Second-order dissipative magneto-hydrodynamics

Dirk H. Rischke

with

Gabriel S. Denicol, Xu-Guang Huang, Etele Molnár, Gustavo M. Monteiro,

Harri Niemi, Jorge Noronha, Qun Wang

PRD 98 (2018) 076009 [arXiv:1804.05210 [nucl-th]]

The 7th Asian Triangle Heavy-Ion Conference ATHIC 2018, USTC, Hefei, China, Nov. 3 – 6, 2018

Novel transport phenomena have been suggested in systems with chiral fermions:

- Chiral Magnetic Effect (CME): $j_V^\mu \sim \mu_A B^\mu$

  K. Fukushima, D.E. Kharzeev, H.J. Warringa, PRD 78 (2008) 074033

- Chiral Separation Effect (CSE): $j_A^\mu \sim \mu_V B^\mu$

  M.A. Metlitski, A.R. Zhitnitsky, PRD 72 (2005) 045011

- Chiral Magnetic Wave (CMW): interplay of CME and CSE

  D.E. Kharzeev, H.U. Yee, PRD 83 (2011) 085007

- Chiral Vortical Effect (CVE): $j_V^\mu \sim \omega^\mu$

  D.T. Son, P. Surowka, PRL 103 (2009) 191601

- . . .

(see also A. Vilenkin, PRD 20 (1979) 1807; PRD 21 (1980) 2260; PRD 22 (1980) 3080)

Motivation

$\Longrightarrow$ quantitative understanding of these novel phenomena requires:
relativistic magneto-hydrodynamics (MHD) for spin-1/2 particles

However: dissipation important in small systems such as QGP created in HIC's

$\Longrightarrow$ requires dissipative relativistic MHD for spin-1/2 particles

However: first-order dissipative fluid dynamics is acausal and unstable

$\Longrightarrow$ requires second-order dissipative relativistic MHD for spin-1/2 particles

For massless particles (chiral limit): "chiral" (or "anomalous") MHD

$\Longrightarrow$ macroscopic derivation via 2^{nd} law of thermodynamics
D.E. Kharzeev, H.U. Yee, PRD 84 (2011) 045025

$\Longrightarrow$ Goal of this work:

Microscopic derivation of second-order dissipative relativistic MHD for spin-0 particles

(not restricted to chiral limit)

Definitions

CRC-TR 211

"Dictionary"

- Non-resistive: electric conductivity $\sigma_E \to \infty$ $\Longrightarrow$ "ideal" MHD
- Resistive: electric conductivity $0 < \sigma_E < \infty$ $\Longrightarrow$ resistive MHD
- Fluid-dynamical transport coefficients: $\sim \lambda_{\rm mfp}$ mean free path
- Non-dissipative: all fluid-dynamical transport coefficients vanish
  $\Longrightarrow$ "ideal" fluid dynamics
- Dissipative: (some) fluid-dynamical transport coefficients non-zero
  $\Longrightarrow$ dissipative/viscous fluid dynamics
- Second-order dissipative: relaxation equations for dissipative currents

Strategy: successively reduce idealizing constraints

$\Longrightarrow$ Non-resistive, second-order dissipative MHD for spin-0 particles
G.S. Denicol, X.-G. Huang, E. Molnár, G.M. Monteiro, H. Niemi, J. Noronha, DHR, Q. Wang, PRD 98 (2018) 076009 [arXiv:1804.05210 [nucl-th]]

$\Longrightarrow$ Resistive, second-order dissipative MHD for spin-0 particles
G.S. Denicol, E. Molnár, H. Niemi, DHR, in preparation

$\Longrightarrow$ Resistive, second-order dissipative MHD for spin-1/2 particles
N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, DHR, in preparation

CRC-TR 211

Maxwell's equations

$$\partial_\mu F^{\mu\nu} = \mathfrak{J}^\nu$$
$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

- $\mathfrak{J}^\mu$ electric charge current
- $F^{\mu\nu}$ field-strength tensor
- $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ dual field-strength tensor

