Superconducting Accelerator Magnets

Reference: Yukikazu Iwasa < Case Studies in Superconducting Magnets >
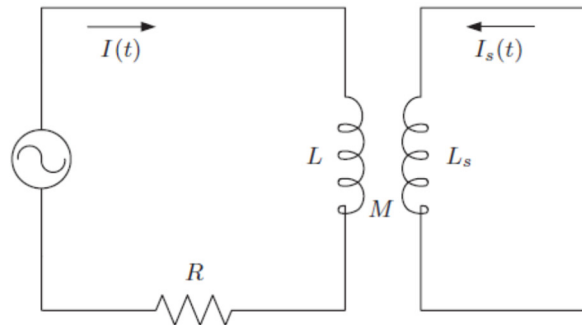
Problem 1: A superconducting loop

Fig. 1.8 Circuit of a superconducting coil inductively coupled to another loop that is connected to a current source.

This problem demonstrates that it is impossible to induce a "persistent" current in a *closed* superconducting loop, coil, or disk by means of an external current source. This may be obvious to some; it is proven here using a circuit model. Figure 1.8 shows a superconducting loop coupled inductively to another loop that is connected to a current source. The superconducting loop, with current $I_s(t)$, is represented by an inductor of self inductance L_s. The loop with the current source of $I(t)$ is represented by an inductor of self inductance L and a resistor of resistance R. The two circuits are coupled inductively through a mutual inductance M.

a) Write two circuit equations, relating voltage in each of the two circuits.

b) Solve the above voltage equations for $I_s(t)$ and show that it is not possible to establish a current in a *closed* superconducting circuit with a current source whose current $I(t)$ is zero at the beginning and at the end, i.e., $I(t=0) = I(t=\infty) = 0$. The closed superconducting circuit may be a magnet with its terminals joined by superconducting splices, a bulk disk or a stack of disks, or a disk with a hole in the middle or a stack of such disks.

Problem 2: The field far from a cluster of four dipoles

This problem considers the field far from a cluster of four *ideal* dipoles, 1–4, arranged as shown in Fig. 2.7, in which the direction of each dipole is indicated by the arrow within a circle. The center-to-center distance between two opposing dipoles is $2\delta_d$. The field of each jth dipole of zero winding thickness, diameter $2r_d$, and overall length ℓ_d in the y-direction, at a radial location (r_j) far from the dipole $(r_j \gg \ell_d)$ may be modeled as a spherical dipole field, $\vec{B}_j$:

$$\vec{B}_j = \frac{r_d^2 \ell_d B_\circ}{2r_j^3} (\cos \vartheta_j \vec{\imath}_{r_j} + \tfrac{1}{2} \sin \vartheta_j \vec{\imath}_{\theta_j}) \tag{2.51}$$

where r_j is measured from the center of *each* dipole and ϑ_j in each dipole is defined such that the field inside the winding points in the r_j-direction when $\vartheta_j = 0°$. Figure 2.7 indicates the direction of the field inside each dipole. Also defined in Fig. 2.7 are r-θ coordinates and z-x coordinates common to all the dipoles. Note that for $r \gg \delta_d$, we have $\vartheta_1 = \theta + 180°$, $\vartheta_2 = \theta - 90°$, $\vartheta_3 = \theta$, and $\vartheta_4 = \theta + 90°$.

Show that an approximate expression for the far field ($\vec{B}$ for $r/\delta_d \gg 1$) of the combined system is given by:

$$\vec{B} \simeq \frac{3r_d^2 \ell_d B_\circ \delta_d}{r^4} (-\sin 2\theta \vec{\imath}_r + \tfrac{1}{2} \cos 2\theta \vec{\imath}_\theta) \tag{2.52}$$

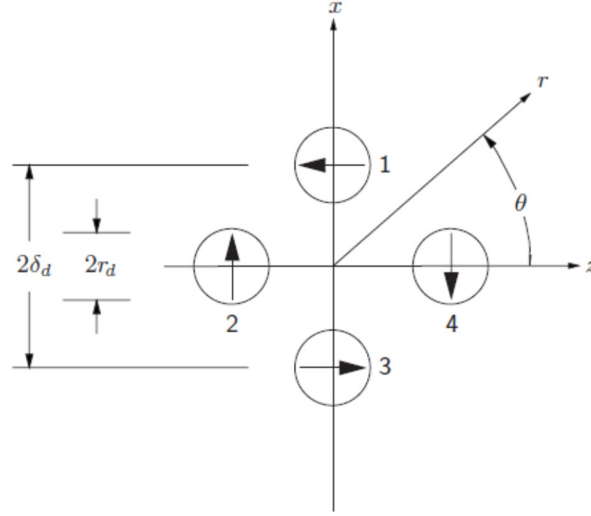Neglect end effects of each dipole, i.e., consider only the plane $y = 0$.

Fig. 2.7   Cross-sectional view of a four-dipole arrangement. The arrow in each dipole indicates the field direction inside the winding.

