Simple analytical solutions of relativistic hydrodynamics with longitudinal accelerated flow

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Presented at "the 4th CLHCP 2018 (第四届中国LHC物理工作会议)", Dec 20th 2018 (Wuhan, China).

Special thanks to:

Xin-Nian Wang, T. S. Biró, Long-Gang Pang, She Duan et al.

Motivation

The next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

-- The frontiers of nuclear science, a long range plan

- What is the initial temperature and thermal evolution of the produced matter?

- What is the viscosity of the produced matter? ... http://www.bnl.gov/physics/rhiciiscience/

Results of perfect fluid

- The exact solutions and results of the perfect fluid with longitudinal accelerated flow. (CNC, CKCJ solutions)

Csörgő, Nagy, Csanád (CNC) arXiv: 0605070, 0710.0327, 0805.1562,
Csanád, et. arXiv:1609.07176.
Z. F. Jiang, et. arXiv: 1711.10740, 1806.05750.
Csörgő, et. arXiv: 1805.01427, 1806.11309, 1810.00154.
(See Z. F. Jiang, C. B. Yang, M. Csanád, T. Csörgő, Phys. Rev. C 97,(2018) 064906,
T. Csörgő, G. Kasza, M. Csanád, Z. F. Jiang, Universe 4, (2018), 69.)

Outline

The next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

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Outline

1. Simple, perturbative solutions of relativistic viscous hydrodynamics with longitudinal accelerated flow.
2. Final state observations and results.
3. Summary and outlook.

Z. F. Jiang, et al. arXiv:1808.10287. CPC 42 (2018) no.12, 123103

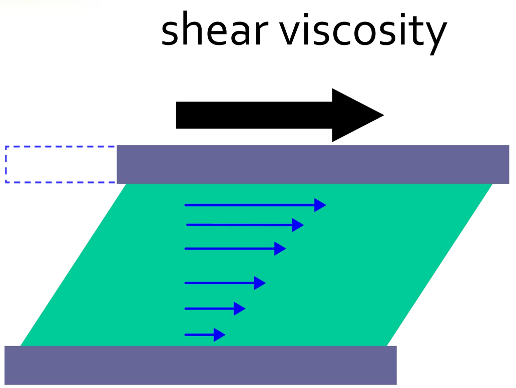
Relativistic viscous hydrodynamic

Longitudinal accelerated effect make the fluid cool faster. (CNC, CKCJ solutions)
The viscosity will generate heat and make the fluid cool slower.

PLB, Hans. Bantilan et al. arXiv:1803.10774

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

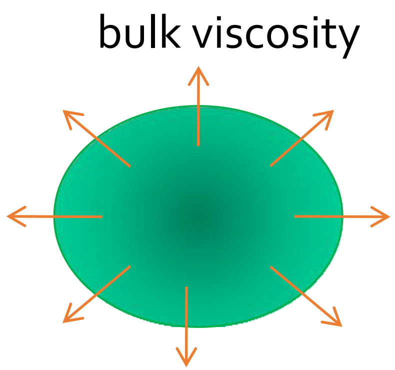
$$u^{\mu} = (\cosh\Omega, 0, 0, \sinh\Omega) \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$$



Shear viscosity tensor: $\pi^{\mu\nu}$ **Bulk viscosity:** Π.

Shear tensor:

$$\sigma^{\mu\nu} \equiv \partial^{\langle\mu}u^{\nu\rangle} \equiv \left(\frac{1}{2}(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta}\Delta^{\nu}_{\alpha}) - \frac{1}{d}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right)\partial^{\alpha}u^{\beta}.$$



The fundamental equations of the viscous fluid:

$$e = \kappa p, \quad \partial_{\mu}T^{\mu\nu} = 0.$$

Equations of viscous hydrodynamic

The second law of thermodynamics: $\partial_\mu S^\mu \geq 0$

$$\tau_\pi \Delta^{\alpha\mu} \Delta^{\beta\nu} \dot{\pi}_{\alpha\beta} + \boxed{\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}} - \frac{1}{2}\pi^{\mu\nu} \frac{\eta T}{\tau_\pi} \partial_\lambda \left(\frac{\tau_\pi}{\eta T} u^\lambda \right)$$

$$\tau_\Pi \dot{\Pi} + \boxed{\Pi = -\zeta (\partial \cdot u)} - \frac{1}{2}\Pi \frac{\zeta T}{\tau_\Pi} \partial_\lambda \left(\frac{\tau_\Pi}{\zeta T} u^\lambda \right)$$

