# Question of Parity Conservation in Weak Interactions

Xin Shi

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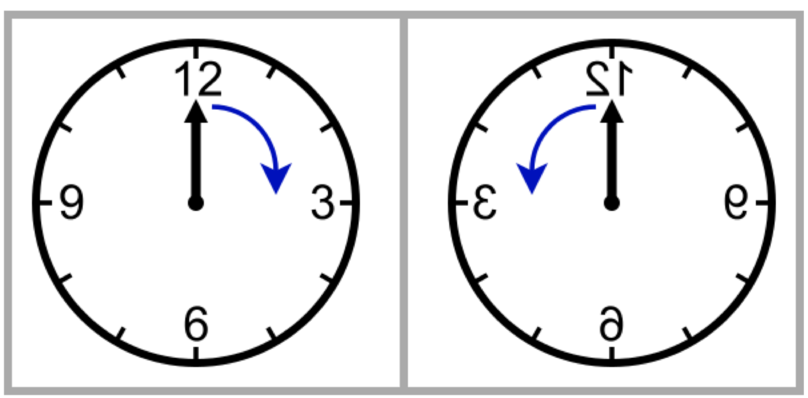
# Parity

- A parity transformation (parity inversion) is the flip in the sign of one spatial coordinate.

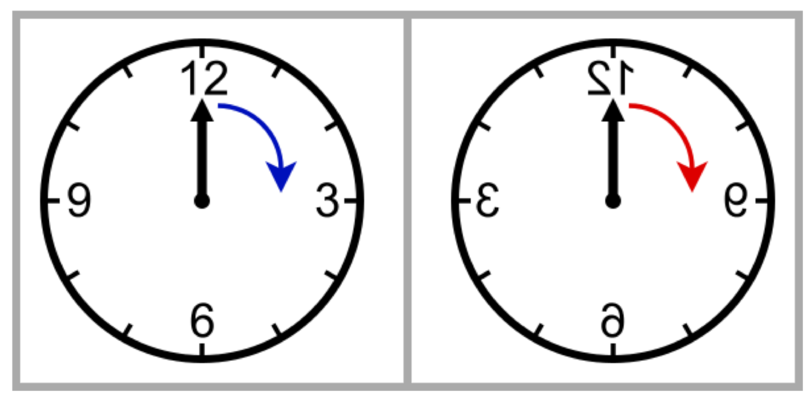
$$\mathbf{P}: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

- Parity is conserved in electromagnetism, strong interactions and gravity, but not in weak interactions.

P-symmetry: A clock built like its mirrored image will behave like the mirrored image of the original clock

