From :

“Measurement of the Negative Muon Anomalous Magnetic Moment to 0.7 ppm”

G. W. Bennett, et. al., PRL 92 161802 (2004)

Reference : arXiv: 0602035 “Final Report of the Muon E821Anomalous Magnetic Moment Measurement at BNL”

12/21/2018 JC90

Anomalous (muon) magnetic moment

anomalous magnetic moment :
$a \equiv \frac{g-2}{2}$

Dirac theory , g is exactly 2

$$\vec{\mu}_\mu = g_\mu \left(\frac{q}{2m}\right) \vec{S}$$

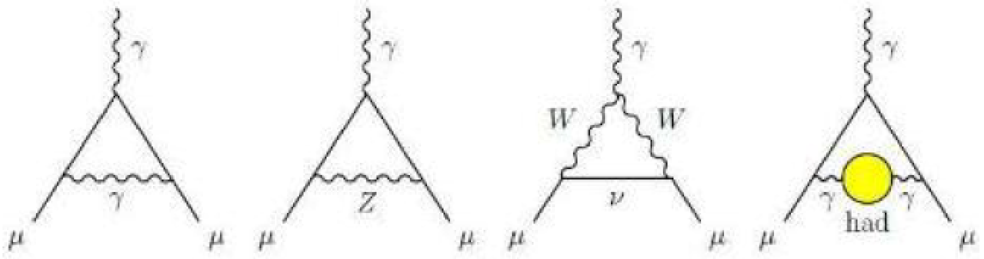


Table 1: Standard model components of the anomaly, taken directly from [1]
HVP to reflect two recent estimates. The terms lo and ho indicate lower order and higher order, respectively.
Other terms are defined in the text.

	Values in 10^{-11} units
QED ($\gamma + l$)	$116584718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077$
HVP(lo) [7]	6923 ± 42
HVP(lo) [8]	6949 ± 43
HVP(ho) [8]	-98.4 ± 0.7
HLbL	105 ± 26
EW	153.6 ± 1.0
Total SM [7]	$116591802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}$ ($\pm 49_{\text{tot}}$)
Total SM [8]	$116591828 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}$ ($\pm 50_{\text{tot}}$)

Higher order can cause a shift !

TABLE I: Summary of a_μ results from CERN and BNL, showing the evolution of experimenta precision over time. The average is obtained from the 1999, 2000 and 2001 data sets only.

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]	Reference
CERN I	1961	μ^+	11 450 000(220 000)	4300	[2]
CERN II	1962-1968	μ^+	11 661 600(3100)	270	[3]
CERN III	1974-1976	μ^+	11 659 100(110)	10	[5]
CERN III	1975-1976	μ^-	11 659 360(120)	10	[5]
BNL	1997	μ^+	11 659 251(150)	13	[6]
BNL	1998	μ^+	11 659 191(59)	5	[7]
BNL	1999	μ^+	11 659 202(15)	1.3	[8]
BNL	2000	μ^+	11 659 204(9)	0.73	[9]
BNL	2001	μ^-	11 659 214(9)	0.72	[10]
Average			11 659 208.0(6.3)	0.54	[10]

