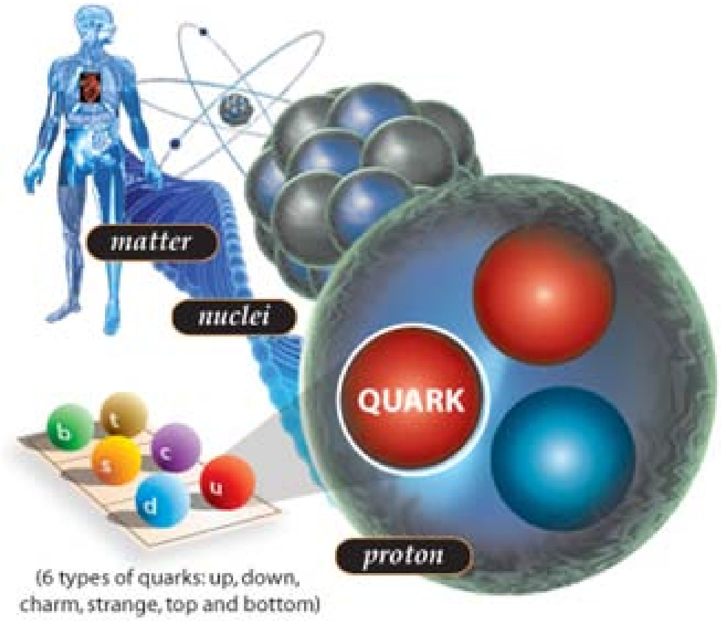
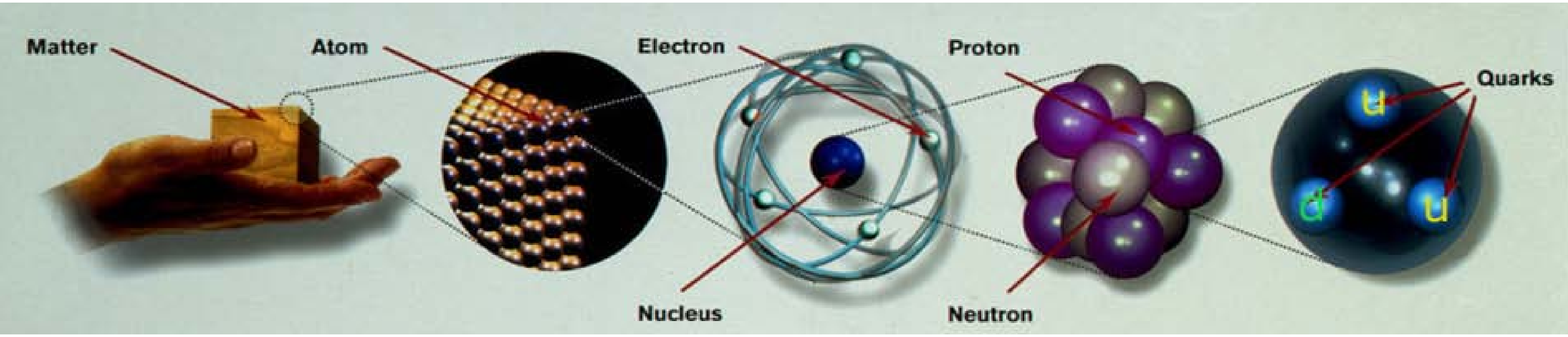
QCD Bases of Low Energy Effective Lagrangian

More then Thirty Years Investigations ......