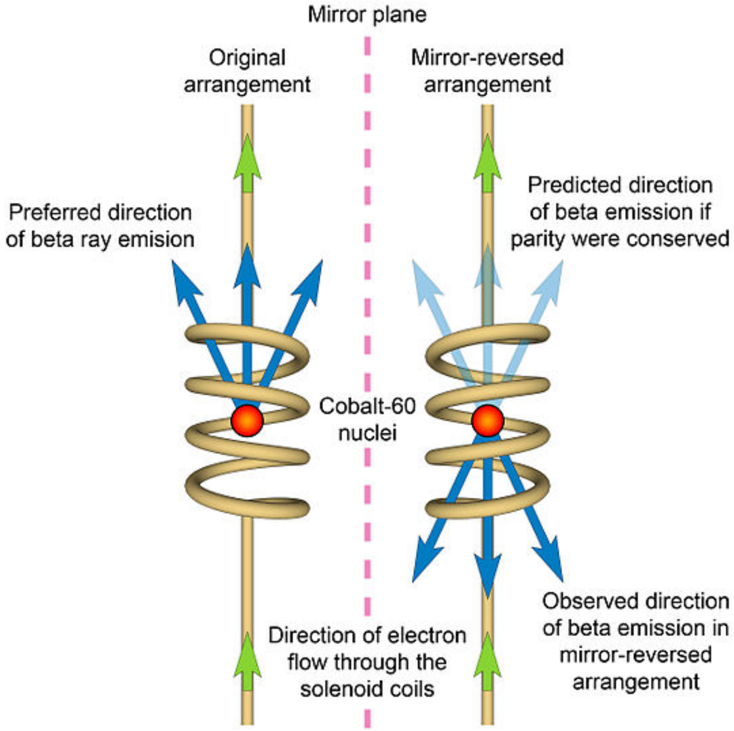


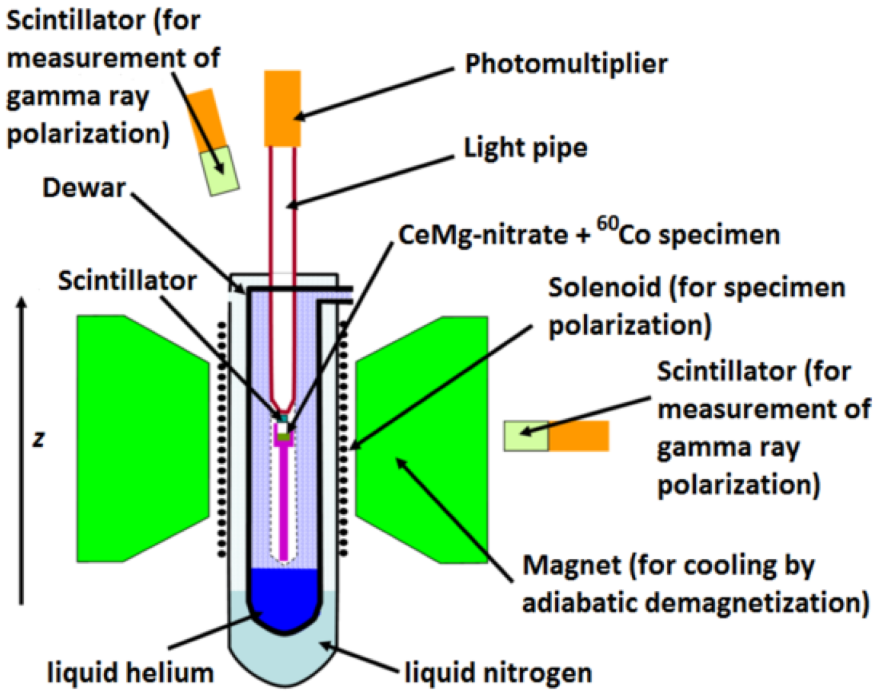

P-asymmetry: A clock built like its mirrored image will *not* behave like the mirrored image of the original clock.



# Wu Experiment

$$ {}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e + 2\gamma $$



# Parity

★The parity operator performs spatial inversion through the origin:

$$\psi'(\vec{x},t) = \hat{P}\psi(\vec{x},t) = \psi(-\vec{x},t)$$

•applying $\hat{P}$ twice: $\hat{P}\hat{P}\psi(\vec{x},t) = \hat{P}\psi(-\vec{x},t) = \psi(\vec{x},t)$

so $\hat{P}\hat{P} = I$ $\Rightarrow$ $\hat{P}^{-1} = \hat{P}$

•To preserve the normalisation of the wave-function

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|\hat{P}^{\dagger}\hat{P}|\psi\rangle$$

$\hat{P}^{\dagger}\hat{P} = I$ $\Rightarrow$ $\hat{P}$ Unitary

• But since $\hat{P}\hat{P} = I$ $\hat{P} = \hat{P}^{\dagger}$ $\Rightarrow$ $\hat{P}$ Hermitian

which implies Parity is an observable quantity. If the interaction Hamiltonian commutes with $\hat{P}$, parity is an observable conserved quantity

• If $\psi(\vec{x},t)$ is an eigenfunction of the parity operator with eigenvalue $P$

$\hat{P}\psi(\vec{x},t) = P\psi(\vec{x},t)$ $\Rightarrow$ $\hat{P}\hat{P}\psi(\vec{x},t) = P\hat{P}\psi(\vec{x},t) = P^2\psi(\vec{x},t)$

since $\hat{P}\hat{P} = I$ $P^2 = 1$

$\Rightarrow$ Parity has eigenvalues $P = \pm 1$

★ QED and QCD are invariant under parity

★ Experimentally observe that Weak Interactions do not conserve parity

## Intrinsic Parities of fundamental particles:

### Spin-1 Bosons

- From Gauge Field Theory can show that the gauge bosons have $P=-1$

$$P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1$$

### Spin-½ Fermions

- From the Dirac equation showed (handout 2):