E821 exp. : Beam Line

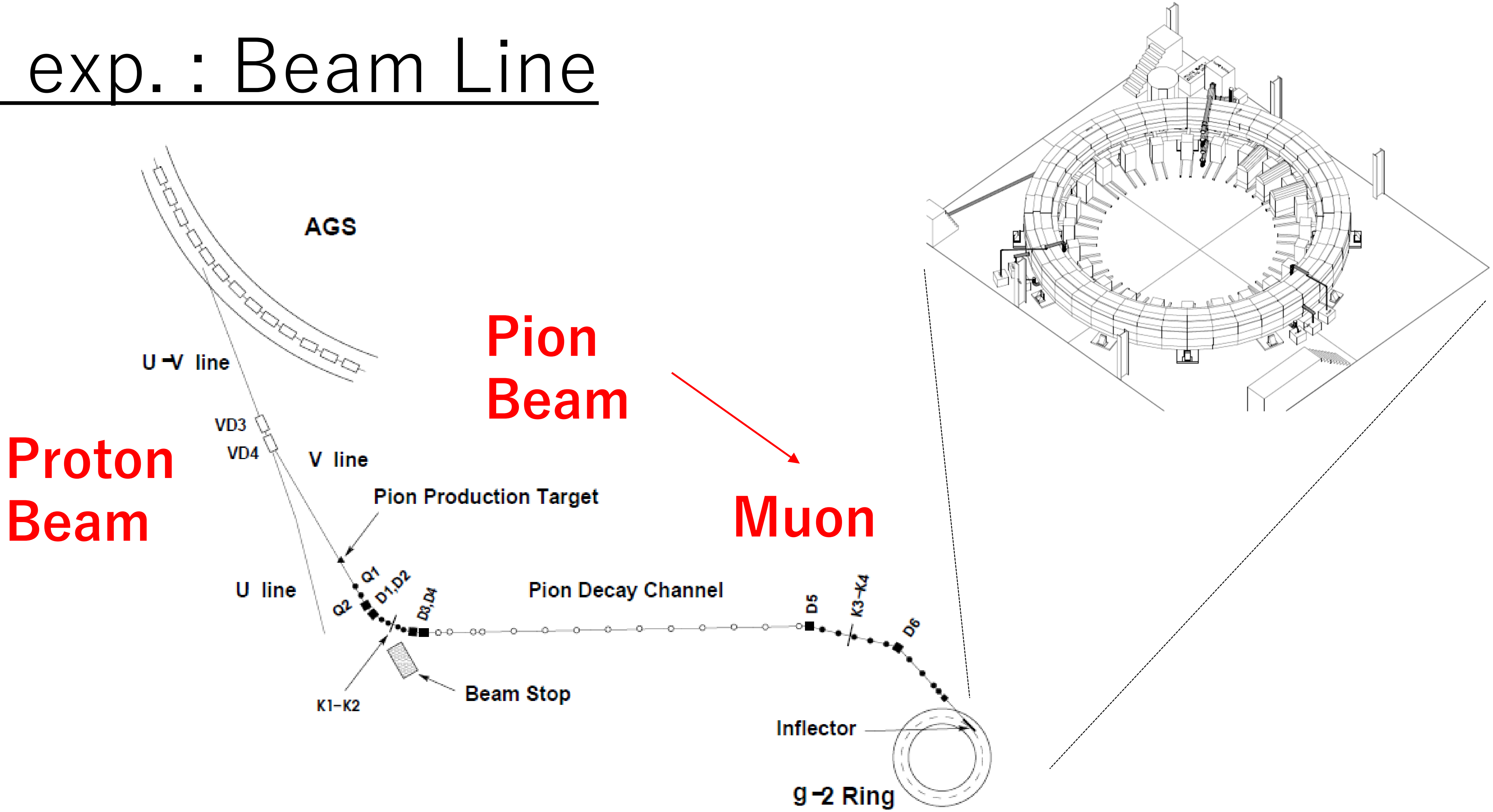


FIG. 3: Plan view of the pion/muon beamline. The pion decay channel is 80 m and the ring diameter is 14.1 m.

Calorimeters

-- electrons/positrons decayed from muons are measured by calorimeters

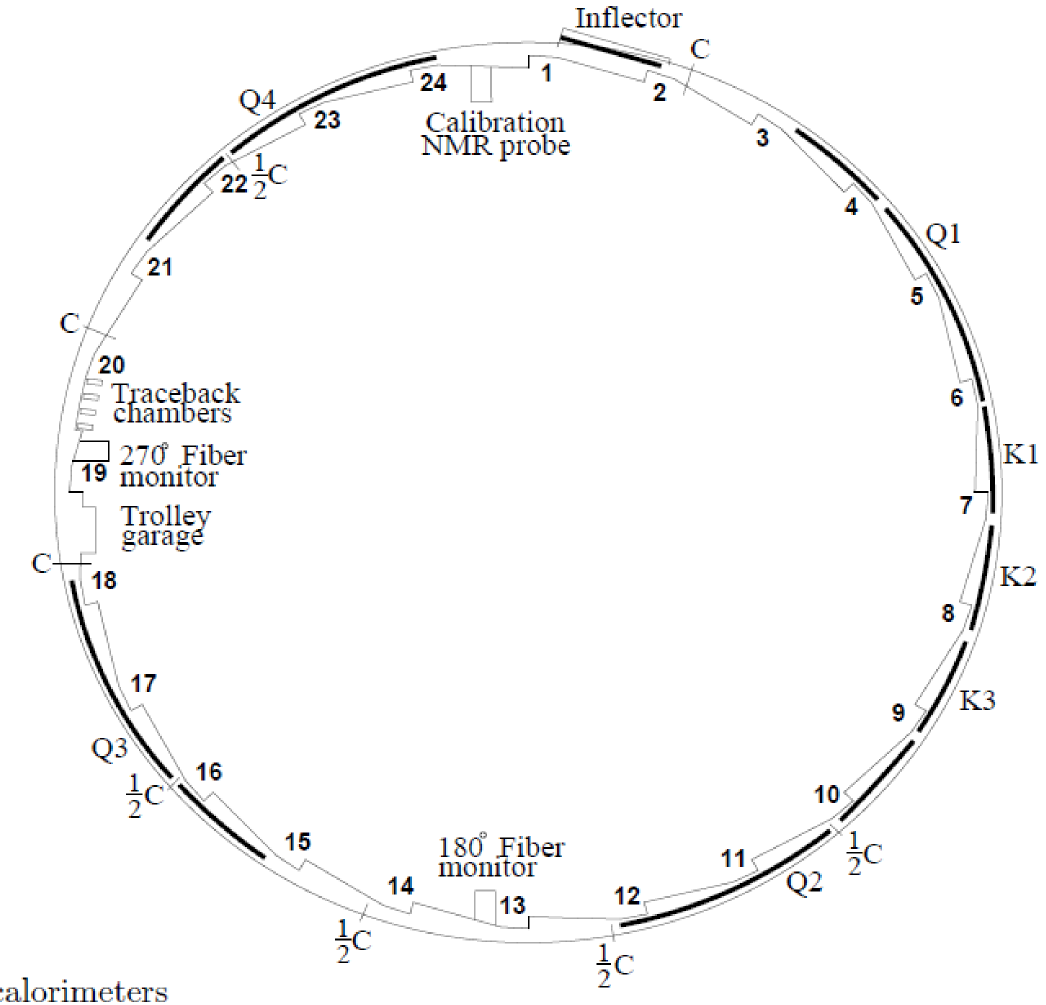


FIG. 8: The $(g-2)$ storage ring layout. The 24 numbers represent the locations of the calorimeters immediately downstream of the scalloped vacuum chamber subsections. Inside the vacuum are four quadrupole sections (Q1-Q4), three kicker plates (K1-K3) and full-aperture (C) and half-aperture ($\frac{1}{2}$C) collimators. The traceback chambers follow a truncated scalloped vacuum chamber subsection.

