Phase diagram of two-color QCD matter at finite baryon and axial isospin densities

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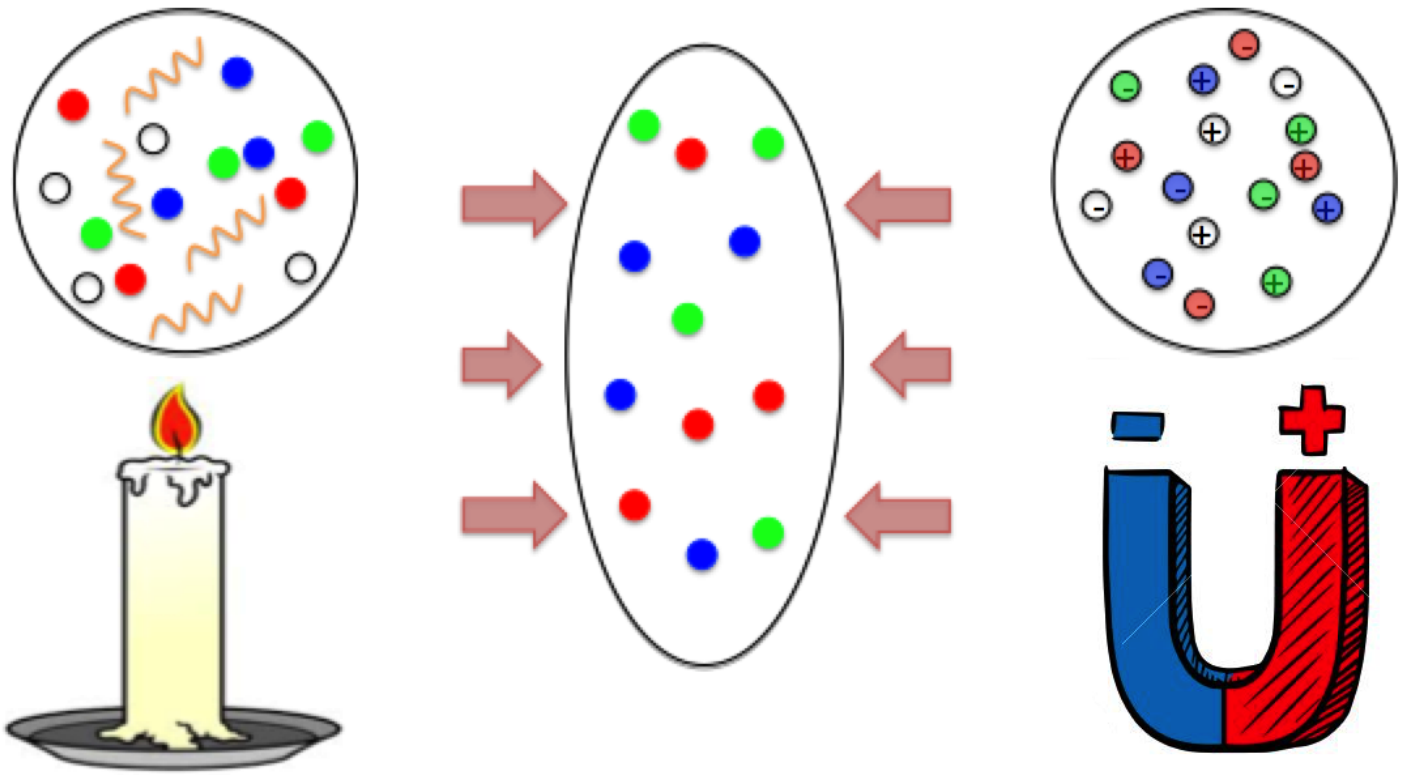
Outline

- Motivations
  - ✓ Different phases of QCD occur in the universe
  - ✓ QCD simplifies in extreme environments
  - ✓ The behaviors of different matter can be similar at the regime of transition
- QCD like theories
- Phase diagram in the plane of $\mu - \nu_5$

Why Electromagnetic Fields

Heavy ion collisions create the strongest magnetic fields in the Laboratory.



