R^4 Corrections to Holographic Schwinger Effect

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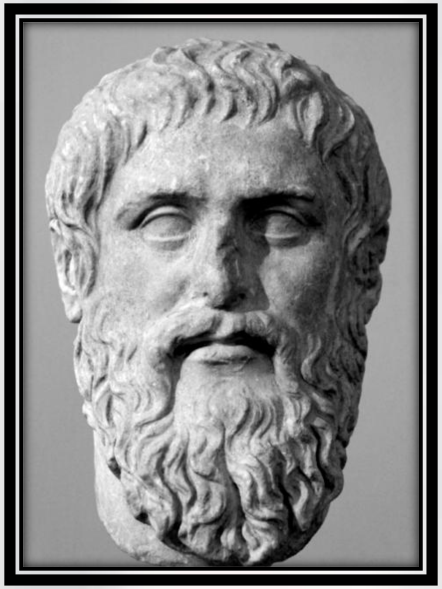
Ⅰ. History

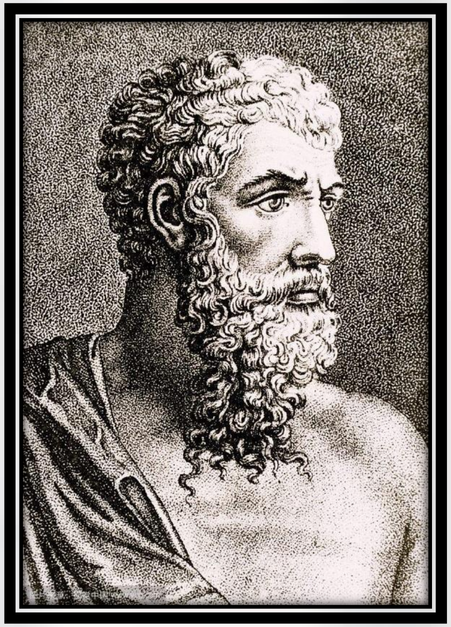


Plato: It was quite literally nothing at all, which cannot rightly be said to exist.

All the debate about Vacuum was on the context of Atomism!

PLATO
柏拉图

Aristole: No void could occur naturally.



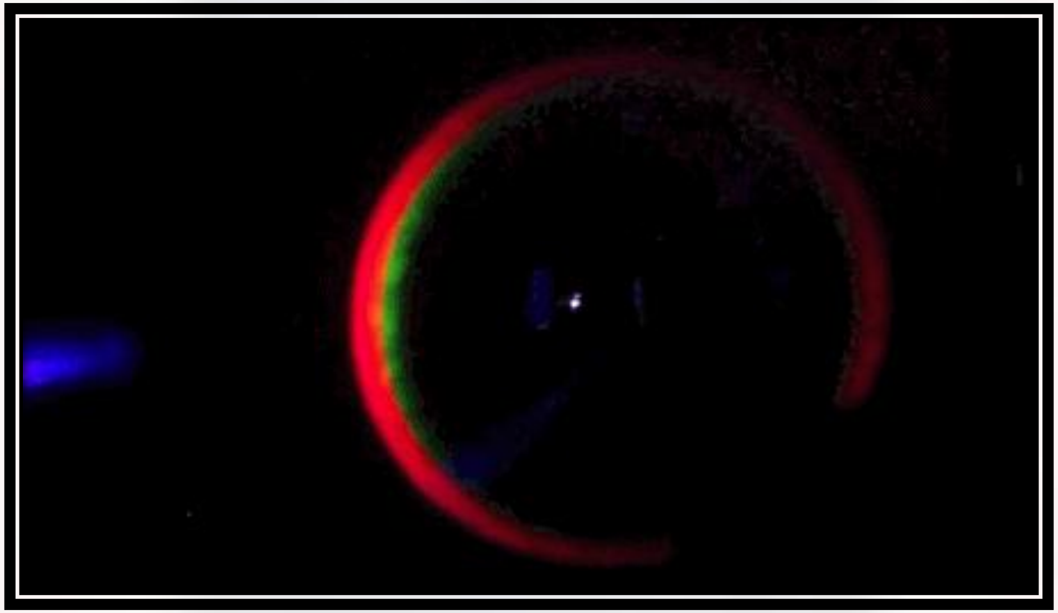
ARISTOLE
亚里士多德

Ⅰ. History

DESCARTES
笛卡尔

Rene Descartes: He proposed a geometrically based alternative theory of atomism, without the problematic nothing–everything dichotomy of void and atom.

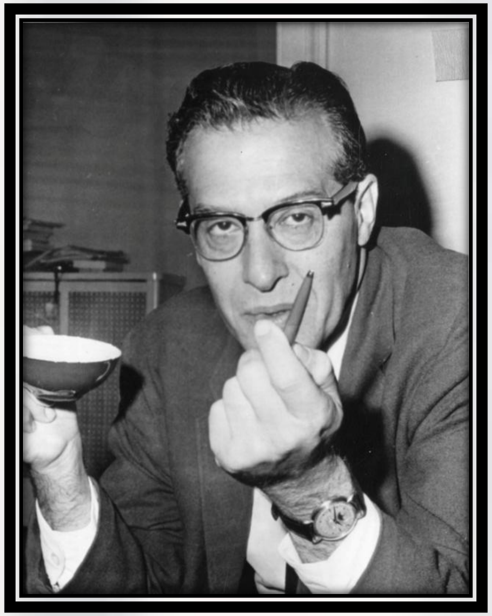
Ⅰ. History



Vacuum fluctuation

In modern particle physics, the vacuum state is considered the ground state of a field.
In QED vacuum, the electric and magnetic fields have zero average values, but their variances are not zero.

Ⅰ. History



Julian Schwinger
施温格

《On Gauge Invariance and Vacuum polarization》
---J. S. Schwinger, PRL,82:664(1951)

The Schwinger effect is a **non-perturbative** phenomenon in QED, which is described that the virtual electron-position pairs can become real particles in a strong electric-field.