Key component

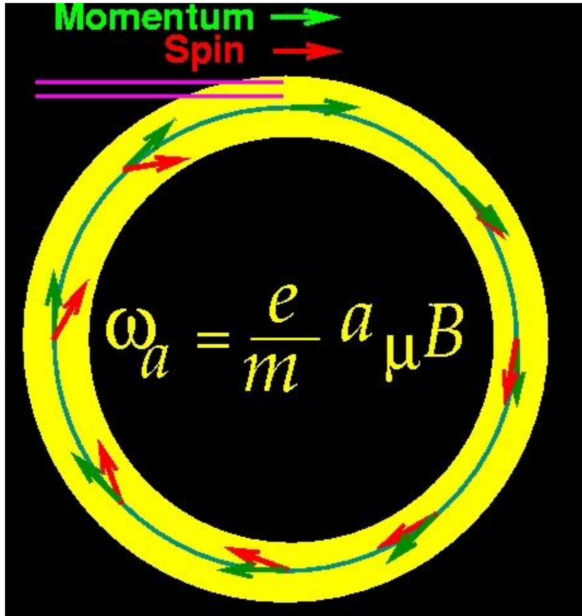
(1) (2)

The cyclotron ω_c and spin precession ω_s frequencies for a muon moving in the horizontal plane of a magnetic storage ring are given by:

$$\vec{\omega}_c = -\frac{q\vec{B}}{m\gamma}, \qquad \vec{\omega}_s = -\frac{gq\vec{B}}{2m} - (1-\gamma)\frac{q\vec{B}}{\gamma m}. \tag{3}$$

The anomalous precession frequency ω_a is determined from the difference

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\left(\frac{g-2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}.$$



if g==2, the anomalous precession frequency = 0

[Question from Xin]

The dependence of !a on the electric field is eliminated by storing muons with the ‘‘magic’’ gamma?
Why this gamma is “magic”?

Precisely determine ω & B is essential !

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]. \qquad (1)$$

vanish

α_μ ~ 1.16*10^-3 => $\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)$ ~ 0 , if $\gamma = 29.3$

but ,,, the systematics are considered since the beam momentum has distribution

Explanation I.

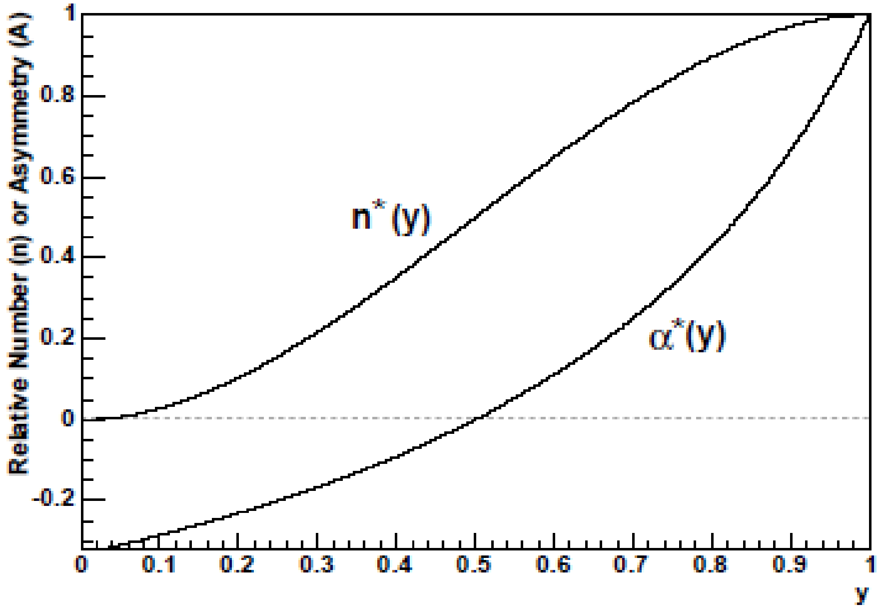
Because of parity violation in the weak decay of the muon, a correlation existsbetween the muon spin and decay electron direction. This correlation allows the spin direction to be measured as a function of time. In the rest frame of the muon—indicated by starred quantities—the differential probability for the electron to emerge with a normalized energy $y = E^*/E_{max}$ ($E_{max} = 52.8$ MeV) at an angle θ^* with respect to the muon spin is [11]

$$\frac{dP(y,\theta^*)}{dy\, d\Omega} = (1/2\pi) n^*(y)[1 - \alpha^*(y)\cos\theta^*] \quad \text{with} \tag{6}$$

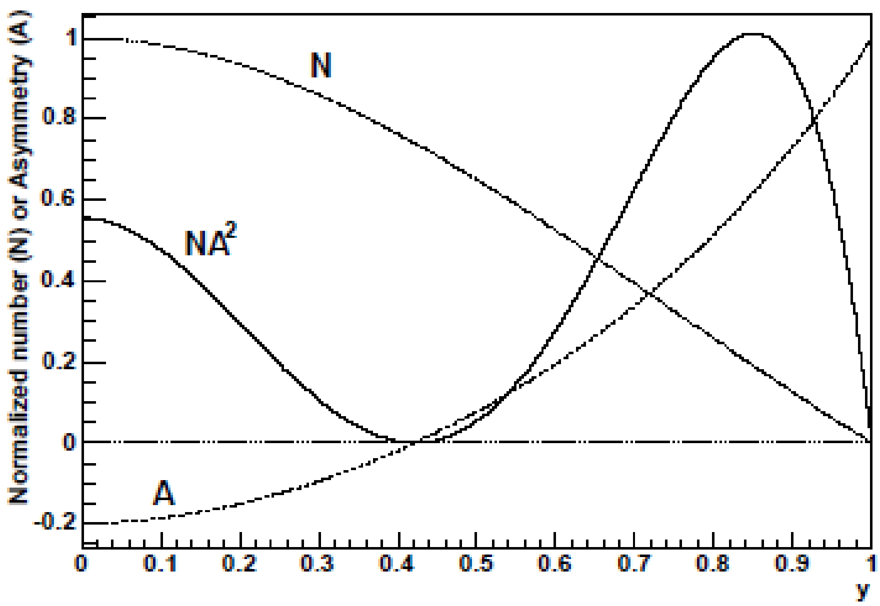
$$n^*(y) = y^2(3 - 2y) \quad \text{and} \tag{7}$$