Different excited freedoms at different environments

Propagators at Constant Magnetic fields

- spin-0 c.f. (A. Erdas and G. Feldman, Nucl. Phys. B 343, 597)

$$D(k) = \int_0^\infty \frac{ds}{\cos(eBs)} \exp\left[is\left(k_\parallel^2 + k_\perp^2 \frac{\tan(eBs)}{eBs} - m^2\right)\right]$$

where $k_\parallel = (k_0, 0, 0, k_3)$ and $k_\perp = (0, k_1, k_2, 0)$.

- spin-$\frac{1}{2}$ c.f. (J. Schwinger, Phys. Rev. 82, 664)

$$S(k) = \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left(k_\parallel^2 + k_\perp^2 \frac{\tan(eBs)}{eBs} - m^2\right)} \left[(m + \not{k}_\parallel)\, e^{-ieBs\gamma_3} + \frac{\not{k}_\perp}{\cos(eBs)}\right]$$

- spin-1 c.f. (A. Erdas and G. Feldman, Nucl. Phys. B 343, 597)

$$G^{\mu\nu}(k) = \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left(k_\parallel^2 + k_\perp^2 \frac{\tan(eBs)}{eBs} - m^2\right)} \left[F^{\mu\nu}\frac{\sin(2eBs)}{B} - g_\parallel^{\mu\nu} - g_\perp^{\mu\nu}\cos(2eBs)\right]$$

where $g_\parallel^{\mu\nu} = \mathrm{diag}(+1, 0, 0, -1)$; $g_\perp^{\mu\nu} = \mathrm{diag}(0, -1, -1, 0)$ and $F^{\mu\nu}$ is the electromagnetic field tensor. 3 different tensor structures !

Landau Levels

- Another expression of fermion propagator in the B field:

$$S(k) = i \exp\left[-\frac{k_\perp^2}{eB}\right] \sum_{n=0}^{\infty} \frac{(-1)^n D_n(eB, k)}{k_0^2 - k_3^2 - M^2 - 2neB}$$

- Free energy spectrum

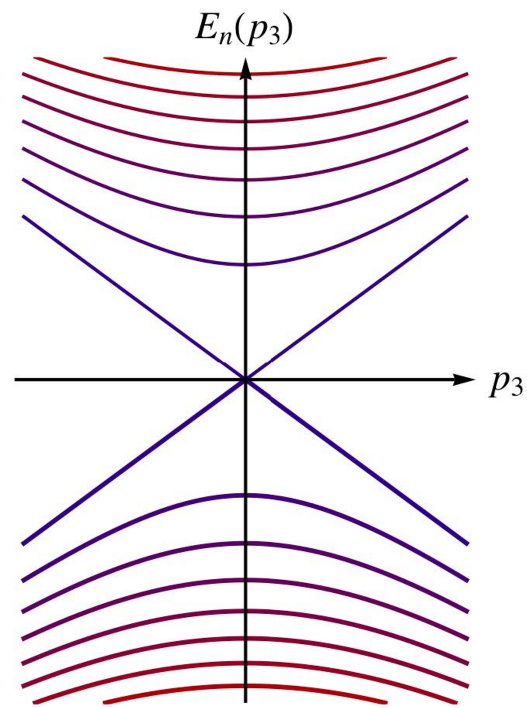
$$E_n^{3+1}(k_3) = \pm\sqrt{(2n + 2s_3 + 1)|eB| + k_3^2 + m^2}$$

where s_3 is the projection of the spin on the B field and $n = 0, 1, 2, \ldots$ is the orbital quantum number.

- Lowest Landau Level assumption of fermions in 3-direction while $B \longrightarrow \infty$

$$E_0^{3+1}(k_3) = \pm|k_3|$$

- Is the effective mass of fermions able to treat as $m_{\text{eff}}^2 = m^2 + 2neB$ at n-th level ?



Equivalent Replacement

How to get a quick physics answer in phenomena?

