

Test Naturalness sum rule (maximal symmetry)?

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C. Csaki, F. Ferreira De Freitas, L. Huang, T. Ma, M.
Perelstein, J. Shu., arxiv: 1811.01961

简介

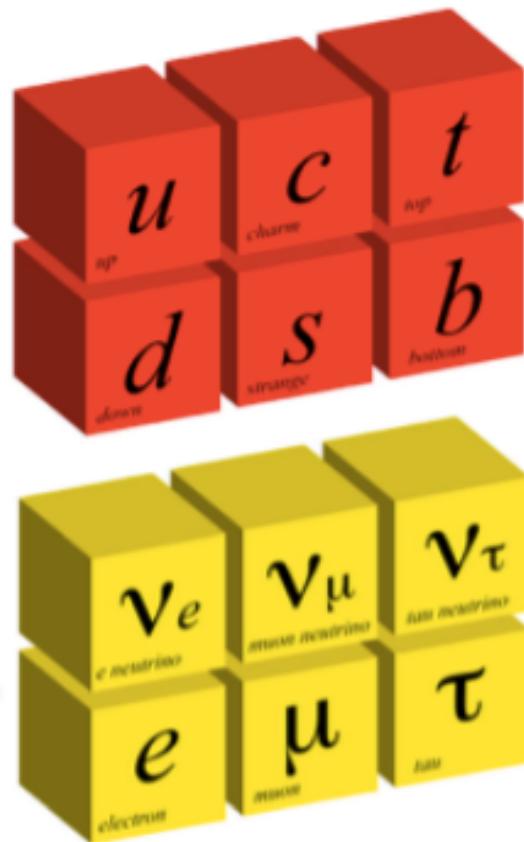
- 电弱对称性破缺? 自然性问题?
- Non-SUSY情况的自然性sum rule (可以推广到SUSY)。
- 如何在未来对撞机上探测相关的问题。
- 展望



Naturalness

我们已知的“旧”物理

Quarks
Leptons



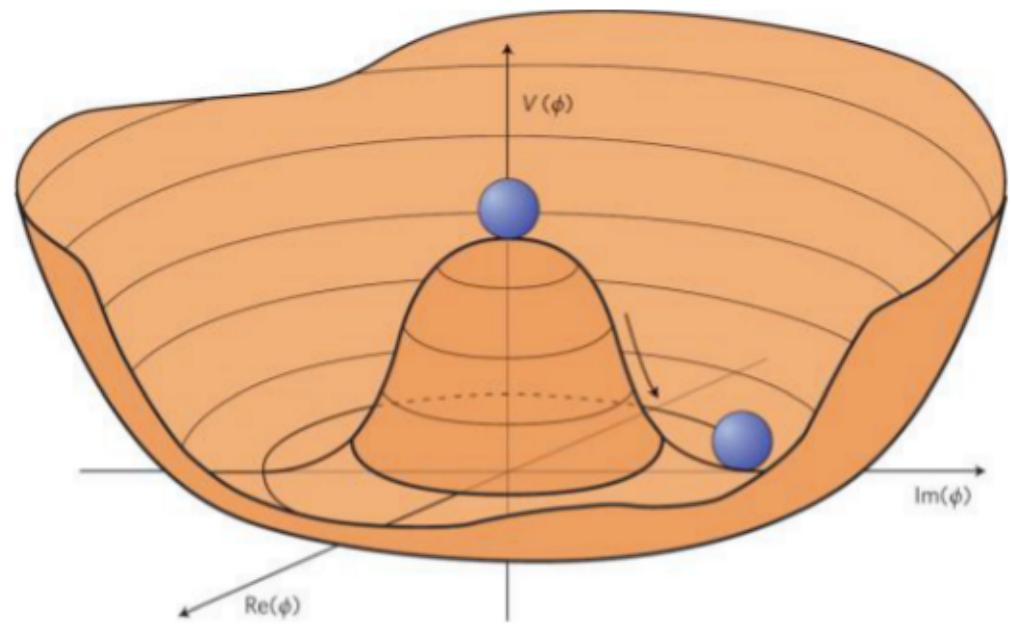
The Weinberg-Salam Model

$$\begin{aligned}\mathcal{L} = & \bar{E}_L(i\partial)E_L + \bar{e}_R(i\partial)e_R + \bar{Q}_L(i\partial)Q_L + \bar{u}_R(i\partial)u_R + \bar{d}_R(i\partial)d_R \\ & + g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + e A_\mu J_{EM}^\mu,\end{aligned}$$
$$J_W^{\mu+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L);$$
$$J_W^{\mu-} = \frac{1}{\sqrt{2}}(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L);$$
$$J_Z^\mu = \frac{1}{\cos \theta_w} \left[\bar{\nu}_L \gamma^\mu \left(\frac{1}{2} \right) \nu_L + \bar{e}_L \gamma^\mu \left(-\frac{1}{2} + \sin^2 \theta_w \right) e_L + \bar{e}_R \gamma^\mu \left(+\frac{2}{3} \sin^2 \theta_w \right) e_R \right. \\ \left. + \bar{u}_L \gamma^\mu \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_L + \bar{u}_R \gamma^\mu \left(-\frac{2}{3} \sin^2 \theta_w \right) u_R \right. \\ \left. + \bar{d}_L \gamma^\mu \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) d_L + \bar{d}_R \gamma^\mu \left(\frac{1}{3} \sin^2 \theta_w \right) d_R \right]$$
$$J_{EM}^\mu = \bar{e} \gamma^\mu (-1) e + \bar{u} \gamma^\mu (+\frac{2}{3}) u + \bar{d} \gamma^\mu (-\frac{1}{3}) d.$$

基本粒子图谱

The chosen one!

为什么Higgs是上帝粒子



Higgs机制给予所有基本粒子质量

Higgs 粒子的势能项

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

自发性电弱对称性破缺
(Higgs机制)

规范对称性自发破缺

$$\langle h \rangle \equiv v \neq 0 \rightarrow m_W = g_W \frac{v}{2}$$

发现Higgs让我们对质量起源了解更多

为什么这是一个问题？

.....



$$m_{phys}^2 = m_0^2 + c\Lambda^2 + \dots$$

- 如果Lambda是我们的某些基本高能标 (Plank? GUT? etc), 可怕的相消. 规范等级度的问题 (Gauge Hierarchy Problem)

- Lambda如果很低呢? OK 我们的新物理也许并不远!

两种情况都在我们
的世界完美体现

电弱能标



电子质量的启示

我们先看看具体情况：

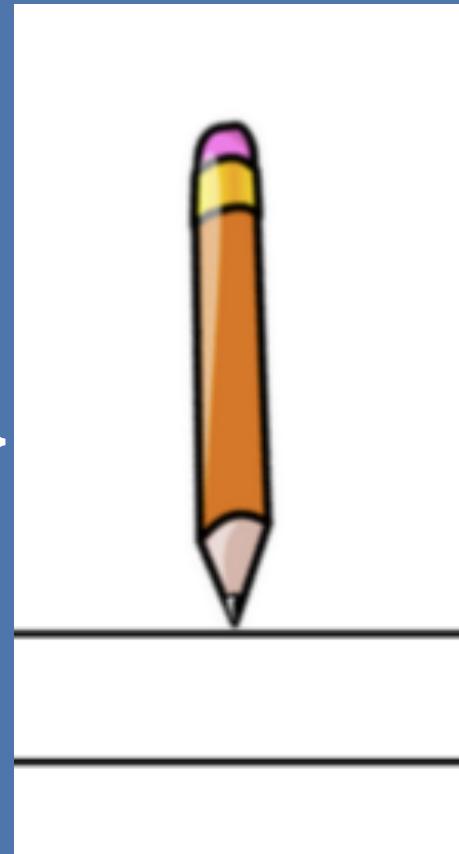
$$m_{phys}^2 = m_0^2 + c\Lambda^2 + \dots$$

如果我们的理论(SM?), 在
能标Lambda下也成立

如果 $\Lambda \sim M_{planck}$ 公式右边相除要达到 10^{32}

Higgs真空期望值对
于量子辐射修正大
大的不稳定

Huge!!!