$$\alpha^*(y) = \frac{q}{e}\frac{2y - 1}{3 - 2y} \; . \tag{8}$$

Explanation II.



(a) Center-of-mass frame



(b) Lab frame

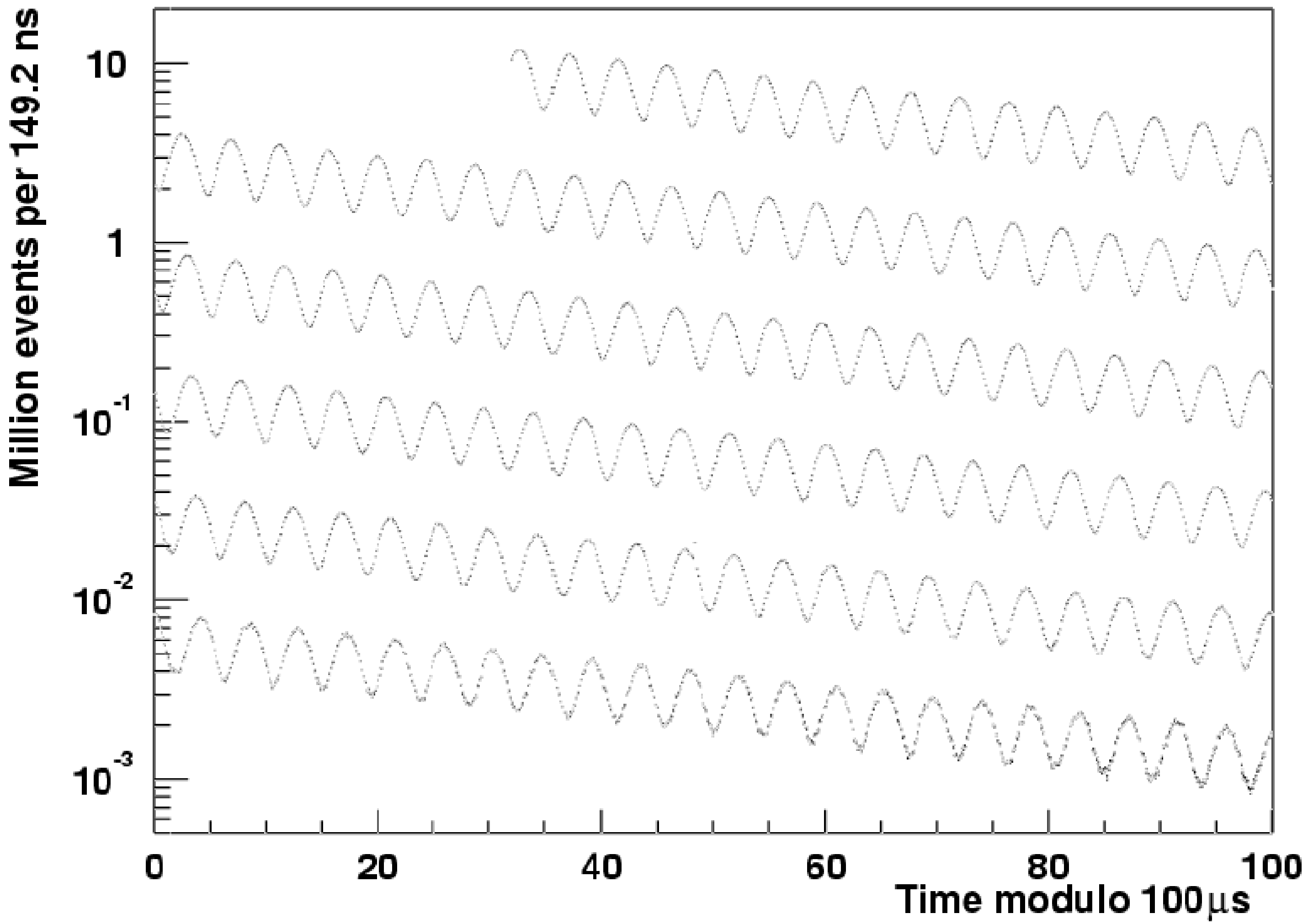
By setting threshold of electron/positron to be detected, the count/energy distribution oscillate

[Question from Shan]