☞ As one of candidate to explain the inverse magnetic catalysis, QCD phase transition has been intensively studied at finite chiral chemical potential μ_5. c.f. (JC, P. Chu and M. Huang, Phys. Rev. D, 88, 054009 (2013).)

Indeed, nonzero n_5 inducing in electromagnetic field is expressed by: c.f. (K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008))

$$\partial_\mu \left(j_5^3\right)^\mu = -\frac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}\cdot \mathrm{tr}\left[\tau^3 Q^2\right]$$

where F is the electromagnetic field strength.

It turns out $\nu_5\bar{\psi}\gamma_0\gamma_5\tau_3\psi$ is added into the Lagrangian because of the coupling of quarks to electromagnetism, i.e., electromagnetic anomaly.

Fermion-Sign Problem

Presented by the γ_5-hermiticity:

$$D(\mu) = \not{D} + m - \gamma_0 \mu$$

$$\gamma_5 \, D^\dagger(\mu) \, \gamma_5 = D(-\mu^*) \Longrightarrow \det D(\mu) = (\det D(-\mu^*))^*$$

Some QCD-like theories (two-color QCD or finite isospin QCD) free of the sign problem, where the analytic continuation can be compared with Monte Carlo determinations obtained directly at real chemical potentials

- QCD with μ_I

  finite isospin QCD $\quad \det D(\mu_I) \cdot \det D(-\mu_I) = |\det D(\mu_I)|^2$

- Dirac operator satisfies the anti-unitary symmetry

  two-color QCD $\quad D(\mu) \, t_2 C \gamma_5 = t_2 C \gamma_5 D(\mu)^*$

QCD-like Theory in $N_c = 2$

The global symmetries of the $\mathrm{SU}(N_f)$ flavor space can be extended to $\mathrm{SU}(2N_f)$

$$\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix}, \quad \Psi^\dagger = \left(\psi_L^\dagger, \tilde{\psi}_R^\dagger\right)$$

at $N_c = 2$ since it allows to rotate ψ_R^C into ψ_L where $\tilde{\psi}_R = -it_2 C \psi_R^*$ and C is the complex conjugate operator in spinor space.

- Symmetry is established by the connection between quarks and antiquarks
- Color-neutral bound states of two quarks, bosonic baryons
  mesons and scalar diquarks become degenerate!
- Similar phase diagram as for N_c=3
  pion and diquark condensation at finite μ_I and μ_B, respectively

Patterns of symmetry breaking

$$\mathcal{L}_{\text{mass}} = \frac{m_0}{2}\left(\Psi^T i t_2 C E_4 \Psi - \Psi^{*T} i t_2 C E_4 \Psi^*\right)$$

The mass operator

$$E_4 = \begin{pmatrix} 0 & \tau_0 \\ -\tau_0 & 0 \end{pmatrix}$$

obey the relation $S_a^T E_4 + E_4 S_a = 0$ for $S^a \in \text{Sp}(2)$

$$\text{SU}(4) \to \text{Sp}(2) \quad \text{at } m \neq 0$$

The chemical potential μ is introduce by $\mu \Psi^\dagger B_0 \Psi$ via the $\mathcal{L}_{\text{kin}} = \Psi^\dagger i \sigma^\mu D_\mu \Psi$

$$B_0 = -\gamma_0 E_4 = \begin{pmatrix} \tau_0 & 0 \\ 0 & -\tau_0 \end{pmatrix}$$

breaks

$$\text{SU}(4) \to \text{SU}_L(2) \times \text{SU}_R(2) \quad \text{at } \mu \neq 0$$

Incorporating External Fields in ChPT

- Including external gauge fields in QCD

$$\mathcal{L}_\psi = \bar{\psi}_L \not{D}_L \psi_L + \bar{\psi}_R \not{D}_R \psi_R$$

$$(D_L)_\mu = \partial_\mu + ig\mathcal{A}_\mu + iL_\mu \quad (D_R)_\mu = \partial_\mu + ig\mathcal{A}_\mu + iR_\mu$$