Israel-Stewart equations.

viscous hydro: near-equilibrium system

The Navier-Stokes approximation,

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad \Pi = -\zeta\left(\partial_\rho u^\rho\right)$$

The shear viscosity and bulk viscosity,

Strongly coupled AdS/CFT prediction: $\eta/s \geq 1/4\pi \approx 0.08$ D.T. Son, et,al. 05

Via lattice calculation: $\zeta/s \leq 0.015\,(\text{for } 3T_c)$ H.B. Meyer, et,al. 0710.3717

Accelerating viscous hydrodynamic equation

In Rindler coordinates (accelerate coordinates), the energy equation and Euler equation reduce to:

$$\begin{cases} \tau \dfrac{\partial T}{\partial \tau} + \tanh(\Omega - \eta_s)\dfrac{\partial T}{\partial \eta_s} + \dfrac{\Omega'}{\kappa} T = \dfrac{\Pi_d}{\kappa}\dfrac{\Omega'^2}{\kappa}\cosh(\Omega - \eta_s), \\ \tanh(\Omega - \eta_s)\left[\tau \dfrac{\partial T}{\partial \tau} + T\Omega'\right] + \dfrac{\partial T}{\partial \eta_s} = \dfrac{\Pi_d}{\kappa}[2\Omega'(\Omega' - 1) + \Omega''\coth(\Omega - \eta_s)]\sinh(\Omega - \eta_s) \end{cases}$$

$$\begin{cases} \Omega' = \dfrac{\partial \Omega}{\partial \eta_s} \\ \Pi_d = \left(\dfrac{\zeta}{s} + \dfrac{2\eta}{s}(1 - \dfrac{1}{d})\right) \end{cases}$$

Bjorken approximation:

$\Pi_d = 0,\ \Omega(\eta_s) = \eta_s$

J.D.Bjorken, Phys.Rev. D27 (1983) 140-151.

Without accelerated flow:

$\Pi_d \neq 0\ \ \Omega(\eta_s) = \eta_s$

A. Muronga, Phys. Rev. C69 (2004), 034904.

Without viscosity effect:

$\Pi_d = 0,\ \Omega(\eta_s) \neq \eta_s$

CKCJ solutions, Universe 4 (2018), 69.

Both are non-zero, a perturbative case.

$\Omega = \lambda \eta_s = (1 + \varepsilon)\eta_s, \qquad |\varepsilon| << 1.$

$\Pi_d \neq 0$

λ: the constant proper acceleration.

ε: the acceleration parameter.

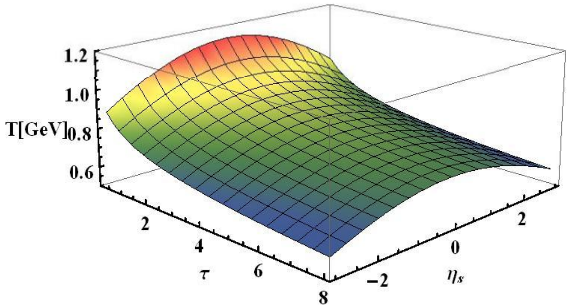
Up to $\mathcal{O}(\epsilon)$,

$$\begin{cases} \tau \dfrac{\partial T}{\partial \tau} + \dfrac{\epsilon + 1}{\kappa} T = \dfrac{\Pi_d}{\kappa}\dfrac{2\epsilon + 1}{\tau} + \mathcal{O}(\epsilon^2). \\ T_1(\eta_s)\left(1 - \dfrac{1}{\kappa}\right)\epsilon\eta_s + \dfrac{\epsilon \Pi_d}{(\kappa - 1)\tau_0}\left(1 - \dfrac{1}{\kappa}\right)\eta_s + \dfrac{\partial T_1(\eta_s)}{\partial \eta_s} + \mathcal{O}(\epsilon^2) = 0 \end{cases}$$

M. Csanád, 2017. Universe. 3. 1-9.
Z. F. Jiang, et. arXiv: 1711.10740

Solution form hydrodynamic equations

The temperature exact solution:



$$T(\tau,\eta_s) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1+\epsilon}{\kappa}} \times \left[\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] + \frac{R_0^{-1}}{\kappa-1}\left(2\epsilon + \exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] - (2\epsilon+1)\left(\frac{\tau_0}{\tau}\right)^{\frac{\kappa-\epsilon-1}{\kappa}}\right)\right]$$

Contribution from ideal terms.