Tensor decomposition

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$$
$$\tilde{F}^{\mu\nu} = B^\mu u^\nu - B^\nu u^\mu - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_\beta$$

- u^μ time-like four vector, $u^\mu u_\mu = 1$
- $E^\mu \equiv F^{\mu\nu} u_\nu$ electric field four-vector
- $B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$ magnetic field four-vector

$\Longrightarrow$ by definition: $E^\mu u_\mu = B^\mu u_\mu = 0$, $E^\mu_{\text{LRF}} = (0, \mathbf{E})$, $B^\mu_{\text{LRF}} = (0, \mathbf{B})$

$\Longrightarrow$ for given $\mathfrak{J}^\mu$, Maxwell's equations determine 6 independent components of E^μ, B^μ

Energy-momentum tensor of electromagnetic field

$$T^{\mu\nu}_{\text{em}} = -F^{\mu\lambda}F^\nu{}_\lambda + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta}F_{\alpha\beta}$$

$\Longrightarrow$ using Maxwell's equations:

Field energy-momentum evolution equation

$$\partial_\nu T^{\mu\nu}_{\text{em}} = -F^{\mu\nu}\mathfrak{J}_\nu$$

Particle current and energy-momentum tensor of the fluid

Single-component fluid of point-like particles with spin zero and mass m

Particle current and energy-momentum tensor of fluid

$$N_{\rm f}^\mu \equiv \int dK\, k^\mu\, f_{\mathbf{k}} = n_{\rm f} u^\mu + V_{\rm f}^\mu$$

$$T_{\rm f}^{\mu\nu} \equiv \int dK\, k^\mu k^\nu\, f_{\mathbf{k}} = \varepsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- $k^\mu = (k^0, \mathbf{k})$ four-momentum of particles, $k^0 = \sqrt{\mathbf{k}^2 + m^2}$ on-shell energy,
- $dK = d^3\mathbf{k}/\left[(2\pi)^3 k^0\right]$
- $f_{\mathbf{k}}$ single-particle distribution function in momentum space
- u^μ fluid four-velocity $\Longrightarrow$ taken to be energy flow (Landau frame), $T_{\rm f}^{\mu\nu} u_\nu = \varepsilon u^\mu$
- $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ 3-space projector orthogonal to u^μ
- $n_{\rm f} \equiv N_{\rm f}^\mu u_\mu$ particle density in LRF
- $\varepsilon \equiv T_{\rm f}^{\mu\nu} u_\mu u_\nu$ energy density in LRF
- $P \equiv -\frac{1}{3} T_{\rm f}^{\mu\nu} \Delta_{\mu\nu}$ isotropic pressure
- $V_{\rm f}^\mu \equiv N_{\rm f}^{\langle\mu\rangle}$ particle diffusion current, where $A^{\langle\mu\rangle} \equiv \Delta^{\mu\nu} A_\nu$
- $\pi^{\mu\nu} \equiv T_{\rm f}^{\langle\alpha\beta\rangle}$ shear-stress tensor, where $A^{\langle\alpha\beta\rangle} \equiv \Delta^{\mu\nu}_{\alpha\beta} A^{\alpha\beta}$

  $\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$ rank-4 symmetric, traceless 3-space projector orthogonal to u^μ

Conservation equations in MHD

Introduce electric charge of particles $\mathfrak{q}$

Charge current of fluid

$$\mathfrak{J}_\mathrm{f}^\mu \equiv \mathfrak{q}\, N_\mathrm{f}^\mu = \mathfrak{n}_\mathrm{f} u^\mu + \mathfrak{V}_\mathrm{f}^\mu$$

- $\mathfrak{n}_\mathrm{f} \equiv \mathfrak{q} n_\mathrm{f}$ charge density in LRF
- $\mathfrak{V}_\mathrm{f}^\mu \equiv \mathfrak{q} V_\mathrm{f}^\mu$ charge diffusion current

To leading order $\mathfrak{V}_\mathrm{f}^\mu \simeq \mathfrak{J}_\mathrm{ind}^\mu = \sigma_E E^\mu$, Ohmic induction current

