$$M_V = \frac{M_V^{OS}}{\sqrt{1 + \left(\frac{\Gamma_V^{OS}}{M_V^{OS}}\right)^2}} \qquad \Gamma_f = \frac{\Gamma_f^{OS}}{\sqrt{1 + \left(\frac{\Gamma_f^{OS}}{M_V^{OS}}\right)^2}}$$

$$\Gamma_f = \frac{N_c^f M_Z}{12\pi} \left[R_V^f F_V^f + R_A^f F_A^f \right]_{s=M_Z^2}$$

- ${\cal R}_V^f$ and ${\cal R}_A^f$ contain QCD and QED corrections:
 - QCD up to $\mathcal{O}(\alpha_s^4)$ and QED up to $\mathcal{O}(\alpha^2)$ for massless fermions
 - mass corrections up to $\mathcal{O}(\alpha_s^3)$
- \bullet F_V^f and F_A^f contain EW and mixed EW-QCD and EW-QED corrections
 - QCD up to $\mathcal{O}(\alpha_s^4)$ and QED up to $\mathcal{O}(\alpha^2)$ for massless fermions
 - mass corrections up to $\mathcal{O}(\alpha_s^3)$

1/2

numerical impacts of higher order contributions

Γ_i [MeV]	Γ_e	Γ_{ν}	Γ_d	Γ_u	Γ_b	$\Gamma_{\rm Z}$
Born	81.142	160.096	371.141	292.445	369.562	2420.19
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$O(\alpha \alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$O(\alpha_t \alpha_s^2, \alpha_t \alpha_s^3, \alpha_t^2 \alpha_s, \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190	1.20
$O(N_f^2 \alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$O(N_f \alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$O(\alpha_{\text{bos}}^2)$	0.017	0.019	0.059	0.058	0.167	0.51

Table 2: Contributions of different orders in perturbation theory to the partial and total Z widths. A fixed value of M_N has been used as input, instead of G_B , M_f and M_f^2 refer to corrections with one and two closed fermion loops, respectively, whereas α_{Don}^2 about contributions without closed fermion loops. Furthermore, $\alpha_L = y_L^2/(4\pi)$. In all rows the radiator functions $\mathcal{R}_{V,N}$ with known contributions through $\mathcal{O}(\alpha_h^2)$, $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha_{OB})$ are included.

estimated theoretical uncertainties

$\Gamma_{e,\mu\tau}$	0.018 MeV	$\Gamma_{u,c}$	0.11 MeV	R_{ℓ}	$6 \cdot 10^{-3}$
Γ_{ν}	0.016 MeV	Γ_b	0.18 MeV	R_c	$5 \cdot 10^{-5}$
$\Gamma_{d,s}$	0.08 MeV	Γ_{Z}	0.4 MeV	R_b	$1 \cdot 10^{-4}$

Table 6: Theory uncertainty estimates for the partial and total Z widths and branching ratios from missing 3-loop and higher orders. See text for details.