The **production rate** of particles:

$$\Gamma \sim \exp\left(-\frac{\pi m^2}{eE}\right)$$

For weak coupling and weak field

Affleck-Alvarez-Manton: $\Gamma \sim \exp\left(-\frac{\pi m^2}{eE}+\frac{e^2}{4}\right)$

---I.K.Affleck, NPB,194:38(1982)

For arbitrary coupling and weak field

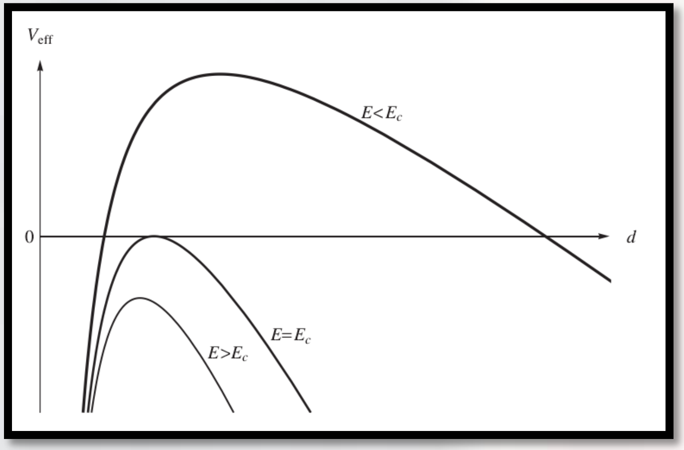
Impossible to calculate the critical field!!

Ⅰ. History

Gordon W. Semenoff

Konstantin Zarembo

AdS/CFT computation: **--- PRL 107 171601(2011)**

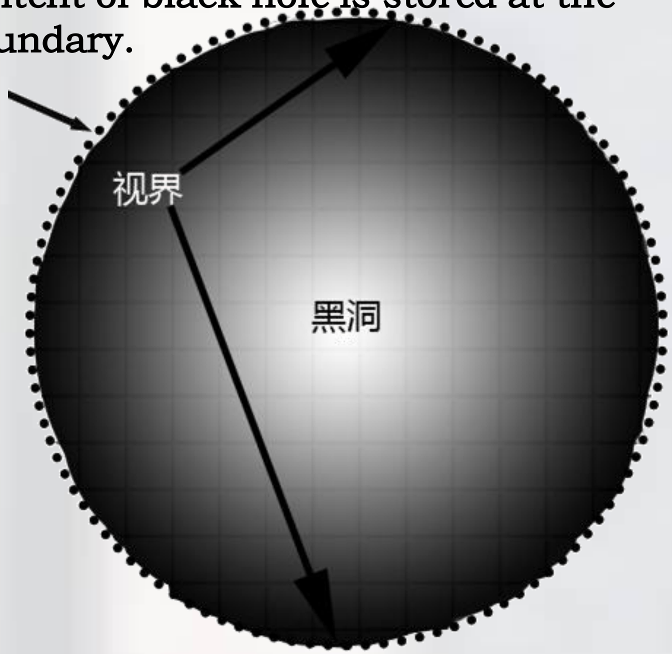


$$\Gamma \sim \exp\left[-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{E_c}{E}}-\sqrt{\frac{E}{E_c}}\right)^2\right] \qquad E_c=\frac{2\pi m^2}{\sqrt{\lambda}}$$

Agreement with the DBI result!

AdS/CFT provides a solution to the problem of strong coupling .

The information which related to the content of black hole is stored at the boundary.



The boundary of Four-dimensional space-time

Holographic Principle:

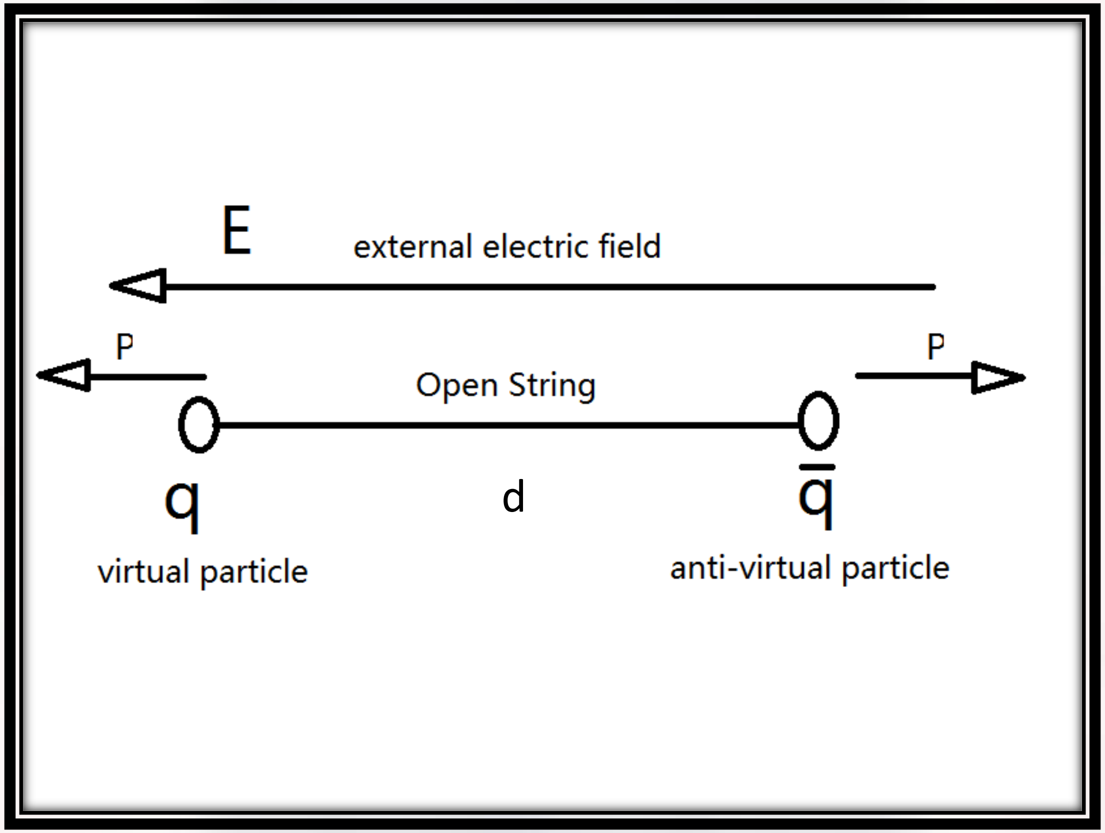
$$\boxed{S_{\max} = \frac{Area}{4G_N}}$$

AdS/CFT:

The gauge field theory on the four-dimensional space - time boundary, corresponding to the superstring theory in five -dimensional of space-time .