Fluid charge conservation

$$\partial_\mu \mathfrak{J}_\mathrm{f}^\mu = 0$$

Introduce
- external charge current $\mathfrak{J}_\mathrm{ext}^\mu \quad \Longrightarrow \quad \mathfrak{J}^\mu = \mathfrak{J}_f^\mu + \mathfrak{J}_\mathrm{ext}^\mu$
- total energy-momentum tensor $T^{\mu\nu} = T_\mathrm{f}^{\mu\nu} + T_\mathrm{em}^{\mu\nu}$

Energy-momentum evolution equation

$$\partial_\nu T^{\mu\nu} = -F^{\mu\nu} \mathfrak{J}_{\mathrm{ext},\nu}$$

$\Longrightarrow$ using energy-momentum evolution of electromagnetic field:

Fluid energy-momentum evolution equation

$$\partial_\nu T_\mathrm{f}^{\mu\nu} = F^{\mu\nu} \mathfrak{J}_{\mathrm{f},\nu}$$

Assume electric conductivity $\sigma_E \to \infty$

(Note: idealization, inconsistent with transport coefficients $\sim \lambda_{\text{mfp}}$!)

$\Longrightarrow$ induced current $\mathfrak{J}^\mu_{\text{ind}} \equiv \sigma_E E^\mu$ needs to stay finite $\Longrightarrow$ $E^\mu \to 0$

$\Longrightarrow$ only magnetic field $B^\mu \neq 0$

Introduce

- $b^\mu \equiv \dfrac{B^\mu}{B}$, $B \equiv \sqrt{-B^\mu B_\mu}$ $\Longrightarrow$ $b^\mu u_\mu = 0$, $b^\mu b_\mu = -1$
- $b^{\mu\nu} \equiv -\epsilon^{\mu\nu\alpha\beta} u_\alpha b_\beta \equiv -\dfrac{F^{\mu\nu}}{B}$ $\Longrightarrow$ $b^{\mu\nu} u_\mu = b^{\mu\nu} u_\nu = 0$
- $\Xi^{\mu\nu} \equiv \Delta^{\mu\nu} + b^\mu b^\nu$ 2-space projector orthogonal to u^μ and b^μ $\Longrightarrow$ $b^{\mu\alpha} b^\nu{}_\alpha = \Xi^{\mu\nu}$

Equation of motion for fluid energy

$$u_\mu \partial_\nu T^{\mu\nu}_{\text{f}} = -B b^{\mu\nu} \mathfrak{J}_{\text{f},\nu} u_\mu = 0$$

$\Longrightarrow$ fluid energy is conserved!

Equation of motion for fluid momentum

$$\Delta^\alpha_\mu \partial_\nu T^{\mu\nu}_{\text{f}} = -B b^{\alpha\nu} (\mathfrak{n}_{\text{f}} u_\nu + \mathfrak{V}_{\text{f},\nu}) = -B b^{\alpha\nu} \mathfrak{V}_{\text{f},\nu}$$

$\Longrightarrow$ fluid momentum changes only through coupling of B with $\mathfrak{V}^\mu_{\text{f}}$!

$\Longrightarrow$ non-resistive, non-dissipative MHD: $\mathfrak{V}^\mu_{\text{f}} \equiv 0$ $\Longrightarrow$ fluid & field evolution decouple!

CRC-TR 211

Boltzmann equation

$\lambda_{\text{mfp}} \gg \ell_{\text{int}}$

- $\lambda_{\text{mfp}} \sim (\sigma n_{\text{f}})^{-1}$, σ cross section
- $\ell_{\text{int}} \sim \sqrt{\sigma/\pi}$ interaction length

Since $n_{\text{f}} \sim \beta_0^{-3}$, where $\beta_0 \equiv 1/T$ thermal wavelength

$\Longrightarrow$ $\lambda_{\text{mfp}} \sim \beta_0^3/\ell_{\text{int}}^2 \gg \ell_{\text{int}}$ $\Longrightarrow$ $\beta_0 \gg \ell_{\text{int}}$ dilute limit

