Principal Component Analysis: Why do we use fourier transformation to analyze flow?

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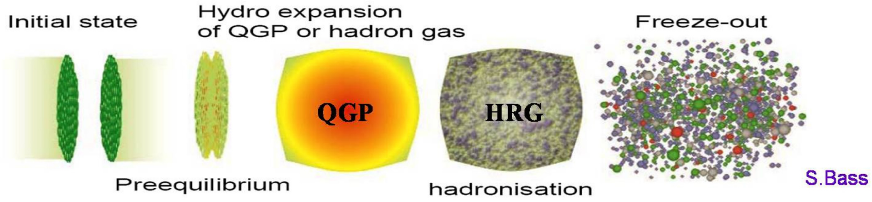
Overview

Simple Review for Flow



Integrated flow is decomposed under Fourier bases:

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} = \frac{1}{2\pi} \sum_{-\infty}^{\infty} \vec{V}_n e^{-in\varphi} = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n\, e^{-in(\varphi - \Psi_n)}\right) \tag{1}$$

- $\vec{V}_n = v_n e^{in\Psi_n}$: n-th order flow-vector
- $v_n = \langle \cos n(\varphi - \Psi_n) \rangle$: n-th flow harmonics
- Ψ_n : corresponding event plane angle

What makes a good flow observable?

Fourier Transformation?

You are right. But, approximately.

Phys.Rev. C64 (2001) 054901
Z.Phys. C70 (1996) 665-672

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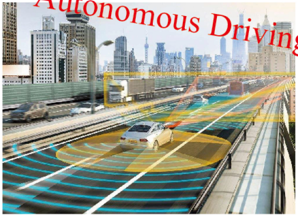
Q: How to find good bases to decompose particle distribution?

Overview

PCA belongs to Machine Learning

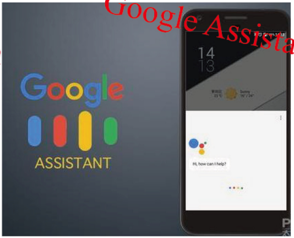
AlphaGo

Autonomous Driving



Machine Translation

Google Assistant

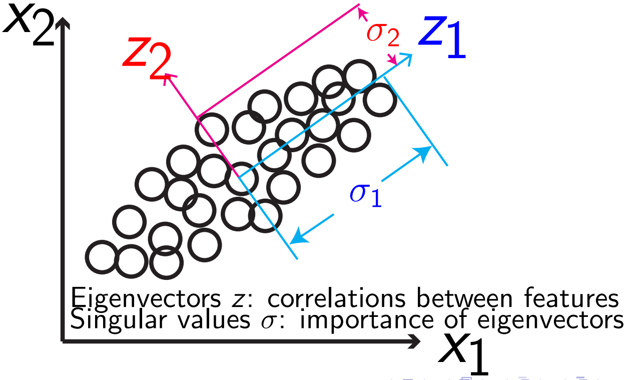


One minute for PCA

PCA transform a set of correlated variables to uncorrelated ones via an orthogonal transformation:

$$X = U\Sigma Z$$

U, Z: orthogonal matrices; Σ: Diagonal matrix.
X: Original variables; Z: transformed variables.



Motivation : Face detection with PCA

Figure: Dataset:different faces

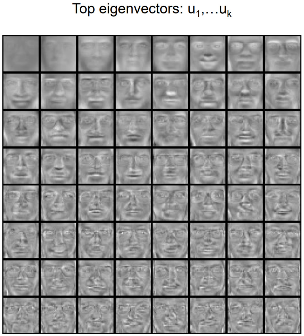
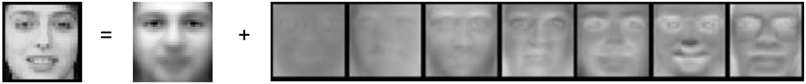


Figure: Eigenfaces

Eigenfaces show interesting correlations:

- More beard/mustache$\rightarrow$ man$\rightarrow$ tanned face
- Round face$\rightarrow$ baby$\rightarrow$ less wrinkle

- Each face is decomposed into superposition of eigenfaces.