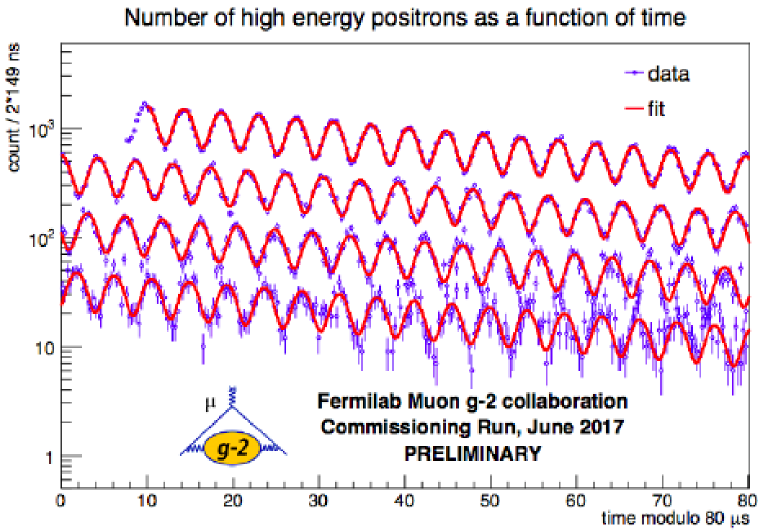
How to determine the initial value of the N0,A and phi_a in Equation 2 ?

$$N(t) = N_0 e^{-t/(\gamma\tau)}[1 + A\sin(\omega_a t + \phi_a)]. \qquad (2)$$

This represent the modulation .
It is explained that the fitting is done by 5 free parameters .....



Million events per 149.2 ns
10
1
10^{-1}
10^{-2}
10^{-3}
0
20
40
60
80
100
Time modulo 100µs



Number of high energy positrons as a function of time
count / 2*149 ns
data
fit
10^3
10^2
10
1
Fermilab Muon g-2 collaboration
Commissioning Run, June 2017
PRELIMINARY
µ
g-2
0
10
20
30
40
50
60
70
80
time modulo 80 µs

Additional :

$$N(t) = \frac{N_0}{\gamma\tau_\mu} e^{-t/\gamma\tau_\mu} \cdot \Lambda(t) \cdot V(t) \cdot B(t) \cdot C(t) \cdot [1 - A(t)\cos(\omega_a t + \phi(t)] \tag{32}$$

with

$$\Lambda(t) = 1 - A_{loss} \int_0^t L(t') e^{-t'/\gamma\tau_\mu} dt' \tag{33}$$

$$V(t) = 1 - e^{-t/\tau_{VW}} A_{VW} \cos(\omega_{VW} t + \phi_{VW}) \tag{34}$$

$$B(t) = 1 - A_{br} e^{-t/\tau_{br}} \tag{35}$$

$$C(t) = 1 - e^{-t/\tau_{cbo}} A_1 \cos(\omega_{cbo} t + \phi_1) \tag{36}$$

$$A(t) = A\left(1 - e^{-t/\tau_{cbo}} A_2 \cos(\omega_{cbo} t + \phi_2)\right) \tag{37}$$

$$\phi(t) = \phi_0 + e^{-t/\tau_{cbo}} A_3 \cos(\omega_{cbo} t + \phi_3). \tag{38}$$

Things are not so simple , , ,

[Question from Kai]

Why they use values of proton in this analysis, such as NMR frequency ¥omega_{p}, and the magnetic moment in ¥lambda

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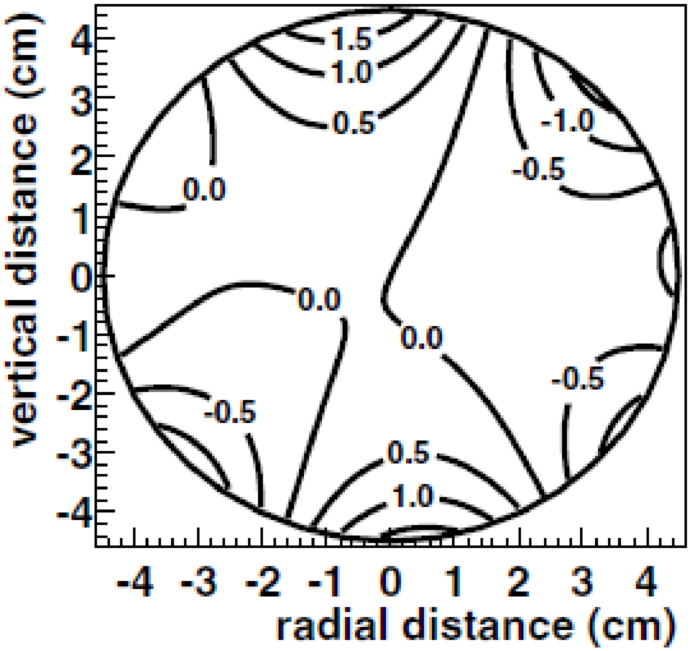
ω_p is related to the measurement of B-field,
which is measured/calibrated by NMR using proton

the very details are not checked yet.

B-field measurement



Multipoles [ppm]

	Normal	Skew
Quad	-0.28	0.11
Sext	-0.72	-0.45
Octu	0.09	0.01
Decu	1.04	0.38

FIG. 1. A two-dimensional multipole expansion of the 2001 field averaged over azimuth from one out of 20 trolley measurements. Half ppm contours with respect to a central azimuthal average field $B_0 = 1.451\,269$ T are shown. The multipole amplitudes relative to B_0 are given at the beam aperture, which had a radius of 4.5 cm and is indicated by the circle.

Need . . .

