Solar Eclipses in the Neutrino Light

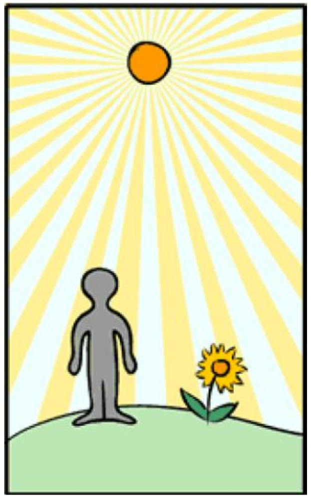
Shun Zhou

(IHEP&UCAS)

Based on _Nucl. Phys. B 931 (2018) 324_
in collaboration with Guo-yuan Huang and Jun-hao Liu

Seminar @ Center for High Energy Physics, Peking University, 2018-7-5

Why the Sun is shining?



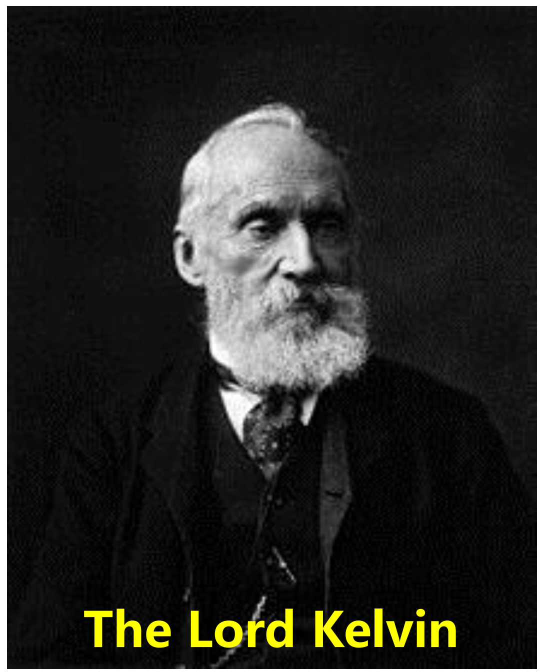
Conflicting Estimates of the Solar Age

In 1854, von Helmholtz proposed that the enormous energy radiated by the Sun is due to the gravitational contraction

In 1859, Darwin estimated the solar age to be 300 million years, based on the biological and geological arguments

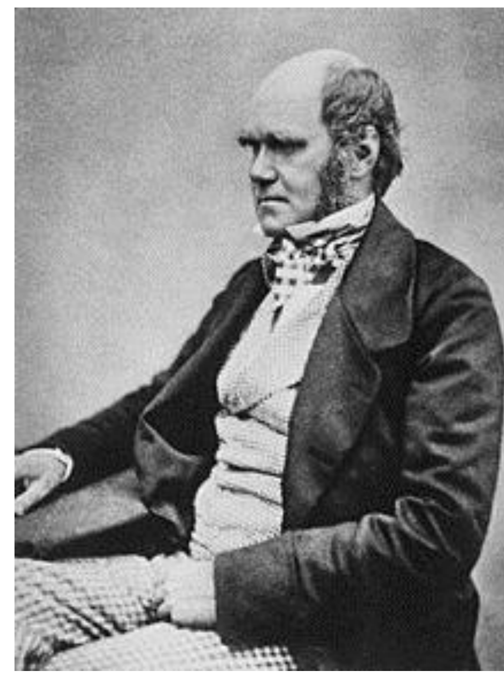
In 1862, Lord Kelvin estimated the solar age as 20 million years, based on the gravitational energy arguments

Hermann von Helmholtz
(1821 - 1894)



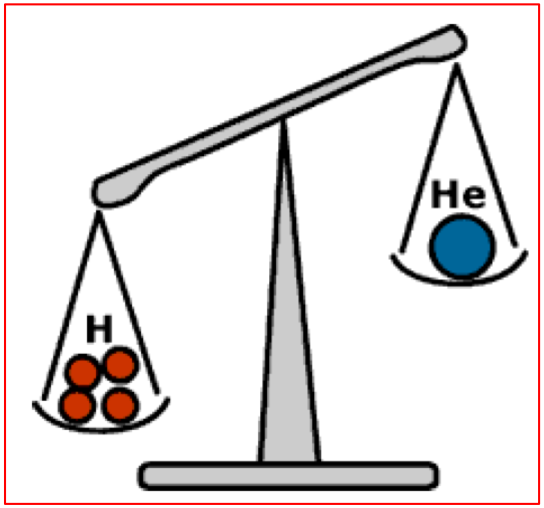
William Thomson
(1824 - 1907)

VS

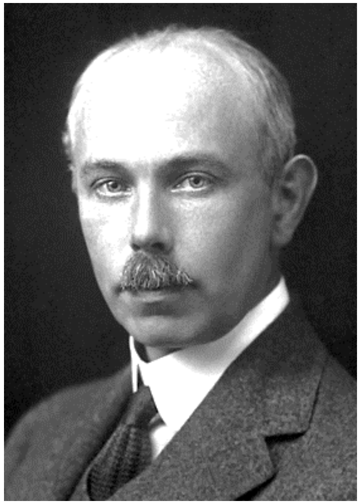


Charles Darwin
(1809 - 1882)

Why the Sun is shining?



Mass (4H) > Mass (He)

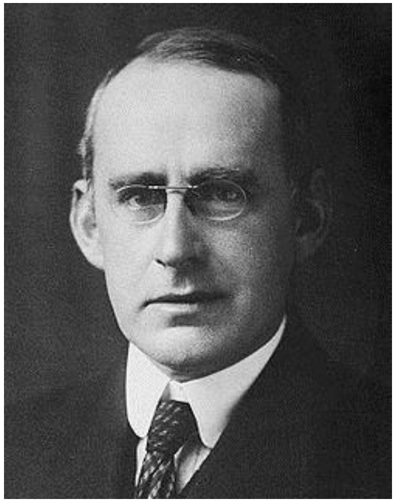


Francis Aston (1877 - 1945)

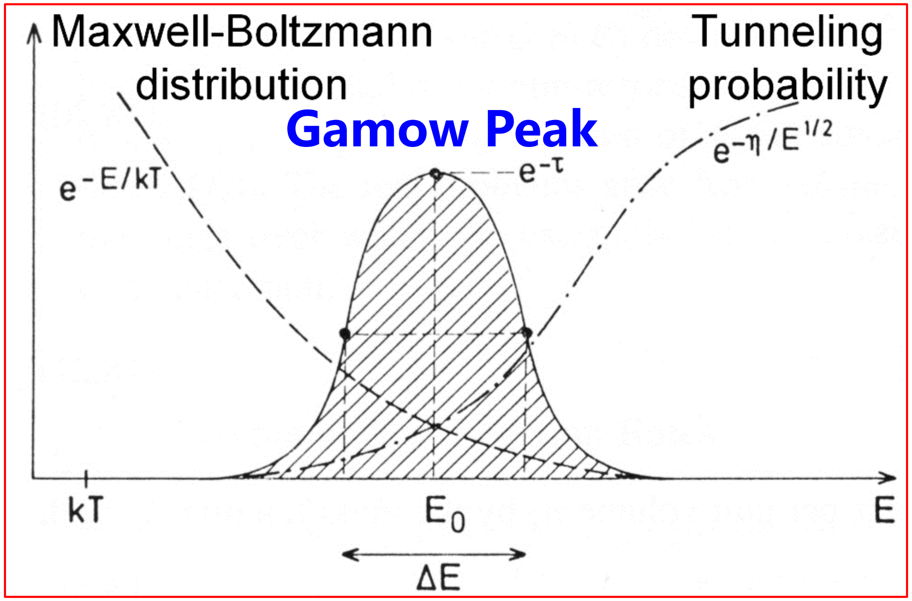
Soon after Aston measured the masses of hydrogen and helium atoms in 1920, Eddington recognized that

4H → He + 0.7% of the mass (lasting for 100 billion years)

But how does this process happen?



Arthur Eddington (1882 - 1944)



George Gamow (1904 - 1968)

Virial Theorem

$$\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$$

$$M_{\rm sun} = 1.99\times10^{33}\,{\rm g}$$

$$R_{\rm sun} = 6.96\times10^{10}\,{\rm cm}$$

$$\langle E_{\rm grav}\rangle = -3.2\ {\rm keV}$$

The estimated temperature $T = 1.1$ keV (cf. $T = 1.34$ keV)

Why the Sun is shining?

Hydrogen Burning: *pp* Chains

"Energy Production in Stars"
H.A. Bethe, PR 55 (1939) 434

$$4p \to {}^4\mathrm{He} + 2e^+ + 2\nu_e + 26.73\ \mathrm{MeV}$$

Hans A. Bethe (1906–2005)

$\mathrm{p} + \mathrm{p} \to {}^2\mathrm{H} + \mathrm{e}^+ + \nu_e$ < 0.420 MeV — 100%

$\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to {}^2\mathrm{H} + \nu_e$ 1.442 MeV — 0.24%

$^2\mathrm{H} + \mathrm{p} \to {}^3\mathrm{He} + \gamma$

85% — **PP-I**: $^3\mathrm{He} + {}^3\mathrm{He} \to {}^4\mathrm{He} + 2\mathrm{p}$

15%: $^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma$

hep: $^3\mathrm{He} + \mathrm{p} \to {}^4\mathrm{He} + \mathrm{e}^+ + \nu_e$ < 18.8 MeV

90%: $^7\mathrm{Be} + \mathrm{e}^- \to {}^7\mathrm{Li} + \nu_e$ 0.862 MeV

10%: $^7\mathrm{Be} + \mathrm{e}^- \to {}^7\mathrm{Li}^* + \nu_e$ 0.384 MeV

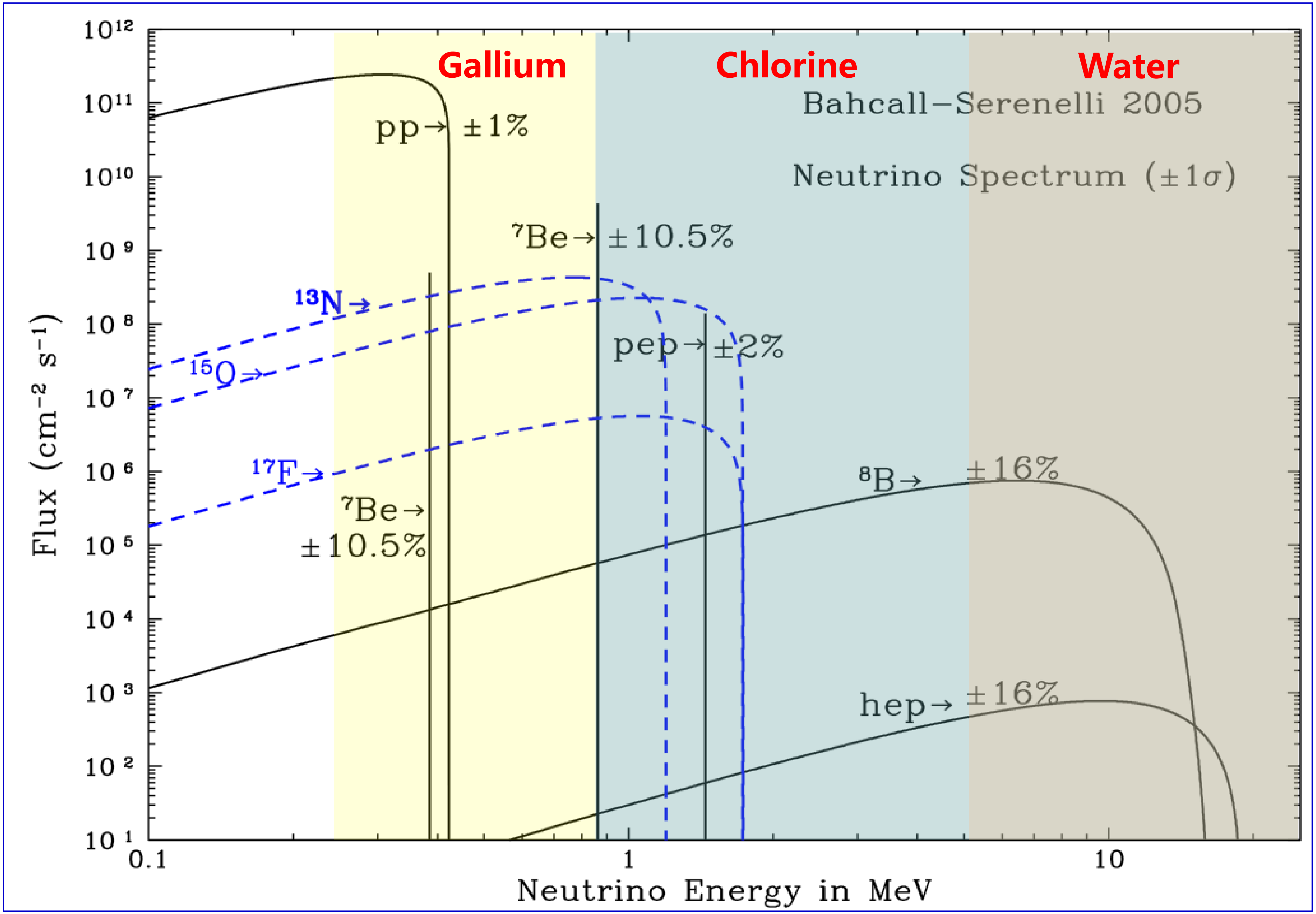
0.02%: $^7\mathrm{Be} + \mathrm{p} \to {}^8\mathrm{B} + \gamma$

$^8\mathrm{B} \to {}^8\mathrm{Be}^* + \mathrm{e}^+ + \nu_e$ < 15 MeV

PP-II: $^7\mathrm{Li} + \mathrm{p} \to {}^4\mathrm{He} + {}^4\mathrm{He}$

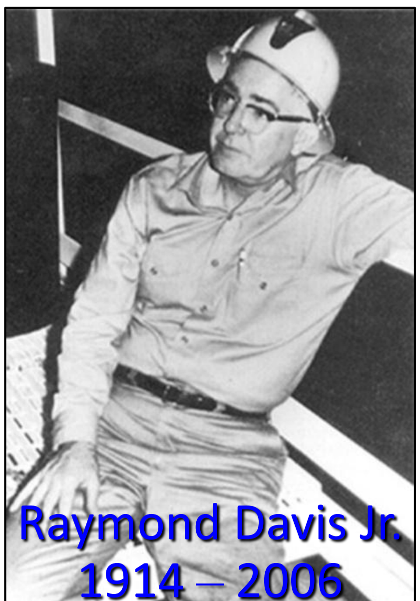
PP-III: $^8\mathrm{Be}^* \to {}^4\mathrm{He} + {}^4\mathrm{He}$

Solar Neutrino Spectrum



Detection of Solar Neutrinos

Solar Neutrinos: **the key to understand thermal nuclear reactions in the Sun**



SOLAR NEUTRINOS. I. THEORETICAL*

John N. Bahcall
California Institute of Technology, Pasadena, California
(Received 6 January 1964)

The principal energy source for main-sequence stars like the sun is believed to be the fusion, in the deep interior of the star, of four protons to form an alpha particle.[1] The fusion reactions are thought to be initiated by the sequence $^1H(p, e^+\nu)^2H(p,\gamma)^3He$ and terminated by the following sequences: (i) $^3He(^3He, 2p)^4He$; (ii) $^3He(\alpha,\gamma)^7Be(e^-\nu)^7Li(p,\alpha)^4He$; and (iii) $^3He(\alpha,\gamma)^7Be(p,\gamma)^8B(e^+\nu)^8Be^*(\alpha)^4He$. No direct evidence for the existence of nuclear reactions in the interiors of stars has yet been obtained because the mean free path for photons emitted in the center of a star is typically less than 10^{-10} of the radius of the star. Only neutrinos, with their extremely small interaction cross sections, can enable us to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation in stars.

