Third Asian School on Superconductivity & Cryogenics for Accelerators (ASSCA2018), IHEP, Beijing, China, December 10-16, 2018

Final Exam

1. (10 points) Introduction
   a) (5 points) What should be the circumference of a proton circular accelerator of 1 TeV energy by using a SC magnet with a central field of 10 Tesla?
   b) (5 points) Which SC material (NbTi or Nb3Sn) and at what temperature we should use for the 10 Tesla magnet and why? Based on their value of T_c (10 K & 18 K) & H_{c2} at 0 K (15 T & 24 T).

2. (10 points) Superconducting Magnet
   a) (3 points) What's the basic superconducting wire and cable applied for accelerator magnets?
   b) (4 points) Why do we need cables? What are the problems in a cable and the characteristic of the magnet cable?
   c) (3 points) What is the specialty of the cables for detector magnets?

3. (15 points) Criterion for flux jumping.

   This problem deals with the derivation of the critical conductor size above which flux jumping will occur. Flux jumping was once a major source of instabilities in the first superconducting magnets of engineering significance in the early 1960s. Flux jumping is a thermal instability peculiar to a Type II superconductor that permits the magnetic field to penetrate its interior. A time-varying magnetic field, $\dot{H}_e$, at the conductor surface induces an electric field $\vec{E}$ in the conductor, which interacts with the supercurrent (density J_c). This $\vec{E}\cdot\vec{J}_c$ interaction heats the conductor. Since J_c decreases with temperature, the field (flux) penetrates further into the conductor, generating more heat, which further decreases J_c. The field penetration and temperature rise can cascade until the conductor loses its superconductivity. This thermal runaway event is called a flux jump.

   a) (5 points) Using the Bean model and computing the $\vec{E}\cdot\vec{J_c}$ interaction over the positive half (0 ≤ x ≤ a) of the slab, show that an expression for the dissipative energy density, $e_\varphi[J/m^3]$, generated within the slab when the critical current density J_c is suddenly decreased by $|\Delta J_c|$ is given by:

$$e_\phi = \frac{\mu_\circ J_c |\Delta J_c| a^2}{3} \quad (1)$$

Note that the entire slab is in the critical state with its surface (±a) exposed to an external field of $H_e\vec{\iota}_y$.

b) (5 points) Derive the last equation by computing the Poynting energy flow into the slab at $x = a$ and equating it with the change in magnetic energy storage and dissipation energy ε_ϕ in the positive half of the slab.

c) (5 points) To relate ΔJ_c to an equivalent temperature rise in the conductor, we may assume a linear temperature dependence for $J_c(T)$:

$$J_c(T) = J_{c_0}\left(\frac{T_c - T}{T_c - T_{op}}\right) \tag{2}$$

where J_{c_0} is the critical current density at the operating temperature T_{op}. T_c is the critical temperature at a given magnetic induction B_o. From last equation (2), ΔJ_c in Eq.(1) may be related to an equivalent temperature rise ΔT:

$$\Delta J_c = -J_{c_0}\left(\frac{\Delta T}{T_c - T_{op}}\right) \tag{3}$$

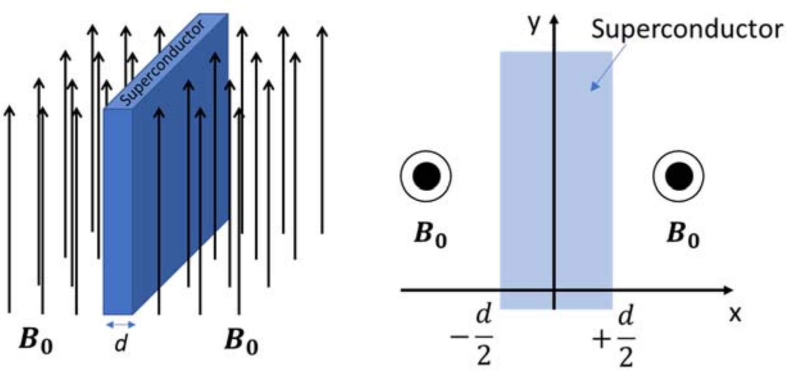
Now, by requiring that $\Delta T_s = e_\phi / \tilde{C}_s \le \Delta T'$, where $\tilde{C}_s$ is the superconductor's average heat capacity [J/m^{3}K] in the range from T_{op} to T_c, show thermal stability implies a critical slab half width a_c of:

$$a_c = \sqrt{\frac{3\tilde{C}_s(T_c - T_{op})}{\mu_o J_{c_o}^2}} \tag{4}$$

4. (10 points) RF Superconductivity.

Consider the system shown in the figure. A superconducting slab with thickness d is placed in the uniform magnetic field parallel to the slab. The field distribution can be written as

$$\boldsymbol{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ B(x) \end{pmatrix} \qquad \begin{aligned} B(-d/2) &= B_0 \\ B(+d/2) &= B_0 \end{aligned}$$



$B(x)$ at - $d/2 \leq x \leq d/2$ is obtained by solving the London equation. Fill the following blanc boxes [1]-[4]:

