Phenomenology for the Simplest Little Higgs Model

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(Dated: December 21, 2018)

Talk@4th CLHCP (CCNU), mainly based on a recent paper:

K. Cheung, S.-P. He, Y.-N. Mao, P.-Y. Tseng, and C. Zhang, "Phenomenology of a Little Higgs Pseudo-Axion", Phys. Rev. **D98**, 075023 (2018) [arXiv: 1809.03809].

Introduction

- Little Higgs mechanism was proposed to solve the "little hierarchy" problem, or equivalently, the stability of EW scale under radiative corrections;

- As an example, the Simplest Little Higgs (SLH) model contains the minimal extended scalar sector, there is only one additional scalar (denoted as η), and its properties are related to the electro-weak symmetry breaking (EWSB);

- An accidental motivation: last year at the same conference (3th CLHCP, NJU), I gave a talk stating that in previous papers, the kinetic part of the SLH model was not canonically-normalized, which leads to wrong interactions, thus the properties of η and its phenomenology must be re-considered;

- I and my collaborators finished that in a series of papers: S.-P. He, Y.-N. Mao, C. Zhang, and S.-H. Zhu, Phys. Rev. **D97**, 075005 (2018) [1709.08929]; K. Cheung, S.-P. He, Y.-N. Mao, C. Zhang, and Y. Zhou, Phys. Rev. **D97**, 115001 (2018) [1801.10066]; S.-P. He, Y.-N. Mao, C. Zhang, and S.-H. Zhu, 1804.11333; K. Cheung, S.-P. He, Y.-N. Mao, P.-Y. Tseng, and C. Zhang, Phys. Rev. **D98**, 075023 (2018), [1809.03809];

1709.08929 1801.10066 1804.11333 1809.03809

Brief Review of the Model

D. E. Kaplan and M. Schmaltz, JHEP 0310 (2003) 039 [hep-ph/0302049]

- Global symmetry breaking $[SU(3) \times U(1)]^2 \to [SU(2) \times U(1)]^2$ at scale $f \gg v$: 10 Nambu-Goldstone bosons are generated;
- Enlarged gauge group $SU(3) \times U(1)$ $\to$additional gauge bosons: there are totally 8 massive gauge bosons thus only 2 Goldstones bosons are left physical (SM-like Higgs boson H and a pseudoscalar η);
- Fermion doublets are also enlarged to triplets: additional heavy fermions.

- Two scalar triplets transform as $(\mathbf{1},\mathbf{3})$ and $(\mathbf{3},\mathbf{1})$ respectively;

- The nonlinear realization:

$$\Phi_1 = \exp\left(\mathrm{i}\frac{\Theta'}{f}\right)\exp\left(\mathrm{i}\frac{t_\beta \Theta}{f}\right)\begin{pmatrix}\mathbf{0}_{1\times 2}\\ f c_\beta\end{pmatrix}, \quad \Phi_2 = \exp\left(\mathrm{i}\frac{\Theta'}{f}\right)\exp\left(-\mathrm{i}\frac{\Theta}{f t_\beta}\right)\begin{pmatrix}\mathbf{0}_{1\times 2}\\ f s_\beta\end{pmatrix};$$

- The matrix fields are defined as:

$$\Theta \equiv \frac{\eta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & h\\ h^\dagger & 0\end{pmatrix}, \quad \Theta' \equiv \frac{\zeta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & k\\ k^\dagger & 0\end{pmatrix};$$

- t_β is the ratio between the VEVs of two scalar triplets;

- The SU(2) doublets are

$$h \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}(v + H - \mathrm{i}\chi) \\ h^- \end{pmatrix}, \quad k \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma - \mathrm{i}\omega) \\ k^- \end{pmatrix}$$

- h is just the SM Higgs doublet, doublet k and singlets η, ζ are expected to be the other 6 Goldstone bosons and 5 of which are eaten by extra gauge bosons (note that after EWSB we need further canonically-normalization, see the details later);

- Gauge boson mass spectrum (to the leading order of v/f)

m_A	$m_{W^\pm}$	m_Z	$m_{X^\pm, Y^0, \bar{Y}^0}$	$m_{Z'}$
0	$gv/2$	$gv/(2c_W)$	$gf/\sqrt{2}$	$\sqrt{\frac{2}{3-t_W^2}}gf$