-- Absolute calibration

-- whole measurement

-- point measurement (continuous)

-- smoothing

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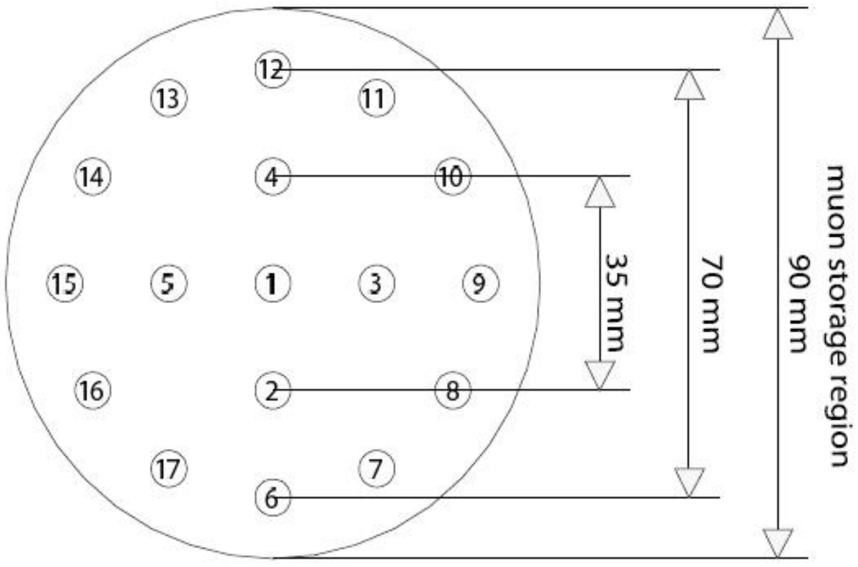
Accomplish this precision is one of key elements for g-2 exp.

NMR Trolley

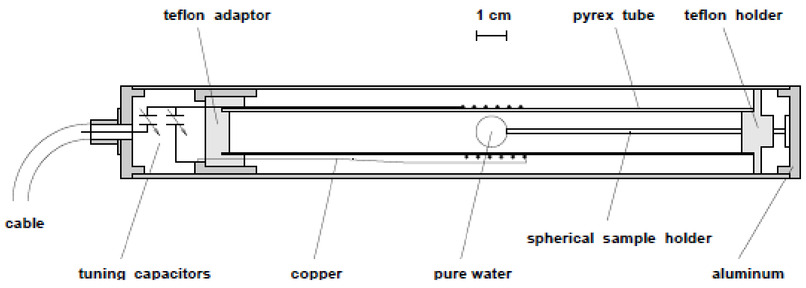
Need calibration of the probes !

(a) NMR Trolley

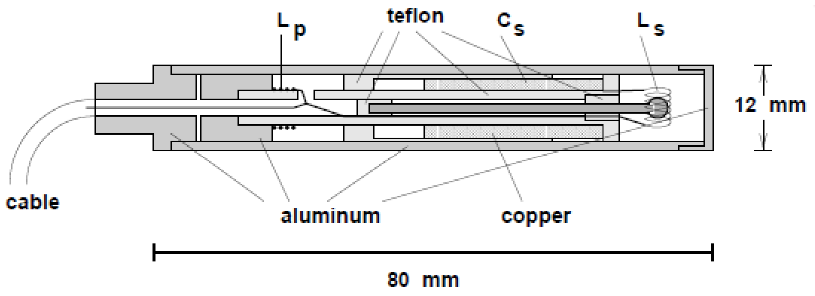
"It can pulled through the storage ring"



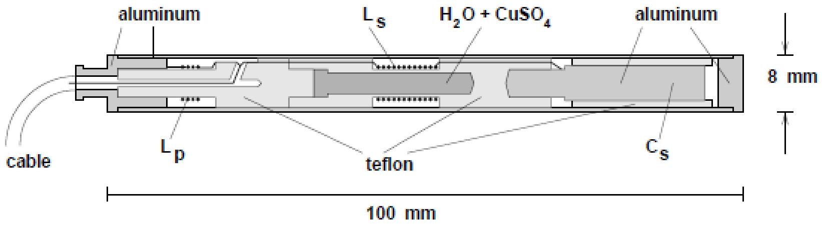
(b) Distribution of NMR probes over a cross section of the trolley



(a) Absolute calibration probe



(b) Plunging probe

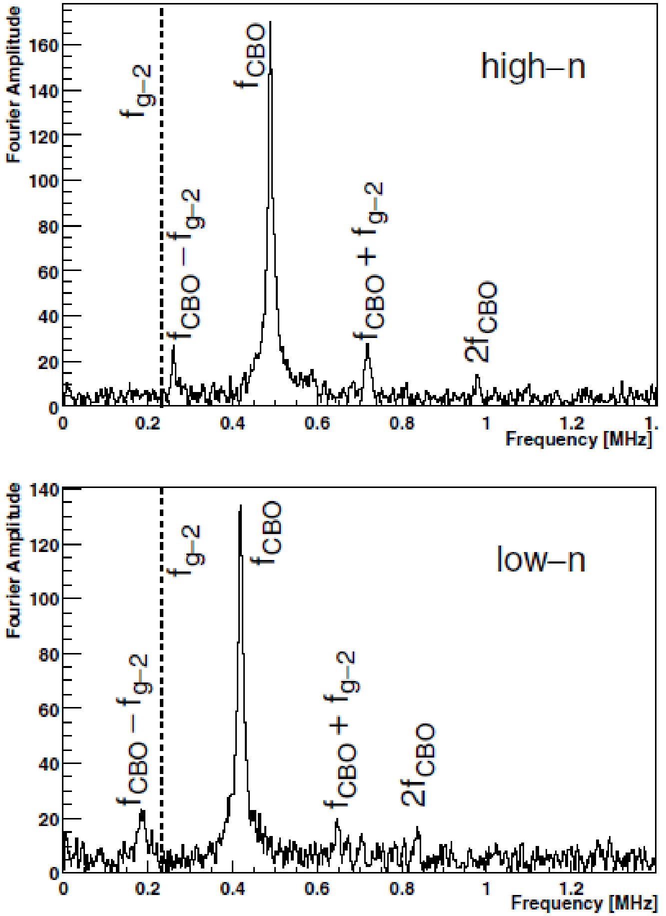


(c) Trolley and fixed probe

[Question from Yuhang]