Qing Wang UCAS/IHEP July 13, 2018



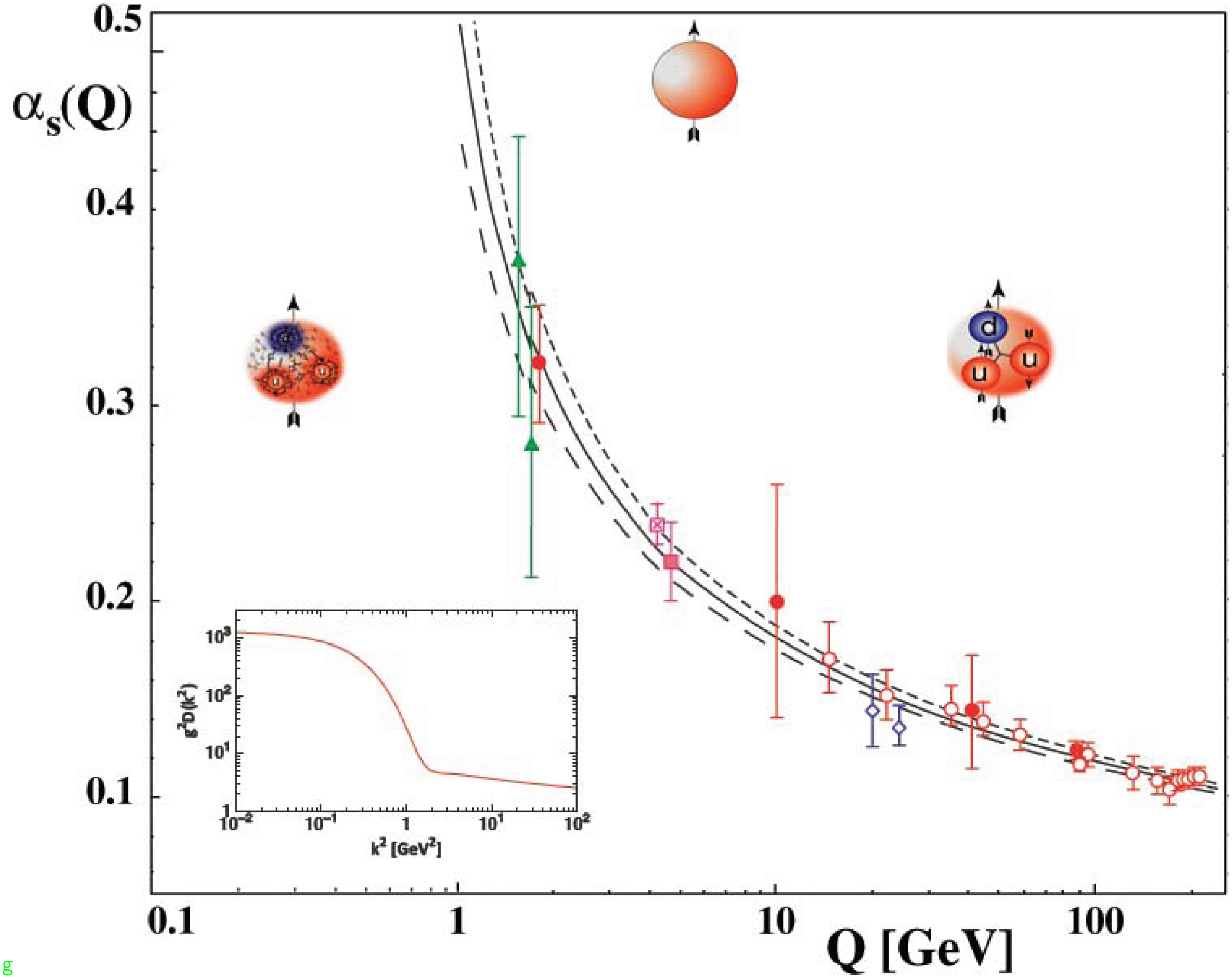


- **There exists phys model or theory at each level of matter** solid, molecue, atom, nulcleus, quark

- **The more small of the scale, the more fundamental of the theory** reductionism/emergentism

- **Interpret Physics in terms of smaller scale theory is difficult** cross scales, **but essential**

- **Report status of inter-relations between EFT** at hadron scale **and QCD** at quark-gluon scale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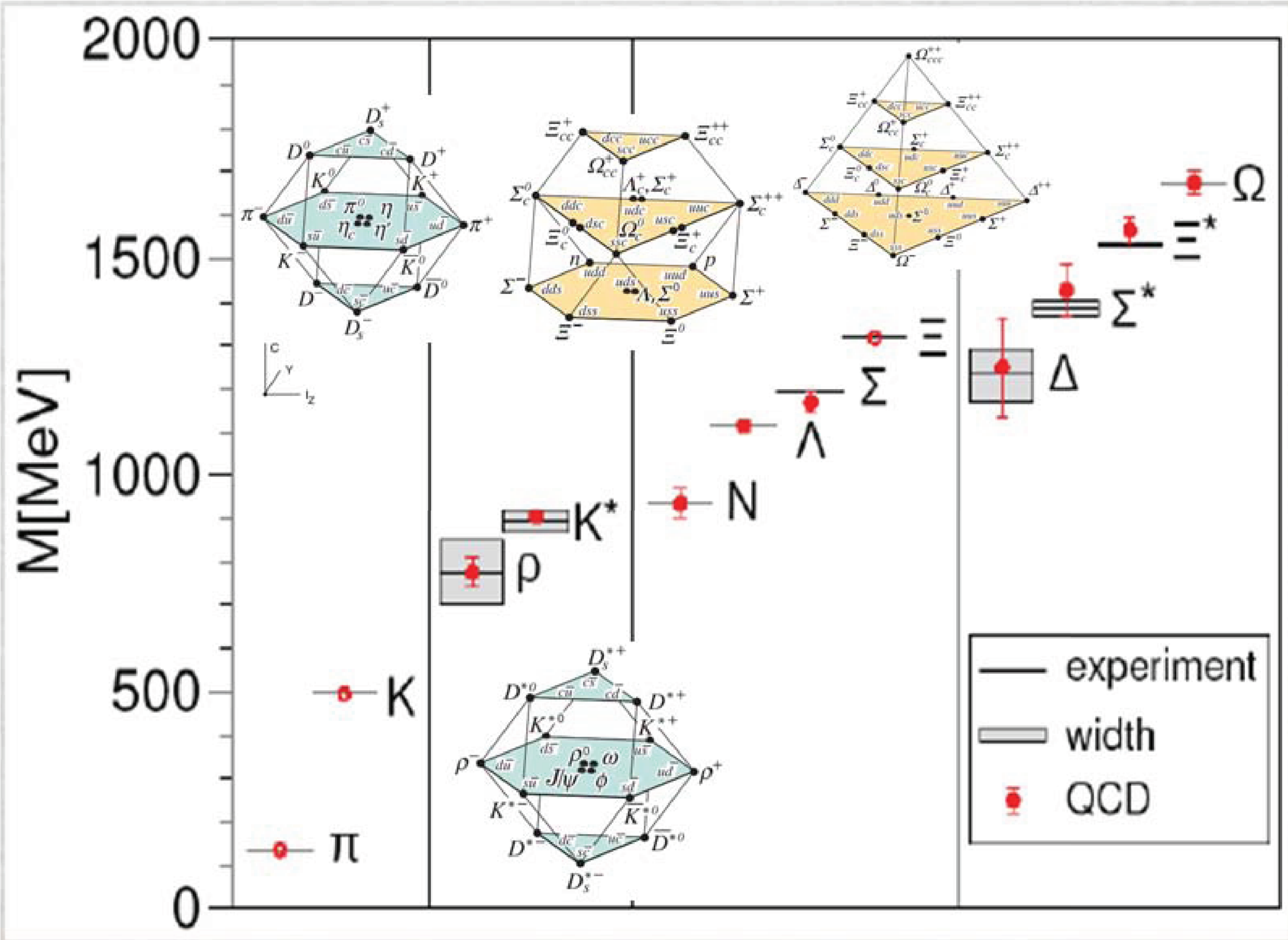
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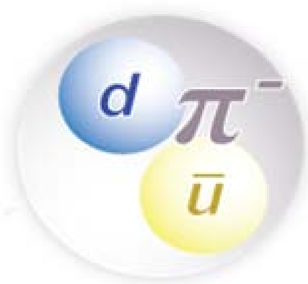
Physics at hadron scale