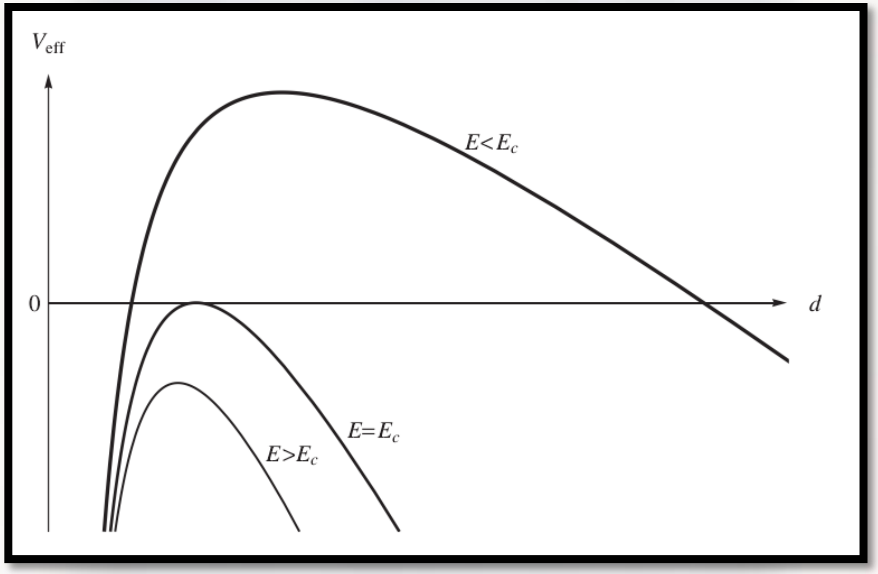
By applying the AdS/CFT, Son[1] obtained a value of η/s which approximately to a certain universal constant, and which was confirmed at the RHIC[2].

[1]PRL,87.081601(2001)/[2]PRC,78,034915(2008)



The electric field pulled the two virtual particles in opposite directions.
The pair gains energy Ed and becomes physical particles when d~2m/E.

I . History



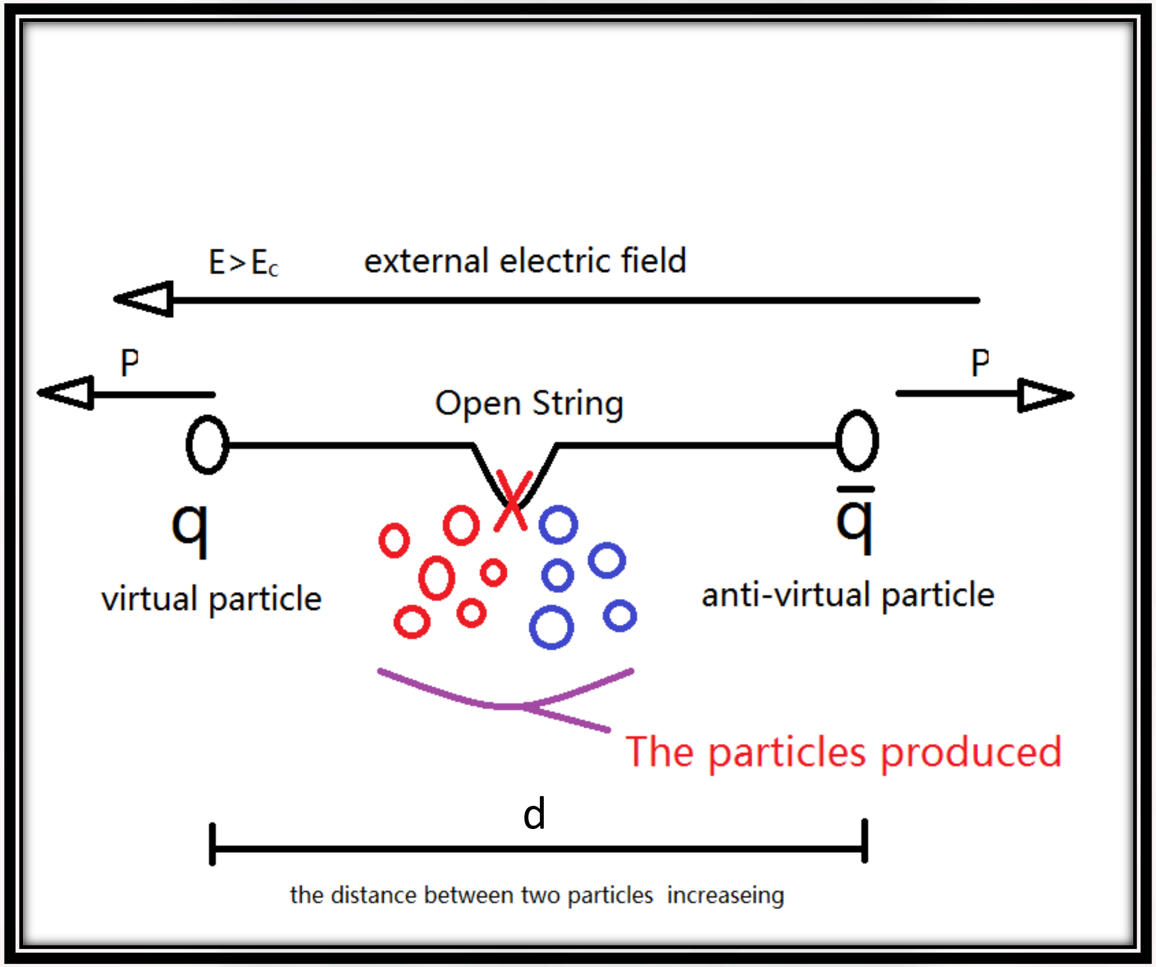
This process is tunneling through a barrier of height ~2m and width ~2m/E.

$$V_{\text{eff}}(d) = 2m - Ed - \alpha/d$$

Coulomb potential, where α contains the electric charge

A virtual particles must gain energy 2m (its combined rest masses) by vacuum fluctuation to become real.