- In this talk, we focus on the phenomenology of η which dominantly interact with t and its partner T, thus we don't discuss the vector properties, vector-scalar interaction, and scalar potential (how to obtain a nonzero v) in details;

- The scalar kinetic term can be expanded as $\mathcal{L} \supset \frac{1}{2}(\partial H^2 + \partial\sigma^2 + \mathbb{K}_{ij}\partial_\mu G_i \partial^\mu G_j)$;

- $G_{i/j}$ runs over $\eta, \zeta, \chi, \omega$, and when $v \neq 0$, $\mathbb{K} \neq \mathbb{I}_{4\times 4}$ $\rightarrow$non-canonically normalized;

- We also have two point transition $\mathbb{F}_{pi} V_p^\mu \partial_\mu G_i$, which must be exactly canceled;

- These two problems can be solved together through a normalization procedure in CP-odd scalar sector: for canonically-normalized basis S_i, define the inner product $\langle S_i|S_j\rangle = \delta_{ij}$, we have $\langle G_i|G_j\rangle = (\mathbb{K}^{-1})_{ij}$, and thus we can find the basis

- $\left(\tilde{\eta}, \tilde{G}_i\right) = \left(\frac{\eta}{\sqrt{(\mathbb{K}^{-1})_{11}}}, \frac{\mathbb{R}_{pq}\mathbb{F}_{qi}G_i}{m_p}\right)$, we can check $\langle\tilde{\eta}|\tilde{\eta}\rangle = 1$, $\langle\tilde{\eta}|\tilde{G}_p\rangle = 0$, $\langle\tilde{G}_p|\tilde{G}_q\rangle = \delta_{pq}$;

- This normalization procedure will modify the interactions including η.

- Choose the "anomaly free" embedding, [O. C. W. Kong, arXiv: hep-ph/0307250; J. Korean Phys. Soc. 45, S404 (2004); Phys. Rev. D 70, 075021 (2004)], the triplets $L^T = (\nu, \ell, \mathrm{i}N)_L, Q_1^T = (d, -u, \mathrm{i}D)_L, Q_2^T = (s, -c, \mathrm{i}S)_L, Q_3^T = (t, b, \mathrm{i}T)_L$;

- $Q_x(Q_1, Q_2, N_R) = 0, Q_x(Q_3) = 1/3, Q_x(u_R) = 2/3, Q_x(d_R) = -1/3, Q_x(L) = -1$;

- Lagrangian [$d_{R,j}$ runs over $(d, D, s, S, b)_R$ and $u_{R,j}$ runs over $(u, c, t, T)_R$]:

$$\mathrm{i}\left(\lambda^a_{d,n}\bar{d}^a_{R,n}\Phi_1^T + \lambda^b_{d,n}\bar{d}^b_{R,n}\Phi_2^T\right)Q_n - \mathrm{i}\frac{\lambda_u^{jk}}{f}\bar{u}_{R,j}\det\left(\Phi_1^*, \Phi_2^*, Q_k\right) - \mathrm{i}\frac{\lambda_{b,j}}{f}\bar{d}_{R,j}\det\left(\Phi_1, \Phi_2, Q_3\right)$$
$$+\mathrm{i}\left(\lambda^a_t\bar{u}^a_{R,3}\Phi_1^\dagger + \lambda^b_t\bar{u}^b_{R,3}\Phi_2^\dagger\right)Q_3 + \mathrm{i}\lambda_{N,j}\bar{N}_{R,j}\Phi_2^\dagger L_j - \mathrm{i}\frac{\lambda_\ell^{jk}}{f}\bar{\ell}_{R,j}\det\left(\Phi_1, \Phi_2, L_k\right) + \mathrm{H.c.}$$

- Fermion Spectrum:

m_{ν_j}	m_{ℓ_j}	m_{N_j}	$m_{u,c}$	m_b	m_q	m_Q
0	$y_{\ell_j}\frac{v}{\sqrt{2}}$	$\lambda_{N_j} f s_\beta$	$y_{u,c}\frac{v}{\sqrt{2}}$	$\lambda_{b,3}\frac{v}{\sqrt{2}}$	$\lambda_q\frac{v}{\sqrt{2}}$	$\sqrt{\left\|\lambda_q^a c_\beta\right\|^2 + \left\|\lambda_q^b s_\beta\right\|^2} f$