Contribution from viscous effect

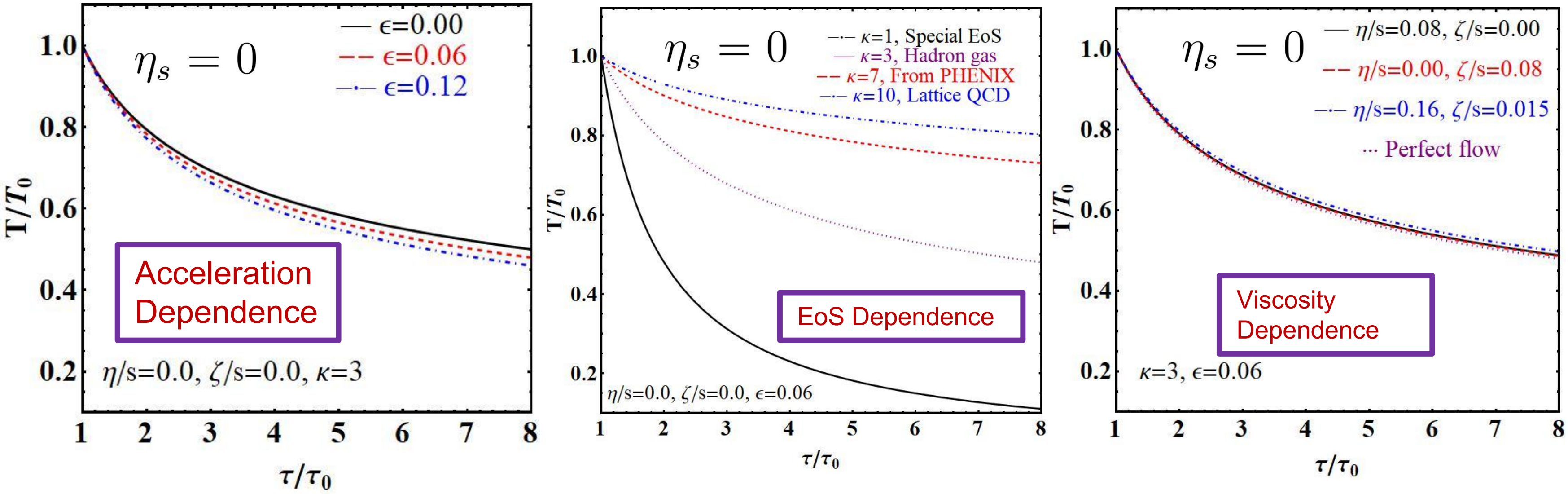
$$R_0^{-1} = \frac{\Pi_d}{T_0\tau_0}$$

Reynolds number
[A. Muronga, arXiv: 0309055]

- A non-zero viscosity makes cooling rate small.
- A non-vanishing longitudinal acceleration makes the cooling ration large.

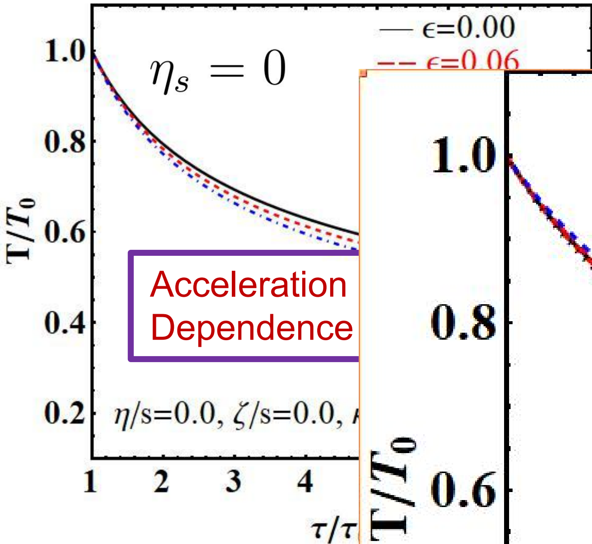
Highlight: first exact solution that include both longitudinal accelerated flow and viscosity effect !!