The energy from electric field.


E>Ec
external electric field
P
P
Open String
q
q̄
virtual particle
anti-virtual particle
The particles produced
d
the distance between two particles increaseing

Ⅱ. Motivation

●In the past, when using AdS/CFT to study the Schwinger effect, the high order term was sacrificed to Einstein field equation, and in order to make the theory more consistent with the actual physical situation, the R^4 correction should be considered.(From theory) [Nucl.Phy.B 534 202]

$$S=-\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{1}{4\cdot 5!}(F_5)^2+\ldots+\frac{\zeta(3)}{8}\lambda^{-\frac{3}{2}}e^{-\frac{3}{2}\phi}\cdot\frac{1}{3\cdot 2^8}(t_8\cdot t_8 RRRR-\frac{1}{4}\varepsilon_8\varepsilon_8 RRRR)+\ldots\right]$$

●The typical values of λ in the experiment ,which have a range (5.5,6π) (From experiment)

●Since Semenoff's production rate formula contains the same coupling constant λ as our string correction, we can verity whether it's consistent with the anticipation of the expression of production rate.

$$\Gamma\sim\exp\left[-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{E_c}{E}}-\sqrt{\frac{E}{E_c}}\right)^2\right]$$

Ⅲ. R⁴ Corrections

The α′-corrected metric:

$$ds^2 = G_{tt}dt^2 + G_{xx}dx_i^2 + G_{rr}dr^2$$

where

$$G_{tt} = -r^2(1-w^{-4})T(w),$$
$$G_{xx} = r^2 X(w),$$
$$G_{rr} = r^{-2}(1-w^{-4})R(w),$$

$$T(w) = 1 - k\left(75w^{-4} + \frac{1225}{16}w^{-8} - \frac{695}{16}w^{-12}\right) + \ldots,$$
$$X(w) = 1 - \frac{25k}{16}w^{-8}(1+w^{-4}) + \ldots,$$
$$R(w) = 1 + k\left(75w^{-4} + \frac{1175}{16}w^{-8} - \frac{4585}{16}w^{-12}\right) + \ldots$$

with

$$w = \frac{r}{r_h}$$

$$k = \frac{\zeta(3)}{8}\lambda^{-3/2} \approx 0.15\lambda^{-3/2}$$

Ⅳ. Potential Analysis

The Nambu-Goto（NG）string action：

$$\mathbf{S} = T_F \int d\tau \int d\sigma \mathcal{L} = T_F \int d\tau \int d\sigma \sqrt{\det G_{ab}} \qquad G_{ab} \equiv \frac{\partial x_\mu}{\partial \sigma_a} \frac{\partial x_\nu}{\partial \sigma_b} g_{\mu\nu}$$

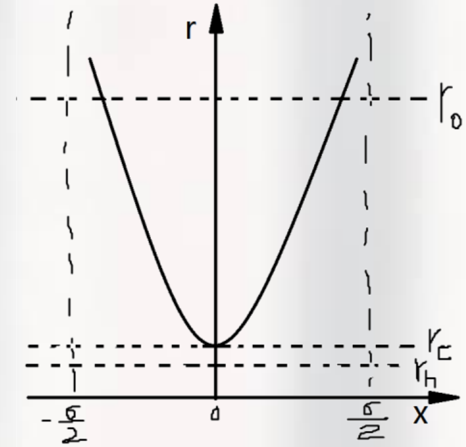
Using the static gauge： $x_0 = \tau \quad x_1 = \sigma \quad x_2 = x_3 = const. \quad \sigma_a = (\tau, \sigma)$

$$ds^2 = -\frac{r^2}{R^2}(1-w^{-4})T(w)d\tau^2 + \frac{r^2}{R^2}X(w)d\sigma^2 + \frac{R^2 R(w)}{r^2(1-w^{-4})}\left(\frac{dr}{d\sigma}\right)^2 d\sigma^2 + L^2 d\Omega_5^2$$

$$= -\frac{r^2}{R^2}(1-w^{-4})T(w)d\tau^2 + \left[\frac{r^2}{R^2}X(w) + \frac{R^2 R(w)}{r^2(1-w^{-4})}\dot{r}^2\right]d\sigma^2 + L^2 d\Omega_5^2$$

$$\boxed{\mathcal{L} = \sqrt{\det G_{ab}}} \longrightarrow \boxed{\mathcal{H} = \mathcal{L} - \dot{r}\frac{\partial \mathcal{L}}{\partial \dot{r}}}$$

From the Hamiltonian conservation, we obtained the derivation of r for $\sigma(x_1)$, Distance x of test particles： $y \equiv \frac{r}{r_c}, \quad a \equiv \frac{r_c}{r_0}, \quad b \equiv \frac{r_h}{r_0}.$



$$\boxed{x = \frac{2R^2}{a r_0}\int_1^{\frac{1}{a}} dy \frac{1}{y^2\sqrt{1-w^{-4}}}\sqrt{\frac{R(w)}{X(w)}}\frac{1}{\sqrt{\frac{y^4(1-w^{-4})T(w)X(w)}{(1-w_c^{-4})T(w_c)X(w_c)}-1}},}$$

The sum of the Coulomb potential(CP) and static energy(SE)：

$$\boxed{V_{CP+SE} = 2T_F\int_0^{x/2} d\sigma \mathcal{L} = 2T_F r_0 a\int_1^{\frac{1}{a}} dy\sqrt{T(w)R(w)}\frac{1}{\sqrt{1-\frac{(1-w_c^{-4})T(w_c)X(w_c)}{y^4(1-w^{-4})T(w)X(w)}}}}$$