Magnetic field

$R_T \equiv (\mathfrak{q} B \beta_0)^{-1} \gg \beta_0$

- R_T Larmor radius for particle with electric charge $\mathfrak{q}$ and transverse momentum $k_T \equiv \beta_0^{-1}$ in magnetic field B ("thermal Larmor radius")

$\Longrightarrow$ $\sqrt{\mathfrak{q} B} \ll T$ weak-field limit $\Longrightarrow$ allows to neglect Landau quantization

$\Longrightarrow$ **Ordering of scales**

$R_T \gg \beta_0 \gg \ell_{\text{int}}$

Define $\xi_B \equiv \lambda_{\text{mfp}}/R_T \equiv \mathfrak{q} B \beta_0 \lambda_{\text{mfp}}$ $\Longrightarrow$ $\xi_B \sim (\beta_0/\ell_{\text{int}})^2(\beta_0/R_T)$

$\Longrightarrow$ study transport coefficients as function of ξ_B

In external electromagnetic field with field-strength tensor $F^{\mu\nu}$, single-particle distribution function $f_{\mathbf{k}}$ satisfies:

Relativistic Boltzmann equation

$$k^\mu \partial_\mu f_{\mathbf{k}} + \mathfrak{q} F^{\mu\nu} k_\nu \frac{\partial}{\partial k^\mu} f_{\mathbf{k}} = C[f_{\mathbf{k}}]$$

Collision term

$$C[f_{\mathbf{k}}] = \frac{1}{2} \int dK' dP dP' W_{\mathbf{kk'}\to\mathbf{pp'}} \left(f_{\mathbf{p}} f_{\mathbf{p'}} \tilde{f}_{\mathbf{k}} \tilde{f}_{\mathbf{k'}} - f_{\mathbf{k}} f_{\mathbf{k'}} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p'}} \right)$$

- $\tilde{f}_{\mathbf{k}} \equiv 1 - a f_{\mathbf{k}}$, with $a = 0, \pm 1$ for Boltzmann, Fermi/Bose statistics
- Transition rate satisfies $W_{\mathbf{kk'}\to\mathbf{pp'}} = W_{\mathbf{kk'}\to\mathbf{p'p}} = W_{\mathbf{pp'}\to\mathbf{kk'}}$

Method of moments

DNMR: G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047

Expansion around local equilibrium

$$f_{\mathbf{k}} = f_{0\mathbf{k}} + \delta f_{\mathbf{k}} \;, \quad f_{0\mathbf{k}} = \left[\exp(\beta_0 E_{\mathbf{k}} - \alpha_0) + a\right]^{-1}$$

- $E_{\mathbf{k}} \equiv k^\mu u_\mu$ LRF particle energy
- $\alpha_0 \equiv \beta_0 \mu$

$\Longrightarrow$ write Boltzmann equation in the form

$$\delta \dot{f}_{\mathbf{k}} = -\dot{f}_{0\mathbf{k}} - E_{\mathbf{k}}^{-1} k_\nu \nabla^\nu \left(f_{0\mathbf{k}} + \delta f_{\mathbf{k}}\right) - E_{\mathbf{k}}^{-1} \mathfrak{q} B b^{\mu\nu} k_\nu \frac{\partial \delta f_{\mathbf{k}}}{\partial k^\mu} + E_{\mathbf{k}}^{-1} C\left[f_{0\mathbf{k}} + \delta f_{\mathbf{k}}\right]$$

- $\dot{A} \equiv u^\mu \partial_\mu A$, $\nabla_\mu \equiv \Delta^\nu_\mu \partial_\nu$

Irreducible moments of $\delta f_{\mathbf{k}}$

$$\rho_r^{\mu_1 \cdots \mu_\ell} \equiv \int dK \, E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \, \delta f_{\mathbf{k}}$$

- $A^{\langle \mu_1 \cdots \mu_\ell \rangle} \equiv \Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell} A^{\nu_1 \cdots \nu_\ell}$, $\Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell}$ rank-2ℓ generalization of $\Delta^{\mu\nu}_{\alpha\beta}$