The most promising method[2] for detecting solar neutrinos is based upon the endothermic reaction ($Q = -0.81$ MeV) $^{37}Cl(\nu_{solar}, e^-)^{37}Ar$, which was first discussed as a possible means of detecting neutrinos by Pontecorvo[3] and Alvarez.[4] In this note, we predict the number of absorptions of

VOLUME 12, NUMBER 11 PHYSICAL REVIEW LETTERS 16 MARCH 1964

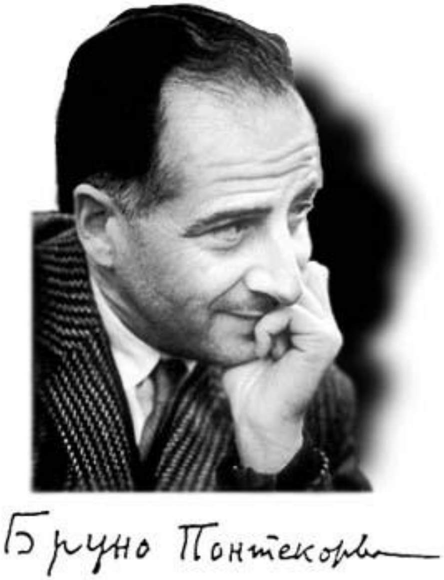
SOLAR NEUTRINOS. II. EXPERIMENTAL*

Raymond Davis, Jr.
Chemistry Department, Brookhaven National Laboratory, Upton, New York
(Received 6 January 1964)

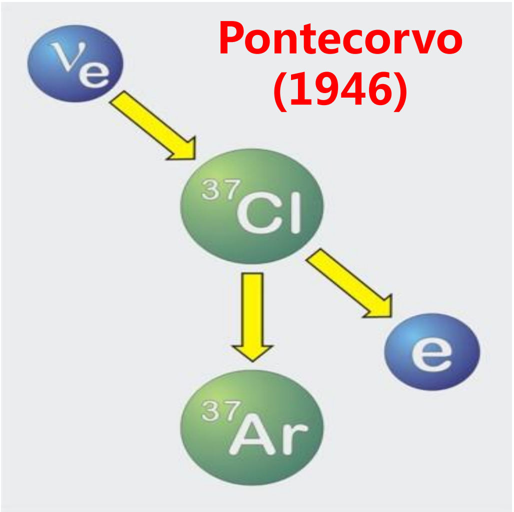
The prospect of observing solar neutrinos by means of the inverse beta process $^{37}Cl(\nu, e^-)^{37}Ar$ induced us to place the apparatus previously described[1] in a mine and make a preliminary search. This experiment served to place an upper limit on the flux of extraterrestrial neutrinos. These 3 counts in 18 days is probably entirely due to the background activity. However, if one assumes that this rate corresponds to real events and uses the efficiencies mentioned, the upper limit of the neutrino capture rate in 1000 gallons of C_2Cl_4 is ≤ 0.5 per day or $\varphi\bar{\sigma} \leq 3\times10^{-34}$ sec^{-1} (^{37}Cl atom)$^{-1}$.

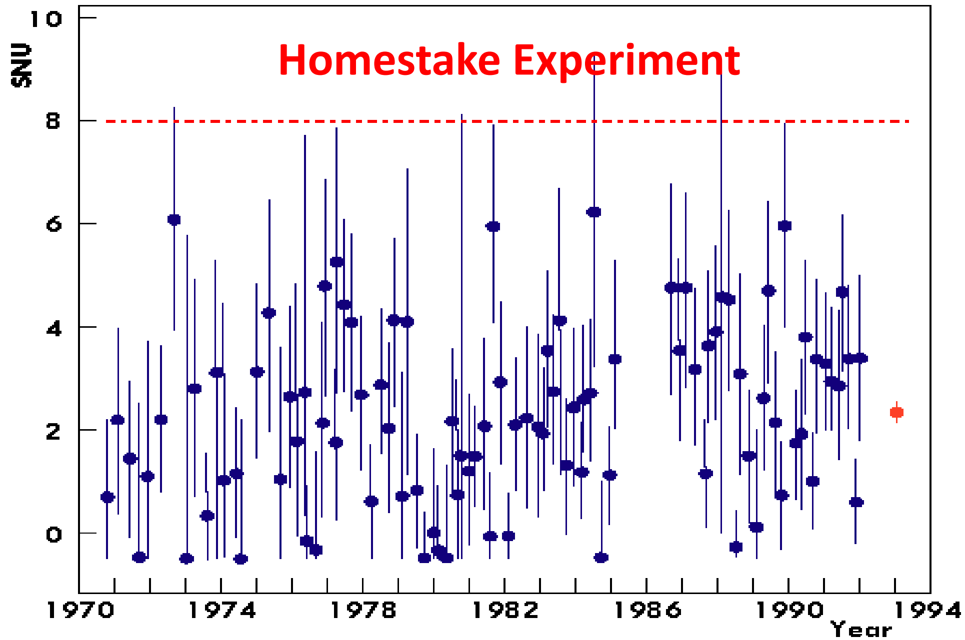
Detection of Solar Neutrinos

Solar Neutrinos: **the key to understand thermal nuclear reactions in the Sun**



Bruno Pontecorvo
(1913 - 1993)



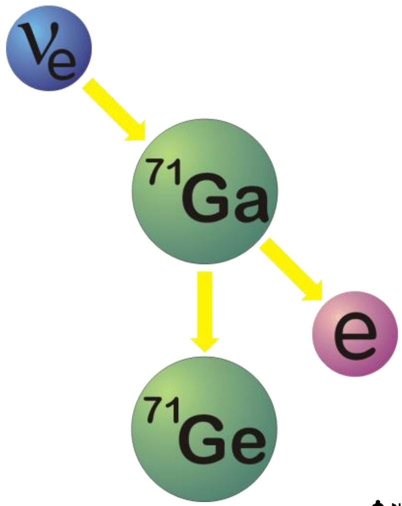


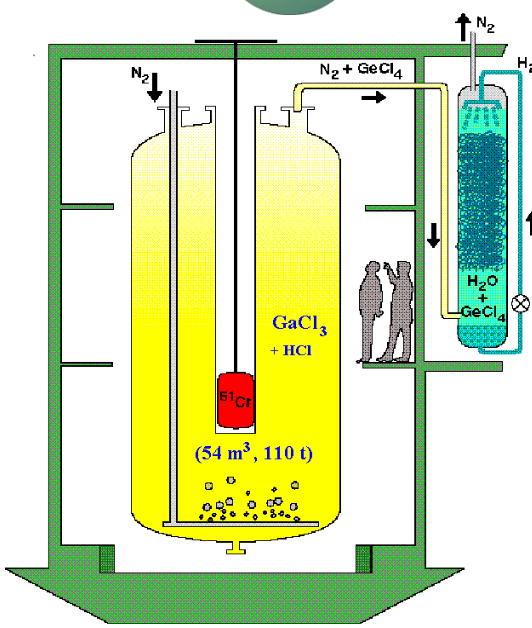
Ray Davis Jr.
(1914 - 2006)

Solar Neutrino Problem

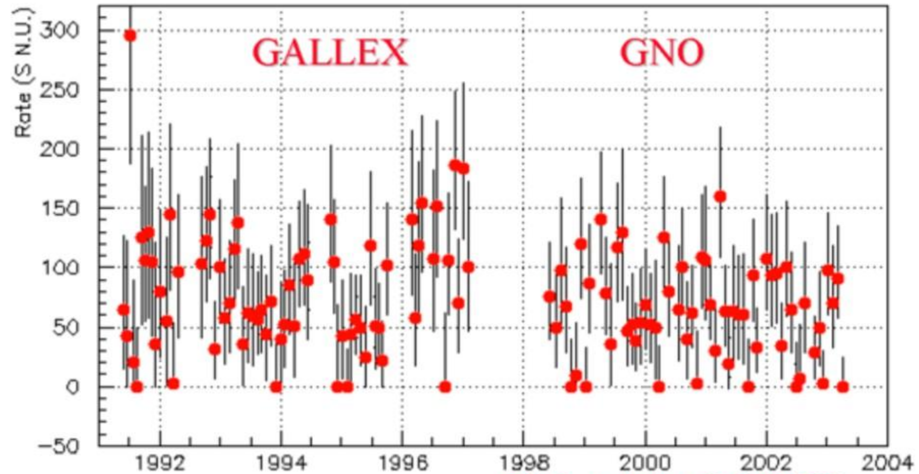
Detection of Solar Neutrinos

Inverse Beta Decay
Gallium → Germanium

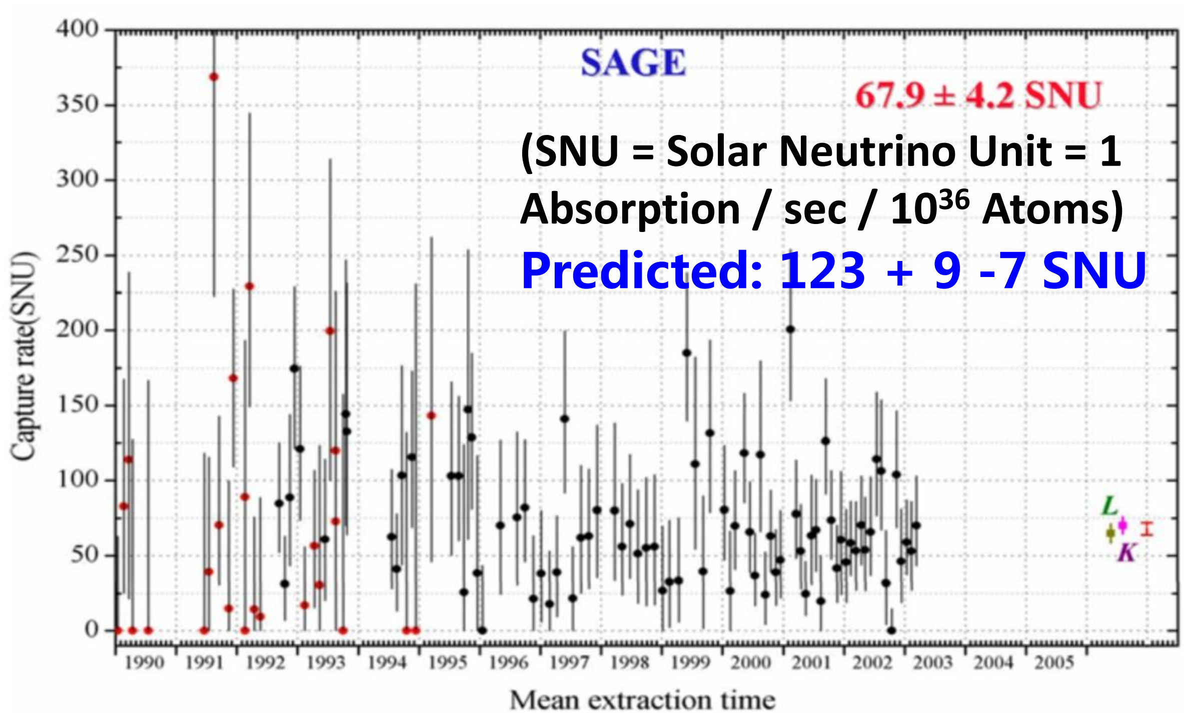




GALLEX/GNO (1991–2003)



69.3 ± 4.1 ± 3.6 SNU

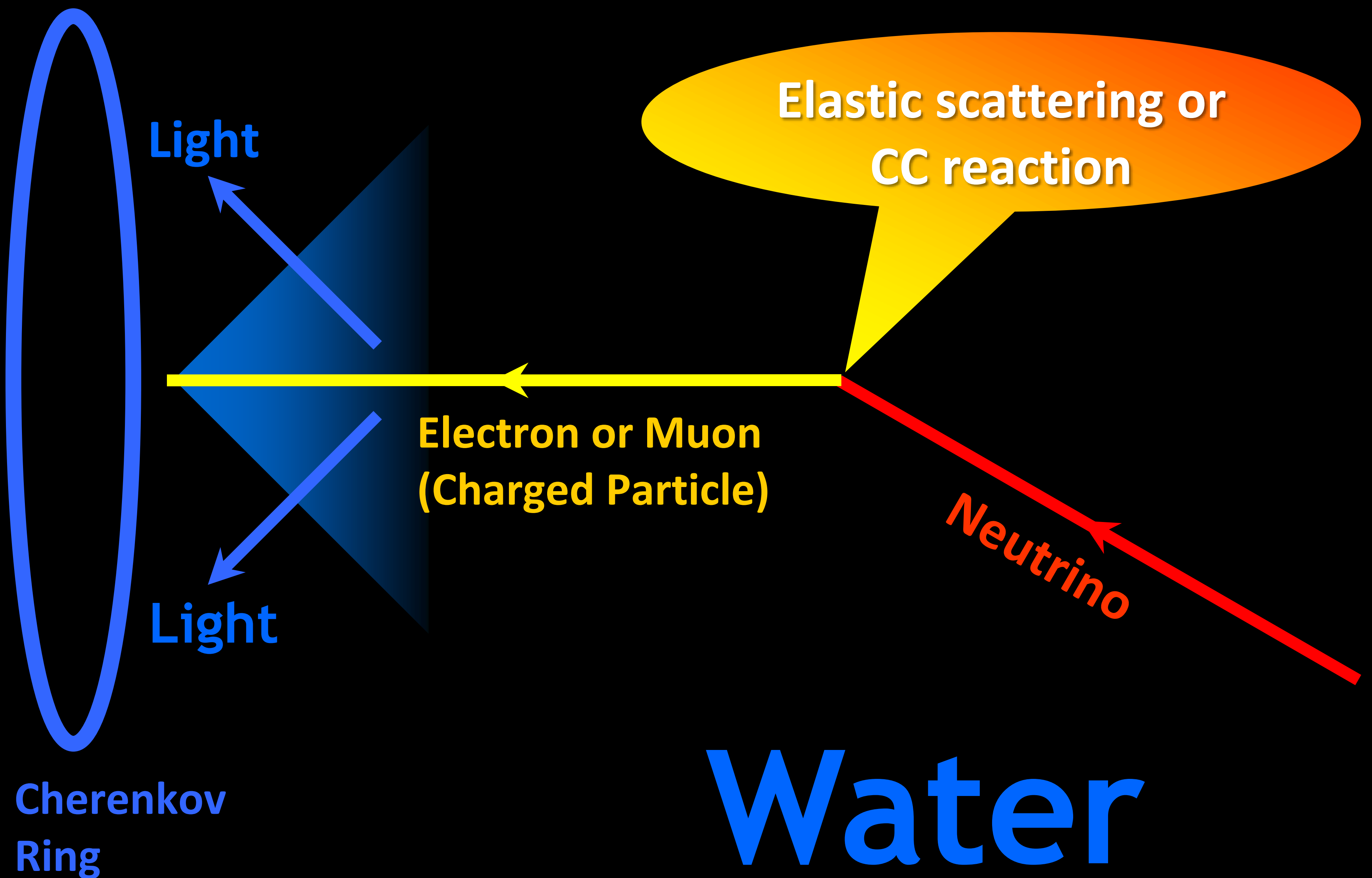


67.9 ± 4.2 SNU

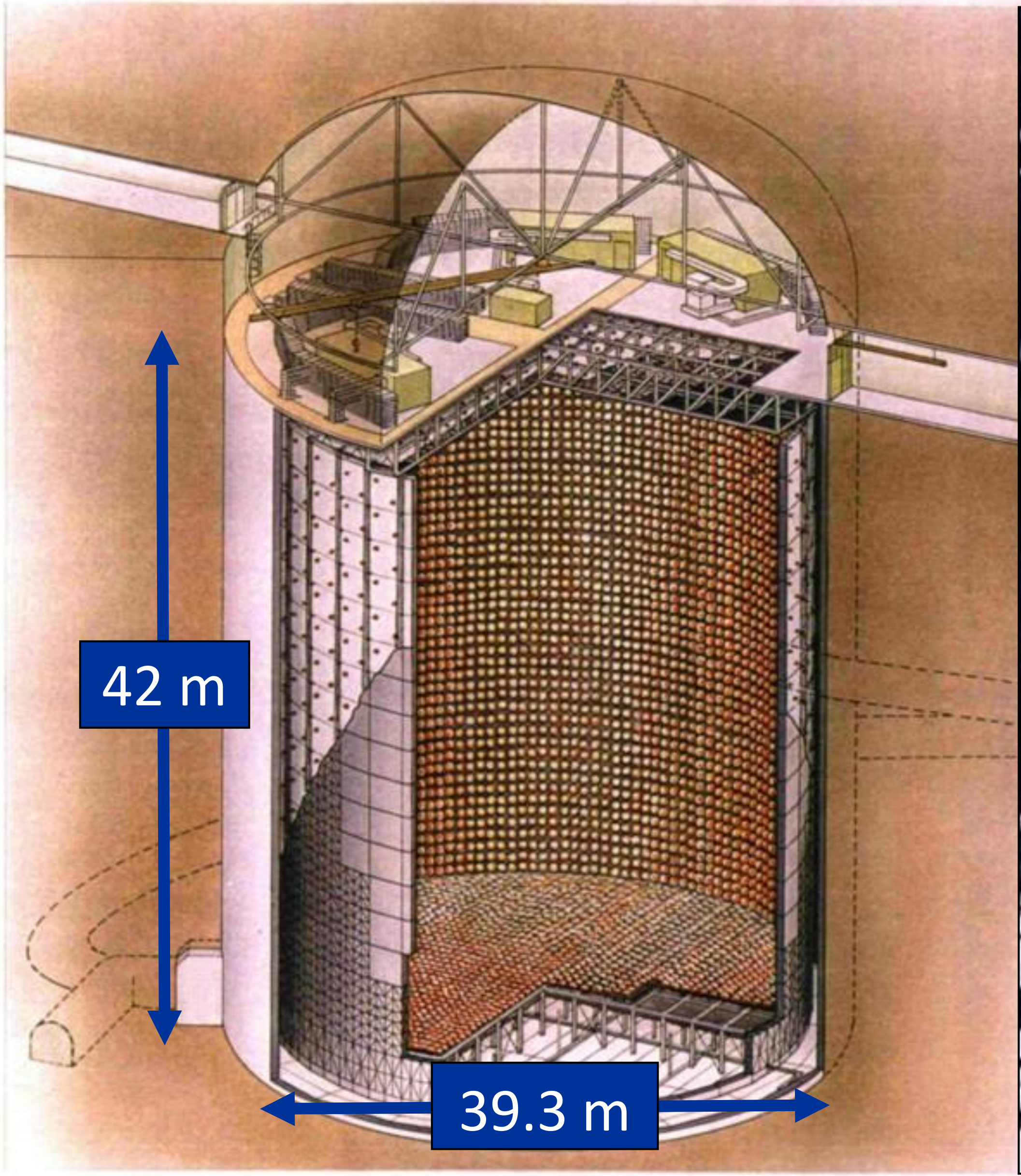
(SNU = Solar Neutrino Unit = 1 Absorption / sec / 10^{36} Atoms)