Covariant derivative is required to compensate the gauge transformation of $\psi_L \to L(x)\psi_L$ and $\psi_R \to R(x)\psi_R$

- Incorporate external gauge fields in ChPT

$$\Sigma \to L(x)\Sigma R^\dagger(x)$$
$$D_\mu\Sigma = \partial_\mu\Sigma + iL_\mu\Sigma - i\Sigma R^\dagger_\mu$$

Bilinear fields Σ are modified to keep local invariance after transformation

Chiral Lagrangian and Its Predictions

The isospin density, $\nu\,\bar{\psi}\gamma_0\tau_3\psi$, is written as $\nu\,\Psi^\dagger I_0\Psi$ with $I_0 = \mathrm{Diag}(\tau_3, -\tau_3)$. Applying the gauge transformation

$$\Psi \to V\,\Psi, \quad V = \exp\left(i\theta^i X_i\right)$$

The Lagrangian term of leading order in ChPT is shown as

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(D_\mu\Sigma\right)^\dagger\left(D_\mu\Sigma\right) - c\,\mathrm{Tr}\left(\Sigma^\dagger + \Sigma\right)$$

The isospin baryon current is embedded in $\pi_\pm$ mesons

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_{\pi_i} \sim \mathrm{Tr}\left[\tau_3\nu, \Sigma\right]\left[\Sigma^\dagger, -\tau_3\nu\right] \sim +\nu^2\mathrm{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right)$$

one has $[X_i, I_0] \neq 0$ for $i = 1, 2$.

Chiral Lagrangian and Its Predictions, cont

Similarly, one has $[X_i, B_0]$ becoming nonzero for $i = 4, 5$ and

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_{\Delta,\Delta^*} \sim \mathrm{Tr}\left\{\tau_0\mu, \Sigma\right\}\left\{\Sigma^\dagger, -\tau_0\mu\right\} \sim -\mu^2\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)$$

The different commutation bracket is corresponding to different composition

It predicts that $\langle\bar{q}q\rangle \neq 0$ and $\langle qq\rangle \neq 0$ for $\mu \geq \frac{m_\pi}{2}$.

$\nu_5\bar{\psi}\gamma_0\gamma_5\tau_3\psi$ is expressed as $\nu_5\Psi^\dagger I_5\Psi$ with $I_5 = \gamma_5 I_0 = \mathbf{Diag}(\tau_3, \tau_3)$ where $[X_i, I_5] \neq 0$ for $i = 1, 2, 4, 5$ and $[S_a, I_5] \neq 0$ for $a = 1, 2, 8, 9$. Therefore,

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_{\pi_i} \sim \mathrm{Tr}\left[\tau_3\nu_5, \Sigma\right]\left[\Sigma^\dagger, \tau_3\nu_5\right] \sim -\nu_5^2\mathrm{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right)$$

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_{\Delta,d} \sim \mathrm{Tr}\left\{\tau_3\nu_5, \Sigma\right\}\left\{\Sigma^\dagger, \tau_3\nu_5\right\} \sim +\nu_5^2\mathrm{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right)$$

Diquarks are energy favored and $\mathbf{Sp(2)}$ reduce to $\mathbf{O(4)}$ at high baryon density

NJL Model at $N_c = 2$ w.r.t. Diquarks

In terms of Nambu-Gorkov bispinors

$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix}, \quad \bar{\psi} = \frac{1}{\sqrt{2}} (\bar{q}, \, q^T C)$$

$$\nu_5 = \mu_L^u - \mu_R^u = \mu_R^d - \mu_L^d \Rightarrow \left(\psi_L^u \psi_R^d \pm \psi_R^u \psi_L^d\right)$$

τ_1 sector locates in the flavor symmetric channel and spin direction of $s = 1, s_z = 0$ is selected due to the Pauli principle.