Equations of motion for irreducible moments

$$\dot{\rho}_r^{\langle \mu_1 \cdots \mu_\ell \rangle} \equiv \Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell} u^\mu \partial_\mu \int dK \, E_{\mathbf{k}}^r k^{\langle \nu_1} \cdots k^{\nu_\ell \rangle} \, \delta f_{\mathbf{k}}$$

CRC-TR 211

Landau matching conditions

- $n_\mathrm{f} \equiv n_{\mathrm{f}0} = \int dK\, E_\mathbf{k}\, f_{0\mathbf{k}} \implies \rho_1 = 0$
- $\varepsilon \equiv \varepsilon_0 = \int dK\, E_\mathbf{k}^2\, f_{0\mathbf{k}} \implies \rho_2 = 0$
- $T_\mathrm{f}^{\mu\nu} u_\nu = \varepsilon u^\mu \implies \rho_1^\mu = 0$

Dissipative currents

$$V_\mathrm{f}^\mu \equiv \rho_0^\mu\,,\; \pi^{\mu\nu} \equiv \rho_0^{\mu\nu}\,,$$

and bulk viscous pressure

$$\Pi \equiv -\frac{m^2}{3}\rho_0 \equiv P - P_0$$

- $P_0 = \frac{1}{3}\int dK (E_\mathbf{k}^2 - m^2) f_{0\mathbf{k}}$ thermodynamic pressure in local equilibrium

Truncation: 14-moment approximation

- $\rho_r^{\mu_1\cdots\mu_\ell} \equiv 0$ for $\ell \geq 3$
- $\rho_r \longrightarrow -\dfrac{3}{m^2}\dfrac{J_{r0}D_{30} + J_{r+1,0}G_{23} + J_{r+2,0}D_{20}}{J_{00}D_{20} + J_{30}G_{23} + J_{40}D_{10}}\,\Pi$
- $\rho_r^\mu \longrightarrow \dfrac{J_{r+2,1}J_{41} - J_{r+3,1}J_{31}}{D_{31}}\, V_\mathrm{f}^\mu$
- $\rho_r^{\mu\nu} \longrightarrow \dfrac{J_{r+2,2}}{J_{42}}\,\pi^{\mu\nu}$

- Thermodynamic integrals: $J_{nq} \equiv \dfrac{1}{(2q+1)!!}\displaystyle\int dK\, E_\mathbf{k}^{n-2q}(E_\mathbf{k}^2 - m^2)^q\, f_{0\mathbf{k}}\tilde{f}_{0\mathbf{k}}$
- $D_{nm} = J_{n+1,m}J_{n-1,m} - J_{nm}^2$
- $G_{nm} = J_{n0}J_{m0} - J_{n-1,0}J_{m+1,0}$

Bulk viscous pressure

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \ell_{\Pi V}\nabla^\mu V_{\mathrm{f},\mu} - \tau_{\Pi V} V_{\mathrm{f},\mu}\dot{u}^\mu - \delta_{\Pi\Pi}\Pi\theta - \lambda_{\Pi V} V_{\mathrm{f},\mu}\nabla^\mu\alpha_0 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

⟹ formally the same as in DNMR, note, however,

$$\dot{u}^\mu = \frac{1}{\varepsilon_0 + P_0}\left[\nabla^\mu P_0 - \Delta^\mu_\nu \partial_\kappa \pi^{\kappa\nu} - \Pi\dot{u}^\mu + \nabla^\mu\Pi - \mathfrak{q}B\, b^{\mu\nu} V_{\mathrm{f},\nu}\right]$$