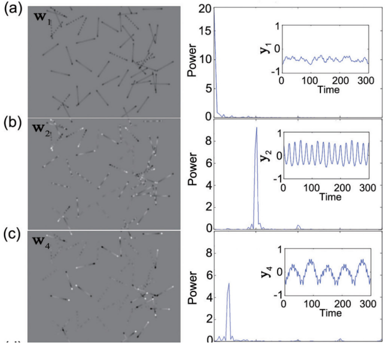
$$\hat{x} = \mu + w_1u_1 + w_2u_2 + w_3u_3 + w_4u_4 + \dots$$

- Each face can be expressed by number of faces far less than pixels of the original image. Correlations play a huge role!

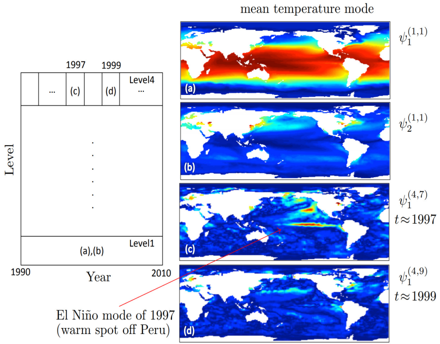
Overview

Classical Mechanics and Atmospheric Sciences

- eigenfrequencies in particle motion



- Multi-resolution PCA to discover El Nino.



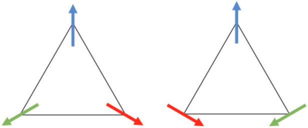
H. Y. Chen, Raphal Ligeois, John R. de Bruyn, and Andrea Soddu Phys. Rev. E 91, 042308 Published 15 April 2015

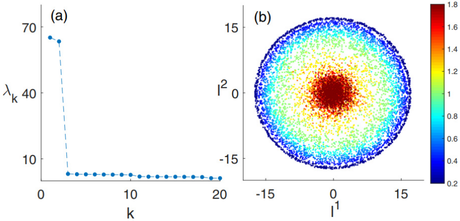
https://arxiv.org/pdf/1506.00564.pdf

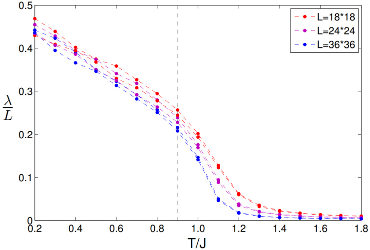
Machine learning helps discover

- Correlations between spin configurations
- Phase transition

$$\mathcal{H} = J \sum_{\langle i j \rangle} \cos(\theta_i - \theta_j)$$



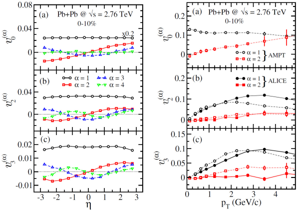




C Wang, H Zhai - Physical Review B,**96(2017)**,14,144432

Flow in Heavy Ion Collisions

- subleading modes of factorization breaking



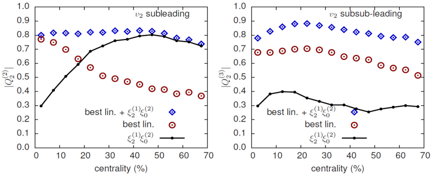
Aleksas Mazeliauskas, Derek Teaney Phys.Rev.**C93** (2016) no.2, 024913

- Nonlinear response coefficients

Piotr Bozek, Phys.Rev. **C97** (2018) no.3, 034905

- Best linear descriptor

$$\zeta^{(a)}_{n,pred} = \varepsilon_{n,n} + c_1 \varepsilon_{n,n+2}$$



Rajeev S. Bhalerao, Jean-Yves Ollitrault, Subrata Pal, Derek Teaney Phys.Rev.Lett. **114** (2015) no.15, 152301

- Experimental data

CMS collaboration, Phys.Rev. **C96** (2017) no.6, 064902

Overview

Previous work and our approach

Previous work[1] utilizes Fourier Transformation in the ϕ direction:

$$\frac{dN}{dp} = \sum_{n=-\infty}^{+\infty} V_n(p) e^{in\phi} \quad p = (p_t, \eta)$$

PCA decomposes $V_n(p)$ into eigenmodes:

$$V_n(p) = \sum_{\alpha=1}^{k} \xi^{(\alpha)} V_n^{(\alpha)}(p)$$

However, we apply PCA directly to $dN/d\phi$ data without FT:

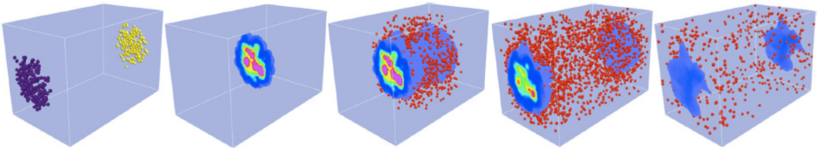
$$\frac{dN}{d\phi} = \sum_{\alpha=1}^{k} \xi^{(\alpha)} (\frac{dN}{d\phi})^{(\alpha)}$$

[1] Rajeev S. Bhalerao, Jean-Yves Ollitrault, Subrata Pal, Derek Teaney Phys.Rev.Lett. **114** (2015) no.15, 152301

Simulations

Pb+Pb collisions at 2.76 A TeV

Trento initial model → Vishnew Hydrodynamics → iss particle sampling



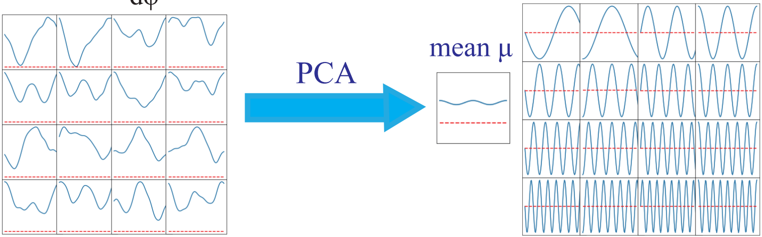
No hadron rescattering or resonance decays to simplify problem settings.

Our approach

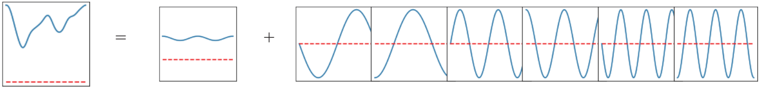
PCA for flow analysis

Data sets: $\dfrac{dN}{d\varphi}$

top eigenvectors: $\sigma_1, \sigma_2, \sigma_3 \ldots\ldots$



With PCA, each flow distribution is decomposed into superposition of eigenmodes.

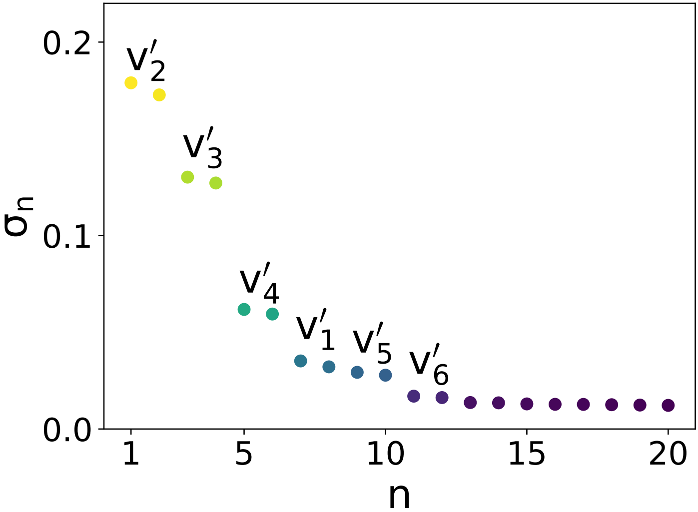


$$\frac{dN}{d\varphi} = \mu + x_1 z_1 + x_2 z_2 + x_3 z_3 + \ldots\ldots$$

Overview

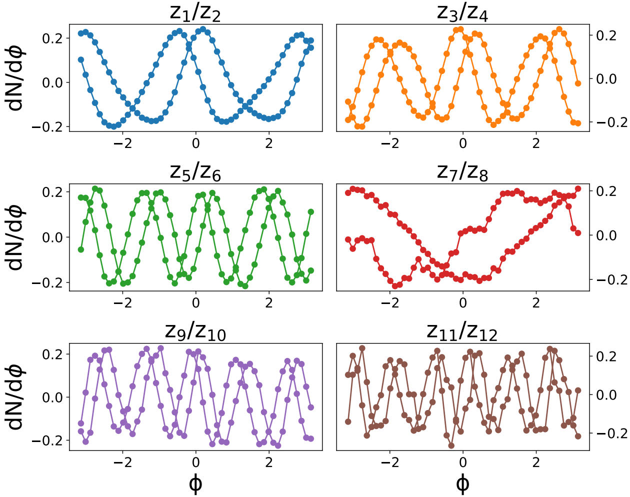
Singular values σ

Singular values σ pairwise matched



Eigenvectors z

Eigenvectors look similar to $\sin(n\phi)$ and $\cos(n\phi)$.



