Question of Parity Conservation in Weak Interactions

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Parity

• A parity transformation (parity inversion) is the flip in the sign of one spatial coordinate.

\[
P : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}
\]

• Parity is conserved in electromagnetism, strong interactions and gravity, but not in weak interactions.

P-symmetry: A clock built like its mirrored image will behave like the mirrored image of the original clock

P-asymmetry: A clock built like its mirrored image will \textit{not} behave like the mirrored image of the original clock.
Wu Experiment

\[ ^{60}_{27}\text{Co} \rightarrow ^{60}_{28}\text{Ni} + e^- + \bar{\nu}_e + 2\gamma \]
Parity

★ The parity operator performs spatial inversion through the origin:

\[ \psi'(\vec{x}, t) = \hat{P} \psi(\vec{x}, t) = \psi(-\vec{x}, t) \]

• applying \( \hat{P} \) twice:

\[ \hat{P} \hat{P} \psi(\vec{x}, t) = \hat{P} \psi(-\vec{x}, t) = \psi(\vec{x}, t) \]

so \( \hat{P} \hat{P} = I \) \( \hat{P}^{-1} = \hat{P} \)

★ To preserve the normalisation of the wave-function

\[ \langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \psi | \hat{P}^\dagger \hat{P} | \psi \rangle \]

\[ \hat{P}^\dagger \hat{P} = I \]

Unitary

But since \( \hat{P} \hat{P} = I \) \( \hat{P} = \hat{P}^\dagger \) \( \hat{P} \) Hermitian

which implies Parity is an observable quantity. If the interaction Hamiltonian commutes with \( \hat{P} \), parity is an observable conserved quantity

★ If \( \psi(\vec{x}, t) \) is an eigenfunction of the parity operator with eigenvalue \( P \)

\[ \hat{P} \psi(\vec{x}, t) = P \psi(\vec{x}, t) \]

\[ \hat{P} \hat{P} \psi(\vec{x}, t) = \hat{P} P \psi(\vec{x}, t) = P^2 \psi(\vec{x}, t) \]

since \( \hat{P} \hat{P} = I \) \( P^2 = 1 \)

Parity has eigenvalues \( P = \pm 1 \)

★ QED and QCD are invariant under parity

★ Experimentally observe that Weak Interactions do not conserve parity
Intrinsic Parities of fundamental particles:

**Spin-1 Bosons**

- From Gauge Field Theory can show that the gauge bosons have $P = -1$

\[
P_{\gamma} = P_{g} = P_{W^+} = P_{W^-} = P_{Z} = -1
\]

**Spin-$\frac{1}{2}$ Fermions**

- From the Dirac equation showed (handout 2):
  - Spin $\frac{1}{2}$ particles have opposite parity to spin $\frac{1}{2}$ anti-particles
  - Conventional choice: spin $\frac{1}{2}$ particles have $P = +1$

\[
P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_{\nu} = P_{q} = +1
\]

and anti-particles have opposite parity, i.e.

\[
P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\bar{\nu}} = P_{\bar{q}} = -1
\]

- For Dirac spinors it was shown (handout 2) that the parity operator is:

\[
\hat{P} = \gamma^0 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]
Parity Conservation in QED and QCD

• Consider the QED process $e^- q \rightarrow e^- q$

• The Feynman rules for QED give:

$$-iM = \left[ \bar{u}_e(p_3) i e \gamma^\mu u_e(p_1) \right] \frac{-i g_{\mu \nu}}{q^2} \left[ \bar{u}_q(p_4) i e \gamma^\nu u_q(p_2) \right]$$

• Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2} g_{\mu \nu} j^\mu_e j^\nu_q = -\frac{e^2}{q^2} j_e \cdot j_q$$

with $j_e = \bar{u}_e(p_3) \gamma^\mu u_e(p_1)$ and $j_q = \bar{u}_q(p_4) \gamma^\mu u_q(p_2)$

★★ Consider the what happen to the matrix element under the parity transformation

• Spinors transform as:

$$[u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u]$$

• Adjoint spinors transform as:

$$\bar{u} = u^\dagger \gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger \gamma^0 = u^\dagger \gamma^0 \gamma^0 = u^\dagger \gamma^0 = \bar{u} \gamma^0$$

$$[\bar{u} \xrightarrow{\hat{P}} \bar{u} \gamma^0]$$

• Hence:

$$j_e = \bar{u}_e(p_3) \gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \bar{u}_e(p_3) \gamma^0 \gamma^\mu \gamma^0 u_e(p_1)$$
Question from Yuhang:

• How to get the conclusion that the present large asymmetry is possible only if both conservation of parity and invariance under charge conjugation are violated from attachments.

• > If $P$-conservation were true in beta decay, electrons would have no preferred direction of decay relative to the nuclear spin.
Question from Shan:

• In fact, the equality of the life times of a charged particle and its charge conjugate against decay through a weak interaction (to the lowest order of the strength of the weak interaction) can be shown to follow from the invariance under proper lorentz transformations.

• Why use Lorentz transformation can obtain that the life times of charged particles through a weak interaction are the same?

• > the Lorentz transformations (or transformation) are linear coordinate transformations between two coordinate frames that move at constant velocity relative to each other.
Question from Ryuta

• Around the first page, the "theta-tau puzzle ( = kaon) " is referred as an inspiration of the discussion. We now know that the weak interaction is cause by (in this case) W-boson, which can only couple to the left-hand particle, s-quark, for the case of the Kaon decay. Then, I'm little bit confusing but, how we can make consistent view from the two issues ?

  1. weak interaction does not conserve the parity, sometimes conserve (3pions), sometimes not (2pions)
  2. W-boson only couples to the left-hand particles
Question from Suyu:

• Is polar angle counter set to some random or certain angle?

• > One in the equatorial plane and one near the polar position. From the paper, it looks in the horizontal position.
Question from Kai:

• In the paper written by Garwin, Lederman, weinrich, why the negative muon shows an asymmetry that different from positive muon?

• > The Fierz-Pauli theory for spin 3/2 particles predicts a g value of 2/3
Question from Amit:

• In the paper written by Ambler, Hayward Hoppes and Hudson, What does it mean by weaker asymmetry which they have mentioned while calculating the peak-to-valley ratio?