Predicted: 123 + 9 -7 SNU

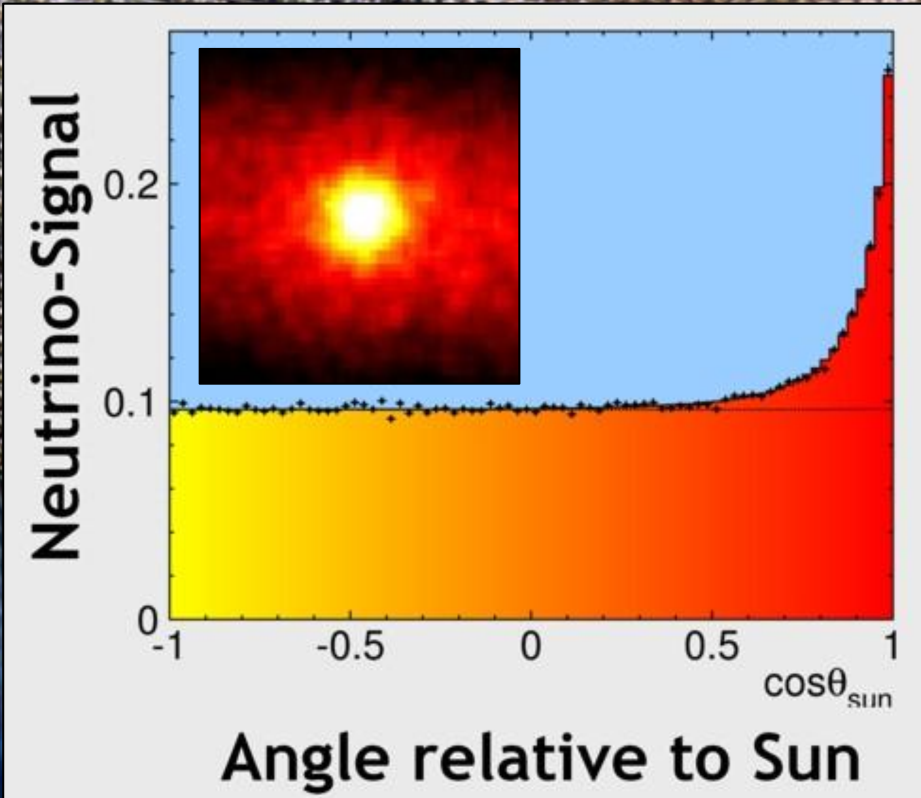
Cherenkov Effect

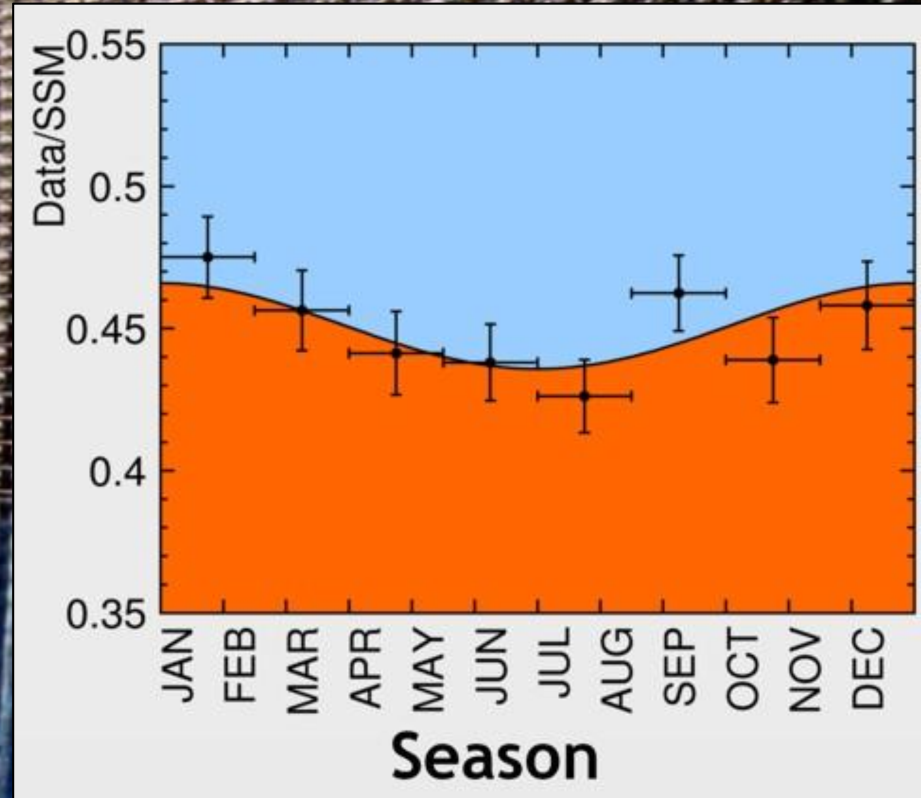


Super-Kamiokande Neutrino Detector (since 1996)

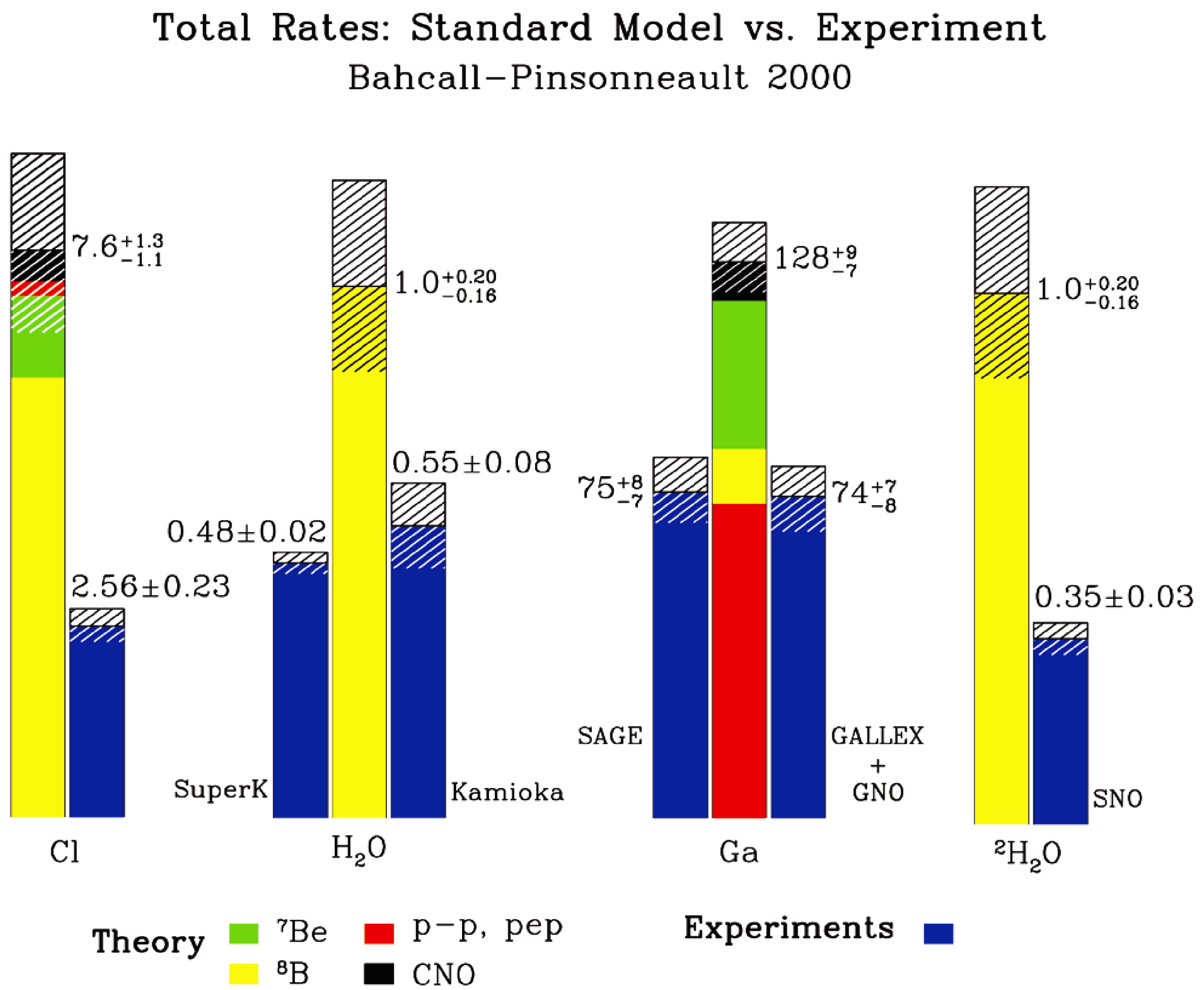


Super-Kamiokande: Sun in the Light of Neutrinos

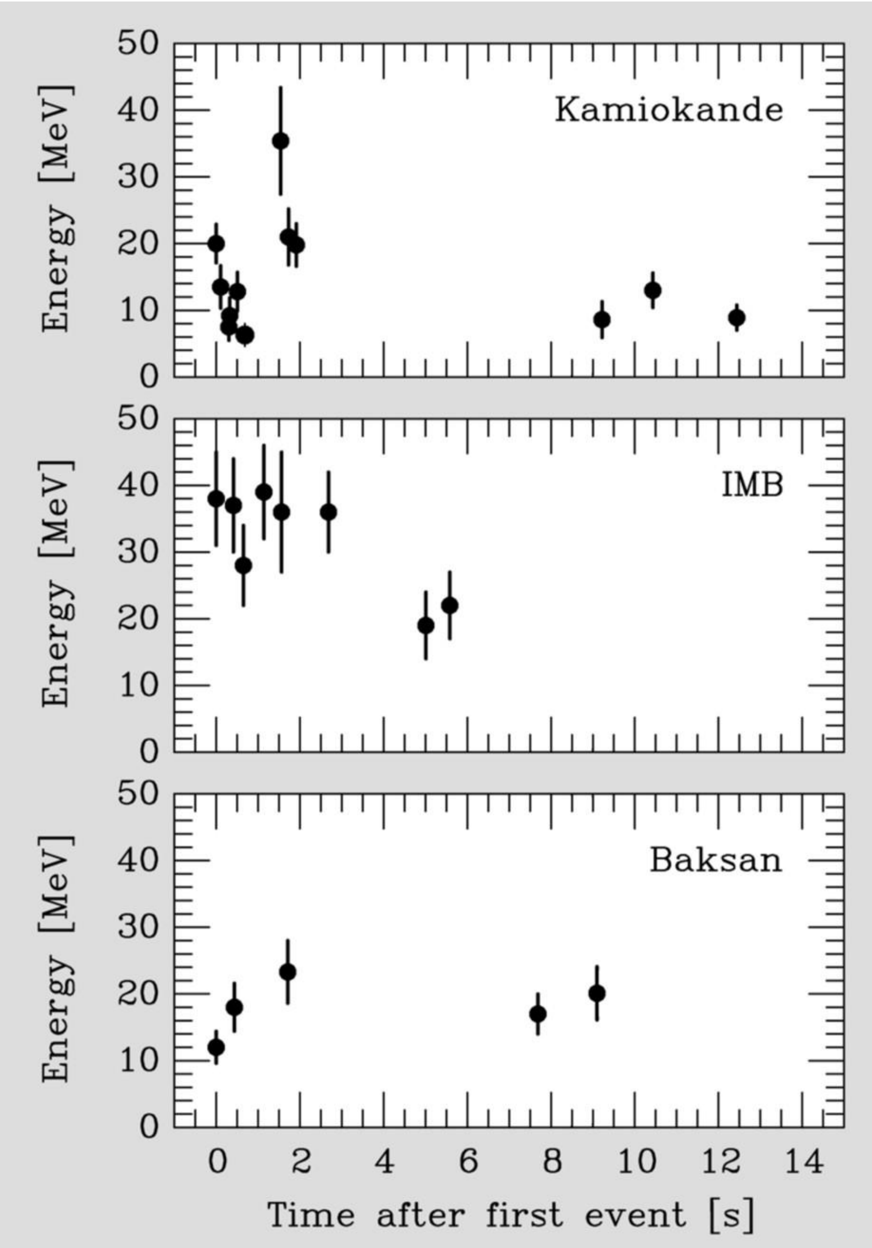


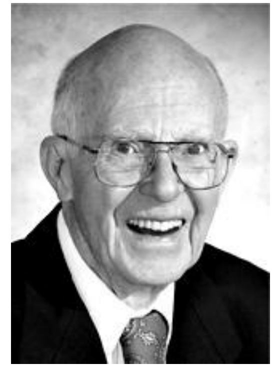


Solar Neutrino Problem



Nobel Prize in Physics 2002





R. Davis

M. Koshiba

R. Giacconi

"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

K-II paper, PRL 58 (1987) 1490
received 10 March, published 6 April

IMB paper, PRL 58 (1987) 1494
received 13 March, published 6 April

Davis and Koshiba made extraordinary contributions in part because “**solar neutrino experiments** have a sensitivity that is not accessible [with neutrinos] from the Earth,” says Bahcall.

Phys. Rev. Focus 10 (2002) 18

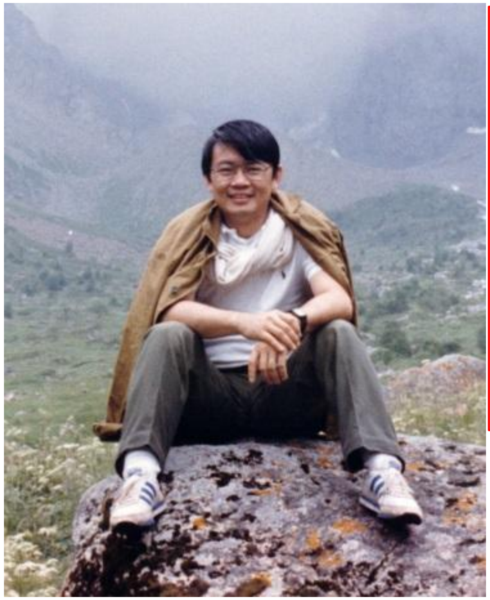
SNO: Solving the Solar Neutrino Problem

VOLUME 55, NUMBER 14 PHYSICAL REVIEW LETTERS 30 SEPTEMBER 1985

Direct Approach to Resolve the Solar-Neutrino Problem

Herbert H. Chen
Department of Physics, University of California, Irvine, California 92717
(Received 27 June 1985)

A direct approach to resolve the solar-neutrino problem would be to observe neutrinos by use of both neutral-current and charged-current reactions. Then, the total neutrino flux and the electron-neutrino flux would be separately determined to provide independent tests of the neutrino-oscillation hypothesis and the standard solar model. A large heavy-water Cherenkov detector, sensitive to neutrinos from ^{8}B decay via the neutral-curent reaction $\nu + d \rightarrow \nu + p + n$ and the charged-current reaction $\nu_e + d \rightarrow e^- + p + p$, is suggested for this purpose.