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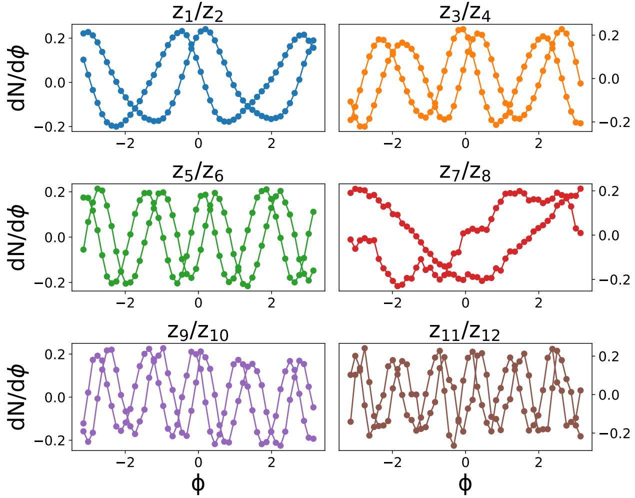


Machines automatically discover fourier transformation for flow!

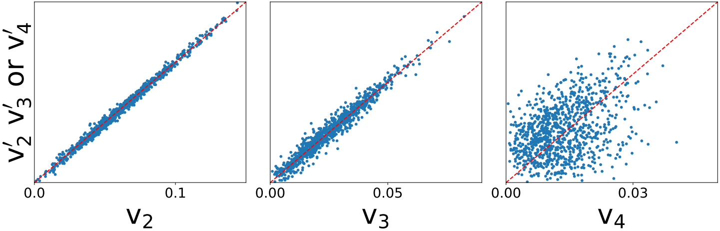
Defining new flow observables v'_n

z_k: k-th (normalized) eigenvector
x_k: amplitude of z_k.

$$\frac{dN}{d\phi} = \mu + \sum_{i=1}^{k} x_k z_k$$

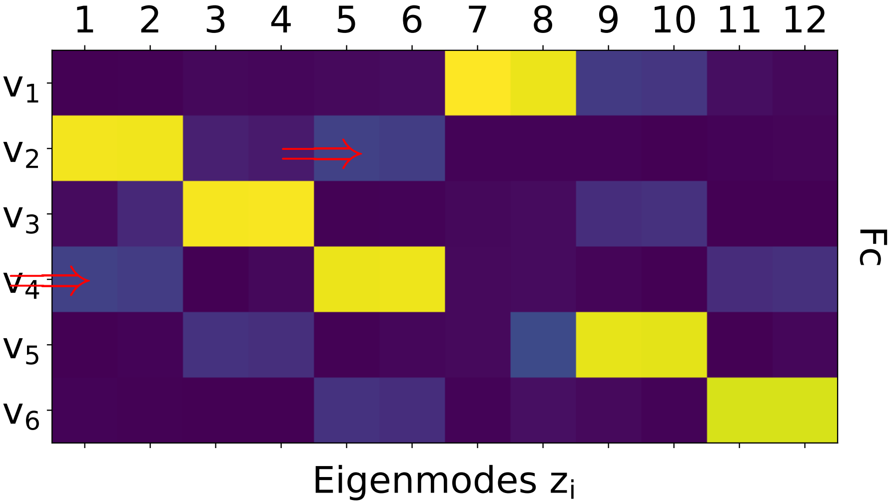
n	v'_n	$\overline{v'_n} \times 10^2$	$\overline{v_n} \times 10^2$
2	$\sqrt{\frac{m}{2}}\sqrt{x_1^2 + x_2^2}$	6.03	6.08
3	$\sqrt{\frac{m}{2}}\sqrt{x_3^2 + x_4^2}$	2.57	2.53
4	$\sqrt{\frac{m}{2}}\sqrt{x_5^2 + x_6^2}$	1.21	1.25
5	$\sqrt{\frac{m}{2}}\sqrt{x_9^2 + x_{10}^2}$	0.57	0.66
6	$\sqrt{\frac{m}{2}}\sqrt{x_{11}^2 + x_{12}^2}$	0.26	0.37

- $v_2^{'}$ fits really well with v_2, and $v_3^{'}$ fits really well with v_3.
- $v_4^{'}$ is deviated from v_4.



