Deuteron v_2 and Synthesis

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Deuteron synthesis

- Stellar synthesis of deuteron: Bethe (1935)
- Big-bang deuteron synthesis: Peebles (1966)
- First observation in nucleus-nucleus collisions: Gosset et al (1977)
- Flurry of models: Mekjian (1977), Lemaire et al (1979) Kapusta (1980) Csernai, Kapusta (1986) Heinz, Zimanyi et al (1991) Danielewicz, Bertsch (1991), Stoecker, Greiner et al (1995), Schiebl, Heinz (1999), Broniowski, Florkowski, Hiller (2003), Andronic et al (2011), Steinheimer et al (2012) ...
- Many modern observations: ALICE, STAR, PHENIX, NA49, NA52, NA44
- At low energy deuteron may come from pre-existing clusters. If fireball enters deconfined phase, then deuteron must be synthesized. Parallel to big bang.

Transport theory of Deuteron synthesis

Since Deuteron is very loosely bound ($\Delta = -2.2$ MeV), it is easily broken. As a result, deuteron produced mainly inside a skin whose depth can be computed in transport theory. Assume that skin depth is small. The most important reaction for synthesis is $np \leftrightarrow d\gamma$.

$$n_d(p_d) = \int d^3p_p d^3p_n \delta^3(p_p + p_n - p_d - p_\gamma) n_p(p_p) n_n(p_n) S(p_n, p_p) P_r(p_n, p_p)$$

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- Neglect recoil photon; less than 1% effect on kinematics
- Useful to introduce CM momentum (p_d) and relative momentum $p_p - p_n = k$. P_r is a function only of k.
- Different models for P_r possible; one is coalescence model.

Outline

Bethe model for P_r

$pn \to D\gamma$ proceeds in the $L = 0$ channel. For NN scattering in the $L = 0$, $I = 0$ channel, S-matrix is one dimensional. Then $S(k) = f(k)/f^*(k)$. Effective range theory: $f(k) = 1 + iak + r^2k^2/2 + \cdots$. Since $a = -23.7$ fm and $r = 2.75$ fm, for $k < 600$ MeV, neglect k^2 term.

Bethe's insight: add in deuteron production by putting an imaginary part to scattering length: $a'' = -2.2$ fm. Then, putting in the corresponding P_r, we find

$$n_d(p_d) = 16\pi(K_{th}a'')^4 n_p\left(\frac{p_d}{2}\right) n_n\left(\frac{p_d}{2}\right)\left[1 - \mathrm{e}^{-K^2/K_{th}^2}\left(1 + \frac{K^2}{K_{th}^2}\right)\right]$$

where $K_{th}^2 = 2M_pT$ and K is any momentum where the linear approximation to the S-matrix works. When $K_{th} \ll K$ the square bracket is 1 and Bethe's theory works well.

Where does it work?

Take $K \simeq$ 200–300 MeV. Then P_r is accurate to 10–15%.

- Stellar cores: $T \simeq$ 100–150 KeV. Then $K_{th} \simeq$ 14–20 MeV, so $K_{th} \ll K$. Bethe's theory works well.
- Big bang nucleosynthesis: $T \simeq 1$ MeV. Then $K_{th} \simeq 45$ MeV, so $K_{th} \ll K$. Bethe's theory works well.
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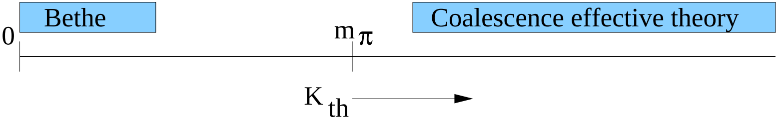


Outline

Coalescence model

Creation of deuteron involves filtering out its wavefunction from the initial pn plane wave. Long distance effect. Well separated from distance scale of $1/K_{th}$. Then first approximation

$$P_r(k) = C\delta^3(k).$$

This gives factorized yield $N_d = \mathcal{B} N_p N_n$, where

$$\mathcal{B} = \int d^3p_d d^3p_p d^3p_n \delta^3(p_p + p_n - p_d) \frac{n_p(p_p)}{N_p} \frac{n_n(p_n)}{N_n} P_r(p_p - p_n),$$