陈华森
Herbert H. Chen (1942-1987)

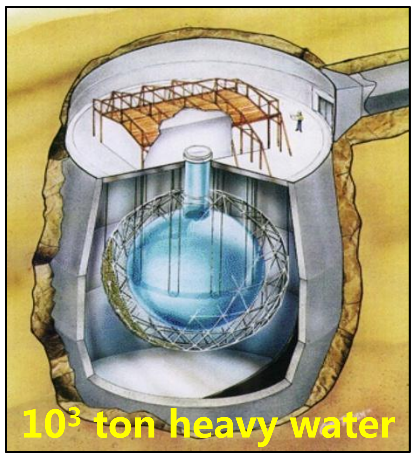
Phys. Rev. Lett. 55 (1985) 1534

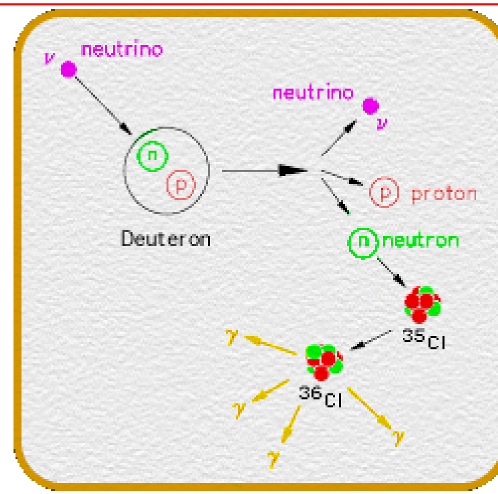
1942: Born in Chongqing

1964: B.Sc. Caltech

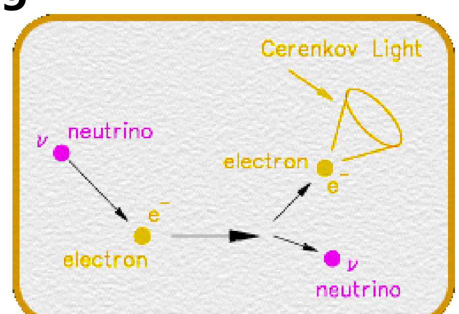
1968: Ph.D. Princeton (Advisor: Sam Treiman)

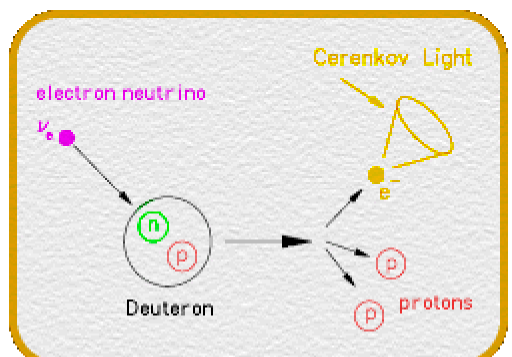
1984: SNO spokesperson





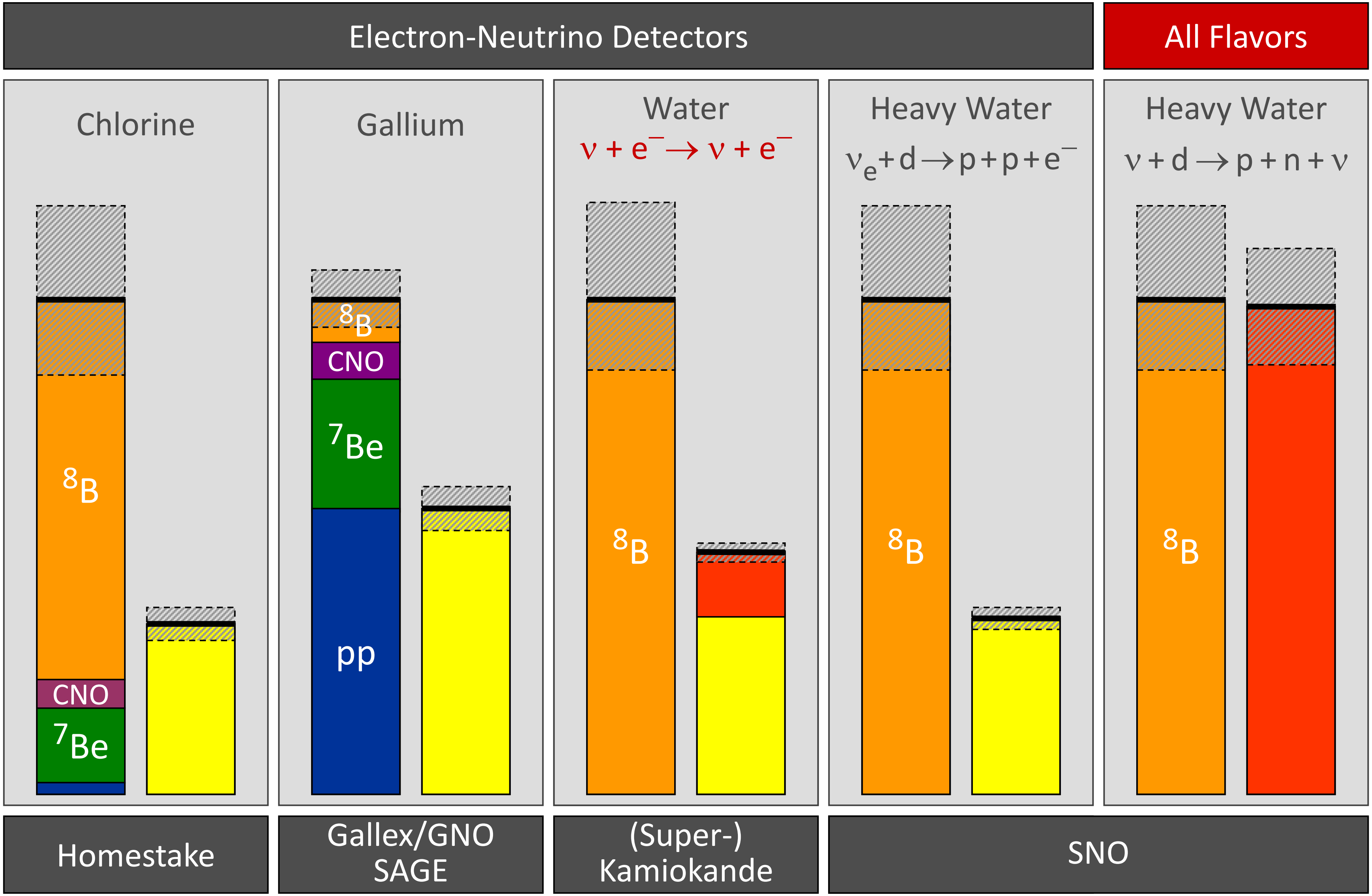
$$\text{CC}: \quad \nu_e + d \rightarrow p + p + e^-$$
$$\text{NC}: \quad \nu_\alpha + d \rightarrow p + n + \nu_\alpha$$
$$\text{ES}: \quad \nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$$



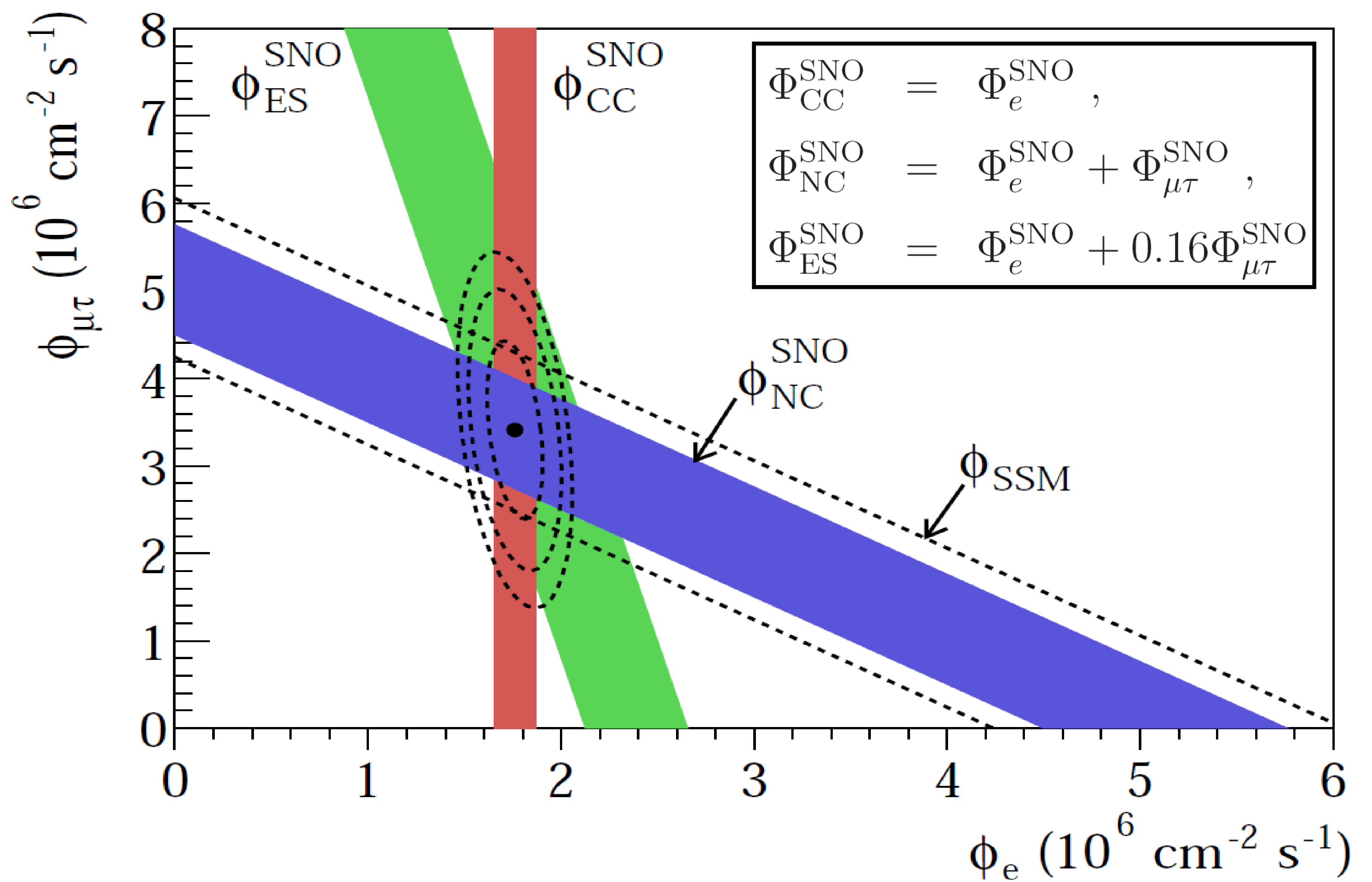


Arthur B. McDonald (SNO Spokesperson since 1987)

SNO: Solving the Solar Neutrino Problem

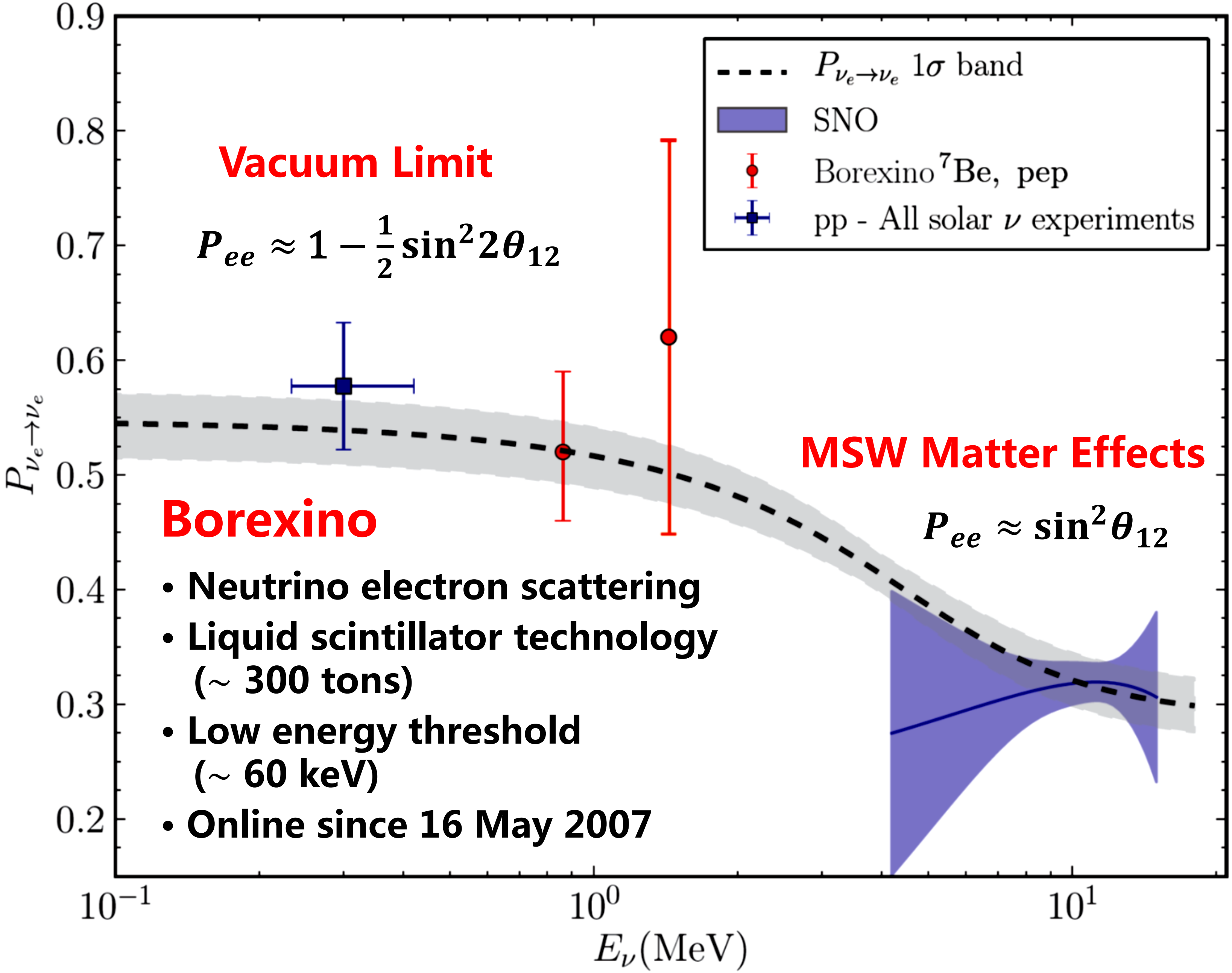


SNO: Solving the Solar Neutrino Problem



Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. 89 (2002) 011301

Flavor Conversions of Solar Neutrinos



Vacuum Limit

$$P_{ee} \approx 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

MSW Matter Effects

$$P_{ee} \approx \sin^2\theta_{12}$$

Borexino

- **Neutrino electron scattering**
- **Liquid scintillator technology (~ 300 tons)**
- **Low energy threshold (~ 60 keV)**
- **Online since 16 May 2007**

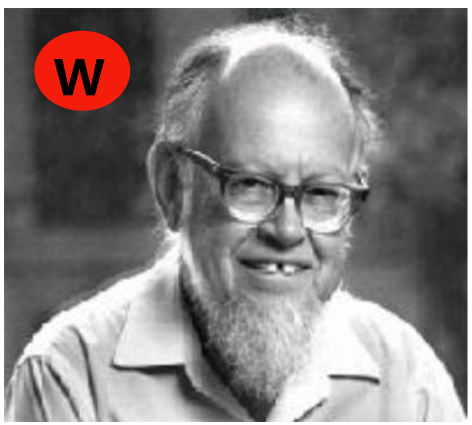
Neutrino Oscillations in Matter

PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

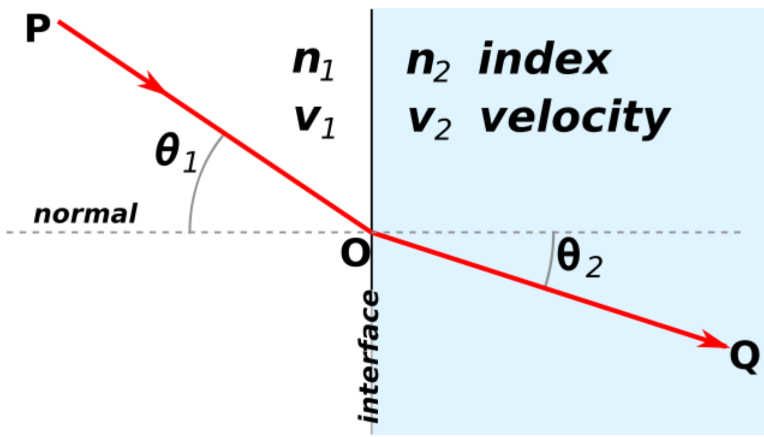
Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein (1923-2015)