Particle diffusion current

$$\begin{aligned} \tau_V \dot{V}_{\mathrm{f}}^{\langle\mu\rangle} + V_{\mathrm{f}}^\mu = \; & \kappa\nabla^\mu\alpha_0 - V_{\mathrm{f},\nu}\omega^{\nu\mu} - \delta_{VV} V_{\mathrm{f}}^\mu\theta - \ell_{V\Pi}\nabla^\mu\Pi + \ell_{V\pi}\Delta^{\mu\nu}\nabla_\lambda\pi^\lambda_\nu + \tau_{V\Pi}\Pi\dot{u}^\mu \\ & - \tau_{V\pi}\pi^{\mu\nu}\dot{u}_\nu - \lambda_{VV} V_{\mathrm{f},\nu}\sigma^{\mu\nu} + \lambda_{V\Pi}\Pi\nabla^\mu\alpha_0 - \lambda_{V\pi}\pi^{\mu\nu}\nabla_\nu\alpha_0 \\ & - \delta_{VB}\,\mathfrak{q}B b^{\mu\nu} V_{\mathrm{f},\nu} \end{aligned}$$

Shear-stress tensor

$$\begin{aligned} \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = \; & 2\eta\sigma^{\mu\nu} + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} \\ & - \tau_{\pi V} V_{\mathrm{f}}^{\langle\mu}\dot{u}^{\nu\rangle} + \ell_{\pi V}\nabla^{\langle\mu} V_{\mathrm{f}}^{\nu\rangle} + \lambda_{\pi V} V_{\mathrm{f}}^{\langle\mu}\nabla^{\nu\rangle}\alpha_0 \\ & - 2\delta_{\pi B}\,\mathfrak{q}B b^{\alpha\beta}\Delta^{\mu\nu}_{\alpha\kappa}\pi^\kappa_\beta \end{aligned}$$

keep only 1st order terms X.-G. Huang, A. Sedrakian, DHR, Annals Phys. 326 (2011) 3075

$$
\begin{aligned}
\Pi &= -\zeta^{\mu\nu}\partial_\mu u_\nu \\
V_f^\mu &= \kappa^{\mu\nu}\nabla_\nu \alpha_0 \\
\pi^{\mu\nu} &= \eta^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}
\end{aligned}
$$

- $\zeta^{\mu\nu} = \zeta_\perp \Xi^{\mu\nu} - \zeta_{||} b^\mu b^\nu - \zeta_\times b^{\mu\nu}$
- $\kappa^{\mu\nu} = \kappa_\perp \Xi^{\mu\nu} - \kappa_{||} b^\mu b^\nu - \kappa_\times b^{\mu\nu}$
- $\eta^{\mu\nu\alpha\beta} = 2\eta_0 \Delta^{\mu\nu\alpha\beta} + \eta_1 \left(\Delta^{\mu\nu} - \frac{3}{2}\Xi^{\mu\nu}\right)\left(\Delta^{\alpha\beta} - \frac{3}{2}\Xi^{\alpha\beta}\right) - 2\eta_2 \left(\Xi^{\mu\alpha} b^\nu b^\beta + \Xi^{\nu\alpha} b^\mu b^\beta\right)$
  $-2\eta_3 \left(\Xi^{\mu\alpha} b^{\nu\beta} + \Xi^{\nu\alpha} b^{\mu\beta}\right) + 2\eta_4 \left(b^{\mu\alpha} b^\nu b^\beta + b^{\nu\alpha} b^\mu b^\beta\right)$

for an alternative decomposition, see J. Hernandez, P. Kovtun, JHEP 1705 (2017) 001

Bulk viscosities

$$\begin{aligned}\zeta_\times &= 0 \\ \zeta_\perp &= \zeta_\parallel \equiv \zeta\end{aligned}$$

$\Longrightarrow$ $\Pi = -\zeta\theta$ as without magnetic field

$\Longrightarrow$ consequence of weak-field limit

For bulk viscosities in strong fields, see
K. Hattori, X.-G. Huang, DHR, D. Satow, PRD 96 (2017) 094009

$\Longrightarrow$ in lowest-Landau-level approximation:

$$\zeta_\perp \ll \zeta_\parallel \sim \mathfrak{q}B\,T\left(\frac{m_q}{T}\right)^2 \frac{1}{g^2\ln(T/m_q)}$$