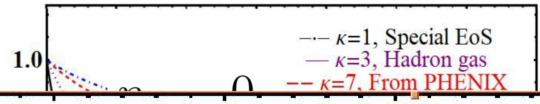
Temperature evolution

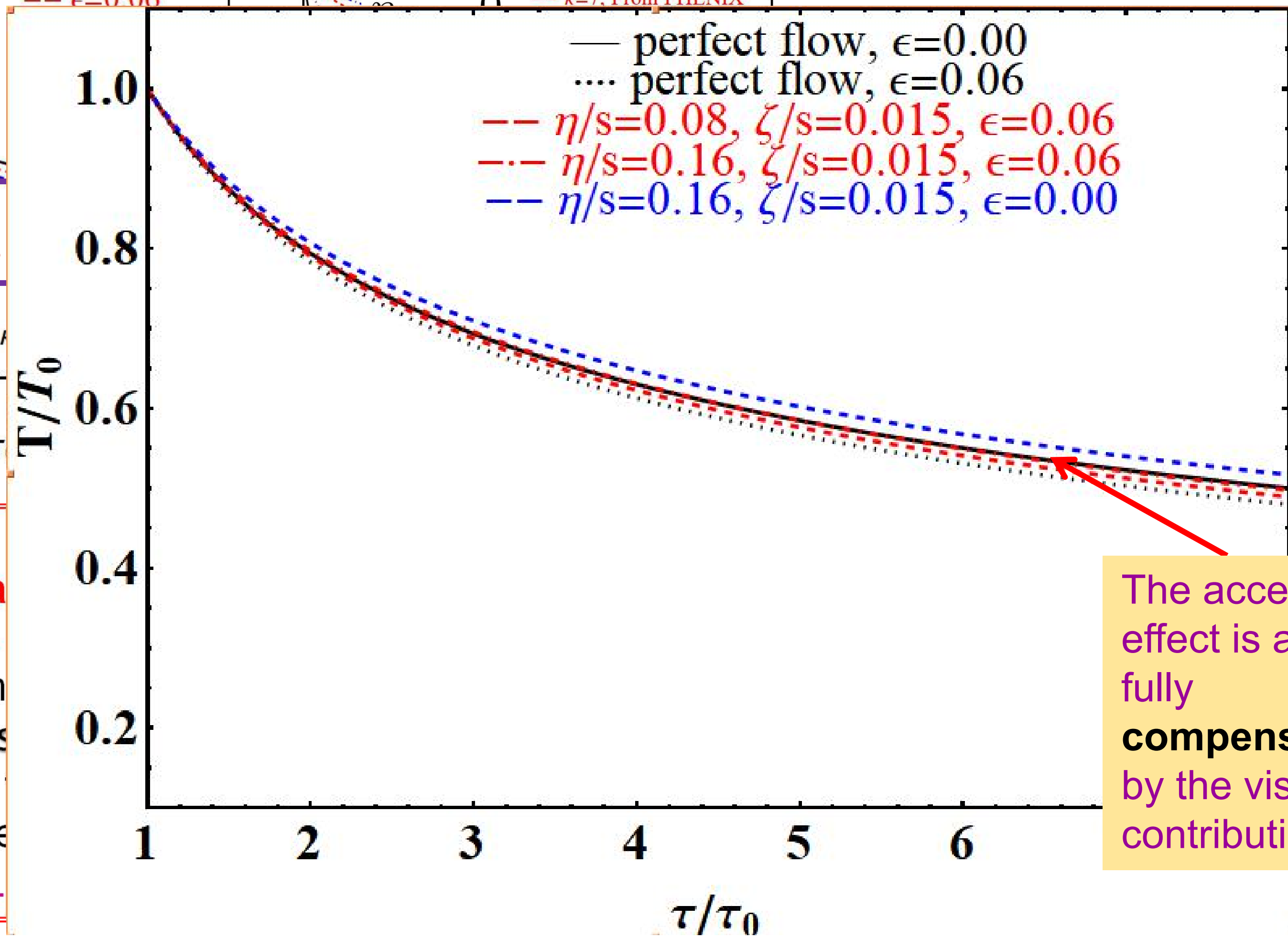


- Longitudinal accelerated flow comes from the pressure gradient, makes the cooling ratio **larger** than non-acceleration flow (Bjorken flow).
  [M. Nagy, T. Csörgő, M. Csanád: arXiv:0709.3677v1
- EoS is an important modified factor.
  κ=1 a very special case, CNC solution.
  κ=7 comes from [PHENIX, arXiv:nucl-ex/0608033v1] .
- Viscosity effect make the cooling rate samller.
  [H. Song, S. Bass, U. Heinz. et, PRL2011]

Temperature evolution







- Acceleration
cooling ratio **la**
[M. Nagy, T.
- EoS is an im
κ=1 a very s
κ=7 comes
- Viscosity effe
Heinz. et, PRL

The acceleration effect is almost fully **compensated** by the viscous contribution.