and flow coefficients

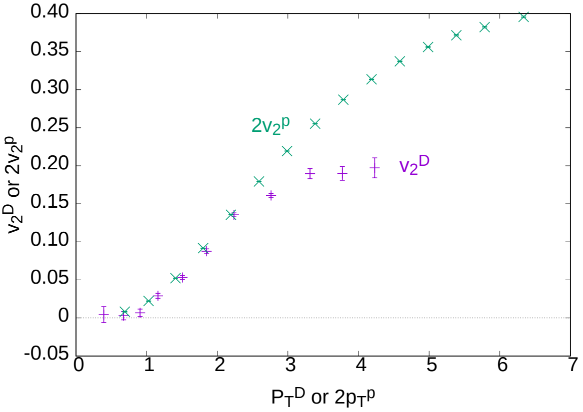
$$v_n^d = \frac{1}{\mathcal{B}} \int d^3p_d d^3p_p d^3p_n \delta^3(p_p + p_n - p_d) \frac{n_p(p_p)}{N_p} \frac{n_n(p_n)}{N_n} P_r(p_p - p_n) \cos n\phi_d.$$

In the model

$$v_2^d = 2v_2^p + 2(v_2^p v_4^p + v_4^p v_6^p + \cdots).$$

When $v_2^p \gg v_4^p, v_6^p \cdots$ then constituent scaling.

Constituent scaling



ALICE 2015

Yield requires unknown parameter C, but constituent scaling is a parameter free test of coalescence model!

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Very complicated physics of a multi-channel S-matrix reduced to the simple matrix element $|T|^2 \propto \delta^3(k)$!

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Coalescence effective theory

$P_r(k)$ occurs inside an integral multiplying thermal distribution functions. These are Gaussian: $\exp(-k^2/K_{th}^2)$. So, a sufficiently well-behaved P_r can be expanded in derivates of delta functions

$$P_r(k) = C_0\delta^3(k) + C_1\delta'^3(k) + C_2\delta''^3(k) + \cdots$$

Definition

$$\int_{-\infty}^{\infty} dx f(x)\frac{d^n}{dx^n}\delta(x) = (-1)^n \left.\frac{d^n}{dx^n}f(x)\right|_{x=0}.$$

Derivatives bring down powers of K_{th}; odd terms vanish.

Not singular, as long the series expansion in C_n/K_{th}^n converges fast enough. Note that dimensionally, each term is same as C_0. So the expansion is roughly in even powers of m_π/K_{th}.

Mathematical niceties: Schwartz spaces, tempered distributions.

Scaling of flow

Inserting the effective theory into the integral for $\mathcal{B}$, one finds an expansion in powers of C_n/K_{th}^n with only even terms.

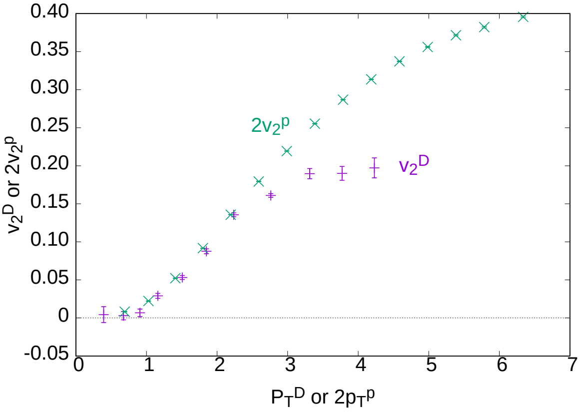
Integral for V_2 contains exactly the same combination of coefficients. As a result, in the effective model

$$v_2^d = 2v_2^p + 2(v_2^p v_4^p + v_4^p v_6^p + \cdots).$$

When $v_2^p \gg v_4^p, v_6^p \cdots$ then constituent scaling. Again!
Constituent scaling says nothing about P_r, but only says that the distributions are Gaussian, *i.e.*, Boltzmann distributed. When distributions are non-Gaussian scaling breaks down.

Large momentum hadrons may be produced by jet or minijet fragmentation. When do nucleon momenta stop becoming thermal?

Limits of thermal distributions



ALICE 2015

Constituent scaling breaks down when $p_T^p > 1.25$ GeV. Proton no longer thermal. Implications for critical point search

Conclusion

Systematic expansion around the coalescence model in powers of $1/K_{th}^2$ gives an effective theory of the transition matrix element.

Systematic alternative to Bethe theory when $K_{th} > m_\pi$. Naturalness, p_T and y distributions, yield, *etc.* to be considered later.

Constituent scaling of v_n persists in this expansion. Tests thermalization of the nucleon distributions. Implies non-thermal production of nucleons dominates for $p_T^p > 1.25$ GeV.