Particle-diffusion coefficients

$$\kappa_{\parallel} \equiv \kappa$$

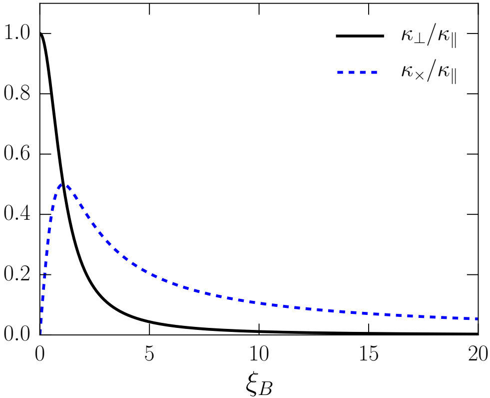
$$\kappa_{\perp} = \kappa \left[1 + (\mathfrak{q} B \delta_{VB})^2\right]^{-1}$$

$$\kappa_{\times} = \kappa_{\perp} \mathfrak{q} B \delta_{VB}$$

For massless Boltzmann gas and constant cross section:

$$\kappa = \frac{3\lambda_{\mathrm{mfp}} n_{f0}}{16}$$

$$\delta_{VB} = \frac{15\beta_0 \lambda_{\mathrm{mfp}}}{16}$$



$\xi_B \to \infty$: Hall diffusion coefficient $\kappa_\times \to \dfrac{n_{f0} R_T}{5}$ becomes dissipationless!

First-order transport coefficients (III)

CRC-TR 211

Shear viscosities

$$\eta_0 = \eta \left[1 + 4\left(\mathfrak{q} B \delta_{\pi B}\right)^2\right]^{-1}$$

$$\eta_1 = \frac{16}{3}\eta_0 \left(\mathfrak{q} B \delta_{\pi B}\right)^2$$

$$\eta_2 = 3\eta_0 \left(\mathfrak{q} B \delta_{\pi B}\right)^2 \left[1 + \left(\mathfrak{q} B \delta_{\pi B}\right)^2\right]^{-1}$$

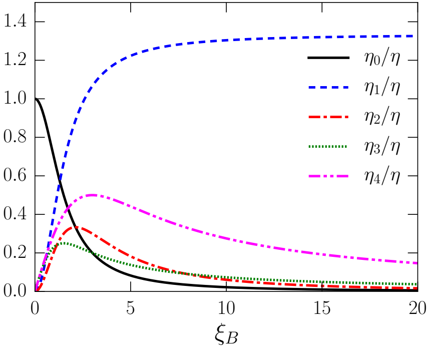
$$\eta_3 = \eta_0\, \mathfrak{q} B \delta_{\pi B}$$

$$\eta_4 = \eta\, \mathfrak{q} B \delta_{\pi B} \left[1 + \left(\mathfrak{q} B \delta_{\pi B}\right)^2\right]^{-1}$$

For massless Boltzmann gas and constant cross section:

$$\eta = \frac{4\lambda_{\text{mfp}} P_0}{3}$$

$$\delta_{\pi B} = \frac{\beta_0 \lambda_{\text{mfp}}}{3}$$



$\xi_B \to \infty$: $\eta_3 = \eta_4/4 \to P_0 R_T$ become dissipationless!

CRC-TR 211

Equation of motion for fluid energy

$$u_\mu \partial_\nu T_{\mathrm{f}}^{\mu\nu} = -E^\mu \mathfrak{V}_{\mathrm{f},\mu}$$

$\Longrightarrow$ fluid gains energy due to acceleration of charges along electric field E^μ

$\Longrightarrow$ resistive, non-dissipative MHD: $\mathfrak{V}_{\mathrm{f}}^\mu \equiv 0$ $\Longrightarrow$ fluid energy is conserved!

Equation of motion for fluid momentum

$$\Delta^\alpha_\mu \partial_\nu T_{\mathrm{f}}^{\mu\nu} = \mathfrak{n}_{\mathrm{f}} E^\alpha - B b^{\alpha\nu} \mathfrak{V}_{\mathrm{f},\nu}$$

$\Longrightarrow$ fluid gains momentum due to acceleration of charges in direction of E^α and due to cyclotron motion!