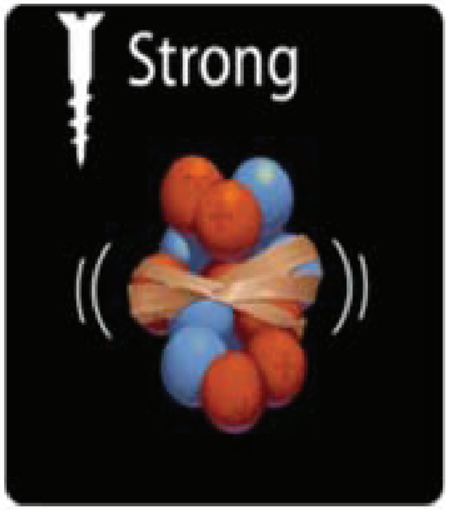
- **Phenomenological model & χEFT $S_{\rm eff}[\phi,\psi]$:** Pseudoscalar, Vector, Scalar,Baryon..
- **Model lacks estimation of error from QCD, use except necessary**
- **Many terms in χEFT for all possible processes** in terms of power counting **!**
- **EOM of each hadron is the stationery Eq for χEFT:**



$$\frac{\delta \mathbf{S}_{\rm eff}[\phi,\psi]}{\delta\phi(\mathbf{x})} = 0 \quad \Rightarrow \quad (\partial^2 + \mathbf{m}_\phi^2)\phi \ + \text{int terms} \ = 0 \qquad \text{boson}$$

$$\frac{\delta \mathbf{S}_{\rm eff}[\phi,\psi]}{\delta\psi(\mathbf{x})} = 0 \quad \Rightarrow \quad (\mathbf{i}\not\partial + \mathbf{m}_\psi)\psi \ + \text{int terms} \ = 0 \qquad \text{fermion}$$

- **Huge No. of LECs in high order χEFT** lost predictability。
- **Need value of LECs in χEFT:**
  - Study and describe hadron physics
  - Examining correctness of the theory



- **QCD action:**
  matter (fermions); gauge (gluons+ghosts)

夸克胶子层次

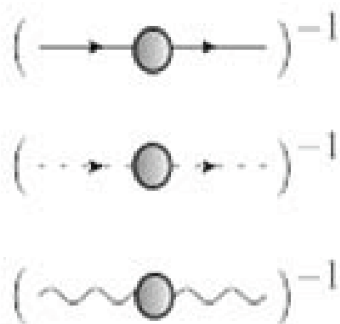
$$S_{\rm QCD} = \int d^4x \, (\mathcal{L}_{\rm I} + \mathcal{L}_{\rm GF+FPG})$$

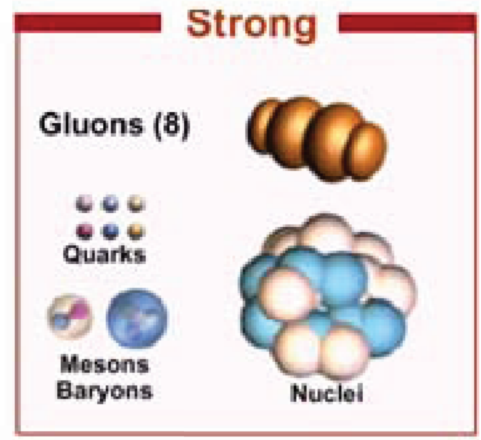
$$\mathcal{L}_{\rm I} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}^i_{\rm f} (i\gamma^\mu D_\mu - m)_{ij} \psi^j_{\rm f}$$