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi} \left(i\gamma^\mu \partial_\mu + \mu\gamma_0 + \nu_5\gamma_0\gamma_5\tau_3 - m_0\right) \psi + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$$

$$\mathcal{L}_{\bar{q}q} = \frac{G}{2} \left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma_5 \vec{\tau} \psi)^2\right]$$

$$\mathcal{L}_{qq} = \frac{H}{2}(\bar{\psi}\, i\gamma_5\tau_2 t_2 C\bar{\psi}^T)(\psi^T C\, i\gamma_5\tau_2 t_2\psi) - \frac{H}{4}(\bar{\psi}\gamma_3\tau_1 t_2 C\bar{\psi}^T)(\psi^T C\gamma_3\tau_1 t_2\psi)$$

Energy Dispersion

Applying $\pi_3 = -\frac{G}{2}\langle\bar{\psi}\, i\gamma_5\tau_3\psi\rangle$, $\Delta = -\frac{H}{2}\langle\psi^T\, i\gamma_5\tau_2 t_2 C\psi\rangle$, $\Delta^* = -\frac{H}{2}\langle\bar{\psi}\, i\gamma_5\tau_2 t_2 C\bar{\psi}^T\rangle$, $d = -\frac{H}{4}\langle\psi^T\gamma_3\tau_1 t_2 C\psi\rangle$ and $d^* = -\frac{H}{4}\langle\bar{\psi}\gamma_3\tau_1 t_2 C\bar{\psi}^T\rangle$,

$$\Omega_{\text{eff}} = \frac{1}{2}\bar{\psi}\mathcal{S}^{-1}\left(p;\pi_3,\Delta,d\right)\psi - \frac{\pi_3^2 + |\Delta|^2 + |d|^2}{4G}$$

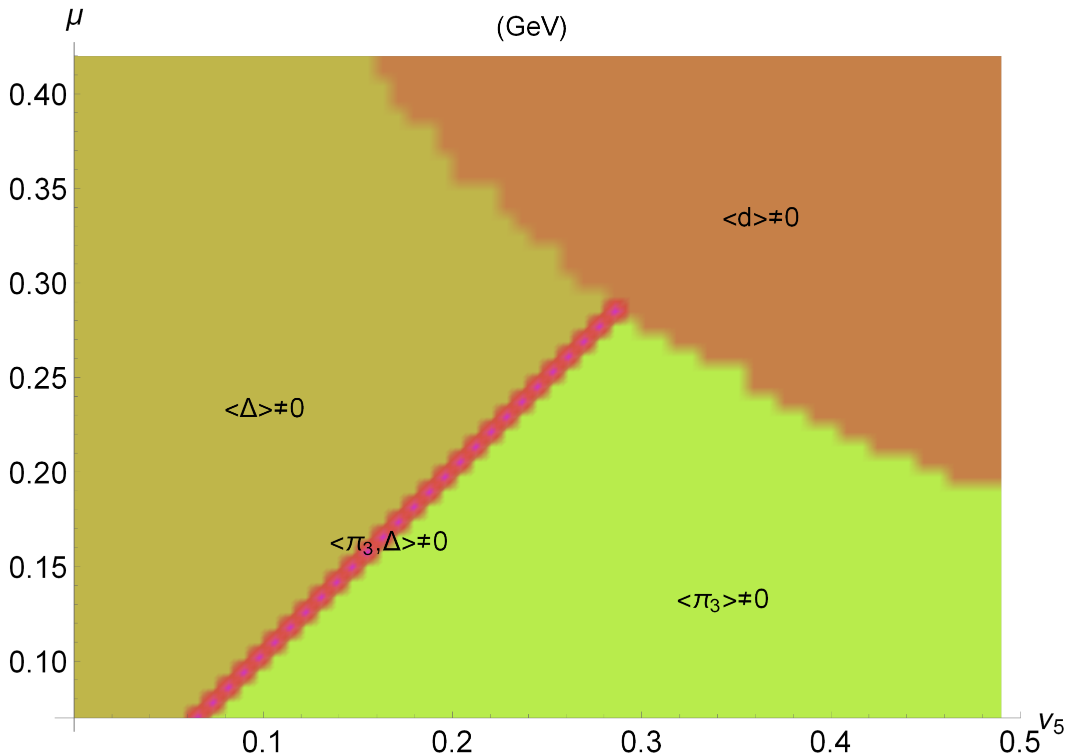
Here, the inverse propagator of fermion is