电子质量的启示



如果我们不能扔很多个这种铅笔的话（人择原理）

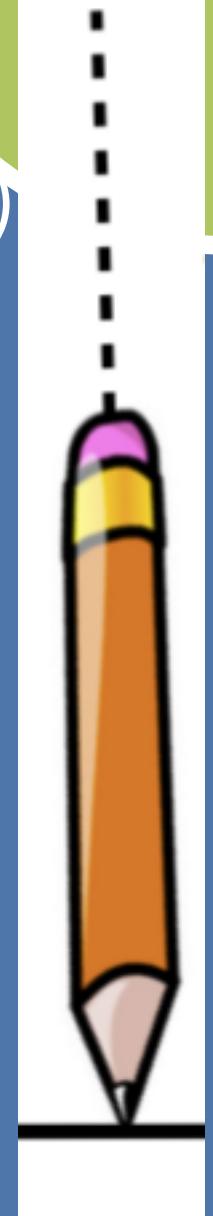
事实上对称性能够保证 (Higgs势能)
对于量子辐射修正的稳定性

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}.$$

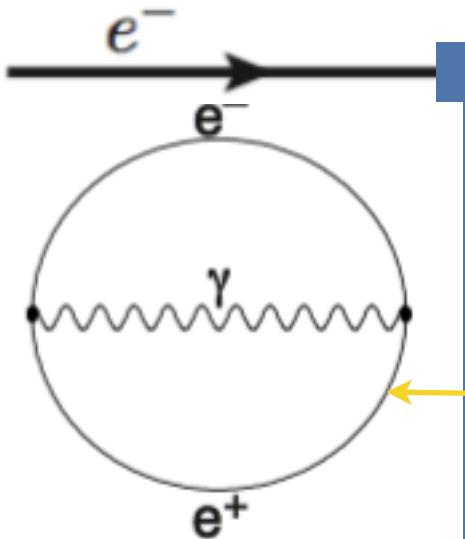
$$\delta m_e = \int_{r=\Lambda^{-1}} d^3 r \vec{E}^2 \simeq \alpha \Lambda$$

$$(m_e c^2)_{\text{obs}} = (m_e c^2)_{\text{bare}} + \Delta E_{\text{Coulomb}}.$$

电子质量项
线性发散



电子质量的启示



$$\Delta E_{\text{pair}} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}.$$

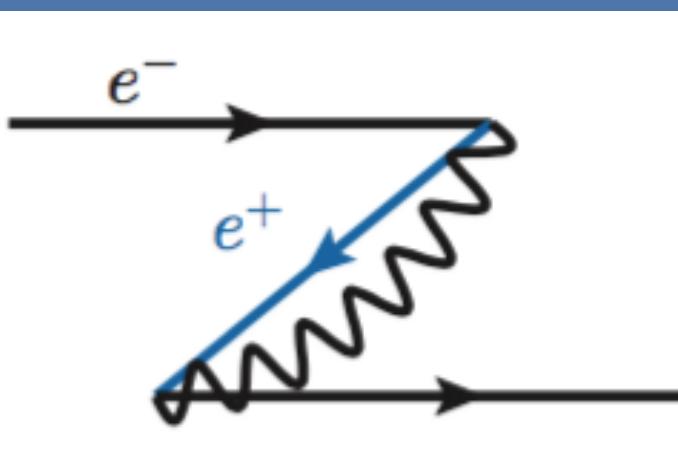
Weisskopf 1939

量子涨落引入了正电子

$$\delta m_e \simeq \frac{\alpha}{\pi} m_e \log \left(\frac{\Lambda}{m_e} \right)$$

忽略质量的情况下，电子正电子满足
Chiral symmetry

Lambda取Plank质量，修正项只有10%



有关Higgs的对称性

手征对称性：

超对称

电子 \Leftrightarrow 正电子

波色子 \Leftrightarrow 费米子

	spin		spin
gluon, g	1	gluino	\tilde{g}
W^\pm, Z	1	gaugino	\tilde{W}^\pm, \tilde{Z}
quark	1/2	squark	\tilde{q}
....		

Standard Model particles superpartners

有关Higgs的对称性



Non-SUSY的情况

top

top partner, t'

W, Z

rho meson (W', Z'), etc

Test Naturalness 问题



以前关于naturalness的test

主要是Little Higgs里面的 t' search

这些都是在gauge basis下的test, 实际check
需要涉及的复杂的质量矩阵的测量

我们需要在mass basis下的test

Higgs as a pNGB

Fermion sum rule



Top sector质量矩阵

1-loop CW 势能

$$\frac{\partial^2}{\partial H^2} \text{Tr}[M_F \cdot M_F^\dagger]|_{H=0} = 0$$

$$H = v + h.$$

$$M_F = M_{F0} + f U\left(\frac{H}{f}\right),$$

$$V(H) \sim \text{Tr}[M_F \cdot M_F^\dagger] \Lambda^2,$$

没有二次发散项的条件

$$\begin{aligned} & \frac{\partial \text{Tr}[M_F \cdot M_F^\dagger]}{\partial h}|_{h=0} \\ &= \left(\frac{\partial(H/f)}{\partial h}|_{h=0} \frac{\partial \text{Tr}[M_F \cdot M_F^\dagger]}{\partial(H/f)}|_{H=0} + \frac{\partial(H/f)^2}{\partial h}|_{h=0} \frac{\partial^2 \text{Tr}[M_F \cdot M_F^\dagger]}{2\partial^2(H/f)}|_{H=0} + \mathcal{O}(v^2/f^2) \right) \\ &= \frac{\partial(H/f)^2}{\partial h}|_{h=0} \frac{\partial^2 \text{Tr}[M_F \cdot M_F^\dagger]}{2\partial^2(H/f)}|_{H=0} + \mathcal{O}(v^2/f^2) \end{aligned} \quad (2.5)$$

No quadratic divergence



不考慮 $1/f^2$ 项高阶贡献

$$\frac{\partial \text{Tr}[M_F \cdot M_F^\dagger]}{\partial h}|_{h=0} = 0 + \mathcal{O}(v^2/f^2).$$

mass basis

$$L^\dagger M_F|_{h=0} R = M_D \quad L^\dagger \left(\frac{\partial}{\partial h} M_F \right) |_{h=0} R = Y_M,$$

$$\frac{\partial \text{Tr}[M_F \cdot M_F^\dagger]}{\partial h}|_{h=0} = \text{Tr}[Y_M \cdot M_D^\dagger + M_D \cdot Y_M] = 0 + \mathcal{O}(v^2/f^2).$$

CP守恒

$$\text{Tr}[Y_D \cdot M_D^\dagger] = 0 + \mathcal{O}(v^2/f^2).$$

No log divergence



Log divergence

$$\text{Tr}[(M_F \cdot M_F^\dagger)^2] \log \Lambda^2$$

不考慮 $1/f^2$ 项高阶贡献

$$\frac{\partial^2}{\partial H^2} \text{Tr}[(M_F \cdot M_F^\dagger)^2]|_{H=0} = 0$$

$$\begin{aligned} & \frac{\partial}{\partial h} \text{Tr}[(M_F \cdot M_F^\dagger)^2]|_{h=0} \\ &= \left(\frac{\partial(H/f)}{\partial h} \Big|_{h=0} \frac{\partial \text{Tr}[(M_F \cdot M_F^\dagger)^2]}{\partial(H/f)} \Big|_{H=0} + \frac{\partial(H/f)^2}{\partial h} \Big|_{h=0} \frac{\partial^2 \text{Tr}[(M_F \cdot M_F^\dagger)^2]}{2\partial^2(H/f)} \Big|_{H=0} + \mathcal{O}(v^2/f^2) \right) \\ &= \frac{\partial(H/f)^2}{\partial h} \Big|_{h=0} \frac{\partial^2 \text{Tr}[(M_F \cdot M_F^\dagger)^2]}{2\partial^2(H/f)} \Big|_{H=0} + \mathcal{O}(v^2/f^2) \\ &= 0 + \mathcal{O}(v^2/f^2) \end{aligned} \tag{2.12}$$

$$\frac{\partial}{\partial h} \text{Tr}[(M_F \cdot M_F^\dagger)^2]|_{h=0} = 2 \text{Tr}[Y_M M_D^\dagger M_D M_D^\dagger + Y_M^\dagger M_D \cdot M_D^\dagger \cdot M_D] = 0 + \mathcal{O}(v^2/f^2)$$