Refraction of light in media

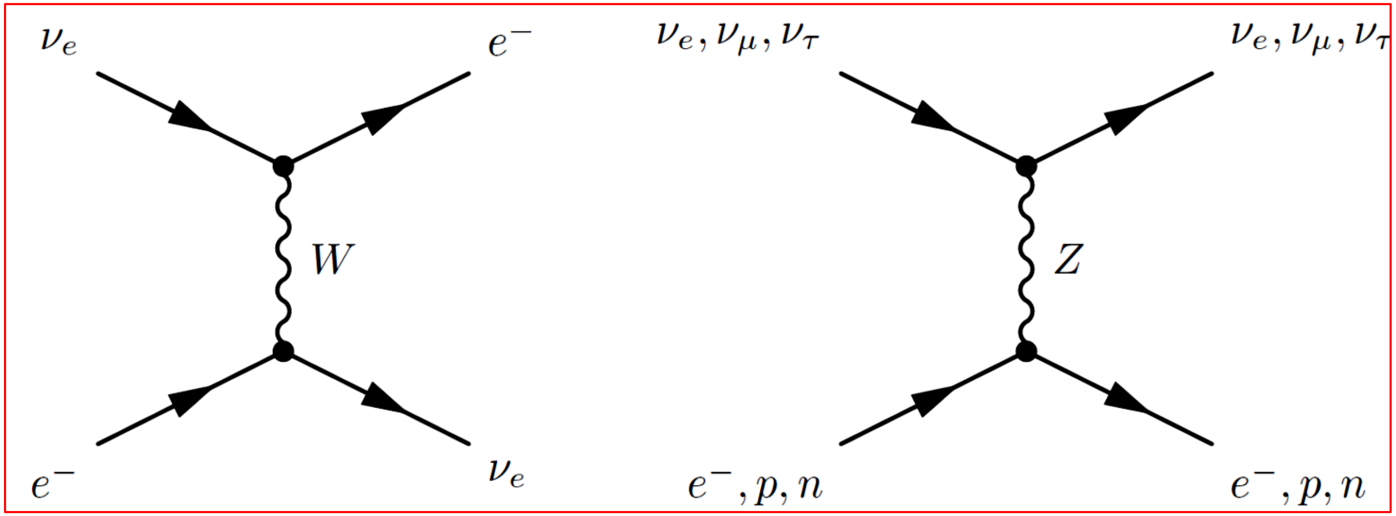
$$\nu_e : \exp[ipx(n_{\rm nc} + n_{\rm cc} - 1)]$$
$$\nu_\mu : \exp[ipx(n_{\rm nc} - 1)]$$
$$\nu_\tau : \exp[ipx(n_{\rm nc} - 1)]$$

Refraction of neutrinos in media, where both CC and NC interactions contribute to refractive indices (not far from 1)

When neutrinos are traveling in matter, the effect of coherent forward scattering with background particles leads to a modification of their energies. Such a modification can be described by a potential energy. The difference between the potentials of distinct neutrino flavors is relevant for neutrino oscillations.

Matter Potentials for Neutrinos

Ordinary matter contains only electrons, neutrons and protons:



Incoherent scattering

$$\ell_{\rm matter} \sim \frac{10^{14}\,{\rm cm}}{(E/{\rm GeV})}$$

We take the number density of particles in normal matter to be about 10^{24} cm^{-3}.

First of all, we look at the Hamiltonian of free neutrinos in vacuum

$$\hat{\mathcal{H}}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix} = \begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix} = \left[E\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} + \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\0 & m_2^2\end{pmatrix}\right]\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$

in mass basis

in flavor basis

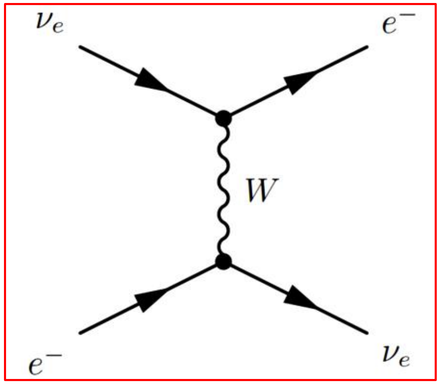
$$\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix} = \begin{pmatrix}c_\theta & s_\theta\\-s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix} \equiv U\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$

$$\hat{\mathcal{H}}\begin{pmatrix}c_\theta & -s_\theta\\s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\0 & m_2^2\end{pmatrix}\begin{pmatrix}c_\theta & -s_\theta\\s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix} \Rightarrow$$

$$\boldsymbol{\mathcal{H} = \frac{1}{2E}U\begin{pmatrix}m_1^2 & 0\\0 & m_2^2\end{pmatrix}U^\dagger}$$

Matter Potentials for Neutrinos

Effective Hamiltonian density for weak interactions



$$\mathcal{H}_{\rm cc}(x) = \frac{G_{\rm F}}{\sqrt{2}}\left[\bar{\nu}_e(x)\gamma^\mu(1-\gamma_5)e(x)\right]\left[\bar{e}(x)\gamma_\mu(1-\gamma_5)\nu_e(x)\right]$$

Fierz transformation

$$\mathcal{H}_{\rm cc}(x) = \frac{G_{\rm F}}{\sqrt{2}}\left[\bar{\nu}_e(x)\gamma^\mu(1-\gamma_5)\nu_e(x)\right]\left[\bar{e}(x)\gamma_\mu(1-\gamma_5)e(x)\right]$$

Averaged Hamiltonian density over the electron background

$$\bar{\mathcal{H}}_{\rm cc}(x) = \frac{G_{\rm F}}{\sqrt{2}}\bar{\nu}_e(x)\gamma^\mu(1-\gamma_5)\nu_e(x)\int d^3p_e\, \boldsymbol{f(p_e)}$$

Distribution function of electrons

Averaged over electron helicities

$$\times\frac{1}{2}\sum_{h_e=\pm}\left\langle e(p_e,h_e)\middle|\bar{e}(x)\gamma_\mu(1-\gamma_5)e(x)\middle|e(p_e,h_e)\right\rangle$$

$$|e(p_e,h_e)\rangle = \frac{1}{2EV}a_e^{(h_e)\dagger}|0\rangle$$

$$\int d^3p_e f(p_e) = N_e V$$

$$\bar{\mathcal{H}}_{\rm cc}(x) = \frac{G_{\rm F}}{\sqrt{2}}\int d^3p_e f(p_e)\,\bar{\nu}_e(x)\,\frac{\boldsymbol{p_{e\mu}}}{\boldsymbol{E_e V}}\gamma^\mu(1-\gamma_5)\nu_e(x)$$

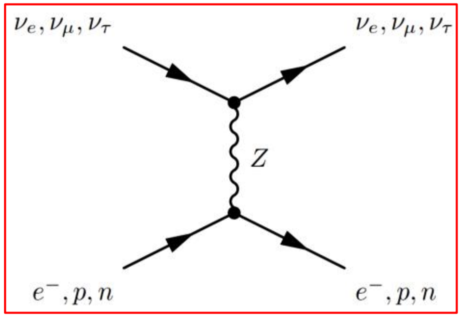
Matter Potentials for Neutrinos

$$\bar{\mathcal{H}}_{\rm cc}(x) = \frac{G_{\rm F}}{\sqrt{2}} \int d^3 p_e f(p_e)\, \bar{\nu}_e(x) \frac{p_{e\mu}}{E_e V} \gamma^\mu (1-\gamma_5) \nu_e(x)$$

$$= \frac{G_{\rm F}}{\sqrt{2} V} \bar{\nu}_e(x) \int d^3 p_e f(p_e) \left(\gamma^0 - \frac{\vec{p}_e \cdot \vec{\gamma}}{E_e} \right) (1-\gamma_5) \nu_e(x)$$

$$= \frac{G_{\rm F}}{\sqrt{2} V} \bar{\nu}_e(x) \gamma^0 (1-\gamma_5) \nu_e(x) N_e V$$

$$= \sqrt{2} G_{\rm F} N_e \nu^\dagger_{e{\rm L}}(x) \nu_{e{\rm L}}(x)$$



NC interactions contribute equally to all flavors, which is irrelevant for oscillations

$$V_e = \sqrt{2} G_{\rm F} N_e$$

Matter Potential (minus sign for $\bar{\nu}$)

Then, we obtain the Hamiltonian for neutrinos travelling in matter

$$A = 2\sqrt{2} G_{\rm F} N_e E$$

$$\mathcal{H}_{\rm m} = \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2E} \left[U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \right]$$

Example: **evaluate the electron number density in Earth matter of a mass density $\rho = 4\ {\rm g/cm^3}$. Note that the normal matter is electrically neutral, so the number fraction of electrons is $Y_e = 0.5$.**

$$N_e = \frac{\rho}{1\ {\rm g/cm^3}} N_A Y_e = 2 N_A\ {\rm cm^{-3}}$$

$$V_e = \sqrt{2} G_{\rm F} N_e = 1.59 \times 10^{-13}\ {\rm eV}$$

Effective Mixing Parameters

The Hamiltonian in a more compact form:

$$\mathcal{H}_{\rm m} = \frac{1}{4E}\left[U\begin{pmatrix} -\Delta m_{21}^2 & 0 \\ 0 & +\Delta m_{21}^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix}\right] + \frac{m_1^2 + m_2^2 + A}{4E}$$

$$\mathcal{H}_{\rm m} = \frac{1}{4E}\begin{pmatrix} A - \Delta m_{21}^2 c_{2\theta} & \Delta m_{21}^2 s_{2\theta} \\ \Delta m_{21}^2 s_{2\theta} & \Delta m_{21}^2 c_{2\theta} - A \end{pmatrix}$$

in flavor basis

Converted into the mass basis

$$\mathcal{H}_{\rm m} = \frac{1}{4E}\widetilde{U}\begin{pmatrix} -\Delta\widetilde{m}_{21}^2 & 0 \\ 0 & +\Delta\widetilde{m}_{21}^2 \end{pmatrix}\widetilde{U}^\dagger$$

Mixing matrix & mass states in matter

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} c_{\widetilde{\theta}} & s_{\widetilde{\theta}} \\ -s_{\widetilde{\theta}} & c_{\widetilde{\theta}} \end{pmatrix}\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix} \equiv \widetilde{U}\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix}$$

$$\Delta\widetilde{m}_{21}^2 = \sqrt{\left(\Delta m_{21}^2 c_{2\theta} - A\right)^2 + \left(\Delta m_{21}^2 s_{2\theta}\right)^2} \qquad \tan 2\widetilde{\theta} = \frac{\Delta m_{21}^2 s_{2\theta}}{\Delta m_{21}^2 c_{2\theta} - A}$$

Relationship between the mixing angle (mass difference) in vacuum and that in matter

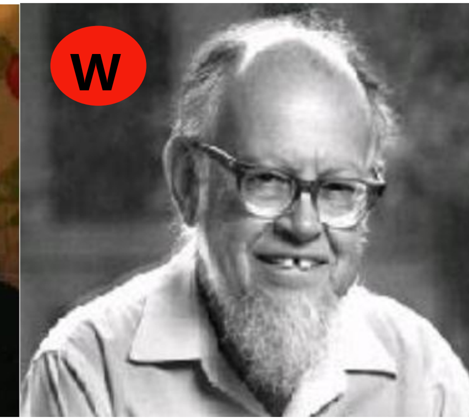
Some Discussions

Mixing angle in matter

$$\tan 2\tilde{\theta} = \frac{\Delta m_{21}^2 s_{2\theta}}{\Delta m_{21}^2 c_{2\theta} - A}$$

MSW resonance $\tilde{\theta} = 45^\circ$

Resonance condition:
$$\Delta m_{21}^2 c_{2\theta} = 2\sqrt{2} G_F N_e E$$



Stanislav Mikheyev (1940-2011) **Alexei Smirnov (1951-)** **Lincoln Wolfenstein (1923-2015)**

Example: assume that the energy of solar ^{8}B neutrinos is $E = 10$ MeV, and take $N_e = 100\, N_A/\mathrm{cm}^3$ for $\rho = 150\ \mathrm{g/cm}^3$ in the solar center. The density decreases from the center to the surface. Check if the MSW resonance can be reached, given $\Delta m_{21}^2 = 7.5 \times 10^{-5}\ \mathrm{eV}^2$.

Solution:

$$A = 2\sqrt{2} G_F N_e E \approx 2\sqrt{2} \cdot \left(1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}\right) \cdot 10\ \mathrm{MeV} \cdot (6 \times 10^{25}\ \mathrm{cm}^{-3})$$

$$\approx 1.5 \times 10^{-4}\ \mathrm{eV}^2$$

$$V_e = \sqrt{2} G_F N_e \approx 7.5 \times 10^{-5}\ \mathrm{eV}^2/\ \mathrm{MeV}$$

A useful relation: $197\ \mathrm{MeV} \cdot 1\ \mathrm{fm} = 1$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

$$\tilde{P}(\nu_e \to \nu_\mu) = \sin^2 2\tilde{\theta} \sin^2 \frac{\Delta \tilde{m}^2 L}{4E}$$

Varying Matter Density

The matter density $\rho(r)$ is varying in astrophysical environments, like the Sun and SNe

$$\tan 2\tilde{\theta}(r) = \frac{\Delta m_{21}^2 s_{2\theta}}{\Delta m_{21}^2 c_{2\theta} - 2\sqrt{2} G_{\rm F} N_e(r) E}$$

$$\mathcal{H}_{\rm m} = \frac{1}{4E} \tilde{U} \begin{pmatrix} -\Delta\tilde{m}_{21}^2 & 0 \\ 0 & +\Delta\tilde{m}_{21}^2 \end{pmatrix} \tilde{U}^\dagger$$

Recall how to calculate neutrino oscillation probabilities in matter of a constant density

$$i\frac{\rm d}{{\rm d}r}\begin{pmatrix} |\nu_e(r)\rangle \\ |\nu_\mu(r)\rangle \end{pmatrix} = \frac{1}{4E} \tilde{U} \begin{pmatrix} -\Delta\tilde{m}_{21}^2 & 0 \\ 0 & +\Delta\tilde{m}_{21}^2 \end{pmatrix} \tilde{U}^\dagger \begin{pmatrix} |\nu_e(r)\rangle \\ |\nu_\mu(r)\rangle \end{pmatrix}$$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \tilde{U} \begin{pmatrix} |\tilde{\nu}_1\rangle \\ |\tilde{\nu}_2\rangle \end{pmatrix}$$

$$i\frac{\rm d}{{\rm d}r}\left[\tilde{U}\begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix}\right] = \frac{1}{4E} \tilde{U} \begin{pmatrix} -\Delta\tilde{m}_{21}^2 & 0 \\ 0 & +\Delta\tilde{m}_{21}^2 \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix}$$

$$i\frac{\rm d}{{\rm d}r}\begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix} = \left[\frac{1}{4E} \begin{pmatrix} -\Delta\tilde{m}_{21}^2 & 0 \\ 0 & +\Delta\tilde{m}_{21}^2 \end{pmatrix} - i\tilde{U}^\dagger \dot{\tilde{U}}\right] \begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix}$$

$$\tilde{U}^\dagger \dot{\tilde{U}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{{\rm d}\tilde{\theta}}{{\rm d}r}$$

$$i\frac{\rm d}{{\rm d}r}\begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\tilde{m}_{21}^2 & -4iE{\rm d}\tilde{\theta}/{\rm d}r \\ 4iE{\rm d}\tilde{\theta}/{\rm d}r & +\Delta\tilde{m}_{21}^2 \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1(r)\rangle \\ |\tilde{\nu}_2(r)\rangle \end{pmatrix}$$

If $\rho(r)$ or $\tilde{\theta}(r)$ changes slowly, no transition between mass states

Adiabatic Evolution

Evolution of mass states in matter

$$i\frac{\mathrm{d}}{\mathrm{d}r}\begin{pmatrix}|\tilde{\nu}_1(r)\rangle\\|\tilde{\nu}_2(r)\rangle\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta\tilde{m}^2_{21} & -4iE\,\mathrm{d}\tilde{\theta}/\mathrm{d}r\\4iE\,\mathrm{d}\tilde{\theta}/\mathrm{d}r & +\Delta\tilde{m}^2_{21}\end{pmatrix}\begin{pmatrix}|\tilde{\nu}_1(r)\rangle\\|\tilde{\nu}_2(r)\rangle\end{pmatrix}$$

express $\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}r}$ in terms of $\frac{\mathrm{d}A}{\mathrm{d}r}$

Adiabaticity parameter: the ratio between the diagonal and off-diagonal elements

$$\gamma=\frac{\Delta\tilde{m}^2_{21}}{4E|\mathrm{d}\tilde{\theta}/\mathrm{d}r|}=\frac{\left(\Delta\tilde{m}^2_{21}\right)^2}{2E\sin 2\tilde{\theta}\,|\mathrm{d}A/\mathrm{d}r|}$$