$$\mathcal{S}^{-1}(p) = \begin{pmatrix} \not{p} - M + \gamma_0\mu + \gamma_0\gamma_5\tau_3\nu_5 & \gamma_5\tau_2\Delta + \gamma_3\tau_1 d \\ -\gamma_5\tau_2\Delta^* - \gamma_3\tau_1 d^* & \not{p} - M^T - \gamma_0\mu + \gamma_0\gamma_5\tau_3\nu_5 \end{pmatrix}$$

The energy dispersion in the chiral limit with $\pi_3 = \Delta = \Delta^* = 0$ for real d

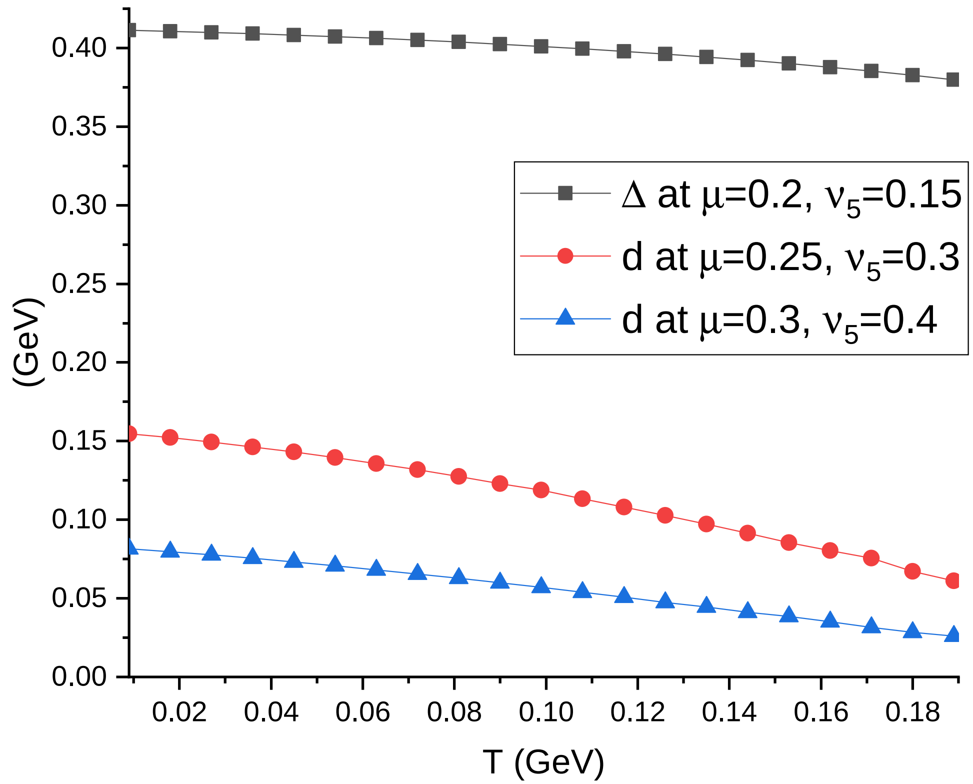
$$E_i(p) = \pm\sqrt{d^2 + \mathbf{p}^2 + \left(\mu \pm \nu_5\right)^2 \pm 2\sqrt{d^2 p_z^2 + \mathbf{p}^2\left(\mu \pm \nu_5\right)^2}}$$