The total potential：

$$\boxed{V_{tot} = V_{CP+SE} - Ex = V_{CP+SE} - \alpha E_c x}$$

$$\boxed{\alpha \equiv \frac{E}{E_c}}$$

The Dirac Born Infeld (DBI) action :

$$S_{DBI} = -T_{D3}\int d^4x\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu})} \qquad T_{D3}=\frac{1}{g_s(2\pi)^3\alpha'^2}$$

where:

$$G_{\mu\nu}+\mathcal{F}_{\mu\nu}=\begin{pmatrix} -\frac{r^2}{R^2}(1-w^{-4})T(w) & 2\pi\alpha' E_1 & 2\pi\alpha' E_2 & 2\pi\alpha' E_3 \\ -2\pi\alpha' E_1 & \frac{r^2}{R^2}X(w) & 0 & 0 \\ -2\pi\alpha' E_2 & 0 & \frac{r^2}{R^2}X(w) & 0 \\ -2\pi\alpha' E_3 & 0 & 0 & \frac{r^2}{R^2}X(w) \end{pmatrix}$$

The critical electric field

$$(1-w_0^{-4})T(w_0)X(w_0)-\frac{(2\pi\alpha')^2R^4}{r_0^4}E^2\geq 0 \longrightarrow E_c = T_F\frac{r_0^2}{R^2}\sqrt{(1-b^4)T(w_0)X(w_0)}.$$

Total Potential :

$$\boxed{\begin{aligned} V_{tot} &= V_{CP+SE}-Ex \\ &= 2T_F r_0\left[\begin{aligned} & a\int_1^{\frac{1}{a}}\sqrt{T(w)R(w)}\frac{dy}{\sqrt{1-\frac{(1-w_c^{-4})T(w_c)X(w_c)}{y^4(1-w^{-4})T(w)X(w)}}} - \\ & \frac{\alpha}{a}\sqrt{(1-b^4)T(w_0)X(w_0)}\times\int_1^{\frac{1}{a}}\frac{dy}{y^2\sqrt{1-w^{-4}}}\sqrt{\frac{R(w)}{X(w)}}\frac{1}{\sqrt{\frac{y^4(1-w^{-4})T(w)X(w)}{(1-w_c^{-4})T(w_c)X(w_c)}-1}}\end{aligned}\right].\end{aligned}}$$