Mass ordering: sign of Δm^2

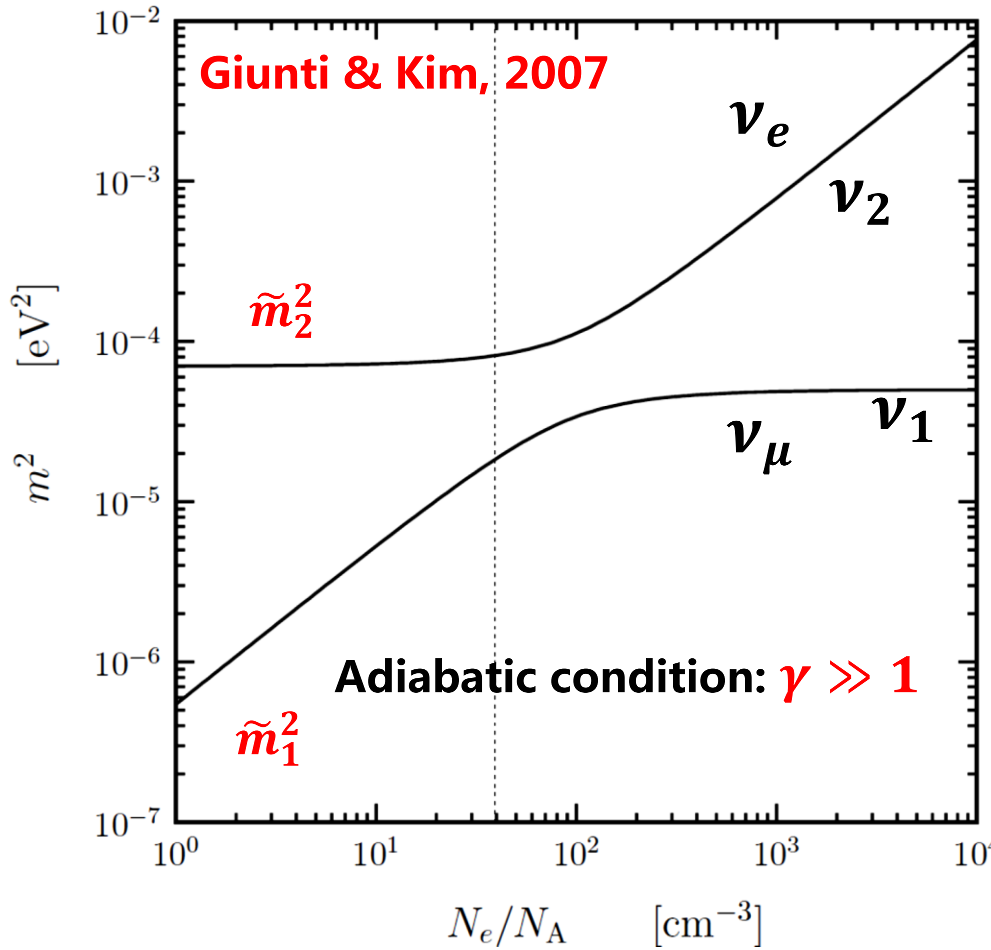
$$\mathcal{H}=U(\theta)\begin{pmatrix}m_1^2 & 0\\0 & m_2^2\end{pmatrix}U^\dagger(\theta)$$

invariant under

$$\theta\Rightarrow\frac{\pi}{2}-\theta \qquad m_1^2\Leftrightarrow m_2^2$$

$$|\nu_e\rangle\Rightarrow+|\nu_e\rangle \qquad |\nu_\mu\rangle\Rightarrow-|\nu_\mu\rangle$$

$\theta\in[0,\frac{\pi}{4}]$, $\Delta m^2_{21}>0$ ⟺ $\theta\in[\frac{\pi}{4},\frac{\pi}{2}]$, $\Delta m^2_{21}<0$



Three-flavor Oscillations in Matter

The effective Hamiltonian in matter

$$\mathcal{H}_{\rm m} = \frac{1}{2E}\left[V\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}V^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

Diagonalize the effective Hamiltonian in matter

$$\tilde{V}^\dagger \mathcal{H}_{\rm m} \tilde{V} = \frac{1}{2E}\begin{pmatrix} \widetilde{m}_1^2 & 0 & 0 \\ 0 & \widetilde{m}_2^2 & 0 \\ 0 & 0 & \widetilde{m}_3^2 \end{pmatrix}$$

Parametrize $\tilde{V}$ in the standard way by three mixing angles $\tilde{\theta}_{ij}$ and one CP-violating phase $\tilde{\delta}$ in matter

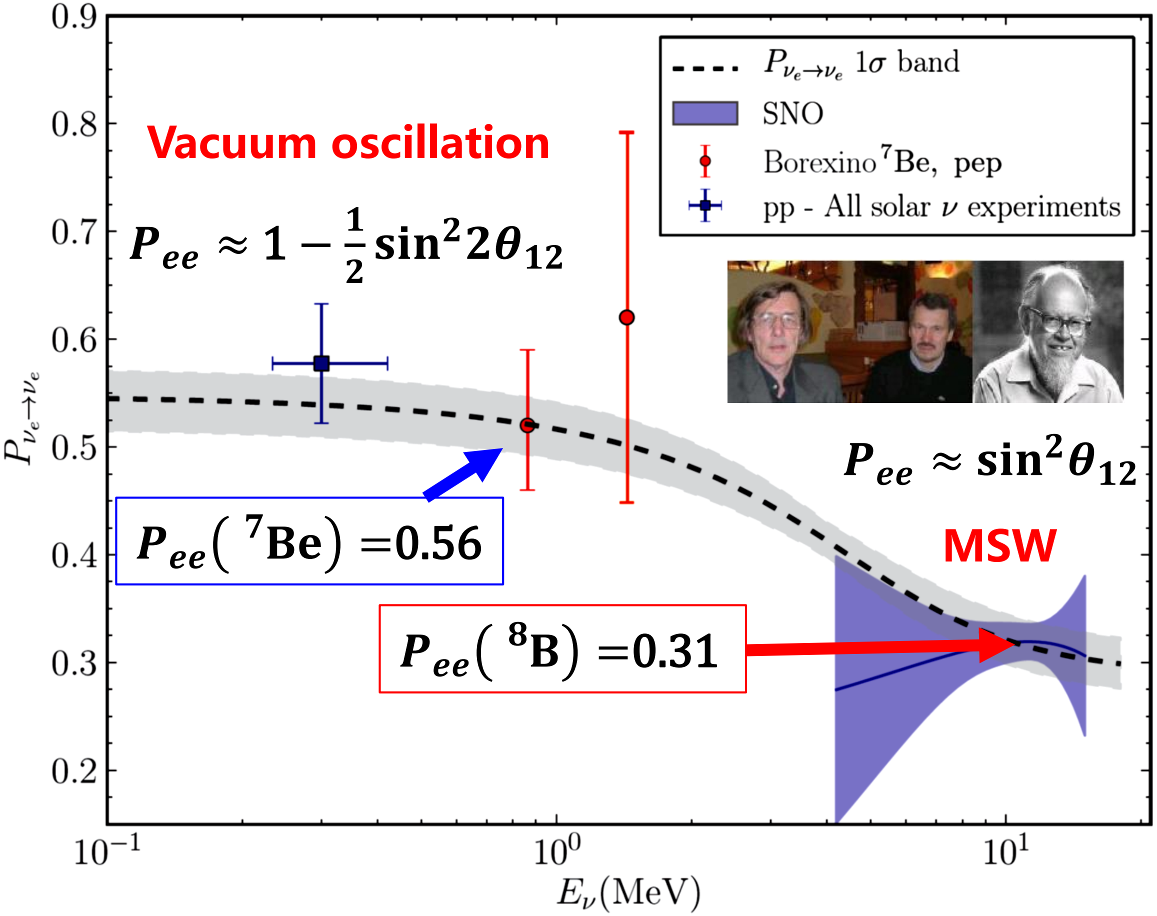
Oscillation probabilities in matter of a constant density

$$\tilde{P}(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} {\rm Re}\left[\tilde{V}_{\alpha i}\tilde{V}_{\beta j}\tilde{V}^*_{\alpha j}\tilde{V}^*_{\beta i}\right]\sin^2\frac{\Delta\widetilde{m}_{ji}^2 L}{4E}$$

$$+8\tilde{\mathcal{J}}\sum_{\gamma}\varepsilon_{\alpha\beta\gamma}\sin\frac{\Delta\widetilde{m}_{21}^2 L}{4E}\sin\frac{\Delta\widetilde{m}_{32}^2 L}{4E}\sin\frac{\Delta\widetilde{m}_{31}^2 L}{4E}$$

The oscillation probabilities for antineutrinos can be obtained by $V \Rightarrow V^*$ and $A \Rightarrow -A$

The LMA-MSW Solution



For high-energy ^{8}B neutrinos

at production $r = 0$

$$\begin{pmatrix} |\tilde{\nu}_1(0)\rangle \\ |\tilde{\nu}_2(0)\rangle \end{pmatrix} = \begin{pmatrix} c_{\hat{\theta}} & -s_{\hat{\theta}} \\ s_{\hat{\theta}} & c_{\hat{\theta}} \end{pmatrix} \begin{pmatrix} |\nu_e(0)\rangle \\ |\nu_\mu(0)\rangle \end{pmatrix}$$

adiabatic evolution

$$\begin{pmatrix} |\tilde{\nu}_1(R)\rangle \\ |\tilde{\nu}_2(R)\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1(0)\rangle \\ |\tilde{\nu}_2(0)\rangle \end{pmatrix}$$

on the solar surface $r = R$

$$\begin{pmatrix} |\nu_e(R)\rangle \\ |\nu_\mu(R)\rangle \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1(R)\rangle \\ |\tilde{\nu}_2(R)\rangle \end{pmatrix}$$

survival probability

$$P_{ee} = c^2_{\hat{\theta}} c^2_\theta + s^2_{\hat{\theta}} s^2_\theta \longrightarrow \sin^2\theta$$

$$\hat{\theta} \longrightarrow \pi/2 \qquad \text{as } A \gg \Delta m^2$$

For the MSW resonance to happen

$$\theta \in [0, \frac{\pi}{4}] \quad \Delta m^2_{21} > 0$$

$\Longleftrightarrow$

$$\theta \in [\frac{\pi}{4}, \frac{\pi}{2}] \quad \Delta m^2_{21} < 0$$

$$\theta_{12} = 34^\circ$$

Normal neutrino mass ordering

For low-energy ^{7}Be neutrinos

$$P_{ee} \approx 1 - \frac{1}{2}\sin^2 2\theta$$

Oscillations in vacuum

What we have learned?

Standard Parametrization of the PMNS Matrix

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta_{23} \sim 45^\circ$

$|\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\ \text{eV}^2$

Atmospheric,
LBL accelerator

$\theta_{13} \sim 9^\circ$

$\delta \sim ?$

Reactor,
LBL accelerator

$\theta_{12} \sim 34^\circ$

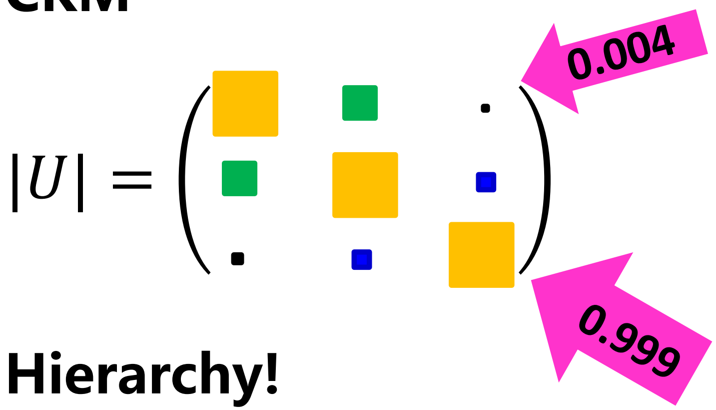
$\Delta m^2_{21} \sim 8 \times 10^{-5}\ \text{eV}^2$

Solar,
KamLAND

0ν2β, LNV?

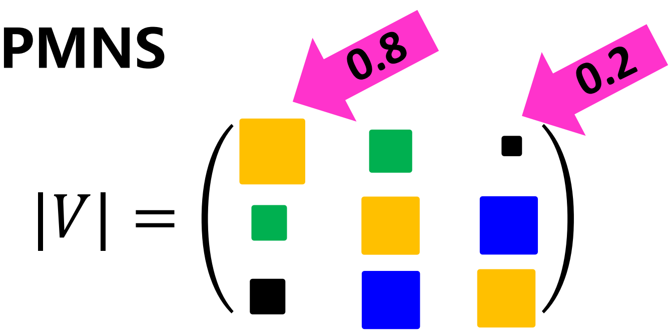
Quarks vs. Leptons: A big puzzle of fermion flavor mixings

CKM

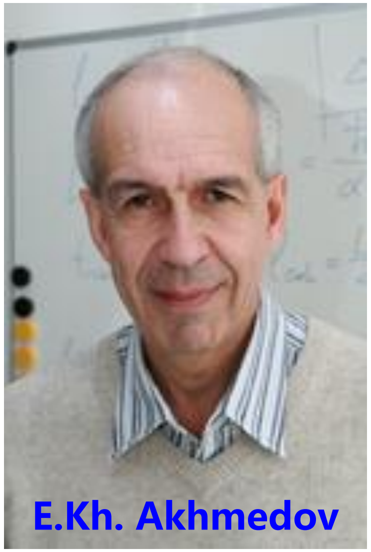


Hierarchy!

PMNS



Approximate μ-τ symmetry?



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Matter effects in oscillations of neutrinos traveling short distances in matter

E.Kh. Akhmedov [1]

Vacuum $$P\left(\nu_e \to \nu_\mu\right) = \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2 L}{4E} \quad \Rightarrow \quad P\left(\nu_e \to \nu_\mu\right) = \sin^2 2\theta \left[\frac{\Delta m_{21}^2 L}{4E}\right]^2$$

Matter $$\tilde{P}\left(\nu_e \to \nu_\mu\right) = \sin^2 2\tilde{\theta} \sin^2 \frac{\Delta \tilde{m}_{21}^2 L}{4E} \quad \Rightarrow \quad \tilde{P}\left(\nu_e \to \nu_\mu\right) = \sin^2 2\theta \left[\frac{\Delta m_{21}^2 L}{4E}\right]^2$$

$$\Delta \tilde{m}_{21}^2 = \sqrt{\left(\Delta m_{21}^2 c_{2\theta} - A\right)^2 + \left(\Delta m_{21}^2 s_{2\theta}\right)^2}$$

$$\sin 2\tilde{\theta} = \frac{\Delta m_{21}^2}{\Delta \tilde{m}_{21}^2} \sin 2\theta$$

Useful relations

In the limit of a short distance, or more precisely a small oscillation phase, the matter effects die out more rapidly than the oscillation effects themselves!!!

Matter Effects: Short-distance Propagation

Look again at the Schrödinger-like equation for the flavor evolution

$$i\frac{\mathrm{d}}{\mathrm{d}r}\begin{pmatrix}|\nu_e(r)\rangle\\|\nu_\mu(r)\rangle\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}A-\Delta m_{21}^2c_{2\theta} & \Delta m_{21}^2 s_{2\theta}\\ \Delta m_{21}^2 s_{2\theta} & \Delta m_{21}^2 c_{2\theta}-A\end{pmatrix}\begin{pmatrix}|\nu_e(r)\rangle\\|\nu_\mu(r)\rangle\end{pmatrix}$$

In the limit *r = L* is small, the perturbation theory is applicable, so one has

$$|\nu_\mu(L)\rangle=-i\frac{\Delta m_{21}^2 L}{4E}s_{2\theta}|\nu_e(0)\rangle \quad \Rightarrow \quad \widetilde{P}(\nu_e\to\nu_\mu)=\sin^2 2\theta\left[\frac{\Delta m_{21}^2 L}{4E}\right]^2$$

The main reasons for the previous observation are

- **The limit for the perturbation theory works well**
- **The matter term *A* appears in the diagonal places**
- **The initial state is a pure flavor eigenstate**

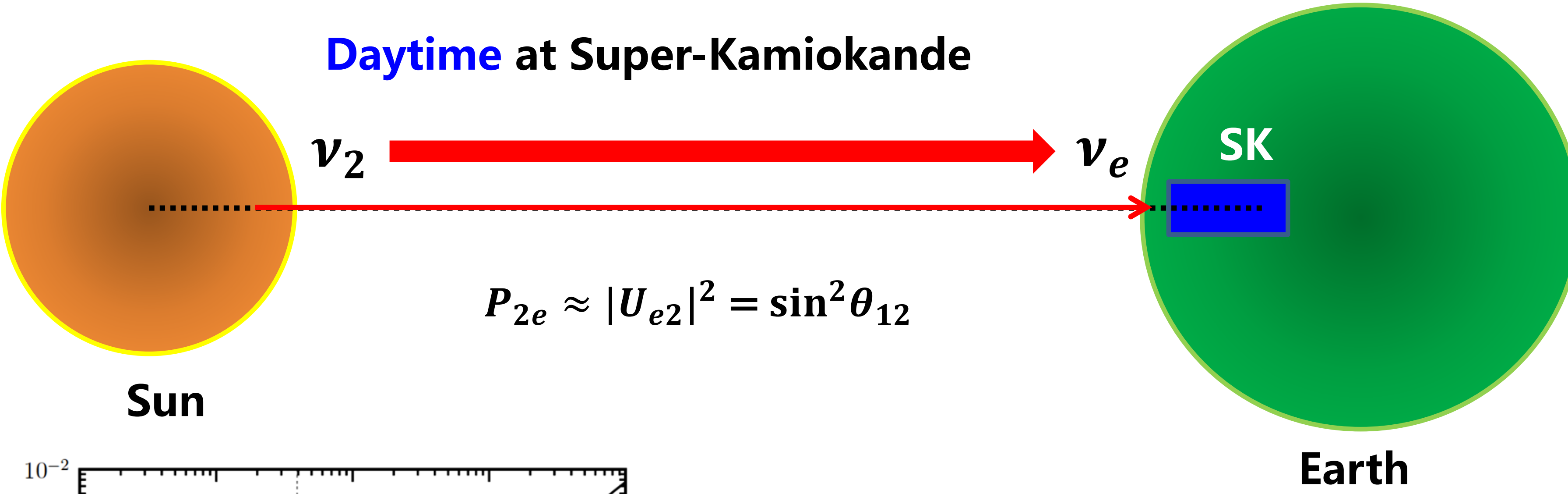
To have large matter effects, take the initial state to not be a pure flavor state

How can we produce a neutrino state that is a coherent superposition of flavor eigenstates?