- y_{ℓ_j} is an eigenvalue of matrix λ_ℓ^{jk}, $y_{u,c}$ is an eigenvalue of matrix λ_u^{jk}, index $q(Q) = d(D), s(S), t(T)$ and $\lambda_q \equiv \left|\lambda_q^a \lambda_q^b\right| / \sqrt{\left|\lambda_q^a c_\beta\right|^2 + \left|\lambda_q^b s_\beta\right|^2}$;

- Right-handed mixing $s_{2\theta_{R,q}} = \sqrt{2} m_q s_{2\beta}/(m_Q \kappa)$;

- The exact mixing angle depends on the relative magnitude of couplings $\lambda_{a,b}$;

- No right handed mixing in lepton sector (since we don't consider ν_R);

- Left-handed mixing [$\sim \mathcal{O}(\kappa)$ to leading order of κ]

$$\begin{pmatrix} \tilde{q} \\ \tilde{Q} \end{pmatrix}_L = \begin{pmatrix} 1 & -\delta_q \\ \delta_q & 1 \end{pmatrix} \begin{pmatrix} q \\ Q \end{pmatrix}_L , \qquad \begin{pmatrix} \tilde{N} \\ \tilde{\nu} \end{pmatrix}_L = \begin{pmatrix} 1 & -\delta_\nu \\ \delta_\nu & 1 \end{pmatrix} \begin{pmatrix} N \\ \nu \end{pmatrix}_L$$

- $\delta_t = \frac{\kappa}{\sqrt{2} s_{2\beta}} \left(\pm |c_{2\theta_{R,t}}| - c_{2\beta} \right)$, $\delta_{d/s} = -\frac{\kappa}{\sqrt{2} s_{2\beta}} \left(\pm |c_{2\theta_{R,d/s}}| - c_{2\beta} \right)$, we denote them as $\delta_q^\pm$;

- If we ignore m_d and m_s, we have $\delta_{d,s}^+ = -\frac{\kappa t_\beta}{\sqrt{2}}$ and $\delta_{d,s}^- = \delta_\nu = \frac{\kappa}{\sqrt{2} t_\beta}$ for simplify;

- Fields with $\tilde{}$ are mass eigenstates, parameterize the Yukawa interaction again as

$$\begin{aligned} \mathcal{L} \supset & -\sum_f \left(y_{H,f} \bar{f} f H + \mathrm{i} y_{\eta,f} \bar{f} \gamma^5 f \eta \right) - \sum_{f/F} H \left(y_{H,fF} \bar{f}_R F_L + y_{H,Ff} \bar{F}_R f_L + \text{H.c.} \right) \\ & -\sum_{f/F} \eta \left(\mathrm{i} y_{\eta,fF} \bar{f}_R F_L + \mathrm{i} y_{\eta,Ff} \bar{F}_R f_L + \text{H.c.} \right) \end{aligned}$$

- F is the corresponding partner of f like above;
- Similarly, Q is the partner of SM quark q;
- The coefficients are listed in the table:

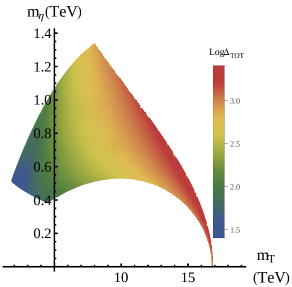
$c_{H,f}\big\|_{f\in \mathrm{SM}}$	$c_{H,N}$	$c_{H,Q}$	$c_{H,Ff}$	$c_{H,tT}$	$c_{H,dD/sS}$
$\frac{m_f}{v}$	0	$-\frac{m_q^2}{v m_Q}$	$\frac{m_Q \delta_q}{v}$	$-\frac{m_t}{v}\left(\frac{\sqrt{2}\kappa}{t_{2\beta}} + \delta_t\right)$	$\frac{m_{d/s}}{v}\left(\frac{\sqrt{2}\kappa}{t_{2\beta}} - \delta_{d/s}\right)$

$c_{\eta,f}\big\|_{f=u,c,b,\ell,\nu}$	$c_{\eta,q}$	$c_{\eta,N}$	$c_{\eta,Q}$	$c_{\eta,qQ}$	$c_{\eta,N\nu}$	$c_{\eta,Qq}$
0	$-\frac{m_q \delta_q}{v}$	$-\frac{m_N}{\sqrt{2} f t_\beta}$	$\frac{m_Q \delta_q}{v}$	$\frac{m_q}{v}$	$\frac{m_N v}{2 f^2 s_\beta^2}$	$-\frac{m_Q}{v}\left(\frac{\kappa^2}{2} + \delta_q^2\right)$

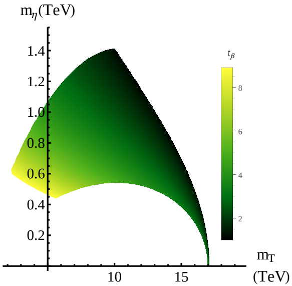
- The red coefficients are different from the results before canonically-normalization.