$$\mathcal{L}_{\rm GF+FPG} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$

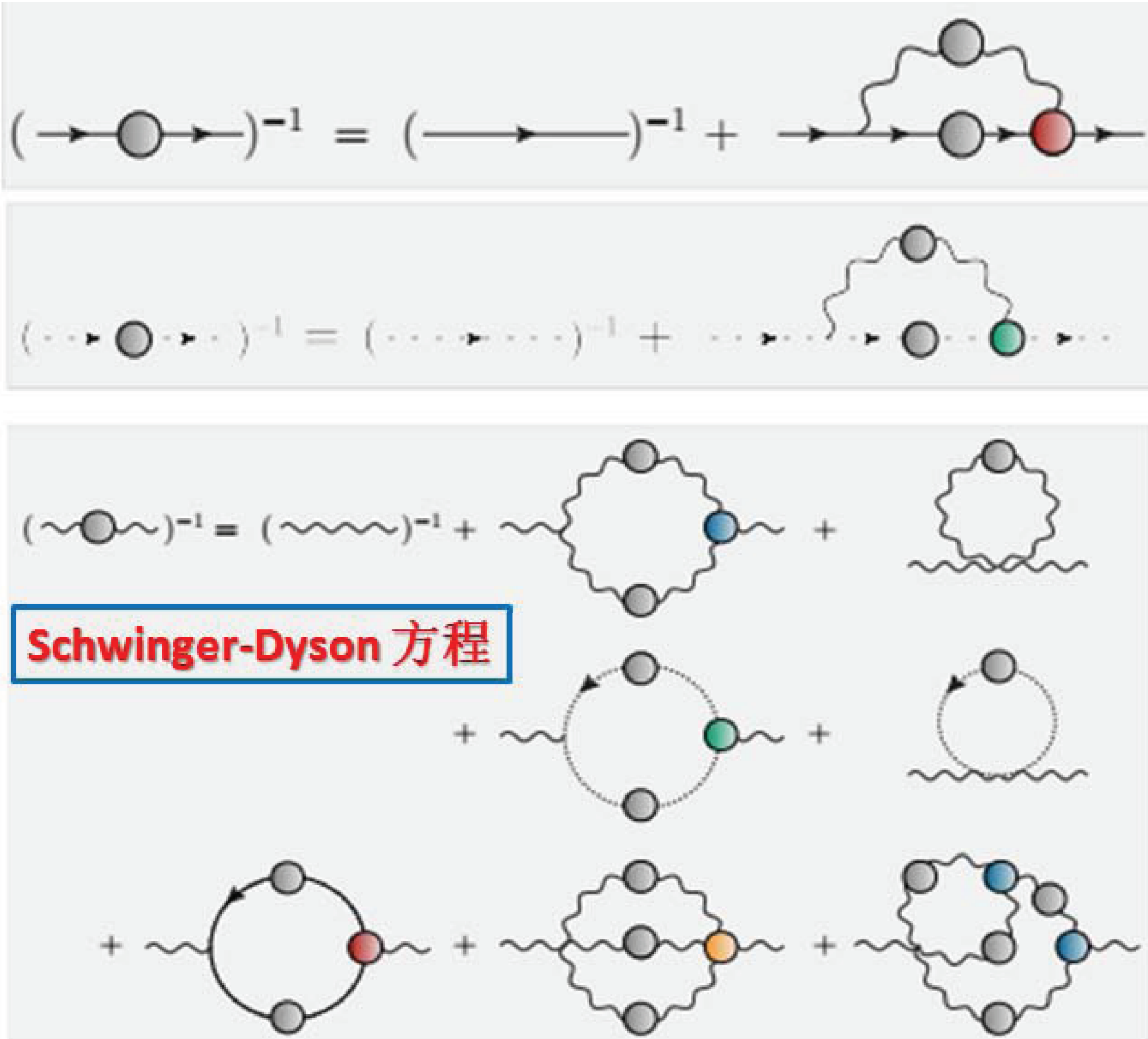
- **Gauge invariant part**
- **Gauge fixing + FP ghost:** BRST exact, does not appear in the spectrum

- **Properties are encoded in Green's functions:**
  Schwinger-Dyson (SD) equations are their quantum eom
  - **Quark propagator:**
  - **Ghost propagator:**
  - **Gluon propagator:**



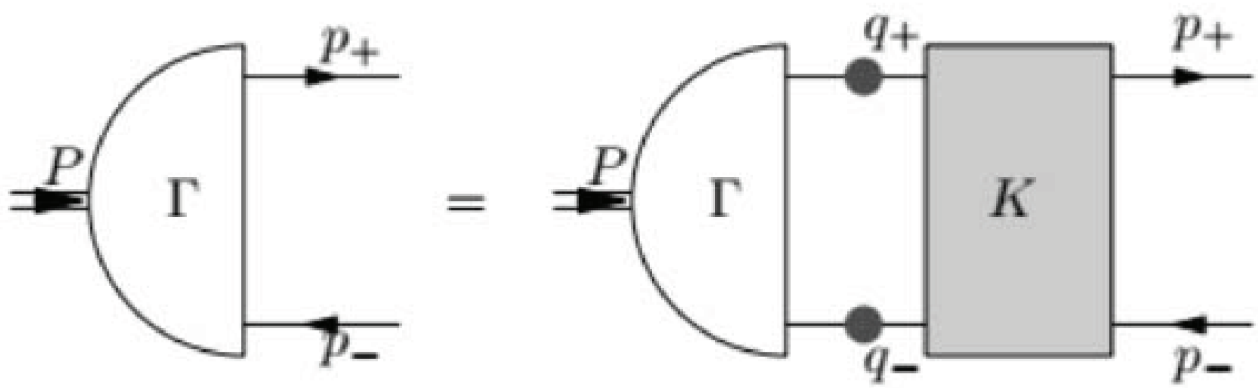


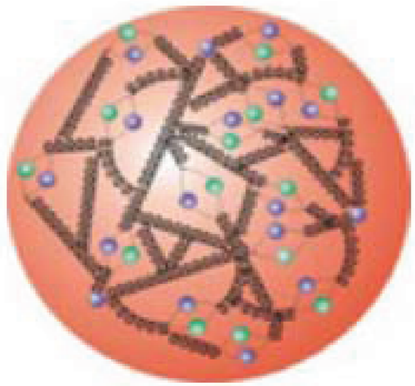
- **Nonperturbative, covariant, IR/UV, light/heavy quarks; but:**
  infinite system of coupled integral equations
  - **Truncations:** possibly gauge invariant
  - **Ansätze/input:** pQCD, lattice, hadron properties