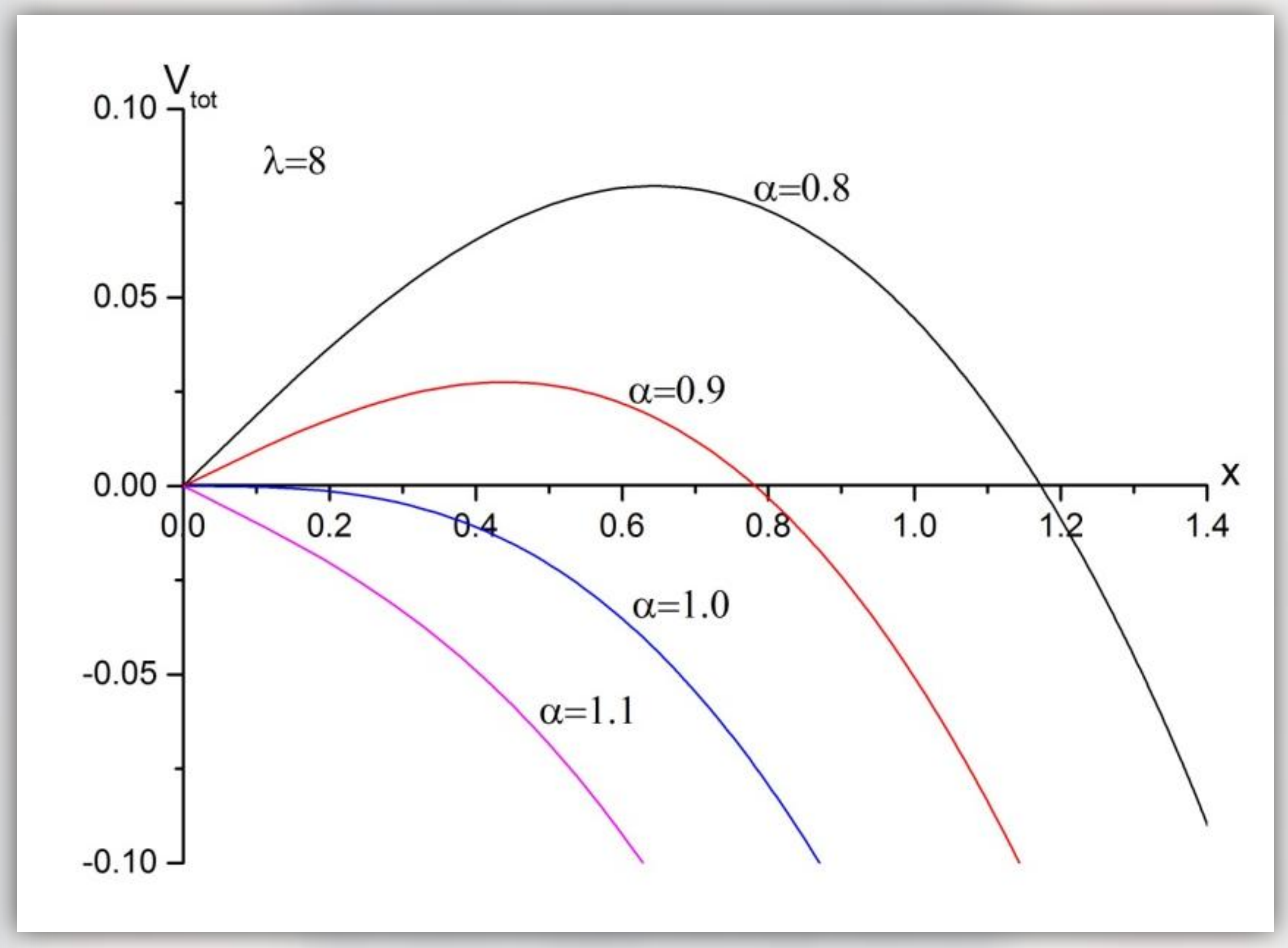


FIG.1:V_{tot} against x

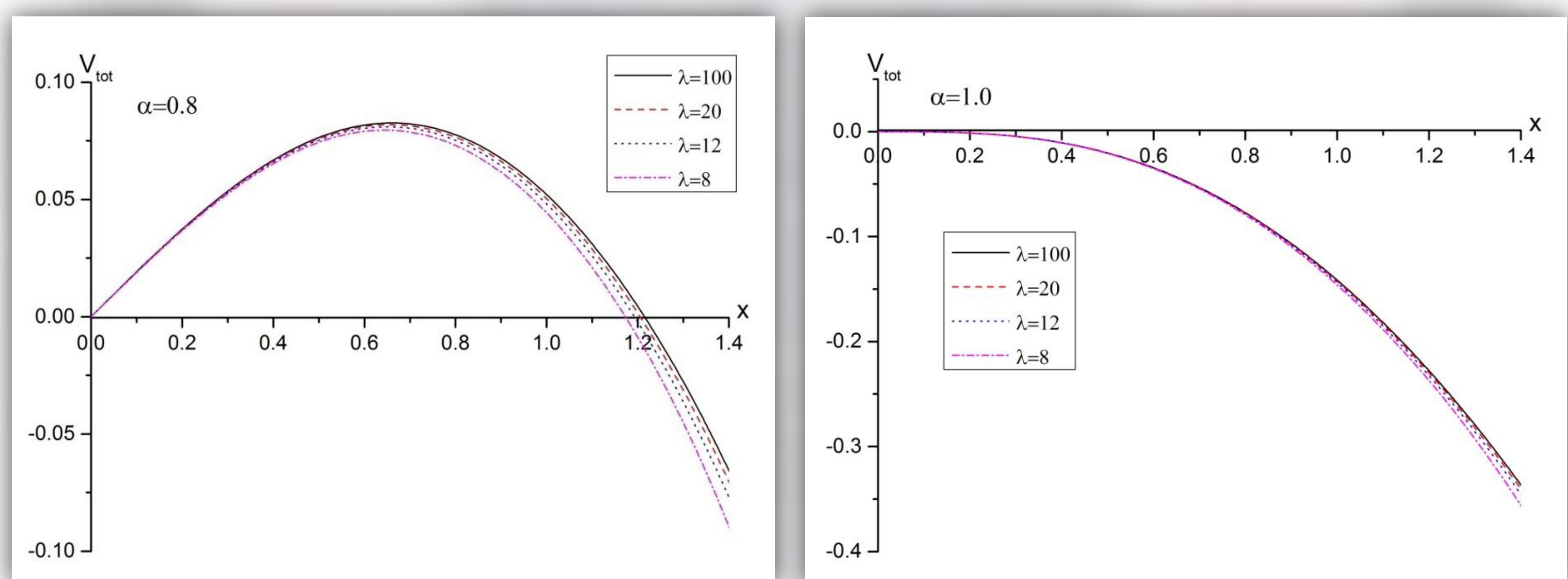
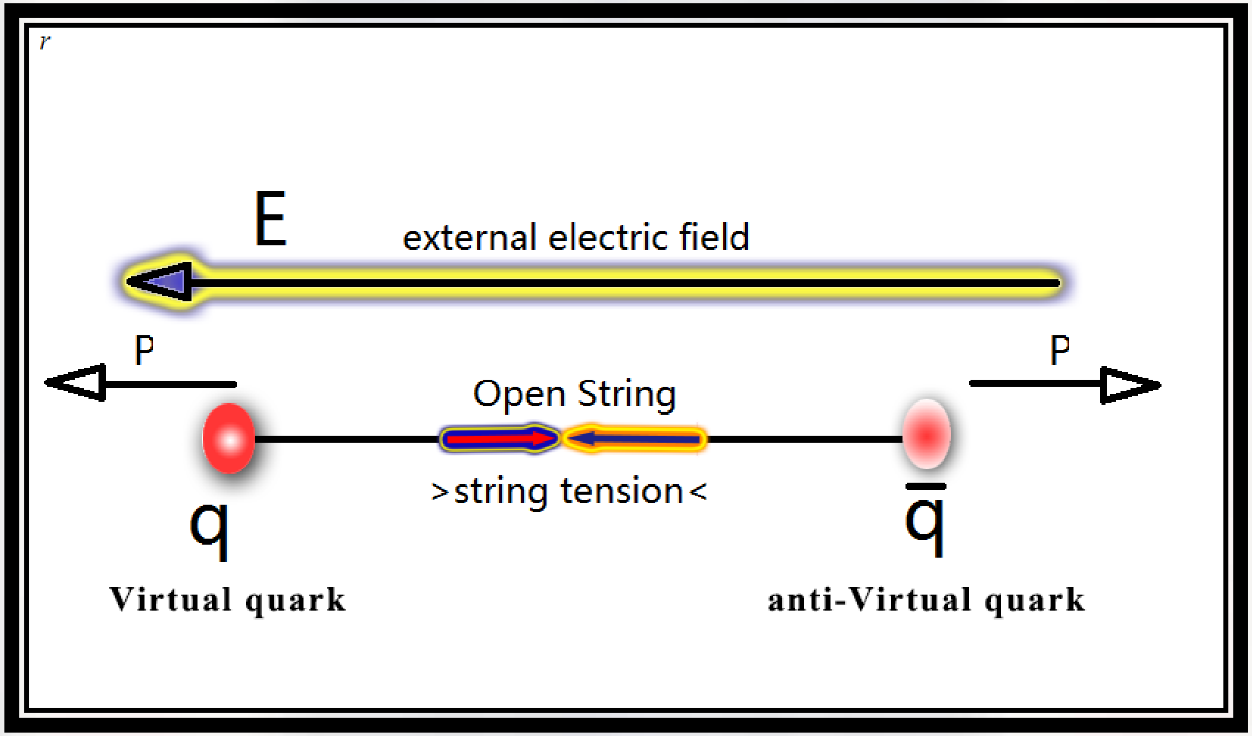


FIG.2: V_{tot} against x in presence of R^4 corrections with different λ.