CP守恒

$$\text{Tr}[Y_M M_D^3] = 0 + \mathcal{O}(v^2/f^2).$$

Gauge sum rule



No quadratic divergence

$$\text{Tr}[g_{VV} h] = 0 + \mathcal{O}(v^2/f^2).$$

No log divergence

$$\text{Tr}[g_{VV} h M_V^2] = 0 + \mathcal{O}(v^2/f^2),$$

SUSY Case

$$\text{Tr}[g_{SSH}] - 2\text{Tr}[Y_M M_D^\dagger + M_D Y_M^\dagger] + 3\text{Tr}[g_{VVH}] = 0,$$

$$\text{Tr}[g_{SSH}] - 4\text{Tr}[Y_M M_D] + 3\text{Tr}[g_{VVH}] = 0.$$

Quadratic divergence

Top sector/stop sector

Gauge/gaugino/Higgs/Higgsino sector

$$\sum_i g_{\tilde{t}_i \tilde{t}_i h} - 4y_t m_t = 0,$$

$$4 \sum_i (y_{C_i^+ C_i^- h} m_{C_i} + y_{N_i N_i h} m_{N_i}) - 3(g_{W^+ W^- h} + g_{ZZ h}) \\ - \sum_i (g_{H_i^0 H_i^0 h} + g_{H_i^+ H_i^- h}) - g_{hh h} = 0$$

Examples



Simplest little Higgs SU3/SU2

$$\mathcal{L}_t = -\lambda_1 f \bar{\Psi}_q \mathcal{H} u_{3R} - \lambda_2 f \tilde{T}_L \tilde{T}_R + h.c.,$$

$$M_F = -f \begin{pmatrix} \lambda_1 \sin(\frac{H}{\sqrt{2}f}) & 0 \\ \lambda_1 \cos(\frac{H}{\sqrt{2}f}) & \lambda_2 \end{pmatrix}.$$

$$\text{Tr}[M_F M_F^\dagger] \Lambda^2 = f^2 (\lambda_1^2 + \lambda_2^2) \Lambda^2$$

first sum rule

After EWSB

$$m_t = \frac{\lambda_1 \lambda_2 v}{\sqrt{\lambda_1^2 + \lambda_2^2}} + \mathcal{O}\left(\frac{v^2}{f^2}\right) \quad m_T = -f \sqrt{\lambda_1^2 + \lambda_2^2} + \mathcal{O}\left(\frac{v^2}{f^2}\right).$$

$$y_t = -\frac{m_t}{v} + \mathcal{O}\left(\frac{v^2}{f^2}\right) \quad y_T = \frac{y_t^2 v}{m_T}.$$

$$y_t m_t + y_T m_T = 0 + \mathcal{O}\left(\frac{v^2}{f^2}\right).$$

Examples

Maximally symmetric composite Higgs SO5/SO4

$$\begin{aligned}\mathcal{L}_f = & \bar{q}_L i \not{D} q_L + \bar{t}_R i \not{D} t_R + \bar{\Psi}_Q i \not{\nabla} \Psi_Q + \bar{\Psi}_S i \not{\nabla} \Psi_S \\ & - \frac{1}{\sqrt{2}} \epsilon_t \bar{\Psi}_{t_R} U \Psi_{+L} - \epsilon_q \bar{\Psi}_{q_L} U \Psi_{+R} - M \bar{\Psi}_{+L} V \Psi_{+R} + h.c,\end{aligned}$$

Before EWSB

$$M_F = \begin{pmatrix} 0 & \epsilon_q \cos^2\left(\frac{H}{2f}\right) & \epsilon_q \sin^2\left(\frac{H}{2f}\right) & -\frac{\epsilon_q \sin\left(\frac{H}{f}\right)}{\sqrt{2}} \\ \frac{1}{2} \epsilon_t \sin\left(\frac{H}{f}\right) & M & 0 & 0 \\ -\frac{1}{2} \epsilon_t \sin\left(\frac{H}{f}\right) & 0 & M & 0 \\ \frac{\epsilon_t \cos\left(\frac{H}{f}\right)}{\sqrt{2}} & 0 & 0 & -M \end{pmatrix}.$$

$$\text{Tr}[M_F M_F^\dagger] \Lambda^2 = (3M^2 + \epsilon_q^2 + \frac{\epsilon_t^2}{2}) \Lambda^2$$

$$\text{Tr}[(M_F M_F^\dagger)^2] \log \Lambda^2 = (3M^4 + \epsilon_q^4 + \frac{\epsilon_t^4}{4} + (2\epsilon_q^2 + \epsilon_t^2)M^2) \log \Lambda^2.$$

Examples



Maximally symmetric composite Higgs SO5/SO4

After EWSB

$$Y = \left(\frac{\partial}{\partial H} M_F \right) |_{H=v} = \begin{pmatrix} 0 & -\frac{\epsilon_q \sin\left(\frac{v}{f}\right)}{2f} & \frac{\epsilon_q \sin\left(\frac{v}{f}\right)}{2f} & -\frac{\epsilon_q \cos\left(\frac{v}{f}\right)}{\sqrt{2}} \\ \frac{\epsilon_t \cos\left(\frac{v}{f}\right)}{2f} & 0 & 0 & 0 \\ -\frac{\epsilon_t \cos\left(\frac{v}{f}\right)}{2f} & 0 & 0 & 0 \\ -\frac{\epsilon_t \sin\left(\frac{v}{f}\right)}{\sqrt{2}f} & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Tr}[Y_D \cdot M_D^\dagger] = \text{Tr}[Y \cdot (M_F^\dagger) |_{H=v}] = 0$$

$$\text{Tr}[Y_D \cdot M_D^3] = \text{Tr}[Y \cdot (M_F^\dagger \cdot M_F \cdot M_F^\dagger) |_{H=v}] = 0$$