η-related Phenomenology

We choose the region $m_\eta \sim (450 - 750)$ GeV, reasons:



(f=8 TeV) Naturalness



(f=10 TeV) t_β distribution

- $Z' \to \ell^+\ell^-$ direct search set a limit $f > 8$ TeV, Goldstone scattering unitarity requires $t_\beta < 8.9$ and UV cutoff $\Lambda = \sqrt{8\pi} f c_\beta$;
- Naturalness defined as $\Delta^\mu_{\text{TOT}} = \left|\frac{\partial \ln m_h^2}{\partial \ln \mu_U^2}\right|$, $\Delta^\lambda_{\text{TOT}} = \left|\frac{\partial \ln m_h^2}{\partial \ln \lambda_U}\right|$, Δ_{TOT} is the maximal one, μ_U and λ_U are defined at Λ;
- We choose the natural region with $f = 8$ TeV and lighter m_T;
- Mass relation can be used to derive t_β distribution:

$$m_\eta^2 s_\alpha^2 \approx m_h^2 - 2v^2(\Delta_A - 2A), \text{ where } \alpha \equiv \sqrt{2}(v/f)/s_{2\beta} \text{ and}$$

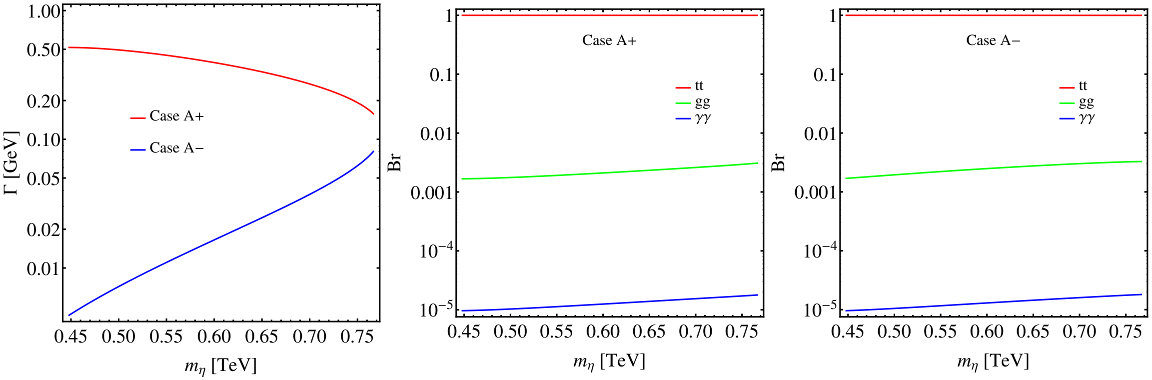
$$\begin{cases} \Delta_A = \frac{3}{16\pi^2}\left(\lambda_t^4 \ln\frac{m_T^2}{m_t^2} - \frac{g^4}{8}\ln\frac{m_X^2}{m_W^2} - \frac{g^4}{16c_W^4}\ln\frac{m_{Z'}^2}{m_Z^2}\right) \\ A = \frac{3}{16\pi^2}\left(\lambda_t^4 - \frac{g^4}{8} - \frac{g^4}{16c_W^4}\right) \end{cases}$$