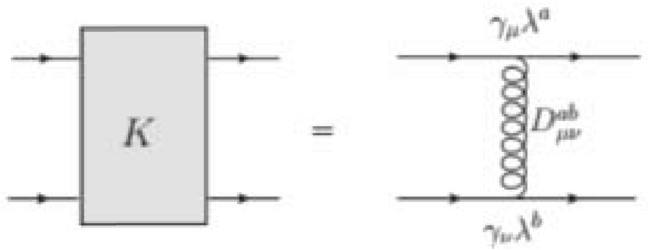


Schwinger-Dyson 方程

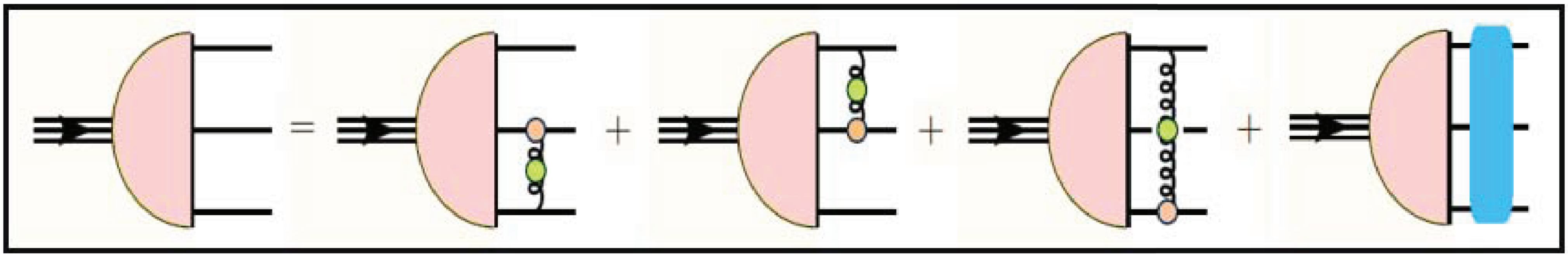
Bethe-Salpeter方程







Faddeev方程



PHYSICAL REVIEW C, VOLUME 60, 055214

Bethe-Salpeter study of vector meson masses and decay constants

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(Received 27 May 1999; published 21 October 1999)

The masses and decay constants of the light vector mesons ρ/ω, ϕ, and $K^\star$ are studied within a ladder-rainbow truncation of the coupled Dyson-Schwinger and Bethe-Salpeter equations of QCD with a model two-point gluon function. The approach is consistent with quark and gluon confinement, reproduces the correct one-loop renormalization group behavior of QCD, generates dynamical chiral symmetry breaking, and preserves the relevant Ward identities. The one phenomenological parameter and two current quark masses are fixed by requiring that the calculated f_π, m_π, and m_K are correct. The resulting f_K is within 3% of the experimental value. For the vector mesons, all eight transverse covariants are included and the dominant ones are identified; the complete angle dependence of the amplitudes is also retained. The calculated values for the masses m_ρ, m_ϕ, and $m_{K^\star}$ are within 5%, while the decay constants f_ρ, f_ϕ, and $f_{K^\star}$ for electromagnetic and leptonic decays are within 10% of the experimental values. [S0556-2813(99)04511-2]

PACS number(s): 14.40.Cs, 24.85.+p, 11.10.St, 12.38.Lg

I. INTRODUCTION

A realistic description of vector mesons at the quark-

example, the axial Ward identity dictates that the chiral limit Bethe-Salpeter (BS) amplitude for a pseudoscalar $\bar{q}q$ bound state in the dominant γ_5 channel is given by $B_0(p^2)/f_P$

and $m(\mu)$ depend on the quark flavor, although we have not indicated this explicitly. However, in our analysis we assume, and employ, a flavor independent renormalization scheme and hence all the renormalization constants are flavor-independent.

A. Meson Bethe-Salpeter equation

The renormalized, homogeneous BSE for a bound state of a quark of flavor a and an antiquark of flavor b having total momentum P is given by

$$\Gamma_M^{ab}(p;P)=\int^{\Lambda}\frac{d^4q}{(2\pi)^4}K(p,q;P)\times S^a(q+\eta P)\Gamma_M^{ab}(q;P)S^b(q-\bar{\eta}P), \tag{4}$$

where $\eta+\bar{\eta}=1$ describes momentum sharing, $\Gamma_M^{ab}(p;P)$ is the BS amplitude, and M specifies the meson type: pseudoscalar, vector, axial-vector, or scalar. In this paper we consider the pseudoscalar and vector amplitudes only. The kernel K operates in the direct product space of color and Dirac

B. Ladder-rainbow truncation

We use a ladder truncation for the BSE

$$K_{tu}^{rs}(p,q;P)\to-\mathcal{G}[(p-q)^2]D_{\mu\nu}^{\text{free}}(p-q)\left(\frac{\lambda^a}{2}\gamma_\mu\right)^{ru}\otimes\left(\frac{\lambda^a}{2}\gamma_\nu\right)^{ts}, \tag{7}$$

which is consistent with a rainbow truncation for the quark DSE

$$Z_1\int_q^{\Lambda}g^2D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma_\nu^a(q,p)\to\int_q^{\Lambda}\mathcal{G}[(p-q)^2]D_{\mu\nu}^{\text{free}}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu. \tag{8}$$

Here $D_{\mu\nu}^{\text{free}}(k)$ is the perturbative gluon propagator in Landau gauge. The model is completely specified once a form is chosen for the "effective coupling" $\mathcal{G}(k^2)$.