Ⅴ. Confining D3-brane background



$$V_{tot}(x) = 2m \quad + \sigma_{st} x \quad - Ex$$

Static energy

String tension

The energy from electric field.

Ⅴ. Confining D3-brane background

The induce metric on confining D3-brane background:

$$\boxed{ds^2 = -\frac{r^2 T(w) dt^2}{R^2} + \frac{r^2 X(w)}{R^2}\left[(dx^1)^2 + (dx^2)^2 + (1-w^{-4})(dx^3)^2\right] + \frac{R^2 R(w) dr^2}{(1-w^{-4}) r^2}}$$

where

$$T(w) = 1 - k\left(75 w^{-4} + \frac{1225}{16} w^{-8} - \frac{695}{16} w^{-12}\right) + \ldots,$$

$$X(w) = 1 - \frac{25k}{16} w^{-8}(1 + w^{-4}) + \ldots,$$

$$R(w) = 1 + k\left(75 w^{-4} + \frac{1175}{16} w^{-8} - \frac{4585}{16} w^{-12}\right) + \ldots$$

with

$$\boxed{w = \frac{r}{r_h}}$$

$$\boxed{(1 - w^{-4}) = 1 - \left(\frac{r_h}{r}\right)^4}$$

$$\boxed{k = \frac{\zeta(3)}{8}\lambda^{-3/2} \approx 0.15\lambda^{-3/2}}$$

Ⅴ. Confining D3-brane background

Distance x of test particles：

$$x=\frac{2R^2}{ar_0}\int_1^{\frac{1}{a}}dy\sqrt{\frac{R(w)}{X(w)\left(1-w^{-4}\right)}}\frac{1}{y^2\sqrt{\frac{y^4T(w)X(w)}{T(w_c)X(w_c)}-1}},$$

The sum of the Coulomb potential(CP) and static energy(SE)：

$$V_{CP+SE}=2T_Far_0\int_1^{\frac{1}{a}}\sqrt{\frac{T(w)R(w)}{(1-w^{-4})}}\frac{y^2dy}{\sqrt{y^4-\frac{T(w_c)X(w_c)}{T(w)X(w)}}}$$

The total potential：

$$V_{tot}=V_{CP+SE}-Ex=V_{CP+SE}-\alpha E_cx$$

$$\alpha\equiv\frac{E}{E_c}$$

The Dirac Born Infeld (DBI) action：

$$S_{DBI} = -T_{D3}\int d^4x\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu})} \qquad T_{D3}=\frac{1}{g_s(2\pi)^3\alpha'^2}$$

where：

$$G_{\mu\nu}+\mathcal{F}_{\mu\nu}=\begin{pmatrix} -\frac{r^2}{R^2}T(w) & 2\pi\alpha'E & 0 & 0 \\ -2\pi\alpha'E & \frac{r^2}{R^2}X(w) & 0 & 0 \\ 0 & 0 & \frac{r^2}{R^2}X(w) & 0 \\ 0 & 0 & 0 & \frac{r^2(1-w^{-4})}{R^2}X(w) \end{pmatrix}$$

The critical electric field

$$E_c = T_F\frac{r_0^2}{R^2}\sqrt{T(w_0)X(w_0)}.$$

Total Potential：

$$V_{tot}=V_{CP+SE}-Ex$$
$$=2T_F r_0\left[a\int_1^{\frac{1}{a}}\sqrt{\frac{T(w)R(w)}{(1-w^{-4})}}\frac{y^2dy}{\sqrt{y^4-\frac{T(w_c)X(w_c)}{T(w)X(w)}}} - \frac{\alpha}{a}\sqrt{T(w_0)X(w_0)}\int_1^{\frac{1}{a}}\sqrt{\frac{R(w)}{X(w)(1-w^{-4})}}\frac{dy}{y^2\sqrt{\frac{y^4T(w)X(w)}{T(w_c)X(w_c)}-1}}\right].$$