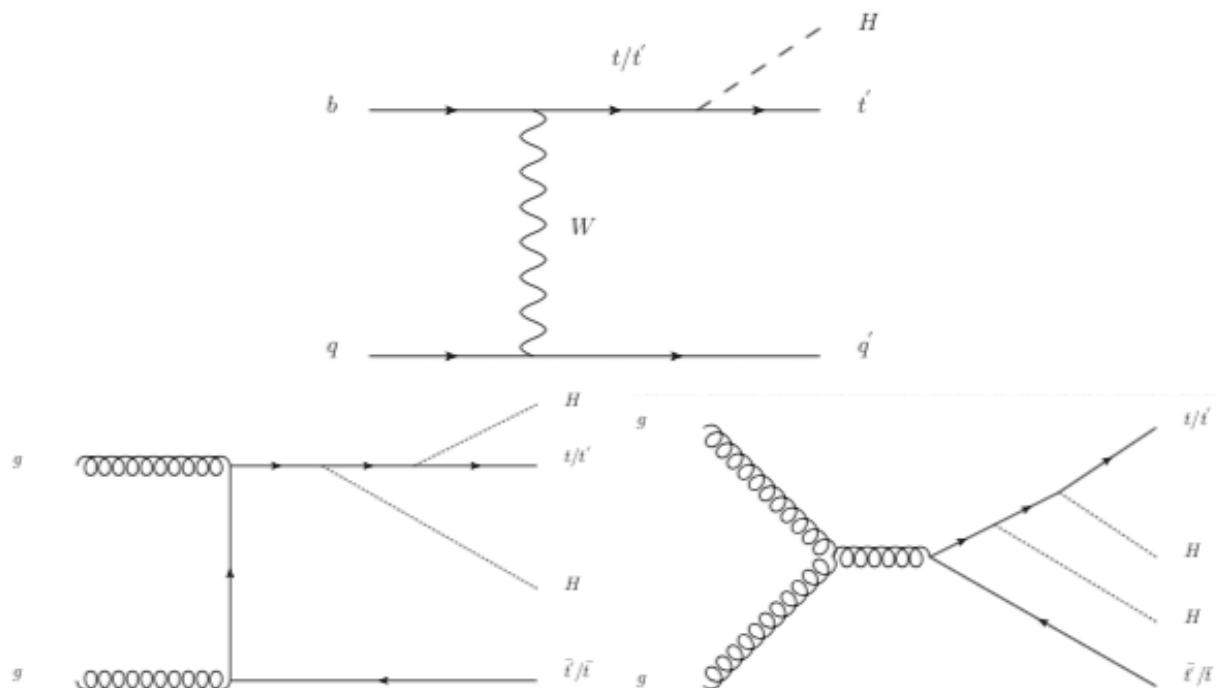
Collider Test



Double production: No sign of the couplings

$$pp \rightarrow t't'h,$$

C.-R. Chen, J. Hajer, T. Liu, I. Low, and H. Zhang, *Testing naturalness at 100 TeV*, *JHEP* **09** (2017) 129, [[arXiv:1705.0774](https://arxiv.org/abs/1705.0774)].



$$pp \rightarrow qht'.$$

single production:
interference

Benchmark-LH

Definition of flipped rate

$$\mathcal{F} = 2 \frac{\sigma(Y_{tt'h}) - \sigma(-Y_{tt'h})}{\sigma(Y_{tt'h}) + \sigma(-Y_{tt'h})}.$$

LH Model $\lambda_1 = 1.48$, $\lambda_2 = 1.11$ and $f = 811$ GeV.

Name	Mass [GeV]	Decays	$\sigma(qht')$, fb	$\mathcal{F}(qht')$	$\sigma(t'thh)$, fb	$\mathcal{F}(t'thh)$
t'	1492	Wb: 50% tZ: 25% tH: 25%	185	-13%	14.4	-26%

Table 1. Top partner parameters at the Benchmark Point in the Little Higgs model.

Benchmark-MSCH

MSH Model $\epsilon_{qQ} = 1.15$, $\epsilon_{tQ} = -1.40$, and $MQ = 1500$ GeV.

Name	Mass [GeV]	Decays	$\sigma(qht')$, fb	$\mathcal{F}(qht')$	$\sigma(t'thh)$, fb	$\mathcal{F}(t'thh)$
t'	1791	Zt: 42.92%	20.9	26%	14.3	9.9%
		Wb: 26.06%				
		Zx ₂ : 12.76%				
		tH : 8.3%				
x_2	1632	tH: 37.51%	5.0	-25%	41.7	-45%
		Wb: 32.84%				
		Zt: 18.33%				
		Wx ₅ : 6.08%				
t_1	1500	tH: 51.45%	XXX	0	YYY	0
		Zt: 32.43%				
		WWt : 11.81%				

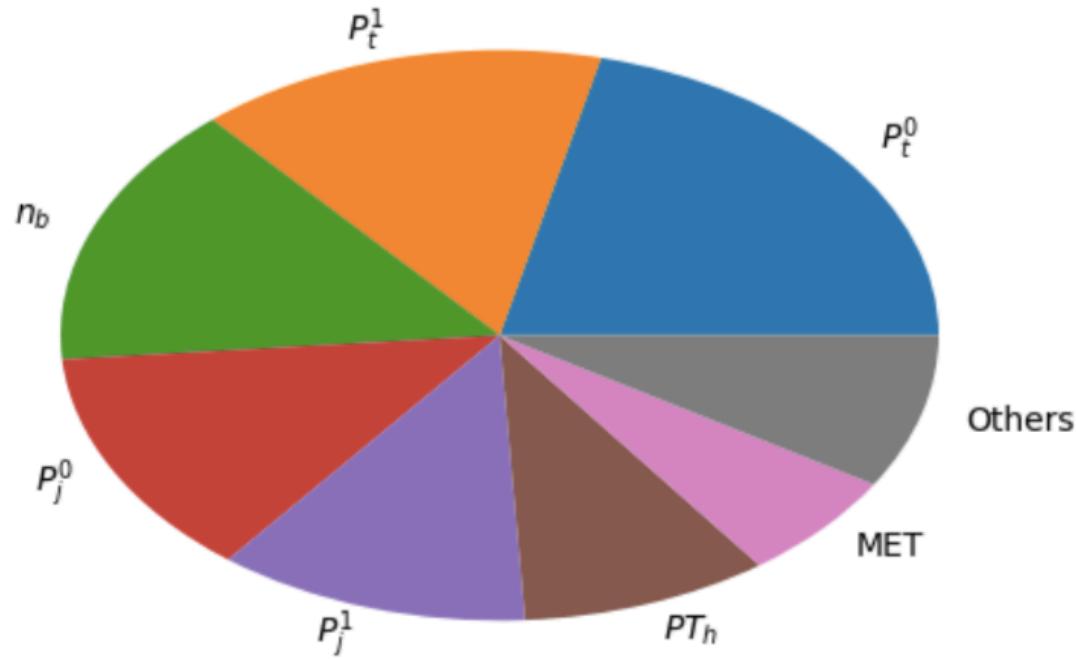
Table 2. Top partner parameters at the Benchmark Point in the Maximally Symmetric Higgs model.

Three channels



We proposed 14 variables in this analysis: $m_{t'}$, HT, MET, n_{fj} , n_j , leading boosted jet PT PT_0 , next leading boosted jet PT PT_1 , n_b , PT_h , m_h , leading boosted jet light boosted jet probability P_j^0 , P_t^0 , P_j^1 , P_t^1 . We already have a lot of data so we do not

RF: Random Forest methods



Three channels

	Event Number	RF	Significance	Sign
<i>signal</i>	4136.09	2644.18	49.01	4.91
<i>hjj</i>	$6.4 \times 10^4 \rightarrow 1.74 \times 10^4$	987.10		
<i>tth</i>	$3.8 \times 10^4 \rightarrow 1.40 \times 10^4$	1162.04		

Table 5. Pre Cut: $m_{H \rightarrow aa} \in [115, 135]$ GeV and $m_{t' \rightarrow jj} \in [1000, 2000]$ GeV, these 2 fat jets' PT > 400 GeV. RF input: $m_{t'}$, HT, MET, n_{fj} , n_j , PT_0 , PT_1 , n_b , PT_h , m_h , P_j^0 , P_t^0 , P_j^1 , P_t^1 .

	Event Number	Updated RF	Updated Significance	Updated Sign
<i>signal</i>	4136.09	2634.01	44.73	4.64
<i>hjj</i>	$6.4 \times 10^4 \rightarrow 1.74 \times 10^4$	1233.80		
<i>tth</i>	$3.8 \times 10^4 \rightarrow 1.40 \times 10^4$	1457.61		

Table 6. Update for Table 5.

简化版的七个参数变化不大

Three channels

	Cut I	Cut II	RF	Significance	Sign Significance
$qht' \rightarrow qhzt \rightarrow q(bb)(ll)(qqb)$	$36634.8 \rightarrow 2234.4$	1836.0	779.8	34.2	3.02
$ttZ \rightarrow (qqb)(qqb)(ll)$	$2.86 \times 10^6 \rightarrow 13585$	5203.2	43.6		
$Zjj \rightarrow (ll)jj$	$3.68 \times 10^7 \rightarrow 56300.$	19253.6	272.1		

Table 7. Cut I: $m_{Z \rightarrow \ell\ell} \in [80, 100]$ GeV and $PT_{Z \rightarrow \ell\ell} > 400$ GeV, only one top fat jet and only one Higgs jet with $PT > 400$ GeV. Cut II: $m_{t' \rightarrow Z_{\ell\ell} j_t} \in [1000, 2000]$ GeV. BDT input: n_{fj} , n_j , PT_Z , PT_t and the leading fat jet PT.