TABLE II. Comparison of the results for the vector mesons for the three different parameter sets for the effective interaction, using all eight BS amplitudes (top), and using the five leading BS amplitudes only (bottom).

	ρ		$K^\star$		ϕ	
	m_ρ	f_ρ	m_{K*}	f_{K*}	m_ϕ	f_ϕ
Experiment	0.770	0.216	0.892	0.225	1.020	0.237
All amplitudes F_1-F_8						
$\omega = 0.3$ GeV, $D = 1.20$ GeV^2	0.747	0.197	0.956	0.246	1.088	0.255
$\omega = 0.4$ GeV, $D = 0.93$ GeV^2	0.742	0.207	0.936	0.241	1.072	0.259
$\omega = 0.5$ GeV, $D = 0.79$ GeV^2	0.74	0.215	0.94	0.25	1.08	0.266
Amplitudes $F_1 \ldots F_5$ only						
Maris–Roberts Ref. [10]	0.71		0.95		1.1	
$\omega = 0.3$ GeV, $D = 1.20$ GeV^2	0.737	0.192	0.942	0.235	1.080	0.247
$\omega = 0.4$ GeV, $D = 0.93$ GeV^2	0.729	0.199	0.919	0.229	1.062	0.250
$\omega = 0.5$ GeV, $D = 0.79$ GeV^2	0.731	0.207	0.926	0.237	1.072	0.259

PHYSICAL REVIEW C VOLUME 56, NUMBER 6 DECEMBER 1997

π- and K-meson Bethe-Salpeter amplitudes

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(Received 18 August 1997)

Independent of assumptions about the form of the quark-quark scattering kernel K, we derive the explicit relation between the flavor-nonsinglet pseudoscalar-meson Bethe-Salpeter amplitude Γ_H and the dressed-quark propagator in the chiral limit. In addition to a term proportional to γ_5, Γ_H necessarily contains qualitatively and quantitatively important terms proportional to $\gamma_5\gamma\cdot P$ and $\gamma_5\gamma\cdot k k\cdot P$, where P is the total momentum of the bound state. The axial-vector vertex contains a bound state pole described by Γ_H, whose residue is the leptonic decay constant for the bound state. The pseudoscalar vertex also contains such a bound state pole and, in the chiral limit, the residue of this pole is related to the vacuum quark condensate. The axial-vector Ward-Takahashi identity relates these pole residues, with the Gell-Mann–Oakes–Renner relation a corollary of this identity. The dominant ultraviolet asymptotic behavior of the scalar functions in the meson Bethe-Salpeter amplitude is fully determined by the behavior of the chiral limit quark mass function, and is characteristic of the QCD renormalization group. The rainbow-ladder *Ansatz* for K, with a simple model for the dressed-quark-quark interaction, is used to illustrate and elucidate these general results. The model preserves the one-loop renormalization group structure of QCD. The numerical studies also provide a means of exploring procedures for solving the Bethe-Salpeter equation without a three-dimensional reduction. [S0556-2813(97)04112-5]

PACS number(s): 14.40.Aq, 24.85.+p, 11.10.St, 12.38.Lg

PHYSICAL REVIEW D **80,** 114010 (2009)

Survey of $J = 0, 1$ mesons in a Bethe-Salpeter approach

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(Received 22 September 2009; published 10 December 2009)

The Bethe-Salpeter equation is used to comprehensively study mesons with $J = 0, 1$ and equal-mass constituents for quark masses from the chiral limit to the b-quark mass. The survey contains masses of the ground states in all corresponding J^{PC} channels including those with "exotic" quantum numbers. The emphasis is put on each particular state's sensitivity to the low- and intermediate-momentum, i.e., long-range part of the strong interaction.

DOI: 10.1103/PhysRevD.80.114010 PACS numbers: 14.40.−n, 11.10.St, 12.38.Lg

I. INTRODUCTION

Mesons offer a prime target for studies of various approaches to quantum chromodynamics (QCD), which is widely accepted as the quantum field theory of the strong interaction. While in terms of the number of constitutents their appearance is simple at first glance, mesons provide a broad range of phenomena and challenges to both theory and experiment. On the theoretical side, the key challenge

tion as in corresponding meson studies (for recent advances, see [26–30], and references therein).

In principle, one would aim at a complete, self-consistent solution of all equations, which is equivalent to a solution of the underlying theory. While this spirit can be held up in investigations of certain aspects of the theory (see, e.g. [31,32], and references therein), numerical studies of hadronic observables require a truncation of the