Decay Widths of η

Channel	Decay Width
$\eta \to t\bar{t}$	$\frac{3m_\eta}{8\pi}\left(\frac{m_t\delta_t}{v}\right)^2\sqrt{1-\frac{4m_t^2}{m_\eta^2}}$ (Domininant)
$\eta \to dD$	$\frac{3m_\eta}{8\pi}\left(\frac{m_D}{v}\right)^2\left(\frac{\kappa^2}{2}+\delta_{Dd}^2\right)^2\left(1-\frac{m_D^2}{m_\eta^2}\right)^2$ (if $m_\eta > m_D$)
$\eta \to N\bar{N}$	$\frac{m_\eta m_N^2}{16\pi f^2 t_\beta^2}\sqrt{1-\frac{4m_N^2}{m_\eta^2}}$ (if $m_\eta > 2m_N$)
$\eta \to N\nu$	$\frac{m_\eta}{8\pi}\left(\frac{\kappa m_N}{2fs_\beta^2}\right)^2\left(1-\frac{m_N^2}{m_\eta^2}\right)^2$ (if $m_\eta > m_N$)
$\eta \to gg$	$\frac{m_\eta^3\alpha_s^2}{128\pi^3v^2}\left\lvert -\delta_t A_{1/2}(\tau_t)+\delta_t A_{1/2}(\tau_T)+\delta_{Dd}A_{1/2}(\tau_D)+\delta_{Ss}A_{1/2}(\tau_S)\right\rvert^2$
$\eta \to \gamma\gamma$	$\frac{m_\eta^3\alpha^2}{2304\pi^3v^2}\left\lvert -4\delta_t A_{1/2}(\tau_t)+4\delta_t A_{1/2}(\tau_T)+\delta_{Dd}A_{1/2}(\tau_D)+\delta_{Ss}A_{1/2}(\tau_S)\right\rvert^2$

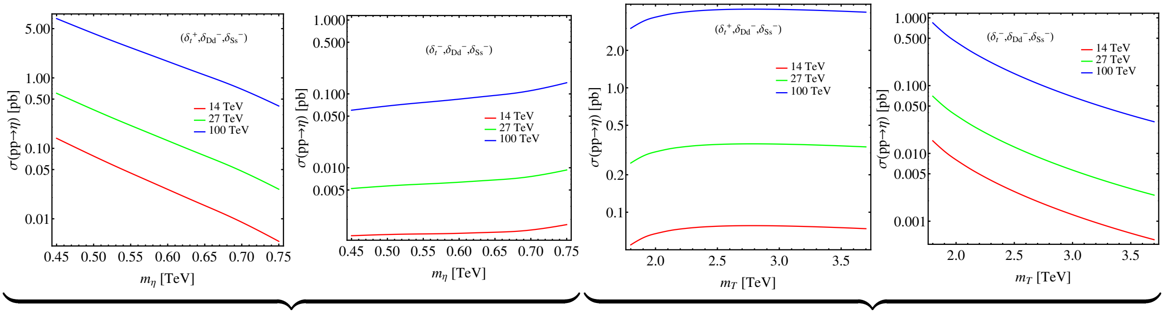
$\tau_f \equiv m_\eta^2/4m_f^2$ and function $A_{1/2}(\tau) \equiv 2f(\tau)/\tau$, where $f(\tau) \equiv \arcsin^2\sqrt{\tau}$ for $\tau \le 1$ and

$$f(\tau) \equiv -\frac{1}{4}\left[\ln\left((1+\sqrt{1-\tau^{-1}})/(1-\sqrt{1-\tau^{-1}})\right)-i\pi\right]^2 \text{ for } \tau > 1.$$



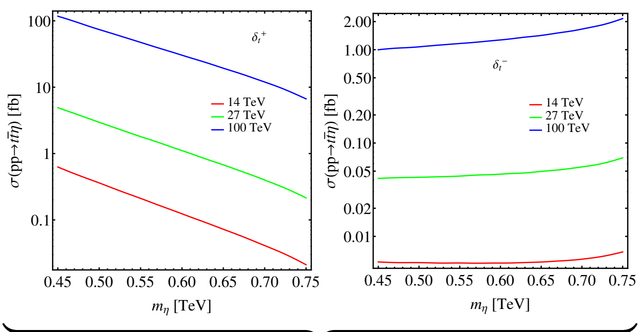
An example: $f = 8$ TeV, $m_T = m_D = m_S = 3$ TeV, $m_{N_i} > m_\eta$;

η Direct Production



$m_T{=}m_D{=}m_S{=}3$ TeV

$m_T{=}m_D{=}m_S, m_\eta{=}500$ GeV



$m_T{=}3$ TeV

- Up: $pp \to \eta$; Down: $pp \to t\bar{t}\eta$;
- For each case, choose $f = 8$ TeV;
- Choose K-factor 1 everywhere;
- Solve β through mass relation;
- Difficult to test at hadron colliders.