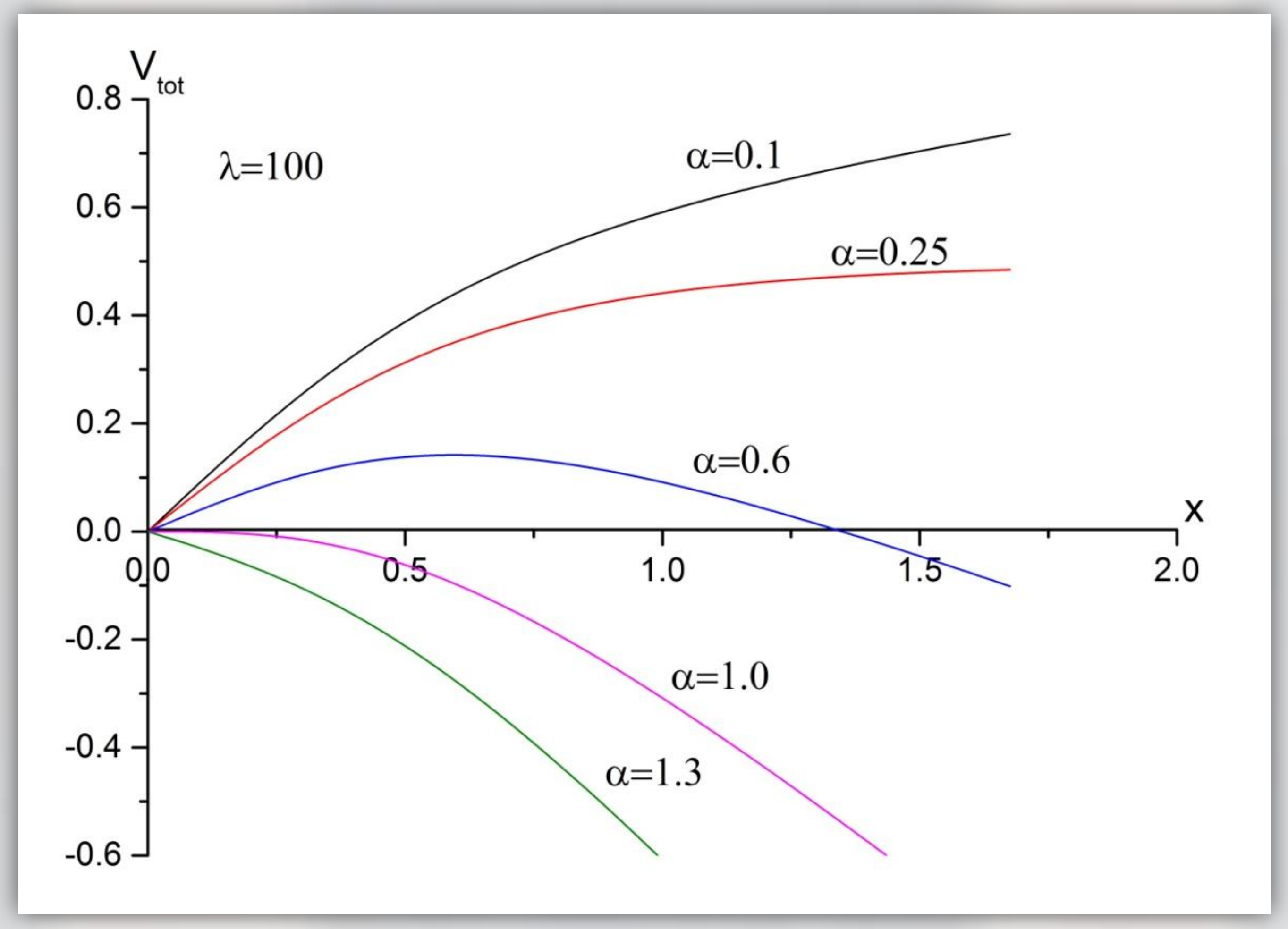
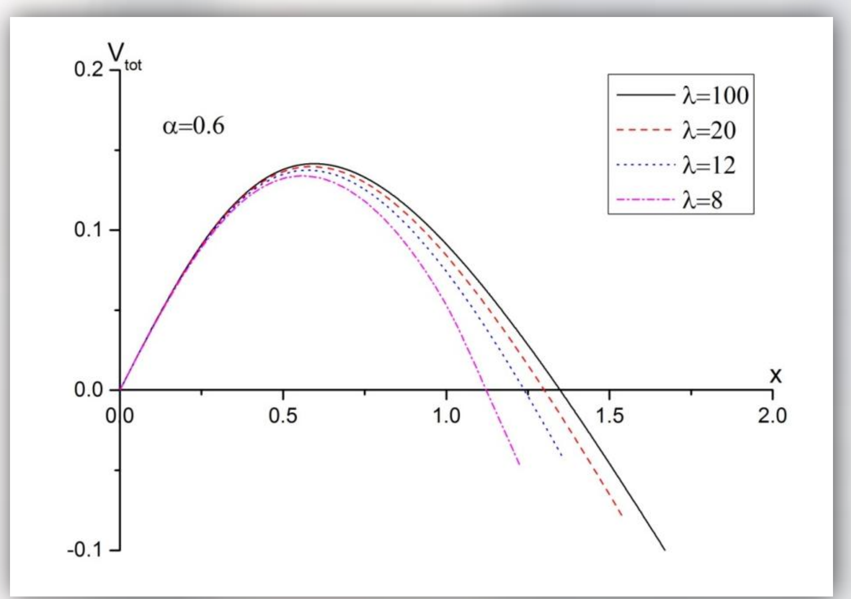


FIG.3: V_{tot} against x on the Confining D3-brane background



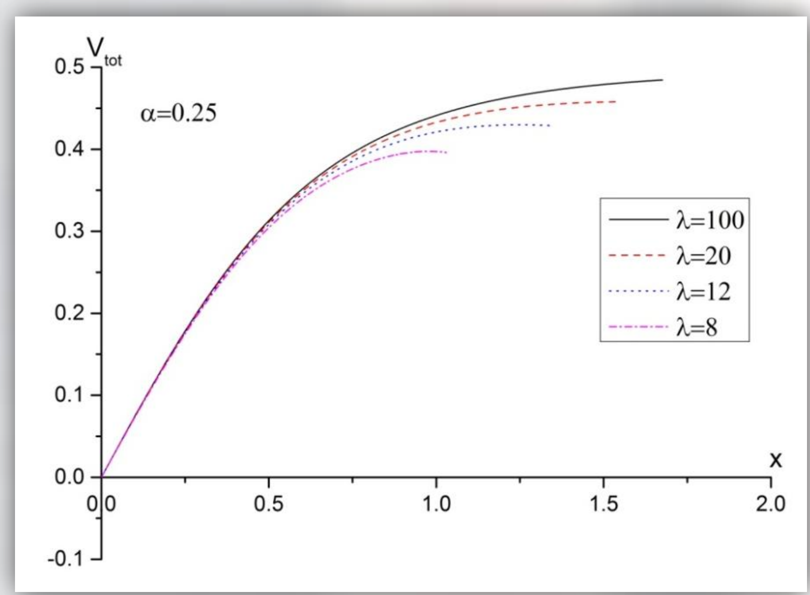


FIG.4:V_{tot} against x on the Confining D3-brane background with different λ.

* When α=0.25,E= E_s= σ_{st} , the electric force equals to string tension.

Ⅵ. Summary

- AdS/CFT provides a efficient tool to the problem of strong coupling .
- Decreasing λ tends to increasing the production rate ,in other words , decreasing λ tends to increasing the Schwinger effect.
- There is exists two critical electric fields. The behavior of parameter λ to the Schwinger effect is similar to the previous analysis in confining D3-brane backgroud.
- By comparing the behavior of λ in η/s and Schwinger effect, one can conclude that the Schwinger effect is enhanced whicle the η/s decrease at strong coupling.

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}}\right)}$$

中国地质大学

THANKS
QUESTION ARE
WELCOME！