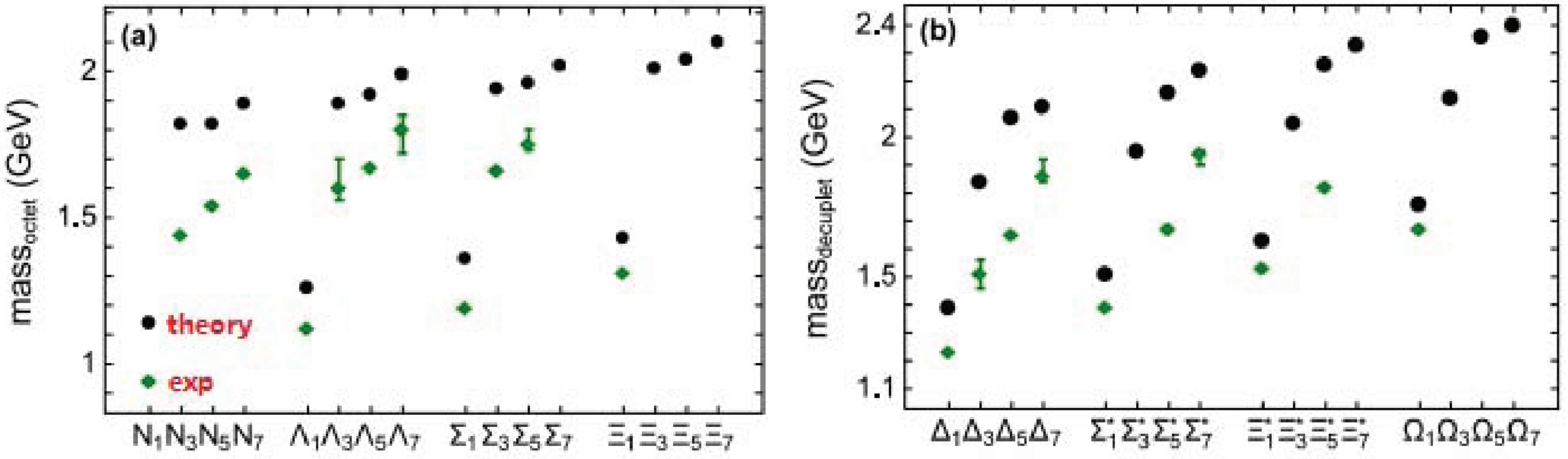
✦ Some properties of mesons in DSE-BSE

Solving the 4-dimenssional covariant B-S equation with the kernel being fixed by the solution of DS equation and flavor symmetry breaking, we obtain

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
"ρ^0"	0.7755	0.7704	-0.66
$\rho^\pm$	0.7755	0.7755	0
"ω"	0.7827	0.7806	-0.27
$K^{*\pm}$	0.8917	0.8915	-0.02
K^{*0}	0.8960	0.8969	0.10
ϕ	1.0195	1.0195	0
D^{*0}	2.0067	1.8321	-8.7
$D^{*\pm}$	2.0100	1.8387	-8.5
$D_s^{*\pm}$	2.1120	1.9871	-5.9
J/ψ	3.0969	3.0969	0
$B^{*\pm}$		4.8543	
B^{*0}		4.8613	
B_s^{*0}		5.0191	
$B_c^{*\pm}$		6.2047	
Υ	9.4603	9.4603	0

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
π^0	0.13498	0.13460	-0.3
$\pi^\pm$	0.13957	0.13499	-3.3
$K^\pm$	0.49368	0.41703	-15.5
K^0	0.49765	0.42662	-14.3
η	0.54751	0.45499	-16.9
η'	0.95778	0.91960	-4.0
D^0	1.8645	1.6195	-13.1
$D^\pm$	1.8693	1.6270	-13.0
$D_s^\pm$	1.9682	1.7938	-8.9
η_c	2.9804	3.0171	1.2
$B^\pm$	5.2790	4.7747	-9.6
B^0	5.2794	4.7819	-9.4
B_s^0	5.3675	4.9430	-7.9
$B_c^\pm$	6.286	6.1505	-2.2
η_b	9.300	9.4438	1.5

(L. Chang, Y. X. Liu, C. D. Roberts, et al., Phys. Rev. C 76, 045203 (2007))



PHYSICAL REVIEW C **96**, 015208 (2017)

Parity partners in the baryon resonance spectrum

Ya Lu,[1,*] Chen Chen,[2,†] Craig D. Roberts,[3,‡] Jorge Segovia,[4,§] Shu-Sheng Xu,[1,∥] and Hong-Shi Zong[1,5,¶]

(Received 10 May 2017; published 28 July 2017)

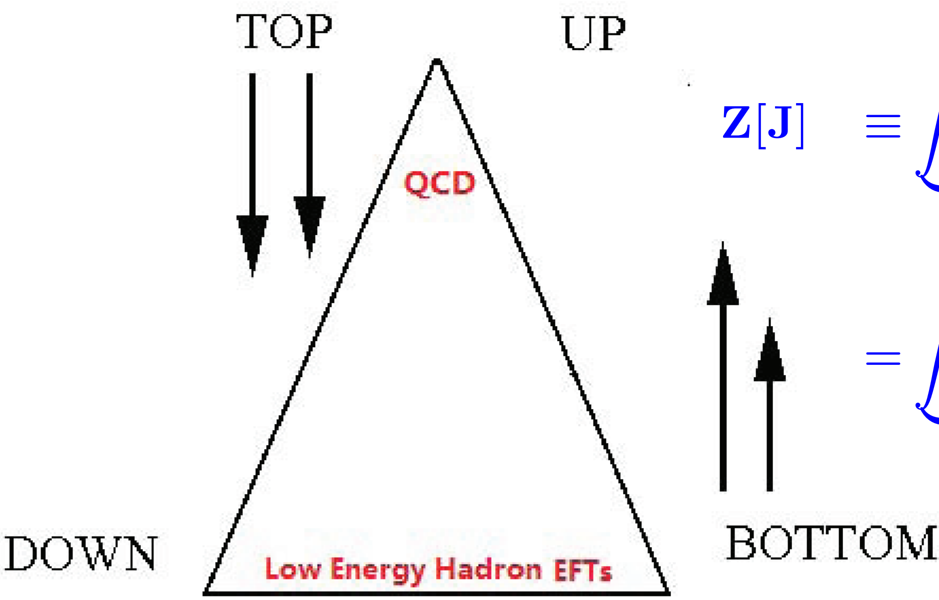
We describe a calculation of the spectrum of flavor-SU(3) octet and decuplet baryons, their parity partners, and the radial excitations of these systems, made using a symmetry-preserving treatment of a vector × vector contact interaction as the foundation for the relevant few-body equations. Dynamical chiral symmetry breaking generates nonpointlike diquarks within these baryons and hence, using the contact interaction, flavor-antitriplet scalar, pseudoscalar, vector, and flavor-sextet axial-vector quark-quark correlations can all play active roles. The model yields reasonable masses for all systems studied and Faddeev amplitudes for ground states and associated parity partners that sketch a realistic picture of their internal structure: ground-state, even-parity baryons are constituted, almost exclusively, from like-parity diquark correlations, but orbital angular momentum plays an important role in the rest-frame wave functions of odd-parity baryons, whose Faddeev amplitudes are dominated by odd-parity diquarks.