In Fig.2.What is the meaning of using high-n and low-n?

in 2000. In 2000, the field focusing index n, which is proportional to the electric field gradient, was $n = 0.137$, corresponding to a horizontal coherent betatron oscillation frequency (CBO) of 466 kHz [3]. This frequency was close to twice the $(g-2)$ frequency of 229 kHz, which resulted in a sizable uncertainty in the fitted ω_a value [3]. In 2001, we used two different n values, $n = 0.122$ and $n = 0.142$, which resulted in CBO frequencies, 419 and 491 kHz, that are further from twice the $(g-2)$ frequency (see Fig. 2). Consequently, the uncertainty caused by CBO is smaller. Furthermore, it also reduced the correlation between the CBO and detector gain effects in the fits to the time spectrum.



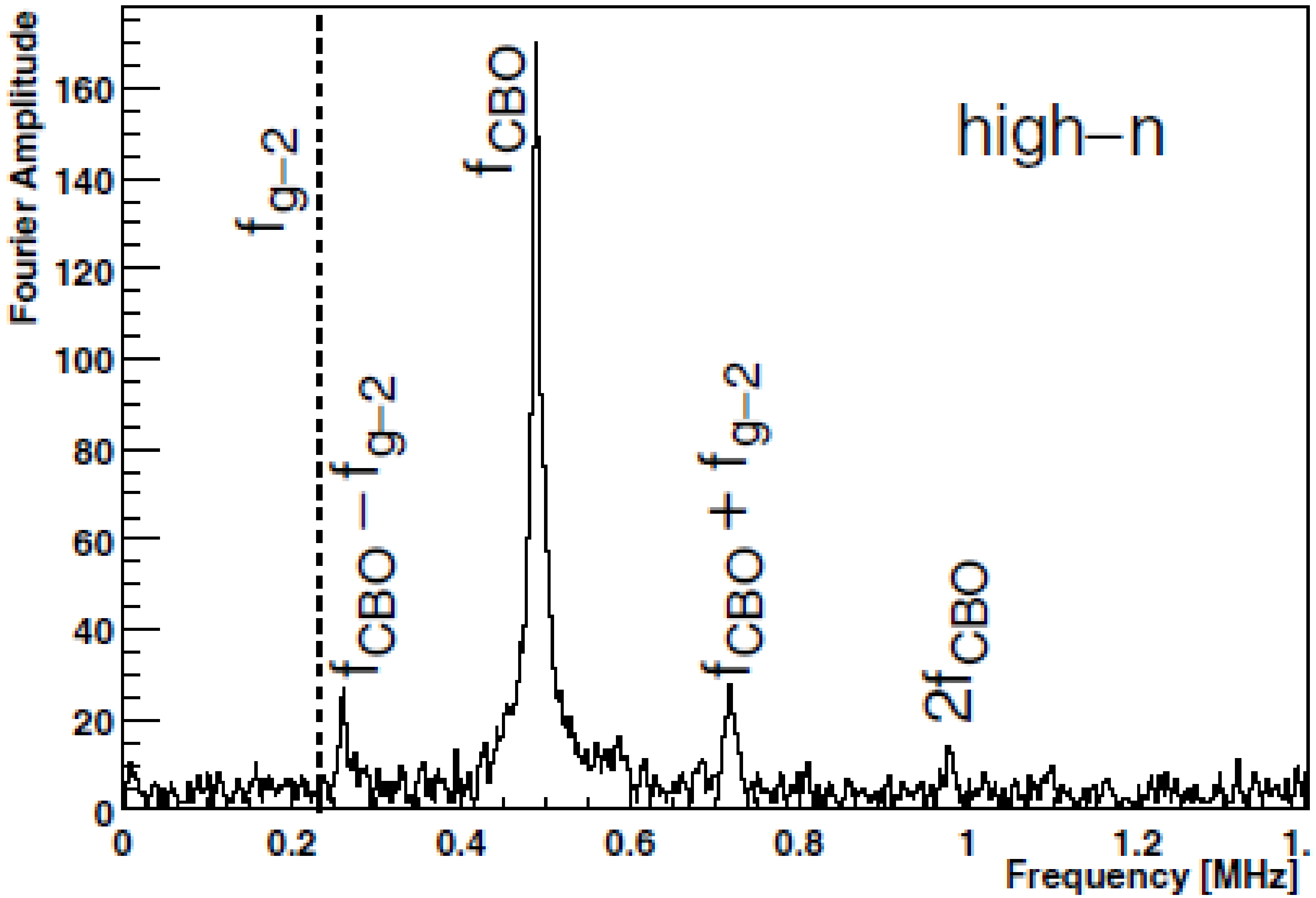
[Question from Amit]

Why is the filed focusing index n is so important? because by changing it's value, also reduced the correlation between the CBO and detector gain effects in the fits to the time spectrum.

It is also related question

Residual frequency

in 2000, n = 0.137, corresponding to CBO frequency of 466 kHz



Close to twice the g-2 frequency ….. bad situation

Change “n” so as the CBO frequency

Ref

TABLE VIII: Important frequencies and periods in the $(g-2)$ storage ring for $n = 0.137$.