	Event Number	RF	Significance	Sign Significance
signal	476.54	308.12	17.99	3.45
$hjjj$	$8752.20 \rightarrow 2367.37$	121.61		
$ttjh$	$3281.35 \rightarrow 1013.02$	84.88		

Table 8. Pre Cut: $m_{H \rightarrow aa} \in [115, 135]$ GeV and $m_{t' \rightarrow jj} \in [1000, 2000]$ GeV, with 3 fat jets' PT > 400 GeV. RF input: $m_{t'}$, HT, MET, n_{fj} , n_j , PT_0 , PT_1 , n_b , PT_h , m_h , P_j^0 , P_t^0 , P_j^1 , P_t^1 , P_j^2 , P_t^2 .

modes, each flip significance is: 4.91, 3.02 and 3.45. So that the combined flip significance is 6.72.

总结展望

- 100TeV, 3000fb^{-1} 可以测量并确定符号
- 方法推广到SUSY-stop里面的测量

物理意义和预言

物理

$\Pi_{0,1}^{q,t}$ 为0 Top动能项：没有非线性修正

$$M_t(h) \sim \sin\left(\frac{2h}{f}\right) \left(1 + \frac{1}{2} \sin^2(h/f) (\Pi_1^q(0) - \Pi_1^t(0))\right)$$

通过测量mt, tth, ttth, etc可以确定 $\Pi_{0,1}^{q,t}$ 是否为0

发现类顶夸克态， 测量它的性质

$$\text{Tr}[Y_m M_D] = 0 + \mathcal{O}(v^2)$$

对角的Higgs Yukawa和质量

类顶夸克态最轻的是exotic charge (5/3)

$$M_Q + M_S = 0$$



Backup
slice

Spin 1/2 Resonances



There are many ways to generate the fermion masses

Bilinear: $\mathcal{L} = \lambda \bar{q} q \langle \bar{\Psi} \Psi \rangle$ techicolor, conformal techicolor, etc

Here we only consider the “partial compositeness”

Linear mixing:

$$\mathcal{O}_i \sim U \Psi_i$$

$$\Psi_i$$

Q_j bi-doublet

$$\mathcal{L}_{mix} = \lambda \bar{q}_i \mathcal{O}_i$$

Composite operators

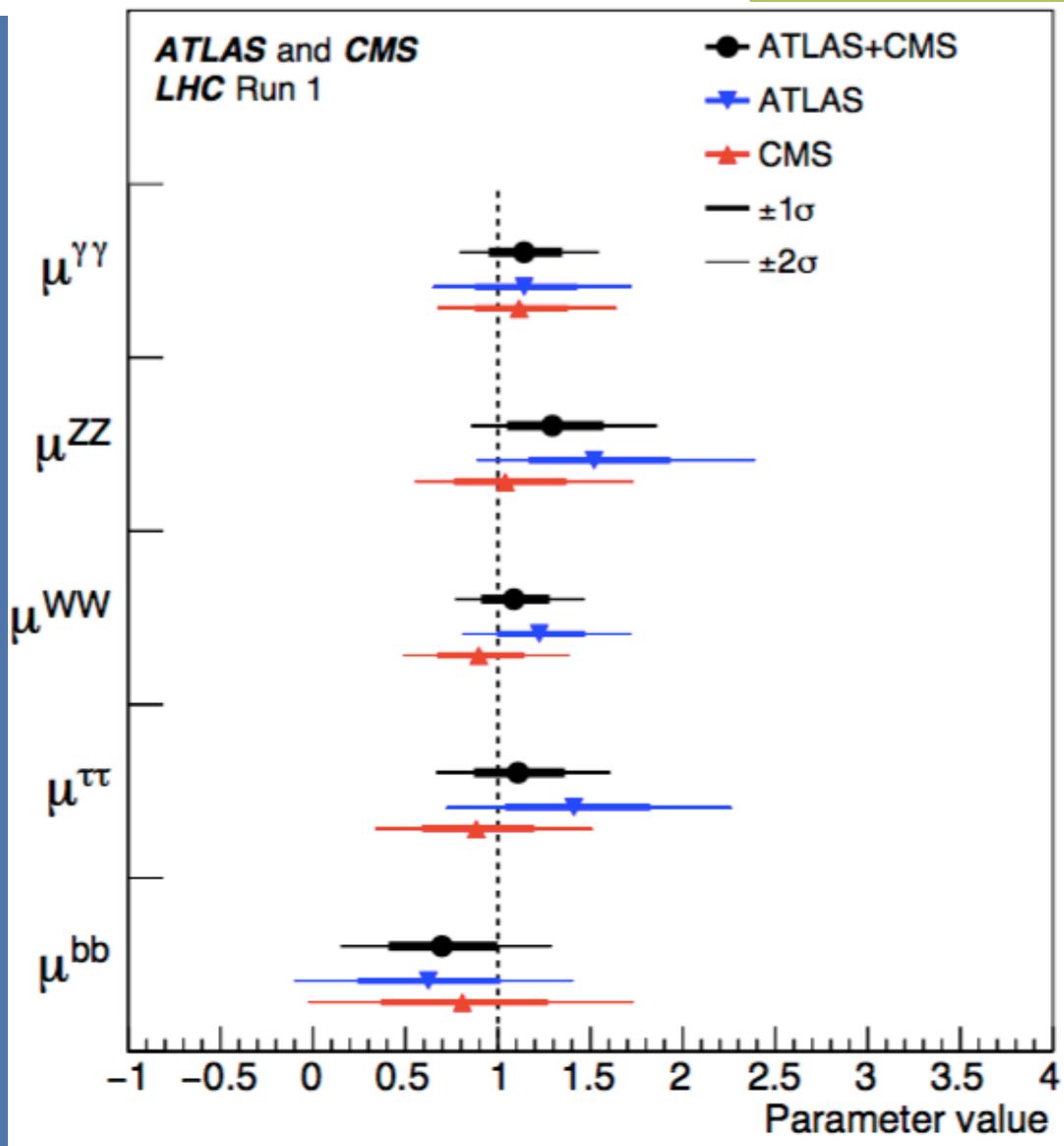
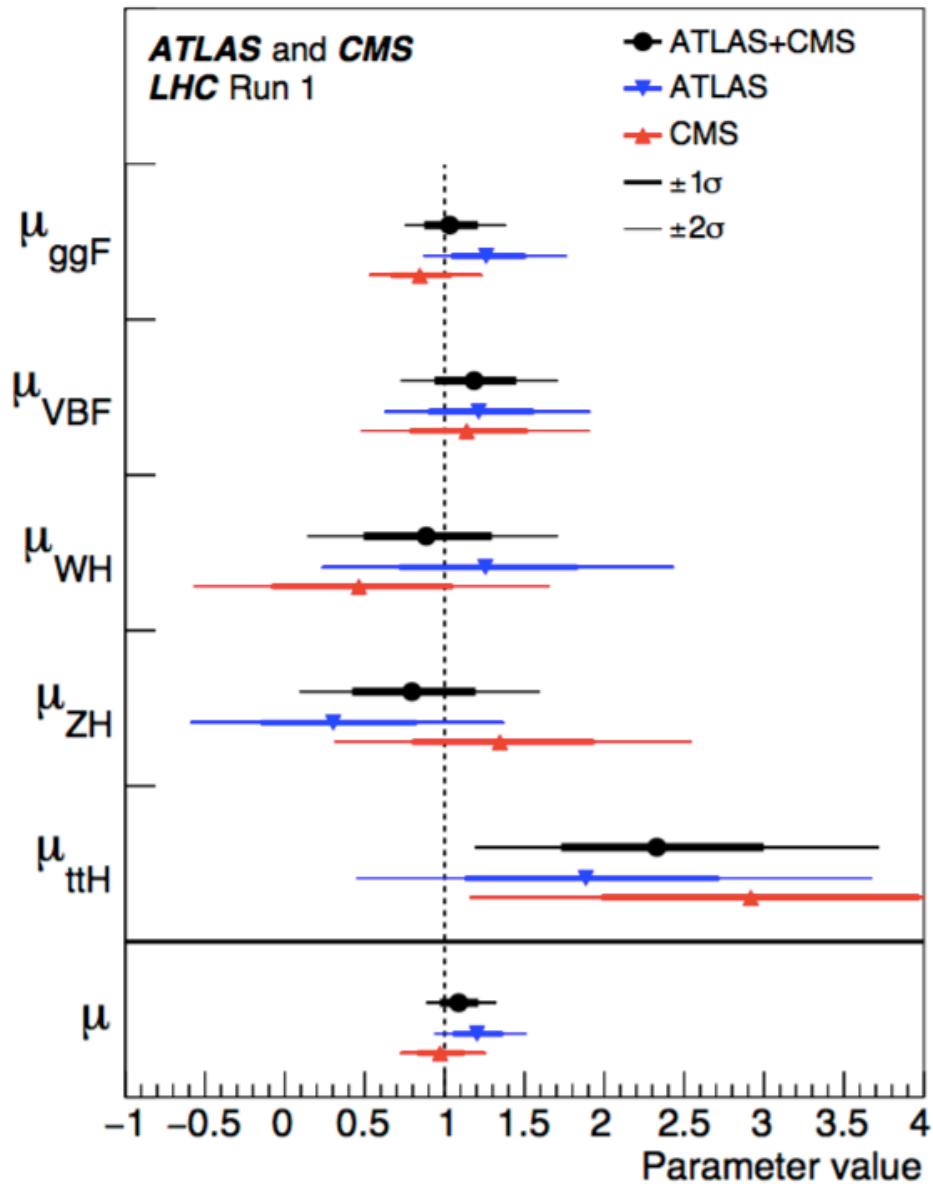
Composite fermions sit in the representation of SO(4)