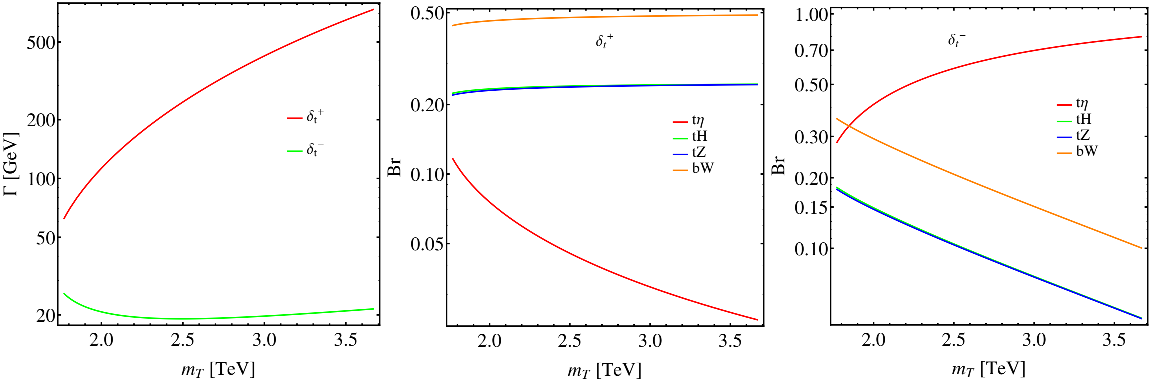
Test η through Cascade Decay

Decay Widths of T

Channel	Decay Width
$T \to Wb$	$\frac{g^2\delta_t^2 m_T^3}{64\pi m_W^2}\left(1+\frac{m_W^2}{m_T^2}-\frac{2m_W^4}{m_T^4}\right) F\left(0, \frac{m_W}{m_T}\right) \simeq \frac{\delta_t^2 m_T^3}{16\pi v^2}$
$T \to Zt$	$\frac{g^2\delta_t^2 m_T^3}{128\pi c_W^2 m_Z^2}\left(1+\frac{m_Z^2-2m_t^2}{m_T^2}+\frac{m_t^4+m_t^2 m_Z^2-2m_Z^4}{m_T^4}\right) F\left(\frac{m_t}{m_T}, \frac{m_Z}{m_T}\right) \simeq \frac{\delta_t^2 m_T^3}{32\pi v^2}$
$T \to Ht$	$\frac{m_T^3\delta_t^2}{32\pi v^2}\left(1+\frac{m_t^2-m_H^2}{m_T^2}\right) F\left(\frac{m_t}{m_T}, \frac{m_H}{m_T}\right) \simeq \frac{\delta_t^2 m_T^3}{32\pi v^2}$
$T \to \eta t$	$\frac{m_T m_t^2}{32\pi v^2}\left(1+\frac{m_t^2-m_\eta^2}{m_T^2}\right) F\left(\frac{m_t}{m_T}, \frac{m_\eta}{m_T}\right) \simeq \frac{m_T m_t^2}{32\pi v^2}\left(1-\frac{m_\eta^2}{m_T^2}\right)^2$

Function $F(x,y) \equiv \sqrt{(1+x+y)(1+x-y)(1-x+y)(1-x-y)}$.

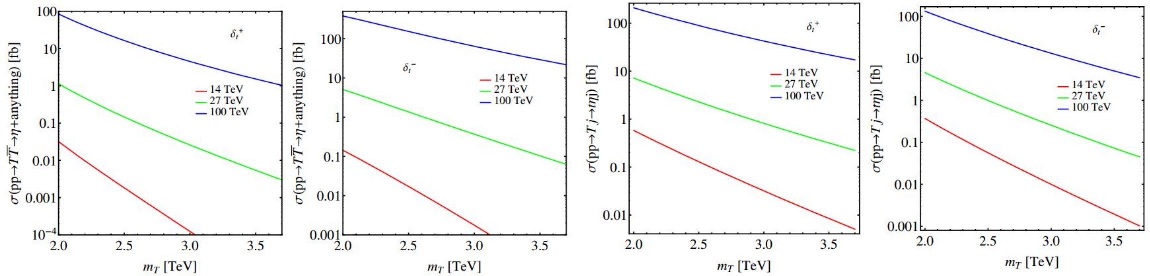
Choose $f = 8$ TeV and $m_\eta = 500$ GeV, note that β can be solved from the mass relation.



