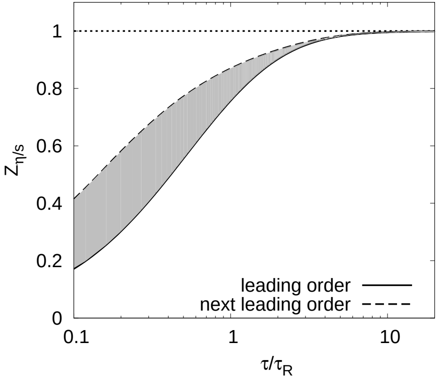
Effects from higher order moments/viscous hydro (leading order):

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_0\mathcal{L}_1)\,,$$
$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_0\mathcal{L}_0) - \underbrace{\left[1 + \frac{c_1\tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}\right]}_{Z^{-1}_{\eta/s}}\frac{\mathcal{L}_1}{\tau_R} \qquad \text{(2nd hydro)}\,,$$

$$g_2(\tau/\tau_R) = -a_2 - b_2\frac{\mathcal{L}_2}{\mathcal{L}_1} - \frac{\tau}{\tau_R}\,, \qquad \text{(Higher order hydro)}$$

- Taking attractor solution for g_2: Borel-resummed gradients.
- Absorb higher order viscous corrections by redefinition of η/s.
- Off-equilibrium effects w.r.t. 2nd order hydro $\Leftrightarrow$ renormalized η/s !

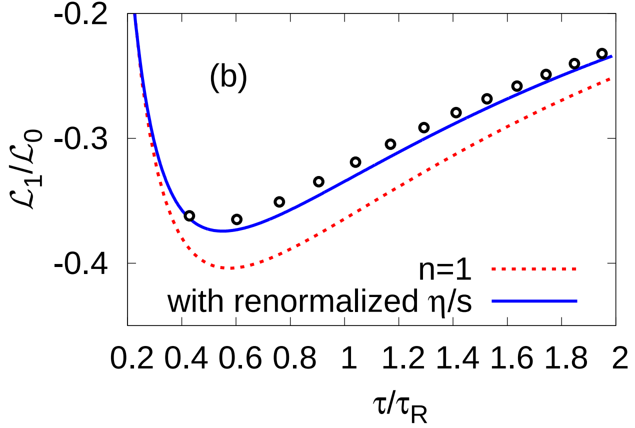
Renormalization of η/s



- Out-of-equilibrium physics can be effectively absorbed into a reduced η/s. [E. Shuryak, M. Lublinsky, P. Romatschke]
- Non-Newtonian fluid with shear thinning: [A. Behtash, S. Kamata, and M. Martinez]

$$\eta = \eta(\nabla) \qquad \text{and} \qquad \frac{d\eta}{d\nabla} < 0$$

Numerical test of η/s renormalization



2nd order viscous hydro using renormalized η/s

Summary

- Hydro has been applied to small colliding systems successfully.
- It is very likely that in small systems the medium is out-of-equilibrium.
- Out-of-equilibrium hydro requires summation of higher order terms.
- Borel resummation lead to attractors, from hydro and kinetic theory.
- Resummed higher orders can be absorbed into redefinition of η/s
- 2nd order viscous hydro works with renormalized η/s.

Back-up slides

Solution: out-of-equilibrium hydro

1/ From the perspective of fluid dynamics,

- MIS hydro with Bjorken symmetry ($\tau = \sqrt{t^2 - z^2}, \xi = \tanh^{-1}(z/t)$)

$$\tau \dot{e} = -\frac{4}{3} e + \pi_\xi^\xi \,, \qquad \tau_\pi \dot{\pi}_\xi^\xi = \frac{4\eta}{3\tau} - \frac{\lambda_1 (\pi_\xi^\xi)^2}{2\eta^2} - \frac{4\tau_\pi \pi_\xi^\xi}{3\tau} - \pi_\xi^\xi$$

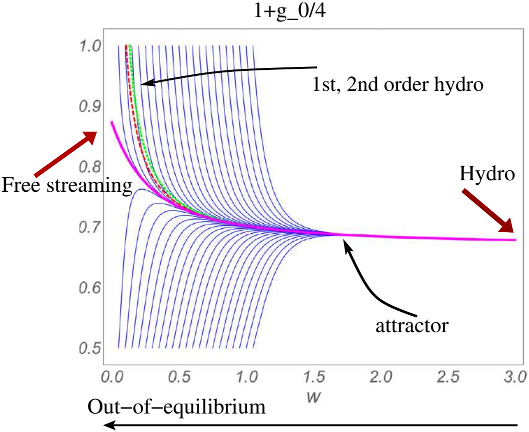
for which $\mathrm{Kn}^{-1} = w = \tau T$.

- Hydro solution with gradient expansion:

$$e(\tau) = e_0 \left(\frac{\tau_0}{\tau}\right)^{4/3} \left[1 + \frac{e_1}{\tau^{2/3}} + \frac{e_2}{\tau^{4/3}} + \ldots\right]$$

or

$$g_0 \equiv \frac{\partial \ln e}{\partial \ln \tau} = \underbrace{-\frac{4}{3}}_{\text{ideal hydro}} + \sum_{n=1}^{\infty} \alpha_n w^{-n} \,, \qquad \alpha_n \sim n!$$



[M. Heller, M. Spalinski, 2015]

- The quantity g_0 has an attractor solution (the purple line).
- Borel resummation and trans-series,

$$g_0(w) = \sum_{n=0}^{\infty} \alpha_n^{(0)} w^{-n} + c e^{-\frac{3w}{2c_{\tau_\pi}}} \left(w^{\frac{c_\eta - 2c_{\lambda_1}}{c_{\tau_\pi}}} \sum_{n=0} \alpha_n^{(1)} w^{-n} \right) + \dots$$

- Non-hydro modes decay exponentially, w.r.t. attractor solutions.

[M. Heller, M. Spalinski, P. Romatschke, A. Kurkela, U. Wiedemann, ...]