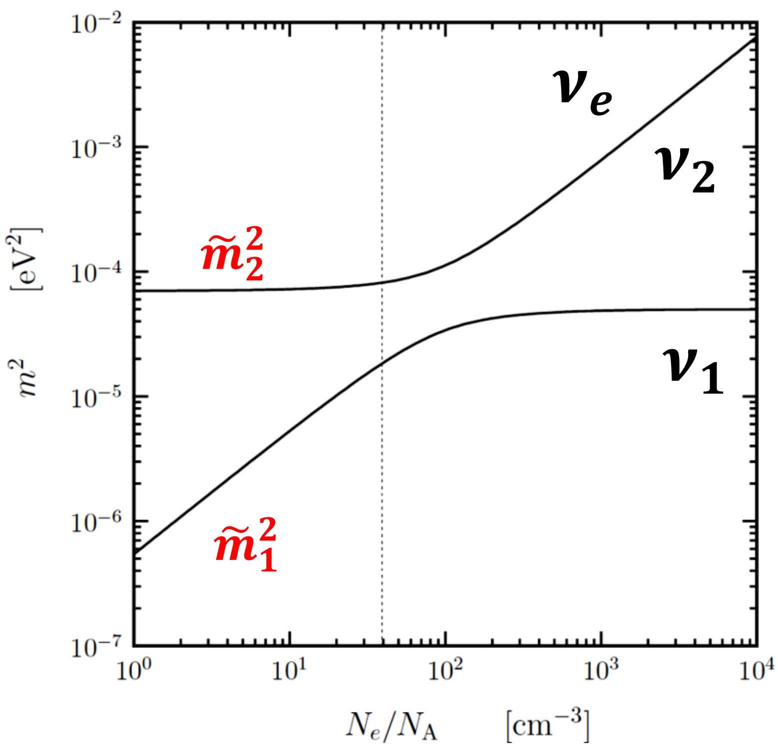
- **Mass eigenstates are coherent superposition of flavor eigenstates**
- **Flavor eigenstates that propagate first in vacuum then enter into matter**

Day-Night Asymmetry of Solar Neutrinos

Daytime at Super-Kamiokande

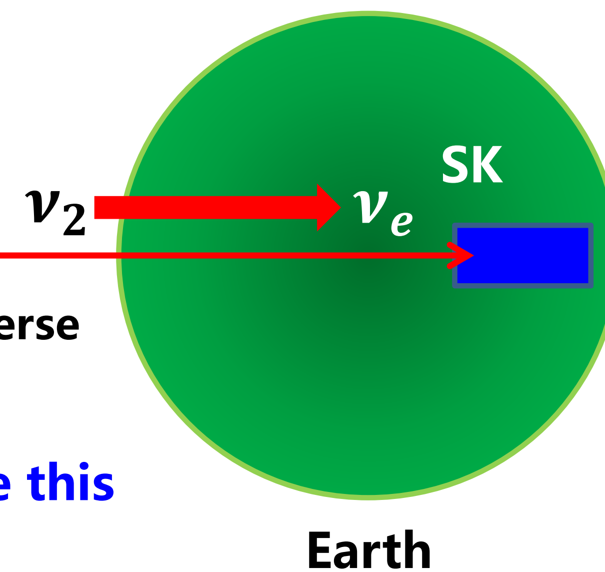


$$P_{2e} \approx |U_{e2}|^2 = \sin^2\theta_{12}$$



Nighttime at Super-Kamiokande

$$\widetilde{P}_{2e} \equiv P(\nu_2 \rightarrow \nu_e)$$



Now neutrinos traverse the Earth matter

How to calculate this probability?

Day-Night Asymmetry of Solar Neutrinos

Two-flavor approximation and a constant matter density

$$i\frac{\mathrm{d}}{\mathrm{d}r}\begin{pmatrix}|\nu_e(r)\rangle\\|\nu_\mu(r)\rangle\end{pmatrix}=\frac{1}{4E}\left[U\begin{pmatrix}-\Delta m_{21}^2 & 0\\0 & +\Delta m_{21}^2\end{pmatrix}U^\dagger+\begin{pmatrix}A & 0\\0 & -A\end{pmatrix}\right]\begin{pmatrix}|\nu_e(r)\rangle\\|\nu_\mu(r)\rangle\end{pmatrix}$$

Transform into the vacuum-mass basis

$$U\equiv\begin{pmatrix}c_\theta & s_\theta\\-s_\theta & c_\theta\end{pmatrix}$$

$$i\frac{\mathrm{d}}{\mathrm{d}r}\begin{pmatrix}|\nu_1(r)\rangle\\|\nu_2(r)\rangle\end{pmatrix}=\frac{1}{4E}\left[\begin{pmatrix}-\Delta m_{21}^2 & 0\\0 & +\Delta m_{21}^2\end{pmatrix}+U^\dagger\begin{pmatrix}A & 0\\0 & -A\end{pmatrix}U\right]\begin{pmatrix}|\nu_1(r)\rangle\\|\nu_2(r)\rangle\end{pmatrix}$$

which describes how the vacuum mass eigenstates evolve in matter

$$\widetilde{\mathcal{H}}_\mathrm{m}=\frac{1}{4E}\left[\begin{pmatrix}-\Delta m_{21}^2 & 0\\0 & +\Delta m_{21}^2\end{pmatrix}+U^\dagger\begin{pmatrix}A & 0\\0 & -A\end{pmatrix}U\right]$$

$$=\frac{1}{4E}\begin{pmatrix}Ac_{2\theta}-\Delta m_{21}^2 & As_{2\theta}\\As_{2\theta} & \Delta m_{21}^2-Ac_{2\theta}\end{pmatrix}$$

$$=\frac{1}{4E}\left[As_{2\theta}\cdot\sigma_1+\left(Ac_{2\theta}-\Delta m_{21}^2\right)\cdot\sigma_3\right]$$

Pauli matrices

$$|\nu_i(L)\rangle=\exp\left(-i\widetilde{\mathcal{H}}_\mathrm{m}L\right)|\nu_i(0)\rangle$$

$$\exp(-i\vec{a}\cdot\vec{\sigma}L)$$
$$=\cos(aL)-i\frac{\vec{a}\cdot\vec{\sigma}}{a}\sin(aL)$$
$$\vec{a}=\frac{1}{4E}\left(As_{2\theta},0,Ac_{2\theta}-\Delta m_{21}^2\right)$$
$$a=\frac{1}{4E}\sqrt{(As_{2\theta})^2+\left(Ac_{2\theta}-\Delta m_{21}^2\right)^2}$$

Day-Night Asymmetry of Solar Neutrinos

$$|\nu_i(L)\rangle = \left[\cos(aL) - i\frac{\vec{a}\cdot\vec{\sigma}}{a}\sin(aL)\right]|\nu_i(0)\rangle$$

in the chosen basis

$$|\nu_e\rangle = U^\dagger \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}c_\theta\\s_\theta\end{pmatrix}$$

$$|\nu_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\widetilde{A}_{2e} \equiv A(\nu_2 \to \nu_e) = \langle\nu_e|\mathcal{T}(L)|\nu_2(0)\rangle$$

$$= (c_\theta \quad s_\theta)\left[\cos(aL) - i\frac{\vec{a}\cdot\vec{\sigma}}{a}\sin(aL)\right]\begin{pmatrix}0\\1\end{pmatrix}$$

$$= s_\theta\left[\cos(aL) - \frac{i}{4Ea}(A + \Delta m^2_{21})\sin(aL)\right]$$

$$\widetilde{P}_{2e} \equiv \left|\widetilde{A}_{2e}\right|^2 = \sin^2\theta + \frac{A\Delta m^2_{21}}{16E^2a^2}\sin^2 2\theta\,\sin^2(aL)$$

A or $L \to 0$ ⟹ $P_{2e} = \sin^2\theta_{12}$

$$a = \frac{1}{4E}\sqrt{(As_{2\theta})^2 + \left(Ac_{2\theta} - \Delta m^2_{21}\right)^2}$$

⬇

$$a = \frac{1}{4E}\sqrt{A^2 - 2\Delta m^2_{21}Ac_{2\theta} + \left(\Delta m^2_{21}\right)^2}$$

⟹

$$\frac{1}{4E}\sqrt{\left(\Delta m^2_{21}s_{2\theta}\right)^2 + \left(A - \Delta m^2_{21}c_{2\theta}\right)^2}$$

⬆

$$\frac{\Delta\widetilde{m}^2_{21}}{4E}$$

Day-Night Asymmetry of Solar Neutrinos

The difference in the probability for $|\nu_2\rangle$ to be detected as $|\nu_e\rangle$

$$\widetilde{P}_{2e} - P_{2e} = \frac{A\Delta m^2_{21}}{\left(\Delta\widetilde{m}^2_{21}\right)^2}\sin^2 2\theta \sin^2\left(\frac{\Delta\widetilde{m}^2_{21} L}{4E}\right)$$

Now we consider the production of ^{8}B neutrinos and their flavor conversion inside the Sun (see the references Carlson, PRD, 1986; Guth/Randall/Serna, JHEP, 1999; Blennow/Ohlsson/Snellman, PRD, 2004)

Daytime survival probability

$$P_S = \sum_{i=1}^{n} k_i |\langle \nu_e|\nu_i\rangle|^2 = \sum_{i=1}^{n} k_i |U_{ei}|^2 \qquad \sum_{i=1}^{n} k_i = 1$$

k_i the fraction of $|\nu_i\rangle$

Nighttime survival probability

$$P_{SE} = \sum_{i=1}^{n} k_i |\langle \nu_e|\tilde{\nu}_i\rangle|^2 \qquad P_{ie} = |\langle \nu_e|\tilde{\nu}_i\rangle|^2 \qquad \sum_{i=1}^{n} P_{ie} = 1$$

Two-flavor approximation $P_S = (1-k_2)\cos^2\theta + k_2 \sin^2\theta = \cos^2\theta - k_2\cos(2\theta)$

$$P_{SE} = P_S + \frac{1-2P_S}{\cos(2\theta)}(P_{2e} - \sin^2\theta)$$

One has to calculate k_2 and P_{2e} for solar neutrinos and the Earth matter

Neutrino production

$$|\nu_e\rangle = \cos\hat{\theta}\,|\nu_{M,1}\rangle + \sin\hat{\theta}\,|\nu_{M,2}\rangle$$

Flavor mixing angle at the production point

Neutrino fraction

$$k_i = \int_0^{R_\odot} \mathrm{d}r\, f(r)(\cos^2\hat{\theta}(r) P^s_{1i} + \sin^2\hat{\theta}(r) P^s_{2i})$$

Transitional probability from i to j

Normalized distribution function for neutrino production

$$P^s_{12} = P^s_{21} = P_{\mathrm{jump}}$$

Non-adiabatic case

$$k_1 = \frac{1+D_{2\nu}}{2}\,, \quad k_2 = \frac{1-D_{2\nu}}{2} \qquad D_{2\nu} = \int_0^{R_\odot} \mathrm{d}r\, f(r)\cos(2\hat{\theta}(r))(1-2P_{\mathrm{jump}})$$

For the Sun, it is easy to estimate the adiabaticity parameter (E = 10 MeV)

$$\gamma_{\mathrm{res}} \simeq 2.6\cdot 10^3$$

$$P_{\mathrm{jump}} \sim 10^{-1700}$$

$$P_S = \frac{1}{2}[(1+D_{2\nu})\cos^2\theta + (1-D_{2\nu})\sin^2\theta] = \frac{1+D_{2\nu}\cos(2\theta)}{2}$$

Day-Night Asymmetry

Final results for two-flavor mixing

$$P_{SE} - P_S = -D_{2\nu} \frac{A \Delta m_{21}^2}{(\Delta \tilde{m}_{21}^2)^2} \sin^2 2\theta \sin^2 \left(\frac{\Delta \tilde{m}_{21}^2 L}{4E} \right)$$

$$D_{2\nu} = \int_0^{R_\odot} \mathrm{d}r \, f(r) \cos(2\hat{\theta}(r))(1 - 2P_{\mathrm{jump}})$$

$P_{SE} - P_S > 0$ regeneration effects

Negative in the production region of ^8B

Final results for three-flavor mixing

$$P_{SE} - P_S = -c_{13}^6 D_{3\nu} \frac{A \Delta m_{21}^2}{(\Delta \tilde{m}_{21}^2)^2} \sin^2 2\theta \sin^2 \left(\frac{\Delta \tilde{m}_{21}^2 L}{4E} \right)$$

$$D_{3\nu} = \int_0^{R_\odot} \mathrm{d}r \, f(r) \cos(2\hat{\theta}_{12}(r))(1 - 2P_{\mathrm{jump}})$$

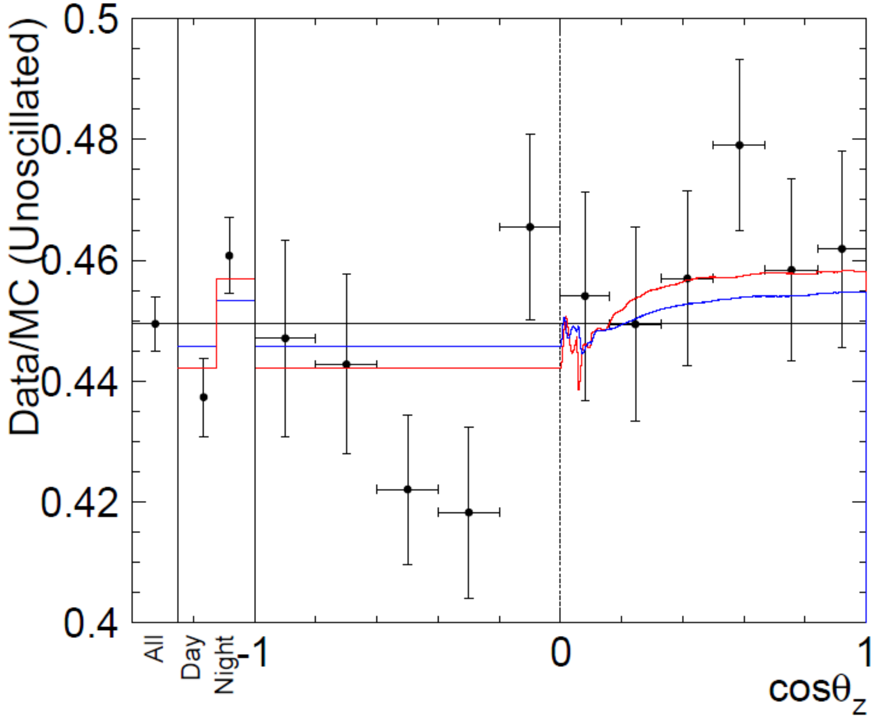
Matter potential multiplied by c_{13}^2

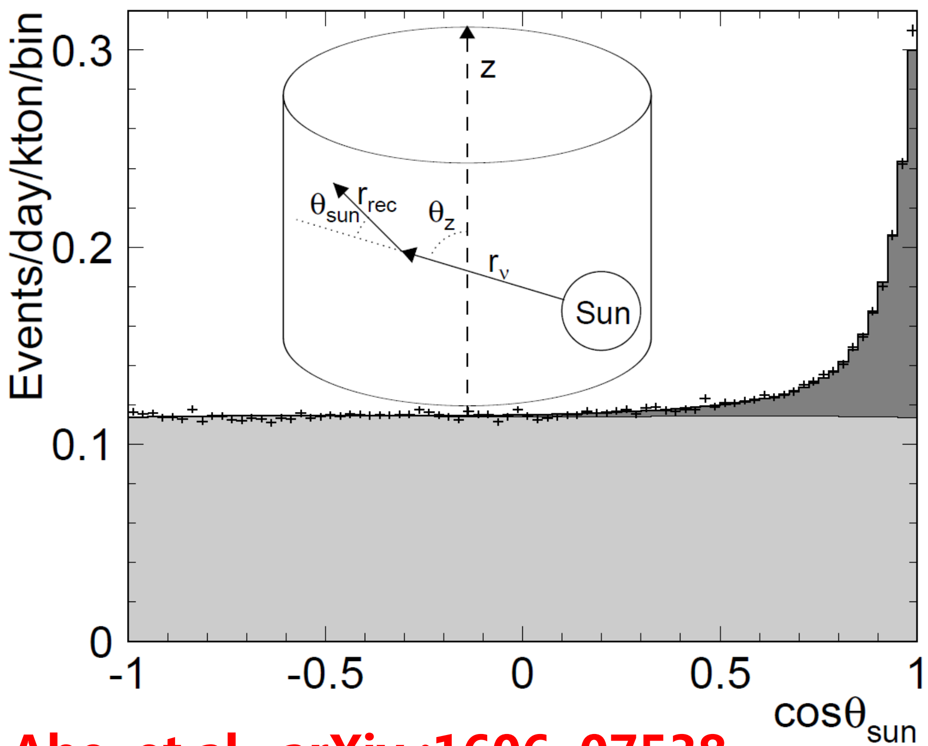
Day-Night Asymmetry

$$A_{\mathrm{DN}} \equiv -2 \frac{P_{SE} - P_S}{P_{SE} + P_S}$$

This can be observed by comparing between the elastic neutrino-electron scattering events in the daytime and those at night.