Phase Diagram in QC_2D



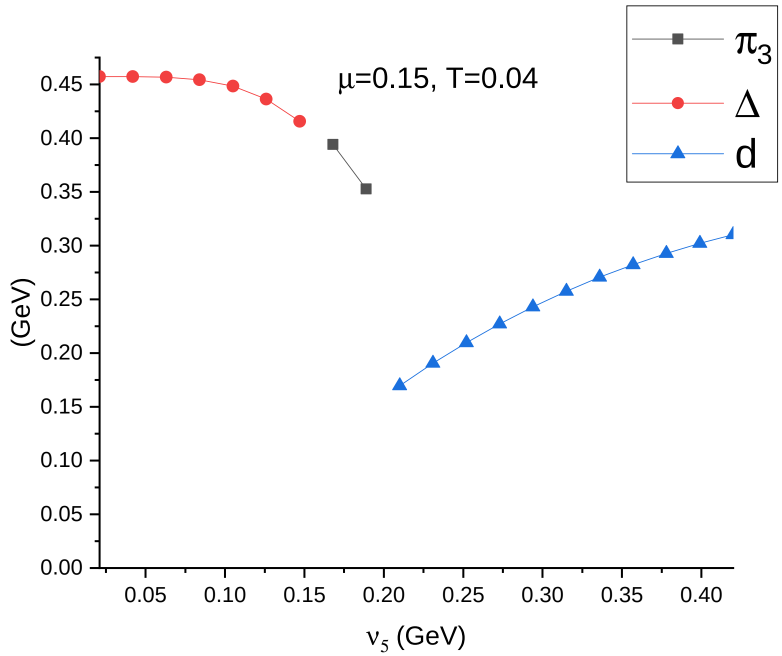
Phase diagram of two-color QCD within two fundamental quarks in the $\mu - \nu_5$ plane. It contains four different states: superfluid pion π_3, condenses of scalar diquark Δ and axial vector diquark d and a mixed state composed by π_3 and Δ. c.f. (JC, arXiv:1808.01928)

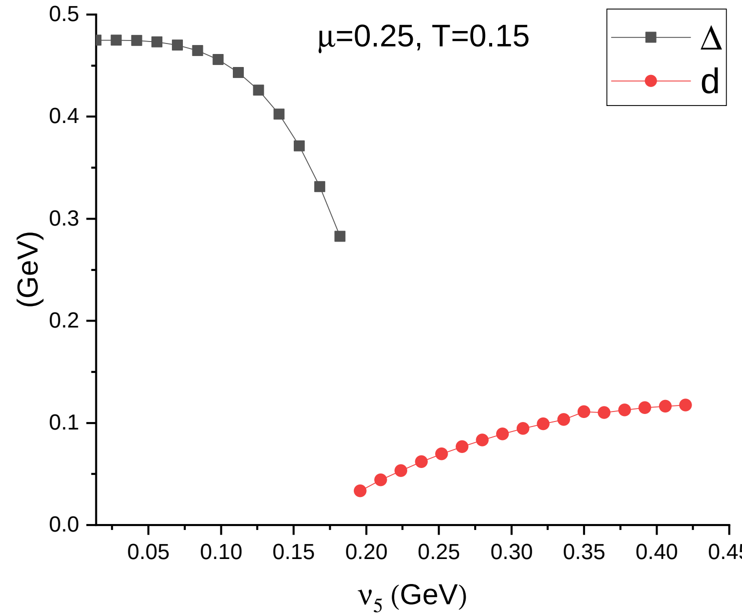
Results at Finite Temperature (I)



Scalar diquark condensate Δ and axial vector diquark d as a function of T at given μ and ν_5. The units of μ and T are GeV. c.f. (JC, arXiv:1808.01928)

Results at Finite Temperature (II)





Neutral pion condensate π_3, scalar diquark condensate Δ and axial vector diquark d as a function of ν_5 at given μ and T. The units of μ and T are GeV. c.f. (JC, arXiv:1808.01928)