Problem 3: Circulating proton in an accelerator

The LHC, the world's largest "atom smasher" (protons), has $\sim$1250 dipole magnets, each $\sim$14 m long and generating a field of 8.3 T within a diameter of 56 mm. The LHC will have two counter-circulating beams of protons, each accelerated to an energy (E_p) of 7 TeV.

a) The oblong-shaped main ring comprises two half circles of radius $R_a = 2.8$ km, connected by 4.5-km nearly straight sections for a proton with an energy E_p of 7 TeV. The dipoles occupy the ring's two half-circle sections. Show that a dipole field of 8.3 T generates a Lorentz force, $\vec{F}_L$, that balances the centripetal force, $\vec{F}_{cp}$, on a 7-TeV proton in the circular section. Assume that the proton speed is equal to the speed of light. Note: $1\,\text{eV} = 1.6 \times 10^{-19}$ J.

b) Show that the proton speed at an energy of 7 TeV is nearly the speed of light.

Problem 4: Criterion for flux jumping

This problem deals with the derivation of the critical conductor size above which flux jumping will occur. Flux jumping was once a major source of instabilities in the first superconducting magnets of engineering significance in the early 1960s [5.10]. Flux jumping is a thermal instability peculiar to a Type II superconductor that permits the magnetic field to penetrate its interior. A time-varying magnetic field, $\dot{H}_e$, at the conductor surface induces an electric field $\vec{E}$ in the conductor, which interacts with the supercurrent (density J_c). This $\vec{E} \cdot \vec{J}_c$ interaction heats the conductor. Since J_c decreases with temperature, the field (flux) penetrates further into the conductor, generating more heat, which further decreases J_c. The field penetration and temperature rise can cascade until the conductor loses its superconductivity. This thermal runaway event is called a flux jump.

a) Using the Bean model and computing the $\vec{E} \cdot \vec{J}_c$ interaction over the positive half $(0 \leq x \leq a)$ of the slab, show that an expression for the dissipative energy density, e_ϕ [J/m^3], generated within the slab when the critical current density J_c is suddenly *decreased* by $|\Delta J_c|$ is given by:

$$e_\phi = \frac{\mu_\circ J_c |\Delta J_c| a^2}{3} \tag{5.37}$$

Note that the entire slab is in the critical state with its surface $(\pm a)$ exposed to an external field of $H_e \, \vec{i}_y$.

b) Derive Eq. 5.37 by computing the Poynting energy flow into the slab at $x = a$ and equating it with the change in magnetic energy storage and dissipation energy $\mathcal{E}_\phi$ in the positive half of the slab.

c) To relate ΔJ_c to an equivalent temperature rise in the conductor, we may assume a linear temperature dependence for $J_c(T)$:

$$J_c(T) = J_{c_\circ} \left(\frac{T_c - T}{T_c - T_{op}} \right) \tag{5.38}$$

where $J_{c_\circ}$ is the critical current density at the operating temperature T_{op}. T_c is the critical temperature at a given magnetic induction $B_\circ$. From Eq. 5.38, ΔJ_c in Eq. 5.37 may be related to an equivalent temperature rise ΔT:

$$\Delta J_c = -J_{c_\circ} \left(\frac{\Delta T}{T_c - T_{op}} \right) \tag{5.39}$$

Now, by requiring that $\Delta T_s = e_\phi / \tilde{C}_s \leq \Delta T$, where $\tilde{C}_s$ is the superconductor's average heat capacity [J/m^3 K] in the range from T_{op} to T_c, show thermal stability implies a critical slab half width a_c of:

$$a_c = \sqrt{\frac{3 \tilde{C}_s (T_c - T_{op})}{\mu_\circ J_{c_\circ}^2}} \tag{5.40}$$

Problem 5: Magnetization of conductors

This problem illustrates the effect of filament size and twisting on magnetization. In the late 1960s, three NbTi composite superconductors of equal volume were subjected to magnetization measurements [5.12]. Conductors 1, 2, and 3, respectively, are: twisted multifilamentary wire with a twist pitch length ℓ_{p1}; twisted multifilamentary wire with a twist pitch length $\ell_{p2} > \ell_{p1}$; and a monofilament.

Figure 5.21 presents three magnetization curves, labeled A, B, and C, for the three NbTi composite conductors. Each conductor was subjected to field pulses indicated by arrows in the figure. Traces A, B, and C do not necessarily correspond to Conductors 1, 2, and 3, respectively. Note that Traces B (B_1, B_2, B_3) show a dependence on field sweep rate; Trace C is independent of field sweep rate; Trace A also is independent of field sweep rate, but shows "partial" flux jumps induced by the field pulses.

a) Identify which magnetization trace corresponds to which conductor.

b) Estimate the ratio of filament diameter in the monofilament conductor to that in the multifilament conductors.