S_i Singlet

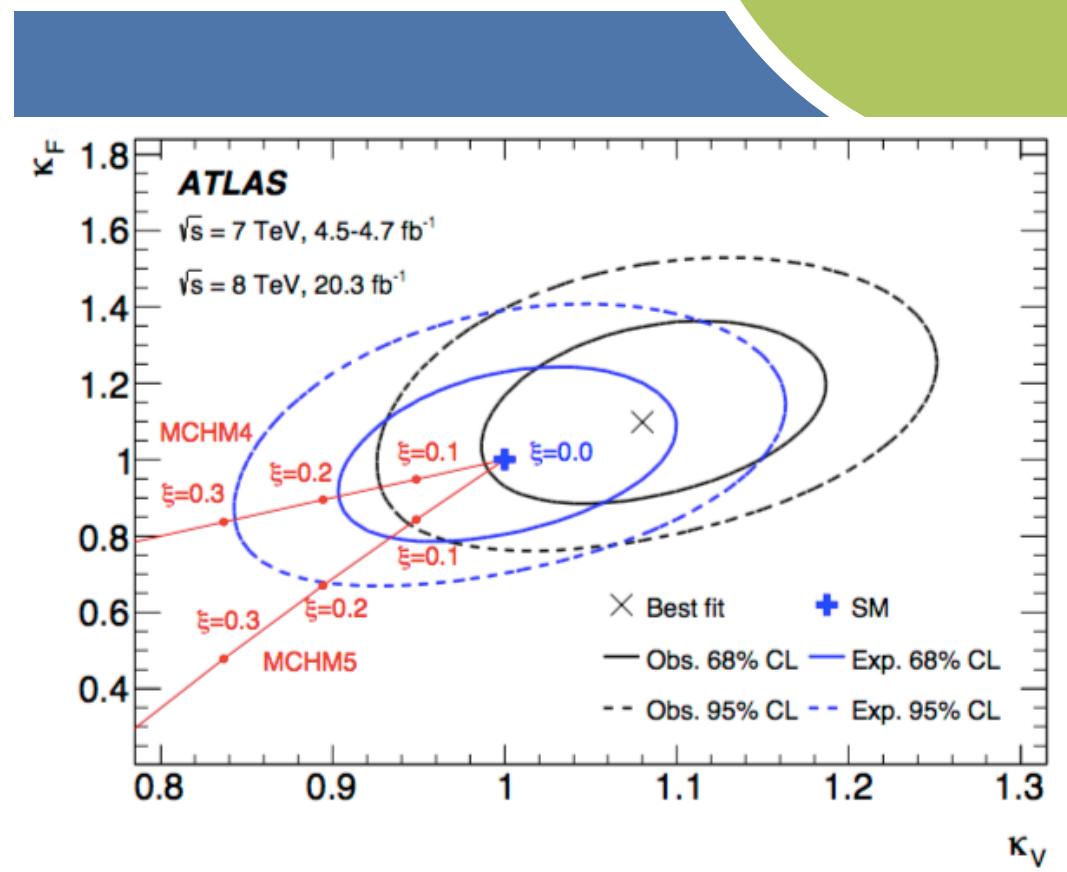
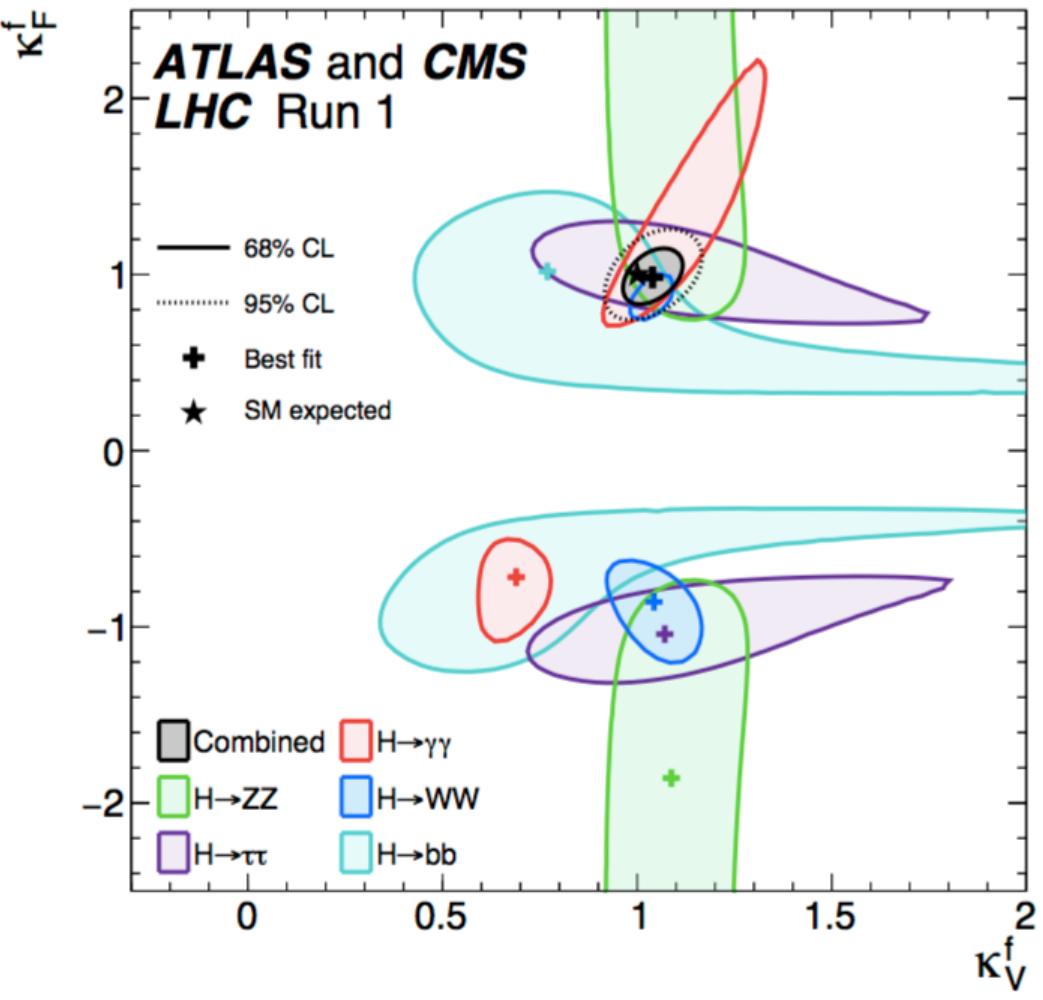
Good for flavor physics