Physical frequency	Variable	Expression	Frequency	Period
Anomalous precession	f_a	$\frac{e}{2\pi m} a_\mu B$	0.23 MHz	4.37 μs
Cyclotron	f_c	$\frac{v}{2\pi R_0}$	6.71 MHz	149 ns
Horizontal betatron	f_x	$\sqrt{1-n} f_c$	6.23 MHz	160 ns
Vertical betatron	f_y	$\sqrt{n} f_c$	2.48 MHz	402 ns
Horizontal CBO	f_{CBO}	$f_c - f_x$	0.48 MHz	2.10 μs
Vertical waist	f_{VW}	$f_c - 2f_y$	1.74 MHz	0.57 μs

Comparison (conclusion)

The standard model (SM) prediction for a_μ consists of QED, hadronic, and weak contributions. The uncertainty on the standard model value is dominated by the uncertainty on the lowest-order hadronic vacuum polarization. This contribution can be determined directly from the annihilation of e^+e^- to hadrons through a dispersion integral [12]. The indirect determination using data from hadronic τ decays, the conserved vector current hypothesis, plus the appropriate isospin corrections, could in principle improve the precision of a_μ(had). However, discrepancies between the τ and the e^+e^- results exist [13,14]. The two data sets do not give consistent results for the pion form factor. Using e^+e^- annihilation data, the corresponding theoretical value is $a_\mu(\mathrm{SM}) = 11\,659\,181(8) \times 10^{-10}$ (0.7 ppm). The value deduced from τ decay is larger by 15×10^{-10} and has a stated uncertainty of 7×10^{-10} (0.7 ppm). The difference between the experimental determination of a_μ and the standard model theory using the e^+e^- or τ data for the calculation of the hadronic vacuum polarization is 2.7σ and 1.4σ, respectively.

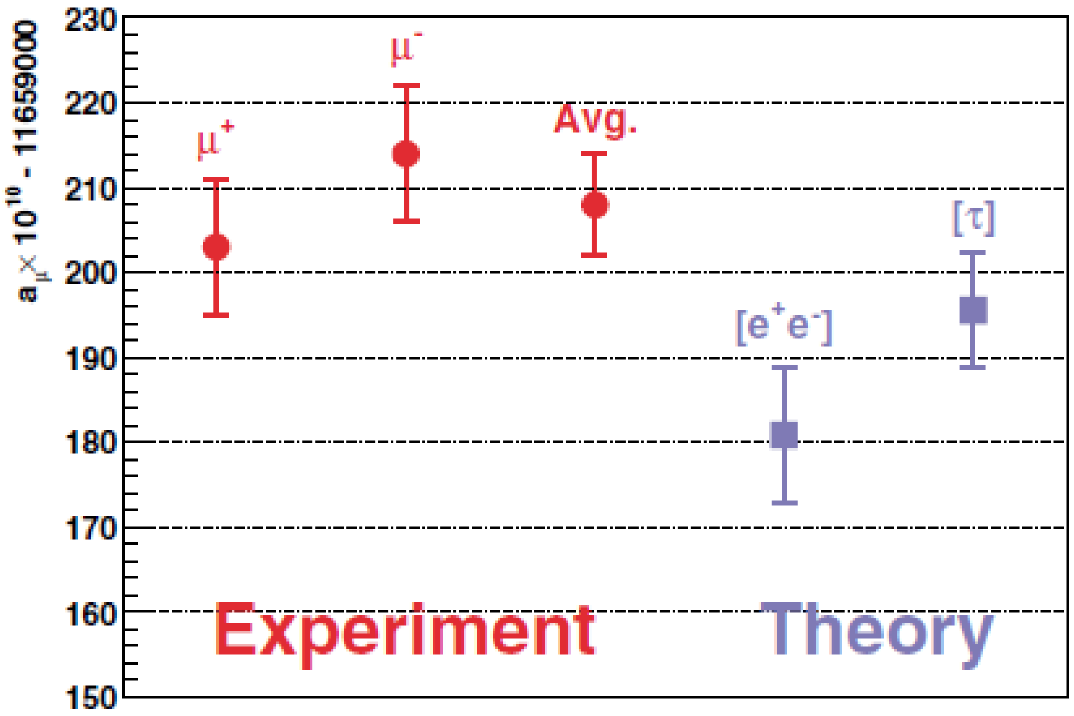


FIG. 4 (color online). Measurements of a_μ by E821 with the SM predictions (see text for discussion). Uncertainties indicated on the measurements are total uncertainties.

From :

“The Muon g-2 experiment at fermilab”

W. Gohn for the Muon g-2 Collaboration, arXiv:1801.00084v1

Abstract. A new measurement of the anomalous magnetic moment of the muon, $a_\mu \equiv (g-2)/2$, will be performed at the Fermi National Accelerator Laboratory with data taking beginning in 2017. The most recent measurement, performed at Brookhaven National Laboratory (BNL) and completed in 2001, shows a 3.5 standard deviation discrepancy with the standard model value of a_μ. The new measurement will accumulate 21 times the BNL statistics using upgraded magnet, detector, and storage ring systems, enabling a measurement of a_μ to 140 ppb, a factor of 4 improvement in the uncertainty the previous measurement. This improvement in precision, combined with recent improvements in our understanding of the QCD contributions to the muon g-2, could provide a discrepancy from the standard model greater than 7σ if the central value is the same as that measured by the BNL experiment, which would be a clear indication of new physics.