Super-Kamiokande: Day-Night Asymmetry





K. Abe, et al., arXiv :1606 .07538

$$A_{\rm DN}^{\rm fit,\,SK} = (-3.3 \pm 1.0({\rm stat.}) \pm 0.5({\rm syst.}))\%$$

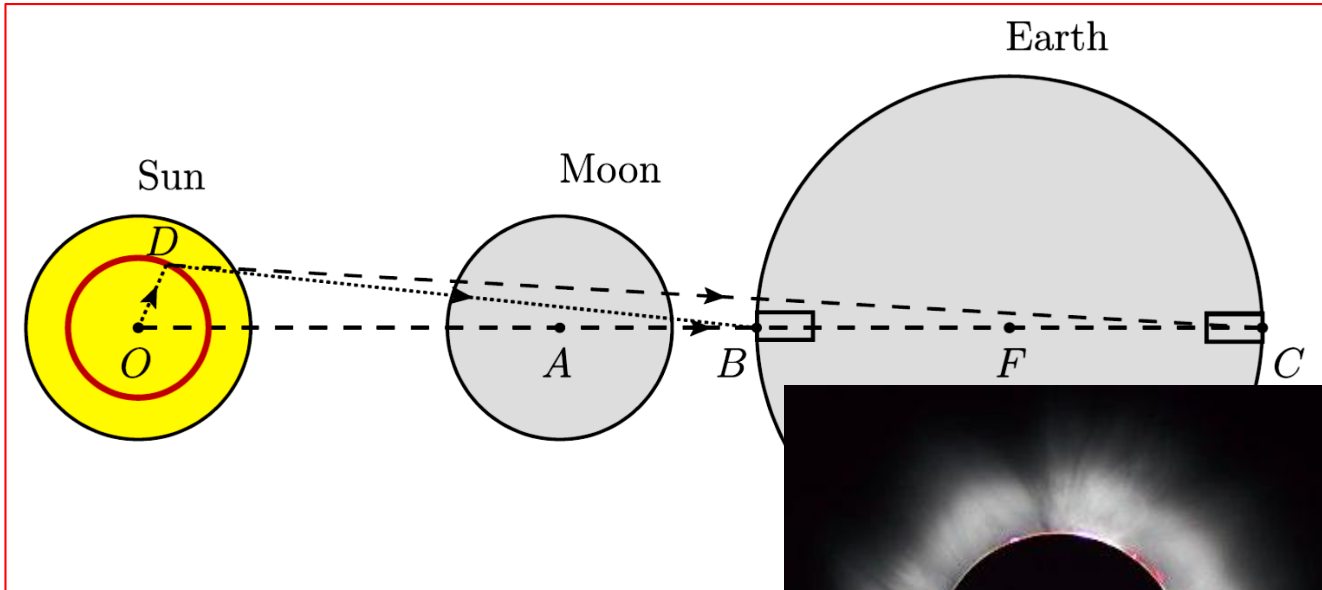
Observed at the 3σ level

Question: **is it possible to observe lunar matter effects on solar neutrinos when the solar eclipses take place?**

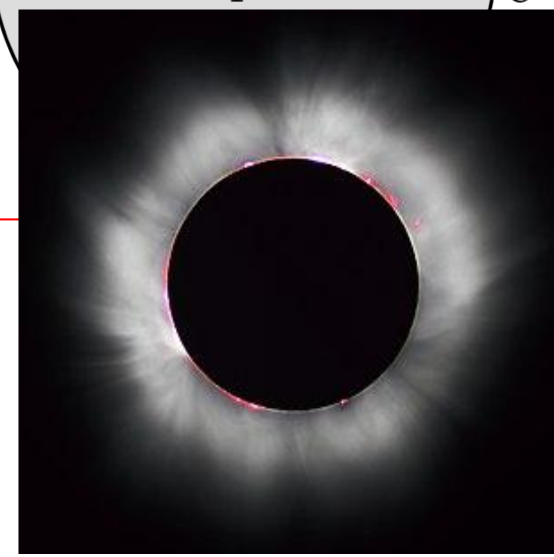
Expectation: **According to the previous discussions and Akhmedov's work, even if neutrinos are traveling in the Moon with a short distance, the matter effects should not be that suppressed.**

Lunar Matter Effects

Solar eclipses



Before reaching the Earth, solar ν's may first traverse the Moon during the solar eclipses



The Moon

Mean radius: 1737.1 km (0.273 of Earth's)

Mean density: 3.344 g/cm^3 (0.606 of Earth's)

Formulation:

Consider the incoherent mass states, they enter into the lunar matter and then propagate in vacuum for a distance L, finally reach the Earth

$$P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) \qquad P_{\alpha\beta} = \sum_{i=1}^{3} k_i^\alpha |U_{\beta i}|^2 \qquad k_1^\alpha + k_2^\alpha + k_3^\alpha = 1$$

Two scenarios:

◆ The distance L between the Moon and the Earth is very long such that the coherence of neutrino mass states is lost before reaching the Earth

$$\widehat{P}^{\rm dec}_{\alpha\beta} = \sum_{i=1}^{3}\sum_{j=1}^{3} k_i^\alpha P(\nu_i \to \nu'_j) |U_{\beta j}|^2$$

$$P(\nu_i \to \nu'_j) \equiv P_{ij}$$

The probability for $|\nu_i\rangle \to |\nu'_j\rangle$ after passing through the Moon

◆ The coherence of neutrino mass states is kept until they reach the Earth

$$\widehat{P}^{\rm coh}_{\alpha\beta} = \sum_{i=1}^{3} k_i^\alpha P(\nu_i \to \nu_\beta)$$

$$P(\nu_i \to \nu_\beta) \equiv P_{i\beta}$$

The probability for $|\nu_i\rangle \to |\nu_\beta\rangle$

Connection

$$\widehat{P}^{\rm coh}_{\alpha\beta} = \sum_{i=1}^{3}\sum_{j=1}^{3} k_i^\alpha \left| \langle \nu_\beta | \nu'_j \rangle \cdot \exp\left[-{\rm i} m_j^2 L/(2E) \right] \cdot \langle \nu'_j | \nu_i \rangle \right|^2 = \widehat{P}^{\rm dec}_{\alpha\beta} + \widehat{I}_{\alpha\beta}$$

$$\widehat{I}_{\alpha\beta} = \sum_{i=1}^{3} k_i^\alpha \sum_{j>k} 2{\rm Re}\left\{ U_{\beta j} U^*_{\beta k} A_{ij} A^*_{ik} \exp\left[-{\rm i}\Delta m^2_{jk} L/(2E) \right] \right\}$$

The amplitude for $|\nu_i\rangle \to |\nu'_j\rangle$

$$A_{ij} \equiv \langle \nu'_j | \nu_i \rangle$$

Transition Amplitude

Evolution in the vacuum mass basis

$$V^\dagger H_{\rm m} V = {\rm diag}\{\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2\}/(2E) \qquad H_{\rm m} = \frac{1}{2E}\left[\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} + U^\dagger \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U\right]$$

Mass eigenvalues

$$\tilde{m}_1^2 = m_1^2 + \frac{1}{3}x - \frac{1}{3}\sqrt{x^2 - 3y}\left[z + \sqrt{3(1-z^2)}\right] \qquad x = \Delta m_{21}^2 + \Delta m_{31}^2 + A$$

$$\tilde{m}_2^2 = m_1^2 + \frac{1}{3}x - \frac{1}{3}\sqrt{x^2 - 3y}\left[z - \sqrt{3(1-z^2)}\right] \qquad y = \Delta m_{21}^2 \Delta m_{31}^2 + A\left[\Delta m_{21}^2(1 - |U_{e2}|^2) + \Delta m_{31}^2(1 - |U_{e3}|^2)\right]$$

$$\tilde{m}_3^2 = m_1^2 + \frac{1}{3}x + \frac{2}{3}z\sqrt{x^2 - 3y} \qquad z = \cos\left[\frac{1}{3}\arccos\frac{2x^3 - 9xy + 27A\Delta m_{21}^2 \Delta m_{31}^2 |U_{e1}|^2}{2(x^2 - 3y)^{3/2}}\right]$$

Unitary matrix

$$V_{ii} = \frac{N_i}{D_i}, \qquad V_{ij} = \frac{A}{D_j}\left(\tilde{m}_j^2 - m_k^2\right) U_{ei}^* U_{ej}$$

$$D_i^2 = N_i^2 + A^2 |U_{ei}|^2 \left[(\tilde{m}_i^2 - m_j^2)^2 |U_{ek}|^2 + (\tilde{m}_i^2 - m_k^2)^2 |U_{ej}|^2\right]$$

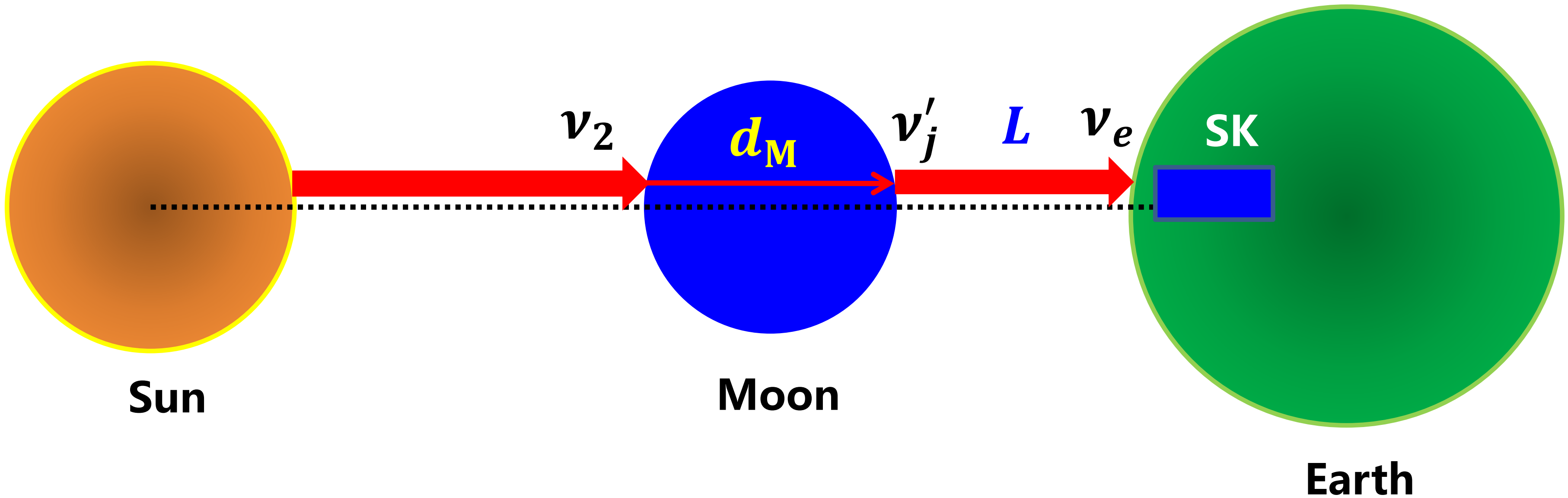
$$N_i = (\tilde{m}_i^2 - m_j^2)(\tilde{m}_i^2 - m_k^2) - A\left[(\tilde{m}_i^2 - m_j^2)|U_{ek}|^2 + (\tilde{m}_i^2 - m_k^2)|U_{ej}|^2\right]$$

$$A_{ij} = \langle \nu_j' | \nu_i \rangle = \sum_{k=1}^{3} V_{jk} V_{ik}^* \exp\left[-{\rm i}\tilde{m}_k^2 d_{\rm M}/(2E)\right]$$

Transition amplitude

$$|\nu_i\rangle \rightarrow |\nu_j'\rangle$$

Solar Eclipses



Oscillation Length

Moon to Earth

$$L_{\rm osc} \sim L_{\rm osc}^{21} \equiv \frac{4\pi E}{\Delta m_{21}^2} \approx 330\ {\rm km}\left(\frac{E}{10\ {\rm MeV}}\right)\cdot\left(\frac{7.5\times10^{-5}\ {\rm eV}^2}{\Delta m_{21}^2}\right) \ll L_{\rm ME} \approx 3.84\times10^5\ {\rm km}$$

Before arriving in the SK detector, neutrinos experience many cycles of oscillations and thus the situation is equivalent to decoherence

$$P_{\rm S}^{\rm M} = \sum_{i=1}^{3}\sum_{j=1}^{3} k_i^e P_{ij}|U_{ej}|^2 \qquad P_{\rm S}^{\rm M} - P_{\rm S} = (k_2^e - k_1^e)(|U_{e1}|^2 - |U_{e2}|^2)P_{12}$$

The calculations of k factors are the same as for the DN asymmetry

Matter density

$$\rho_{\mathrm{M}} \approx 3 \;\mathrm{g\,cm^{-3}}$$

Electron fraction

$$Y_{\mathrm{M}}^{e} \approx 0.5$$

Matter term

$$A \approx 2.28 \times 10^{-6}\ \mathrm{eV}^2 \cdot [E/(10\ \mathrm{MeV})]$$

$$A/\Delta m_{21}^2 \approx 0.03[E/(10\ \mathrm{MeV})]$$

In the limit $A \ll \Delta m_{21}^2 \ll \Delta m_{31}^2$

$$V \approx \begin{pmatrix} 1 & AU_{e1}^* U_{e2}/\Delta m_{21}^2 & AU_{e1}^* U_{e3}/\Delta m_{31}^2 \\ AU_{e1} U_{e2}^*/\Delta m_{21}^2 & 1 & AU_{e2}^* U_{e3}/\Delta m_{32}^2 \\ AU_{e1} U_{e3}^*/\Delta m_{31}^2 & AU_{e2} U_{e3}^*/\Delta m_{32}^2 & 1 \end{pmatrix}$$

Extremely difficult to observe solar eclipses via neutrinos

$$P_{12} \approx \left(\frac{A}{\Delta m_{21}^2}\right)^2 \sin^2 2\theta_{12} \sin^2 \frac{\Delta \tilde{m}_{21}^2 d_{\mathrm{M}}}{4E}$$

$$\frac{P_S^M - P_S}{P_{SE} - P_S} = \frac{A \cos 2\theta_{12}}{\Delta m_{21}^2} \simeq 1.2\%$$

Summary

- **Matter effects are very important for neutrino oscillations. We investigate the possibility to observe solar eclipses in the neutrino light. In principle this is possible due to the lunar matter effects, similar to the Earth matter effects on solar neutrinos**

- **It turns out that the lunar matter effects are smaller by a factor of 1.2% compared to the ordinary day-night asymmetry**

- **The reason for such a suppression is due to the loss of coherence during the propagation between the Moon and the Earth**

- **We set up a general formalism to calculate the impact of any astrophysical objects in the way of neutrino propagation. Other examples include the UHE neutrinos and solar atmospheric neutrinos**

Thanks for your attention!