Maximally suppressed the FCNC by the small fermion mass

Higgs产生和衰变



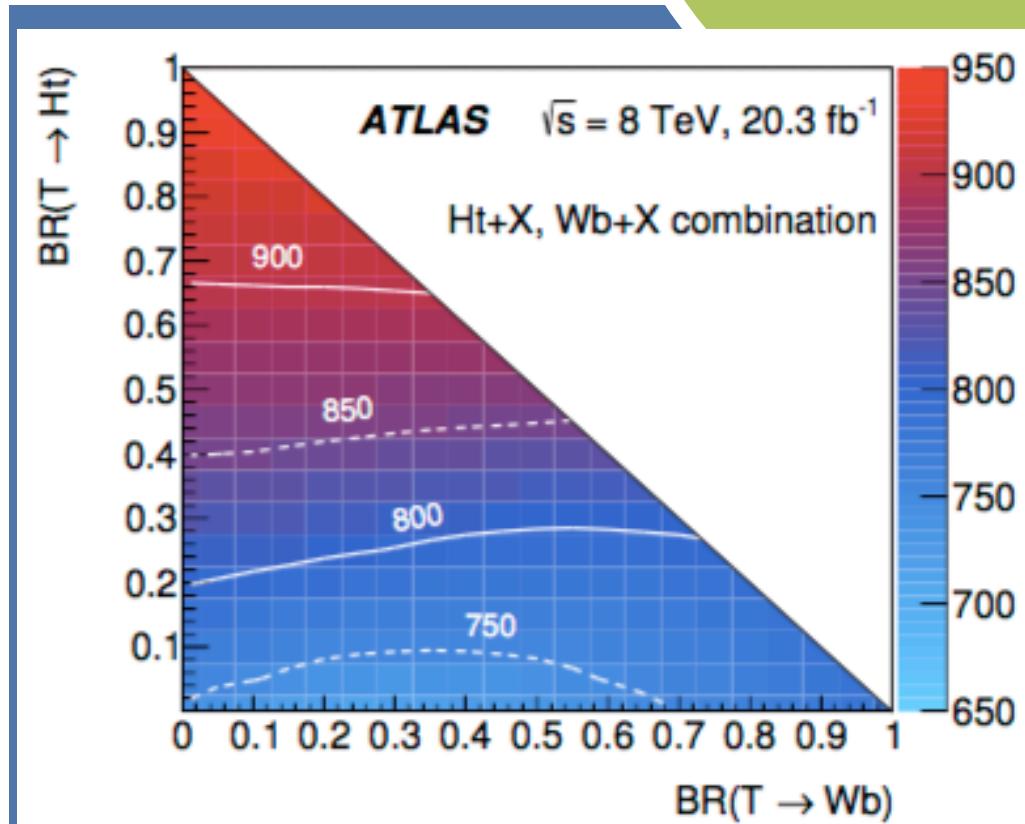
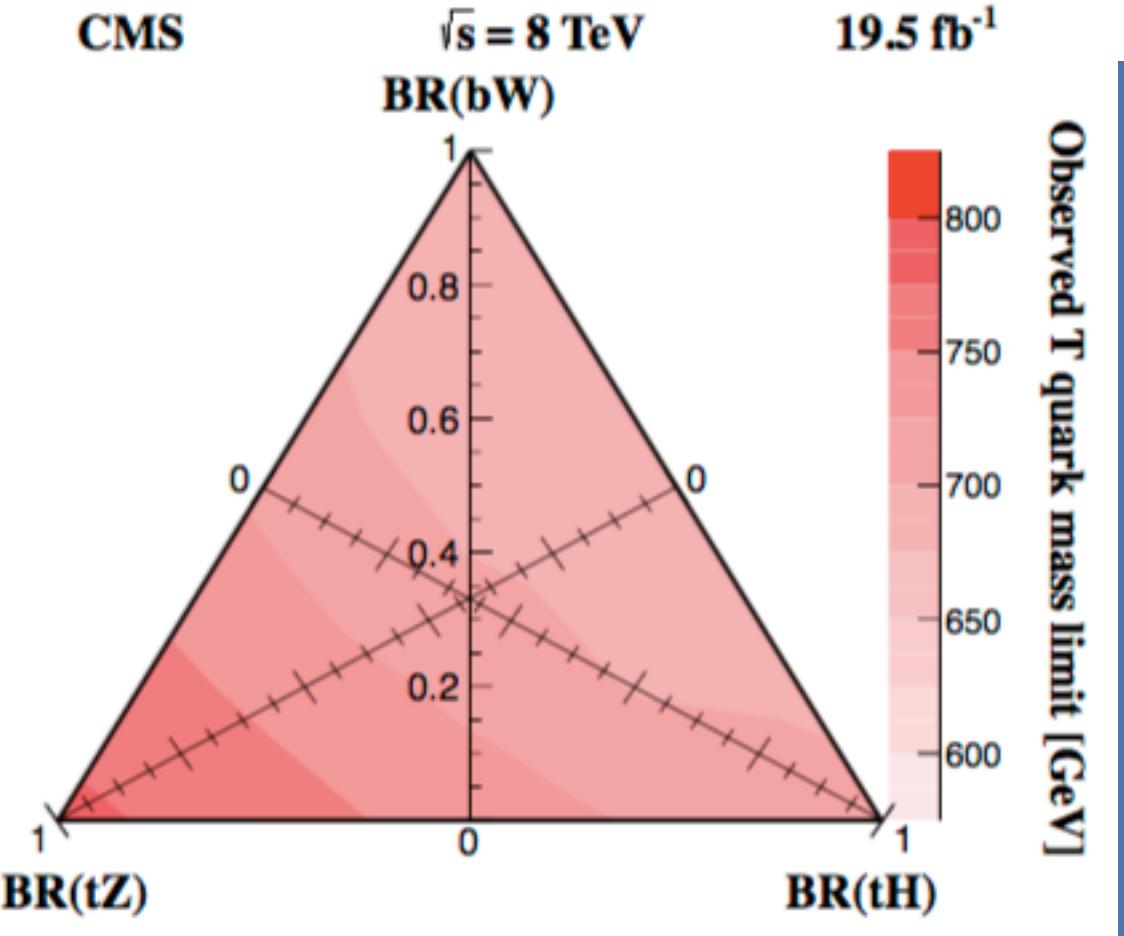
Higgs物理