Spin ½ particles have opposite parity to spin ½ anti-particles

- Conventional choice: spin ½ particles have $P=+1$

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1$$

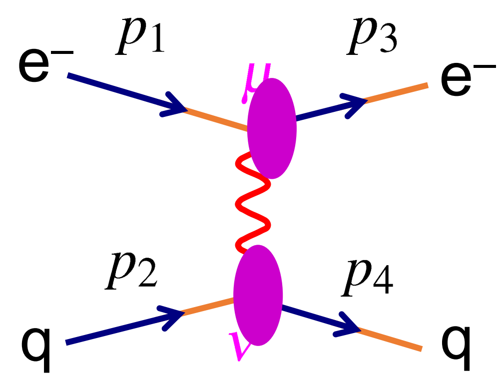
and anti-particles have opposite parity, i.e.

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\overline{\nu}} = P_{\overline{q}} = -1$$

★ For Dirac spinors it was shown (handout 2) that the parity operator is:

$$\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

# Parity Conservation in QED and QCD




- Consider the QED process e⁻q → e⁻q
- The Feynman rules for QED give:

$$-iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_q(p_4)ie\gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2}g_{\mu\nu}j_e^\mu j_q^\nu = -\frac{e^2}{q^2}j_e.j_q$$

with $j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1)$ and $j_q = \overline{u}_q(p_4)\gamma^\mu u_q(p_2)$

★ Consider the what happen to the matrix element under the parity transformation

- Spinors transform as $\left[u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u\right]$
- Adjoint spinors transform as

$$\overline{u} = u^\dagger\gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger\gamma^0 = u^\dagger\gamma^{0\dagger}\gamma^0 = u^\dagger\gamma^0\gamma^0 = \overline{u}\gamma^0$$

$$\left[\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0\right]$$

- Hence $j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \overline{u}_e(p_3)\gamma^0\gamma^\mu\gamma^0 u_e(p_1)$

# Question from Yuhang:

- How to get the conclusion that the present large asymmetry is possible only if both conservation of parity and invariance under charge conjugation are violated from attachments.

- > If *P*-conservation were true in beta decay, electrons would have no preferred direction of decay relative to the nuclear spin.

# Question from Shan:

- In fact, the equality of the life times of a charged particle and its charge conjugate against decay through a weak interaction (to the lowest order of the strength of the weak interaction) can be shown to follow from the invariance under proper lorentz transformations.
- Why use Lorentz transformation can obtain that the life times of charged particles through a weak interaction are the same?
- > the Lorentz transformations (or transformation) are linear coordinate transformations between two coordinate frames that move at constant velocity relative to each other.

# Question from Ryuta

- Around the first page, the "theta-tau puzzle ( = kaon) " is referred as an inspiration of the discussion. We now know that the weak interaction is cause by (in this case) W-boson, which can only couple to the left-hand particle, s-quark, for the case of the Kaon decay. Then, I'm little bit confusing but, how we can make consistent view from the two issues ?

1. weak interaction does not conserve the parity, sometimes conserve (3pions), sometimes not (2pions)
2. W-boson only couples to the left-hand particles

# Question from Suyu:

- Is polar angle counter set to some random or certain angle?

- > One in the equatorial plane and one near the polar position. From the paper, it looks in the horizontal posibion.

# Question from Kai:

- In the paper written by Garwin, Lederman, weinrich,  why the negative muon shows an asymmetry that different from positive muon?
- > The Fierz-Pauli theory for spin 3/2 particles predicts a g value of 2/3

# Question from Amit:

- In the paper written by Ambler, Hayward Hoppes and Hudson, What does it mean by weaker asymmetry which they have mentioned while calculating the peak-to-valley ratio?