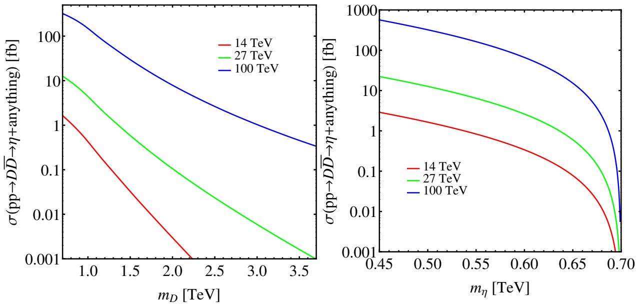
Decay Widths of $D/S/N$

Channel	Decay Width
$D \to Wu$	$\frac{g^2\delta_{Dd}^2 m_D^3}{64\pi m_W^2}\left(1+\frac{m_W^2}{m_D^2}-\frac{2m_W^4}{m_D^4}\right)F\left(0,\frac{m_W}{m_D}\right)\simeq\frac{\delta_{Dd}^2 m_D^3}{16\pi v^2}$
$D \to Zd$	$\frac{g^2\delta_{Dd}^2 m_T^3}{128\pi c_W^2 m_Z^2}\left(1+\frac{m_Z^2}{m_D^2}-\frac{2m_Z^4}{m_D^4}\right)F\left(0,\frac{m_Z}{m_D}\right)\simeq\frac{\delta_{Dd}^2 m_D^3}{32\pi v^2}$
$D \to Hd$	$\frac{m_D^3\delta_{Dd}^2}{32\pi v^2}\left(1-\frac{m_H^2}{m_D^2}\right)F\left(0,\frac{m_H}{m_D}\right)\simeq\frac{\delta_{Dd}^2 m_D^3}{32\pi v^2}$
$D \to \eta d$	$\frac{m_D^3}{32\pi v^2}\left(\frac{v^2}{2f^2}+\delta_{Dd}^2\right)^2\left(1-\frac{m_\eta^2}{m_D^2}\right)^2 \ll \Gamma_{D\to Wu,Zd,Hd}$

The cases for S and N are the same after corresponding replacements.



m_T=m_D=m_S=3 TeV, m_η=500 GeV, $\bar{T}j$ processes are also included for the right two figures.



m_T=3 TeV, m_η=500 GeV(left), m_D=700 GeV(right)

- Choose $f = 8$ TeV as usual;
- $\sigma_{pp\to F\bar{F}\to\eta+X} = \sigma_{pp\to F\bar{F}}\mathrm{Br}_\eta(2-\mathrm{Br}_\eta)$;
- $\sigma_{pp\to Fj\to\eta+X} = \sigma_{pp\to Fj}Br_\eta$;
- LHC very difficult, but we may expect for future pp colliders with larger $\sqrt{s}$.

Summary and Discussion

- The SLH model is not a new model, but some of the interactions about η was incorrect, we re-derived the interactions and re-considered the phenomenology;

- After the correction, $ZH\eta$-vertex is not important $[\sim (v/f)^3]$, thus we should turn to the Yukawa sector: direct production or cascade decay;

- f is now pushed to > 8 TeV and we choose the naturalness favored region $m_\eta \sim (450 - 750)$ GeV, where $\text{Br}_{\eta\to t\bar{t}} \sim 1$;

- The direct production is impossible to test η, while it is possible through T cascade decay: the cross section can reach about $\mathcal{O}(10^2)$ fb at $\sqrt{s} = 100$ TeV proton colliders, though still very difficult at LHC;

- In many models, a similar pseudoscalar can exist, our analysis is just an example, but there may also be similar properties in other models;

- For example, $ZH\eta$-vertex is suppressed by $(v/f)^3$ is a model-independent property if η is a SM-singlet pseudoscalar, based on an EFT analysis; f is also pushed to a similar high scale for models with extra gauge bosons with similar properties [for example, the littlest Higgs model without T-parity];

- Light η scenario is disfavored by naturalness, but not excluded;

- This talk is a simplify version due to the time limit, I will present a complete version at CFHEP, IHEP (Dec. 27, Thursday, 14:30).


The end,
thank you!
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