Constraints in Hybrid Star from Modern Cooling Data

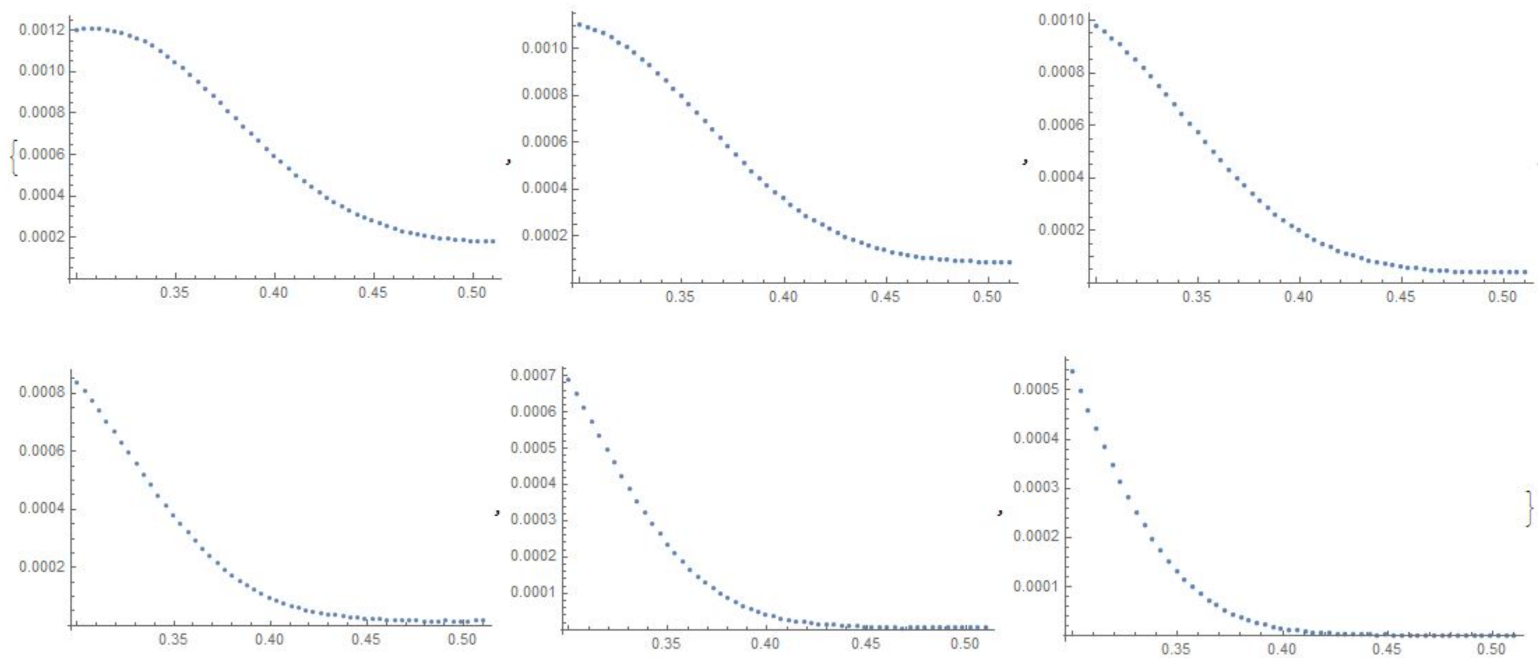
Emission of neutrino processed is applied in quark matter for cooling the star

- All quarks need to be paired
- The smallest gaps should be in the range $10 \sim 100\,\mathrm{keV}$
- The smallest gaps should have a decreasing density dependence in the relevant domain of $\mu_{\mathrm{crit}} \leq \mu \leq 0.5\mathrm{GeV}$

c.f. (H. Grigorian, D. Blaschke and D. Voskresensky, Phys. Rev. C 71, 045801)

Fitted Cooling Behaviors at $N_c = 3$

A linear combination of colors, $\lambda_{A=2,5,7}$ locking with spin, $(C\gamma_{i=1,2,3})$, it remains an unbroken global SO(3) of mixture and the gap is isotropic



The decreasing gap of spin one diquark d in the range of $\mu \in (0.35, 0.5)$ GeV for $\nu_5 = 0.25 \pm 0.025\,\text{GeV}$. c.f. (JC and C. Zhou, Preliminary)

Summary

- Explore the new phases for QCD matter after turning on $\mathbf{E} \cdot \mathbf{B}$.
- Take home message: the possible cooling mechanism in neutron stars.
- Include Polyakov loop dynamics.

Thank You for Your Attention!