Top耦合为负的情况不再存在

Higgs 拟合 $\xi < 0.1$

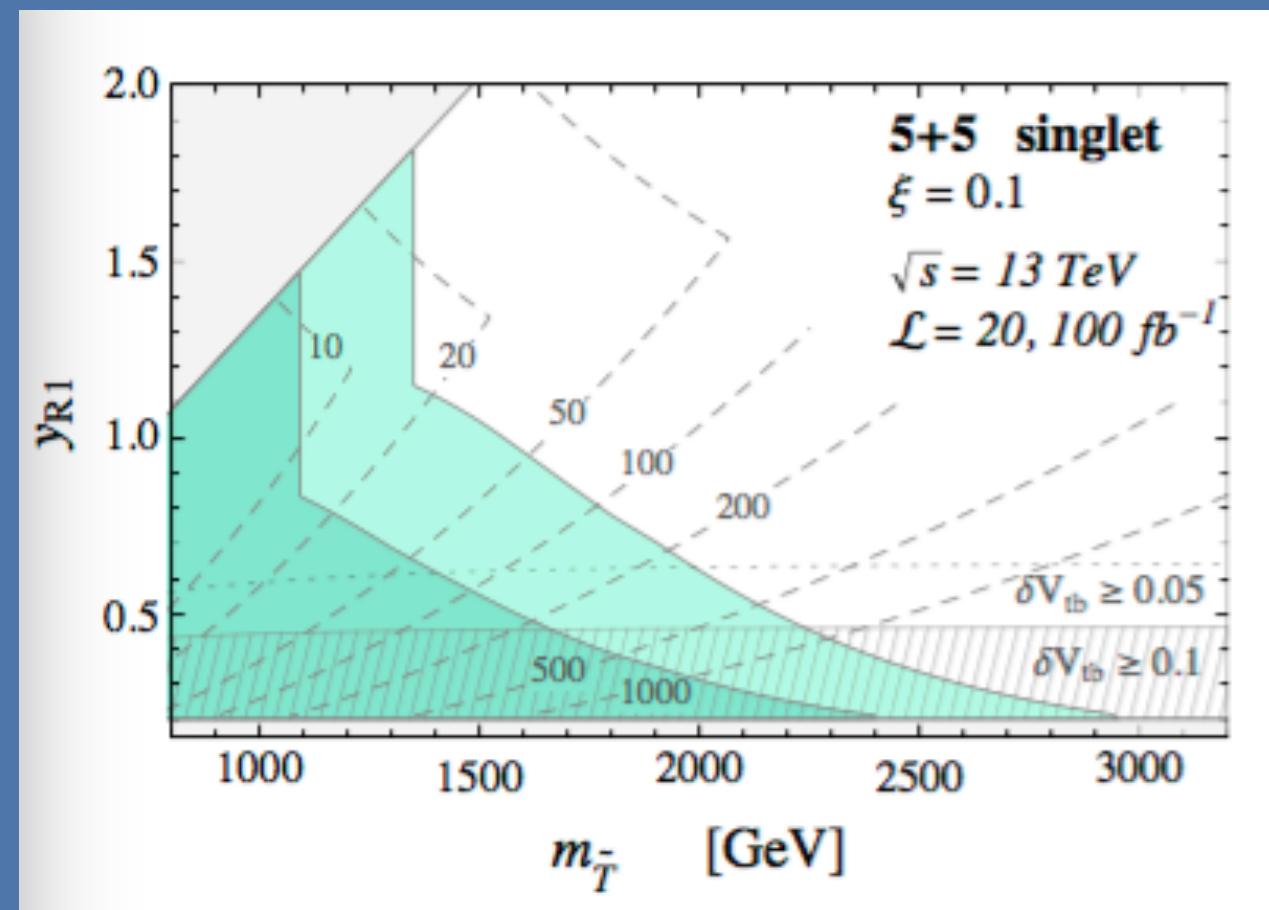
Top 伴随着的寻找



$\text{BR}(t' \rightarrow th) \approx \text{BR}(t' \rightarrow tZ) \approx \text{BR}(t' \rightarrow bW)/2 \approx 0.25$

限制在700~900GeV

Top 伴随子的寻找



D. Matsedonskyi, G. Panico, A. Wulzer, JHEP, 1604, (2016) 003.

当前Top伴随子寻找正在检验原始的复合Higgs模型

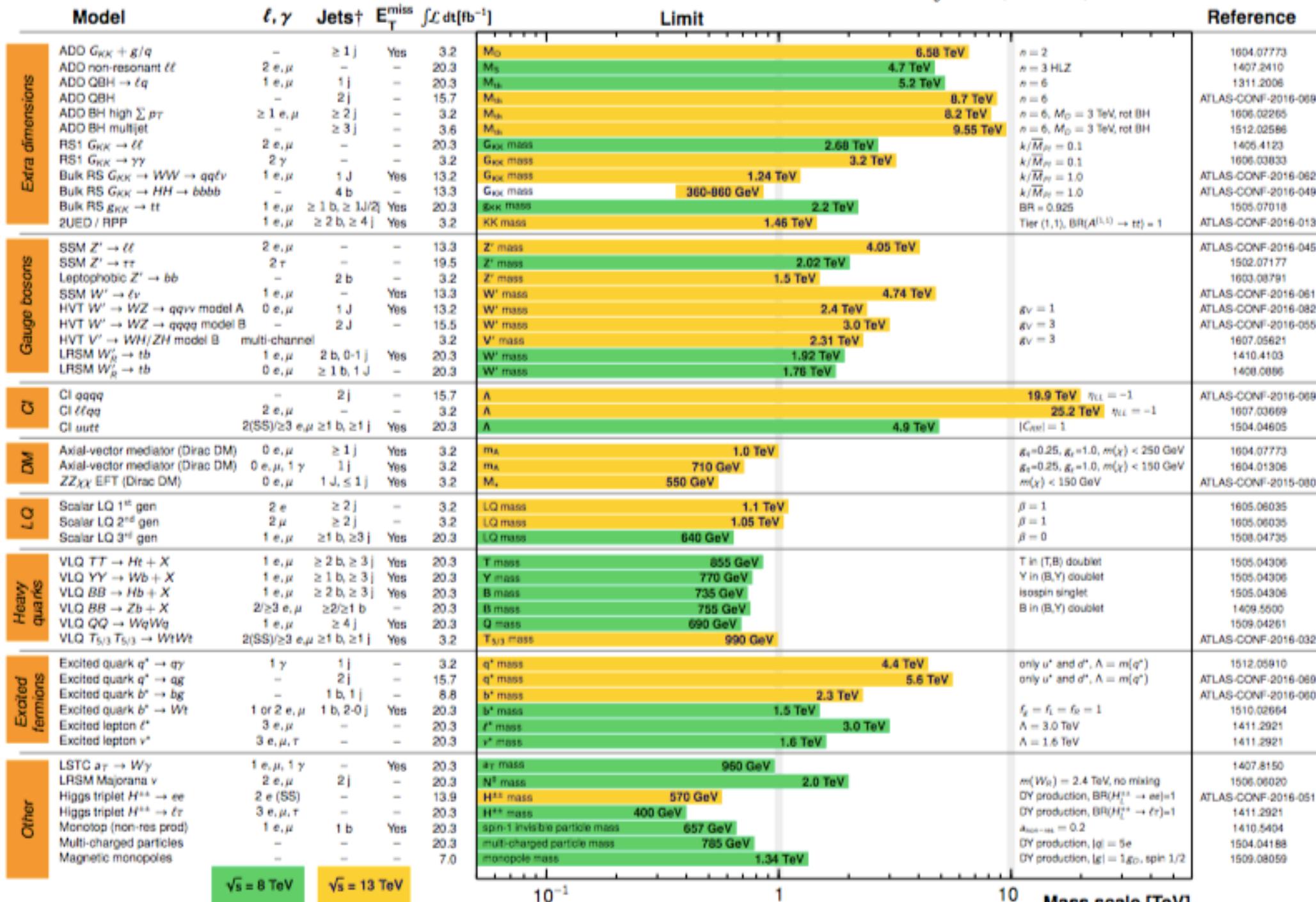
ATLAS Exotics Searches* - 95% CL Exclusion

Status: August 2016

ATLAS Preliminary

$\int \mathcal{L} dt = (3.2 - 20.3) \text{ fb}^{-1}$

$\sqrt{s} = 8, 13 \text{ TeV}$



$\sqrt{s} = 8 \text{ TeV}$

$\sqrt{s} = 13 \text{ TeV}$

10⁻¹

1

10

Mass scale [TeV]

*Only a selection of the available mass limits on new states or phenomena is shown. Lower bounds are specified only when explicitly not excluded.

ttbar Higgs

Summary of the ttH signal strength measurements (left) and upper limits (right).