$\Longrightarrow$ resistive, non-dissipative MHD: $\mathfrak{V}_{\mathrm{f}}^\mu \equiv 0$ $\Longrightarrow$ fluid gains momentum!

CRC-TR 211

Bulk viscous pressure

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \ell_{\Pi V}\nabla^\mu V_{f,\mu} - \tau_{\Pi V} V_{f,\mu}\dot{u}^\mu - \delta_{\Pi\Pi}\Pi\theta - \lambda_{\Pi V} V_{f,\mu}\nabla^\mu\alpha_0 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} - \delta_{\Pi VE}\,\mathfrak{q}E^\mu V_{f,\mu}$$

Particle diffusion current

$$\tau_V \dot{V}_f^{\langle\mu\rangle} + V_f^\mu = \kappa\nabla^\mu\alpha_0 - V_{f,\nu}\omega^{\nu\mu} - \delta_{VV} V_f^\mu\theta - \ell_{V\Pi}\nabla^\mu\Pi + \ell_{V\pi}\Delta^{\mu\nu}\nabla_\lambda\pi^\lambda_\nu + \tau_{V\Pi}\Pi\dot{u}^\mu - \tau_{V\pi}\pi^{\mu\nu}\dot{u}_\nu - \lambda_{VV} V_{f,\nu}\sigma^{\mu\nu} + \lambda_{V\Pi}\Pi\nabla^\mu\alpha_0 - \lambda_{V\pi}\pi^{\mu\nu}\nabla_\nu\alpha_0 - \delta_{VB}\,\mathfrak{q}Bb^{\mu\nu}V_{f,\nu} + \delta_{VE}\,\mathfrak{q}E^\mu + \delta_{V\Pi E}\,\mathfrak{q}E^\mu\Pi + \delta_{V\pi E}\,\mathfrak{q}E_\nu\pi^{\mu\nu}$$

Note: $\delta_{VE} = \kappa\beta_0$,

induced current:

$$\mathfrak{J}^\mu_{\text{ind}} \equiv \mathfrak{q}V^\mu_{f,\text{ind}} \simeq \delta_{VE}\mathfrak{q}^2E^\mu \equiv \sigma_E E^\mu$$

$\Longrightarrow$

Wiedemann-Franz law

$$\sigma_E \equiv \mathfrak{q}^2\delta_{VE} \equiv \mathfrak{q}^2\kappa\beta_0$$

Shear-stress tensor

$$\tau_\pi\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi V} V_f^{\langle\mu}\dot{u}^{\nu\rangle} + \ell_{\pi V}\nabla^{\langle\mu}V_f^{\nu\rangle} + \lambda_{\pi V} V_f^{\langle\mu}\nabla^{\nu\rangle}\alpha_0 - 2\delta_{\pi B}\,\mathfrak{q}Bb^{\alpha\beta}\Delta^{\mu\nu}_{\alpha\kappa}\pi^\kappa_\beta + \delta_{\pi VE}\,\mathfrak{q}E^{\langle\mu}V_f^{\nu\rangle}$$

Conclusions and Outlook

- considered single species of electrically charged, point-like particles with spin zero
- derived equations of motion for non-resistive and resistive, second-order dissipative relativistic MHD from the Boltzmann equation, using method of moments in 14-moment approximation
- identified new transport coefficients due to electromagnetic fields
- computed first-order transport coefficients in constant magnetic field for massless Boltzmann gas with constant cross section
- confirmed (kinetic-theory version of) Wiedemann-Franz law for electric conductivity and particle-diffusion coefficient

- generalize beyond 14-moment approximation via resumming moments
- consider particles and antiparticles (positively and negatively charged particles)
- consider spin-1/2 particles

  $\Longrightarrow$ MHD with non-vanishing polarization, magnetization
- make contact with chiral limit

  $\Longrightarrow$ microscopic calculation of transport coefficients of chiral (anomalous) MHD