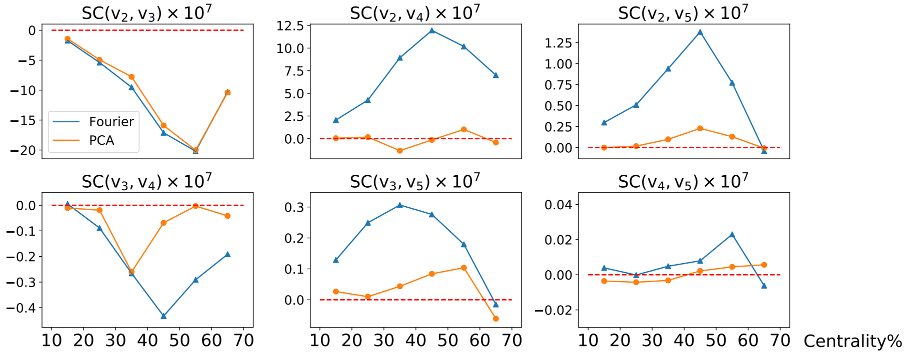
FC of eigenvectors

- z_1/z_2 contain $\sin(4\phi)$ and $\cos(4\phi)$ bases as well.

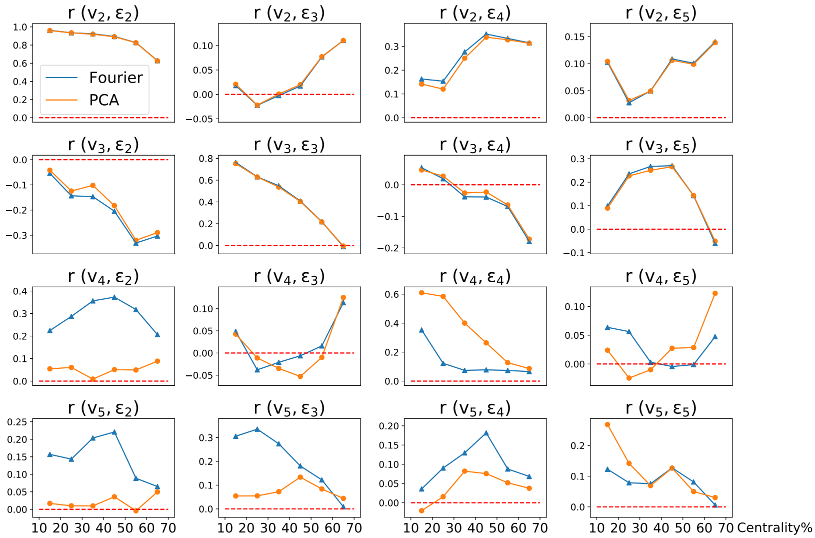


$\mathrm{SC}(v_m, v_n)$

$$\mathrm{SC}(v_m, v_n) = \langle v_m^2 v_n^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$$

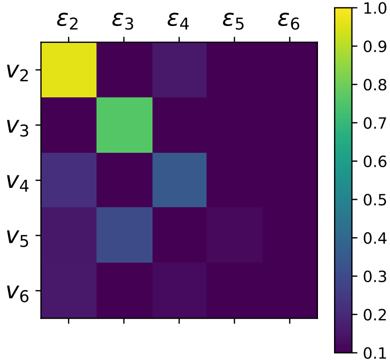


Pearson Coefficient: $r(v_m, \varepsilon_n)$

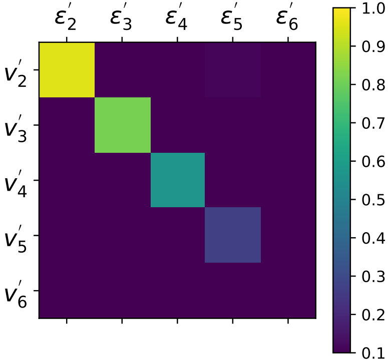


PCA correlators has a more diagonal pattern.

Fourier:



PCA:



Overview

Conclusions

- PCA helps visualize data.
- PCA automatically discovers flow observables.
- PCA provides a new perspective that relates better to initial profile.

Prospectives

- PCA helps reveal structure of data with its strong power of visualization.
- PCA aids in designing observables in complicated systems.
- Carefully applying PCA to real experimental data in relativistic heavy-ion experiments.

Thanks!