c) Estimate the value of ℓ_{p2}. Take $J_c d_f = 4 \times 10^4$ A/m for Conductors 1 and 2. Also comment on ℓ_{p1}.
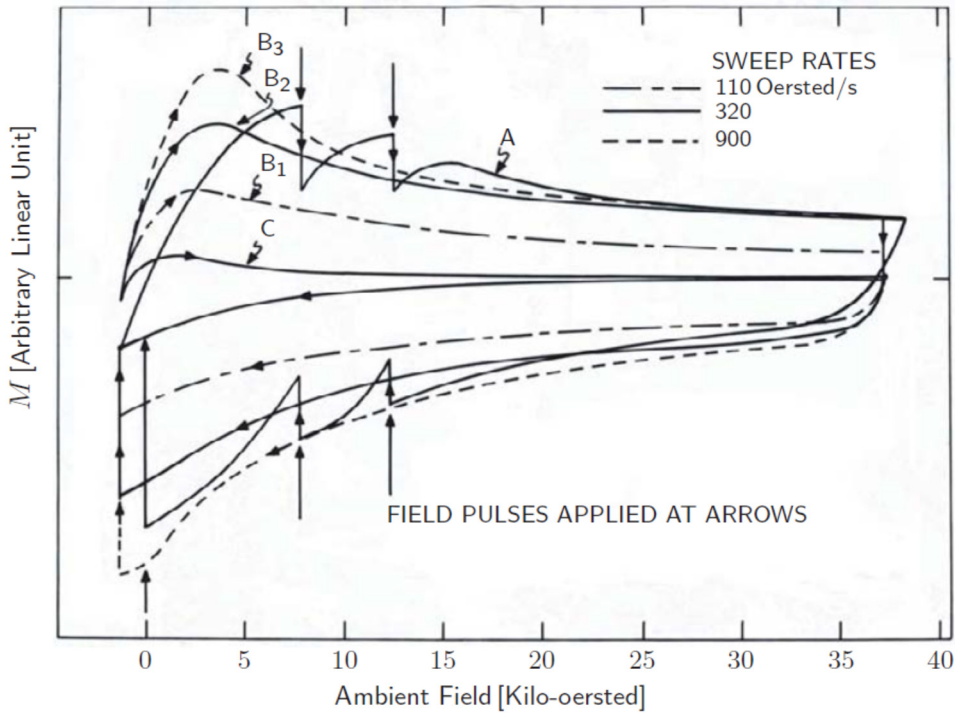

Fig. 5.21   Magnetization traces for Conductors 1, 2, and 3 [5.12].

For a composite superconductor in which the superconductor is characterized by Eq. 6.25a, the equivalent circuit of Fig. 6.5b is modified, as shown in Fig. 6.11. It consists of an ideal voltage source (zero internal resistance) V_S in series with a differential resistance, $R_{dif} \equiv \partial V_s/\partial I_s$, where V_s is the voltage across the superconductor. As in Fig. 6.5b the matrix is represented by a resistor R_m.

a) Show, with $R_c \equiv V_c/I_c$, that $R_s \equiv V_s/I_s$ and R_{dif} are given by:

$$R_s = R_c \left(\frac{I_s}{I_c}\right)^{(n-1)} \tag{6.26a}$$

$$R_{dif} = n R_c \left(\frac{I_s}{I_c}\right)^{(n-1)} \tag{6.26b}$$

Note that $R_{dif} = n R_s$. For a superconductor of $n=1$, its V-I curve becomes similar to a regular resistor's and, as may be expected, $R_s = R_c = R_{dif}$.

b) For a 10-cm long and 1-cm wide composite superconductor with $I_c = 100$ A at 77.3 K, $V_c = 10\,\mu$V, $n=15$, a matrix resistance of $R_m = 0.3$ mΩ, and assuming, for simplicity, that the composite, cooled by boiling nitrogen, always remains at 77.3 K, compute: 1) I_m and I_s; 2) total voltage across the 10-cm long composite; 3) total Joule dissipation in the composite; 4) Joule heat flux over the composite cooling surface area of 10 cm^2 (10 cm$\times$1 cm); and 5) R_{dif}, at transport currents, I_t, of 90 A; 100 A; 120 A; 150 A; 300 A; and 500 A.

c) Discuss the assumption of constant temperature of 77.3 K and discuss, qualitatively, how results are modified if the composite superconductor's temperature increases with increasing Joule dissipation.

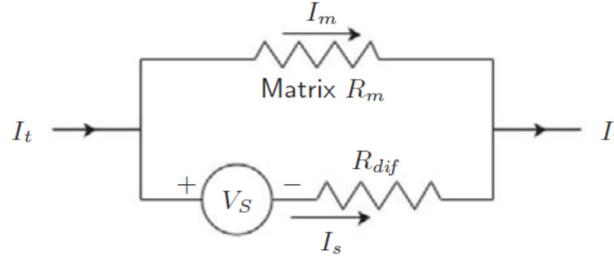d) Repeat b), except with $n=30$.

e) Repeat b), now with $n=60$.

Fig. 6.11   Circuit model for a composite superconductor with a superconductor of the V_s vs. I_s characteristic (Eq. 6.25a), shunted by a matrix resistance R_m. The superconductor consists of V_S, an ideal voltage source, in series with R_{dif}, the differential resistor.