When the London penetration depth is given by λ, London equation is $d^2B/dx^2 = B/\lambda^2$. The solution can be written as

$$B(x) = C_1 \cosh\frac{x}{\lambda} + C_2 \sinh\frac{x}{\lambda}$$

where C_1 and C_2 are constants. By using the boundary conditions, we find

$$C_1 = [1]$$

and

$$C_2 = [2]$$

Thus $B(x)$ is given by

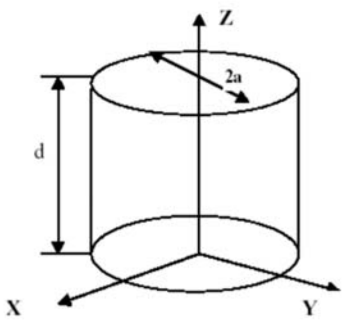
$$B(x) = B_0 \frac{[3]}{\cosh\frac{d}{2\lambda}}$$

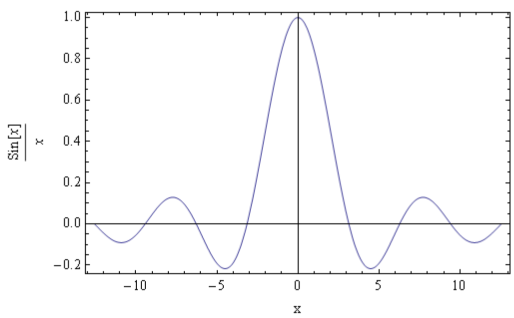
The current distribution in the slab is obtained by using the Maxwell equation, $\mathrm{rot}\,\boldsymbol{B} = \mu_0 \boldsymbol{j}$ or $\boldsymbol{j}(x) = \begin{pmatrix} 0 \\ j_y(x) \\ 0 \end{pmatrix}$. Thus

$$j_y(x) = -\frac{1}{\mu_0}\frac{dB}{dx} = [4]$$

Now consider the case the thickness d is much smaller than the penetration depth λ, namely, $\frac{d}{\lambda} \to 0$. In this case, we have $B(x) \to B_0$ and $j_y \to 0$. This is an example that engineering material geometry allows us to control the current that breaks superconductivity.

5. (15 points) Pillbox cavity at 2 K. Beam is on axis (assuming field is not distorted by the beam ports). 10 mA proton of β=0.75 is accelerated by TM010 mode of this cavity, and the beam gains energy of 1.0 MeV with synchronized phase 0 deg. Given: a = 382.77 mm, d = 500.00 mm.





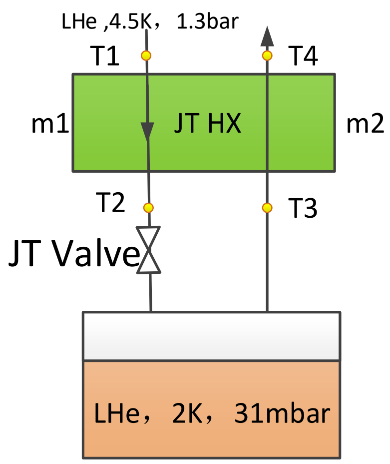
Calculate:

a) (2 points) Frequency of the TM010 mode.
b) (2 points) Transient time factor as function of β, i.e. TTF(β).
c) (2 points) Optimal beta β_0.
d) (3 points) Given R/Q = 196 Ω for particle at β=1, what is the R/Q for particle at β=0.75 ?
e) (3 points) Matched Q_{in}.
f) (3 points) If the microphonics introduces a frequency deviation of 125 Hz, what is the minimum power that the power source has to provide?

6. (10 points) Show the method to measure the accelerating gradient of superconducting cavity at 2 K in a cryomodule. Prove the relation $P_t Q_t = 4 P_g Q_L$.

7. (15 points) In ADS 2 K cryogenic system, the heat exchanger and JT valve are used to achieve 2K saturated helium. The 4.5 K saturated liquid helium is subcooled by the 2K return gas helium with JT heat exchanger. Then the subcooled liquid helium is Isenthalpic Expand by the JT valve. T1=4.5 K, T2=2.37 K, T3=2 K. The enthalpy for different temperature is as follows: H1=11.64 J/g, H2=4.31 J/g, H3= 25.04 J/g. The latent heat of 4.5 K saturated helium is 18.75 W/g. The latent heat of 2K saturated helium is 23.05 W/g. The mass flow of 2 K gas helium is equal to the mass flow of 4.5K. m1=m2=5 g/s.

a) (5 points) Calculate the enthalpy value of T4.
b) (5 points) Calculate the mass flow of 2 K liquid helium after the JT valve. (The enthalpy for 2 K liquid helium is 25.04 J/g, the enthalpy for 2 K gas helium is 1.64 J/g)
c) (5 points) The gas helium doesn't return to the refrigerator. In ADS cryogenic system, 1 g/s 4.5K liquid helium corresponds to the refrigeration capacity of 100 W. Calculate the refrigeration capacity at 2 K.



8. (15 points) In BEPCII cryogenic system, the superconducting cavity was immersed in 4.4 K, 1.2 bar saturated helium. The latent heat of helium is used to cool the cryomodule. (ρ = 121.1 kg/m^3, latent heat = 19.49 kJ/kg)

a) (5 points) The liquid helium supply valve was closed. With zero cavity voltage, the helium level was decreased from 97 % to 86 % with 27 minutes. Please calculate the static heat load.

b) (10 points) The liquid helium supply valve was closed. With cavity voltage 1.5 MV, the liquid helium volume was decreased from 97 % to 86 % with 21 minutes. Calculate cavity Q_0. ($R/Q = 95.3\ \Omega$)

Helium Level (%)	Volume (L)
97	264.69
94	256.91
91	248.68
87	237.25
86	234.35