The final state spectrum

Freeze-out hypersurface:

$$p_\mu d\Sigma^\mu = m_T \tau_f \cosh^{\frac{2-\Omega'}{\Omega'-1}}((\Omega'-1)\eta_s)\cosh(\Omega - y) r dr d\phi d\eta_s$$

[M. I. Nagy, T. Csörgő, M. Csanád: arXiv:0709.3677v1]

The transverse momentum distribution (a toy model):

$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty} m_T \cosh((\epsilon+1)\eta_s - y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s - y)\right]$$
$$\times\left(\tau_f \cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2 - 2m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right.\right.$$
$$\left.\left.-\frac{1}{5}\frac{\zeta}{s}(p_T^2 + m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right]\right)d\eta_s$$

- $f_0 + \delta f$, Boltzmann approximation, from K. Dusling and D. Teaney (2010).
- Temperature solution, viscosity, acceleration parameter, mass, ...

D. Teaney, 2003. P. R. C 68, 034913, blast wave model, presented a special case when there is no acceleration effect (ε=0).

(Pseudo-) Rapidity distribution

Rapidity distribution

Contribution from perfect fluid

$$\frac{dN}{dy} = \frac{\pi R_0^2}{(2\pi)^3}\int_0^{+\infty}\left\{\cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s)\frac{4\tau_f T^3(\tau,\eta_s)}{\cosh^2((\epsilon+1)\eta_s - y)} + \frac{48(1+\epsilon)T^2(\tau,\eta_s)}{\cosh^4((\epsilon+1)\eta_s - y)}\right.$$
$$\left.\times\left[\frac{1}{3}\frac{\eta}{s}(1-2\sinh^2((\epsilon+1)\eta_s - y)) - \frac{1}{5}\frac{\zeta}{s}\cosh^2((\epsilon+1)\eta_s - y)\right]\right\}d\eta_s$$

Rapidity distribution,
- the integral value $error \propto m^3$, this is a good approximation for the particle that mass m is little.

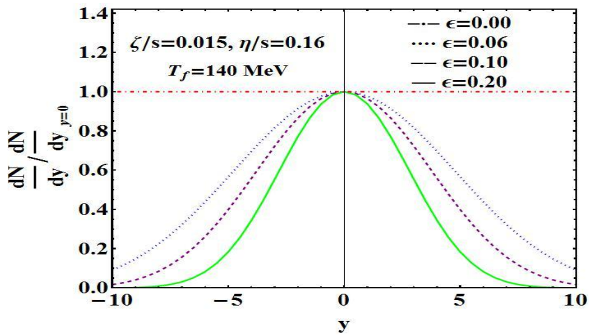
Pseudo-rapidity distribution

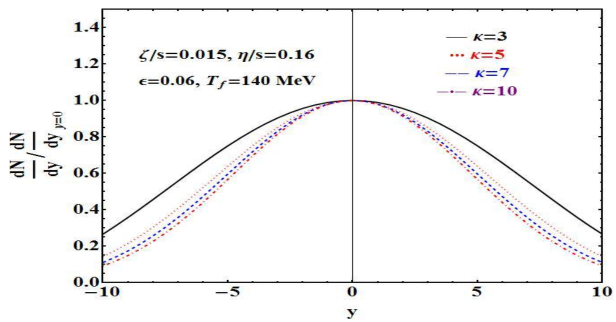
$$\frac{dN}{d\eta} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty} d\eta_s \int_0^{+\infty} dp_T\sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\, m_T p_T \cosh((\epsilon+1)\eta_s - y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s - y)\right]$$
$$\times\left(\tau_f\cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2 - 2m_T^2\sinh^2((\epsilon+1)\eta_s - y)) - \frac{1}{5}\frac{\zeta}{s}(p_T^2 + m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right]\right)$$

Contribution from perfect fluid | **Contribution from viscous effect**

Particle's distribution (toy model)

Numerical results (Rapidity distribution):





Gubser flow, PLB, Hans. Bantilan et al. arXiv:1803.10774

Acceleration parameter extracted from RHIC and the LHC data:

$\sqrt{s_{NN}}$ /[GeV]		$\frac{dN}{d\eta}\Big\vert_{\eta=\eta_0}$	ϵ	χ^2/NDF
130	Au+Au	563.9±59.5	0.076±0.003	9.41/53
200	Au+Au	642.6±61.0	0.062±0.002	12.23/53
200	Cu+Cu	179.5±17.5	0.060±0.003	2.41/53
2760	Pb+Pb	1615±39.0	0.035±0.003	5.50/41
5020	Pb+Pb	1929±47.0	0.032±0.002	33.0/27
5440	Xe+Xe	1167±26.0[41]	0.030±0.003	−/−

$$\epsilon = A\left(\frac{\sqrt{s_{NN}}}{\sqrt{s_0}}\right)^{-B}$$

$$\sqrt{s_0} = 1\ \mathrm{GeV}$$

$$A = 0.045 \text{ and } B = 0.23$$

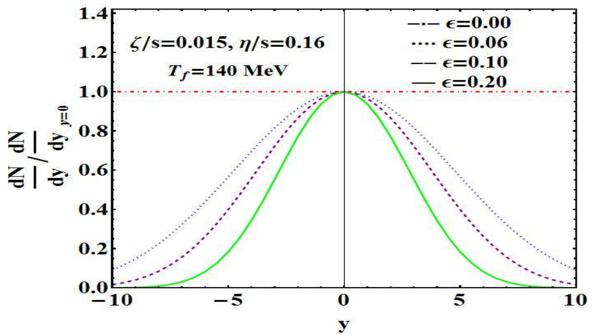
Z.F. Jiang, C.B. Yang, Chi Ding, Xiang-Yu Wu. arXiv: 1808.10287.

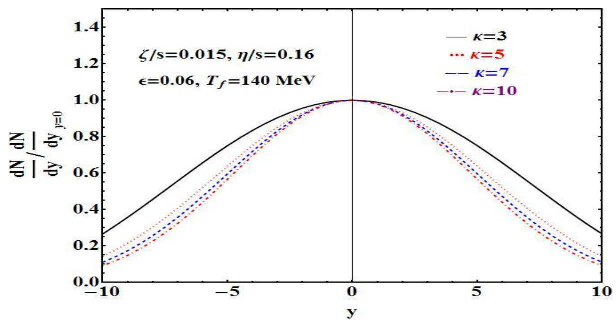
Particle's distribution

- at finial state, the dn/dy and $dn/d\eta$ are effected sensitively by the acceleration parameter.
- This toy model’s prediction for XeXe@5440 GeV works well!
- A simple description of acceleration parameters is obtained.

Particle's distribution (toy model)

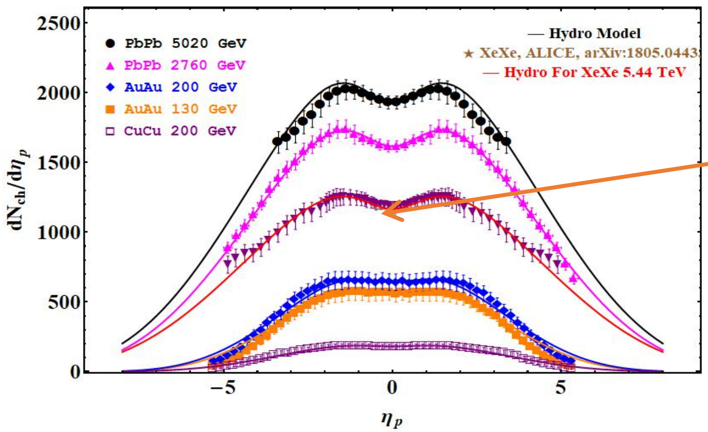
Numerical Results (Rapidity distribution):





Gubser flow, PLB, Hans. Bantilan et al. arXiv:1803.10774

Numerical results (pseudo-rapidity distribution):



Red line: prediction for XeXe @ 5440 GeV.
Data: ALICE@QM2018, 1805.04432.

Z.F. Jiang, et al. arXiv: 1711.10740
Z.F. Jiang, C.B. Yang, Chi Ding, Xiang-Yu Wu. arXiv: 1808.10287.

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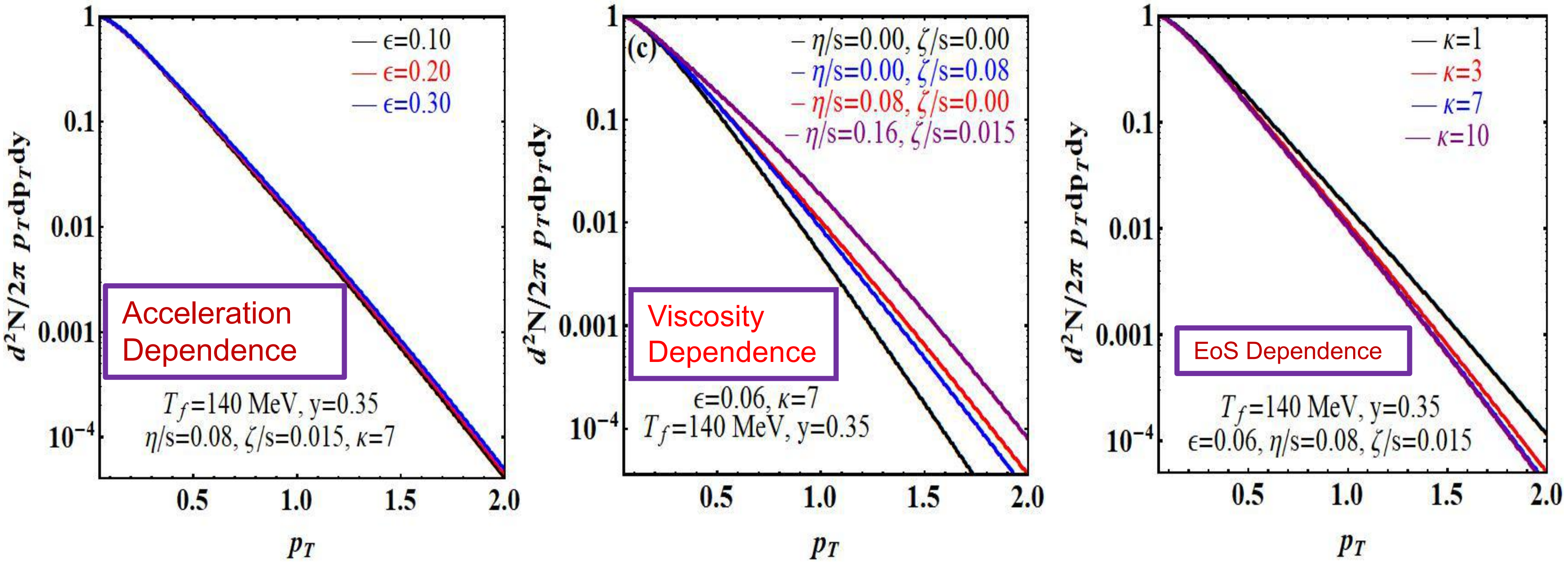
Transverse momentum distribution

Numerical results:



Transverse momentum distribution:
- the longitudinal accelerated effect is little,
- the distribution is sensitive to the EoS,
- the viscous effect play an important role for transverse momentum distribution.

Anisotropic flow ?

An interesting question is, what will happen if we include the longitudinal accelerated effect ?

The anisotropic flow is defined as:

$$v_n(p_T) \equiv \frac{\displaystyle\int_{-\pi}^{\pi} d\phi\, \cos[n(\phi - \Psi_n)]\, \frac{dN}{dy\, p_t\, dp_T\, d\phi}}{\displaystyle\int_{-\pi}^{\pi} d\phi\, \frac{dN}{dy\, p_t\, dp_T\, d\phi}}$$

D. Teaney, 2003, P. R. C 68, 034913. A. Jaiswal et, al, 1508.05878.

Ψ_n is the n-th harmonic event-plane angle.

Preliminary conclusion:

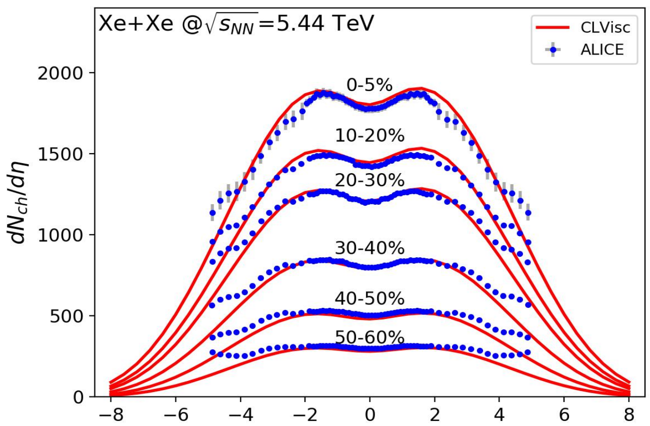
- difficult, because of the new complex velocity field.
- The viscous correction to the distribution function dominates the reduction in v2 at large p_t .
- the longitudinal accelerated effect lead to the increase in v2 at large p_t.

K. Dusling, et al, 2007. K. Dusling, et al, 2009. Sangyong Jeon, et al, 2013.

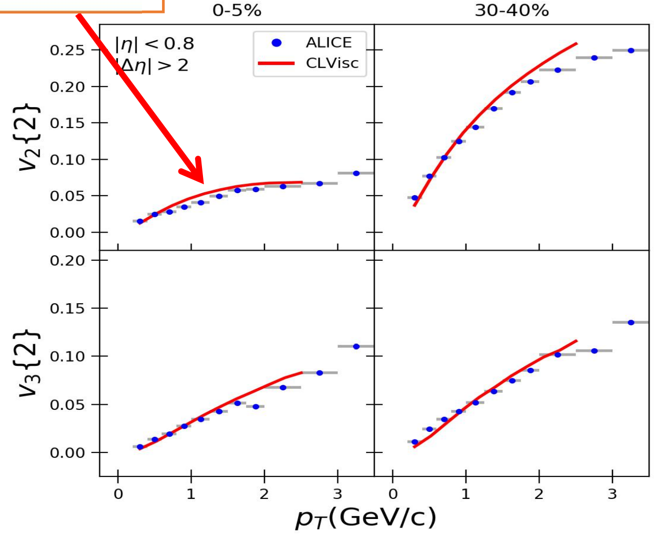
Appear soon ! A testbed for CLVisc.

CLVisc: **v2/v3** for Xe+Xe/Pb+Pb

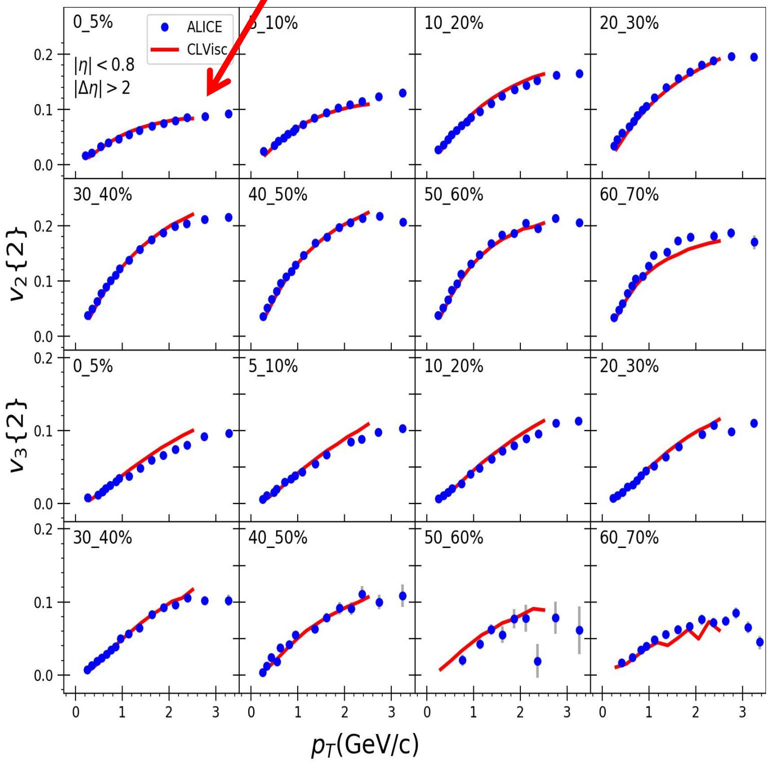
Perliminary results by CLVisc:



Pb+Pb @ 5.02 TeV



Xe+Xe @ 5.44 TeV



initial condition: Trento
EoS: WB 2014
Hydro simulation: 3+1D CLVisc
OpenCL/GPU

Long-Gang Pang, Xin-Nian Wang et.al 1802.04449

Chi Ding, Xiang Yu Wu, Ze Fang Jiang, et al. In preparation

A brief summary and outlook

Summary:
1. A perturbative solution with both longitudinal accelerated flow and viscous correction are obtained.
2. The final state spectrum are obtained, this toy model described well the LHC data.

Outlook:
1. 2rd I-S problem, magnetohydrodynamics (MHD) and CLVisc 3+1D numerical code.
2. Fast parton disturbance evolution on medium background…

Thank you for your attention