DOI: 10.1103/PhysRevC.96.015208

Why talk so much on SD/BS/Faddeev Eqs

- They are QCD non-perturbative bound state equations cannot get rid of them
- They are quark-gluon level first principle determination of hadrons
- Any other equations are all simplified version of them include all pheno models
- Hadrons phenomenologically are described by effective Lagrangian
- Lagrangian Eq of EFFLs leads hadron equation of motions
- They are hadron level phenomenological EOMs
- Need a formalism combining bound state Eqs and EFFLs & their EOMs together
- We build such a formalism based QCD first principle path integral

QCD First Principle Calculation



$$\mathbf{Z[J]} \equiv \underbrace{\int \mathcal{D}\mathbf{G}\mathcal{D}\mathbf{\Psi}\mathcal{D}\overline{\mathbf{\Psi}}\mathcal{D}\mathbf{q}\mathcal{D}\overline{\mathbf{q}}\ \mathbf{e}^{\mathbf{i}\int \mathbf{d^4x}[\mathcal{L}_{\rm QCD}(\mathbf{q},\mathbf{\Psi},\mathbf{G})+\mathbf{F}(\mathbf{q},\mathbf{G},\mathbf{J})]}}_{\mathbf{G},\mathbf{q},\mathbf{\Psi}:\ \text{gluon,light \& heavy quark fields in QCD}}$$

$$= \underbrace{\int \mathcal{D}\phi\ \mathbf{e}^{\mathbf{i}\mathbf{S}_{\rm eff}[\phi,\mathbf{J}]}}_{\phi:\ \text{fields in EFT}} \overset{S_{\rm eff}\propto N_c}{=\!=\!=\!=\!=} e^{iS_{\rm eff}[\phi_c,\mathrm{J}]+\cdots} \qquad \frac{\partial S_{\rm eff}[\phi_c,\mathrm{J}]}{\partial\phi_c} = 0$$

$$\underbrace{\mathbf{e}^{\mathbf{i}\mathbf{S}_{\rm eff}[\phi,\mathbf{J}]} = \int \mathcal{D}\mathbf{G}\mathcal{D}\mathbf{\Psi}\mathcal{D}\overline{\mathbf{\Psi}}\mathcal{D}\mathbf{q}\mathcal{D}\overline{\mathbf{q}}\ \delta[\phi - \mathbf{H}(\mathbf{q},\mathbf{G})]\ \mathbf{e}^{\mathbf{i}\int \mathbf{d^4x}[\mathcal{L}_{\rm QCD}(\mathbf{q},\mathbf{\Psi},\mathbf{G})+\mathbf{F}(\mathbf{q},\mathbf{G},\mathbf{J})]}}_{\text{path integral representation at quark-gluon level for hadron EFT, if integratable: compact EFT can be used at any scale}}$$

Low Energy Hadron EFTs & Models

Lattice QCD Method To Study Hadron Mass Is Not Correct

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(Dated: July 12, 2018)

Abstract

Since the numerical path integration in the lattice QCD involves quark and gluon fields (not hadron fields) the lattice QCD cannot calculate any hadronic observable. Because of this reason the hadronic properties are extracted in the lattice QCD method by inserting complete set of hadron states $\sum_n |n><n| = 1$ in between the partonic operators by assuming $H_{\rm QCD}|n>= E_n|n>$ where E_n is the energy of the hadron. However, in this paper we find $H_{\rm QCD}|n>\neq E_n|n>$ because the QCD hamiltonian H_{QCD} is unphysical but the E_n and $|n>$ of the hadron are physical. We show that this is consistent with $E_{\rm QCD}(t) =< n|H_{QCD}|n>\neq E_n$ due to non-zero energy flux $E_{\rm flux}(t)$ in QCD because of confinement involving non-perturbative QCD. Hence we find that the lattice QCD method to study hadron mass is not correct.

PACS numbers: 12.38.Aw, 12.38.Gc, 12.38.Lg, 11.30.Cp

Tsinghua University

University of Chinese Academy of Sciences

early

Anomaly Approach

Quark flavor anomaly can induce Chiral Lagrangian !

$$S_{\rm eff}\Big|_{\text{anomaly approach}} = -i\ln\frac{\int\mathcal{D}\psi_\Omega\mathcal{D}\overline{\psi}_\Omega\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}}{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}} = i\ln\frac{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J_\Omega)\psi}}{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}}$$

$$= -i\ln\mathrm{Det}[i\partial\!\!\!/+J_\Omega]+i\ln\mathrm{Det}[i\partial\!\!\!/+J]$$

Lead WZW anomaly terms and **nonzero CL LECs:** $8L_1=4L_2=-2L_3=\underbrace{24L_7=-8L_8}_{\text{wrong signs}}=L_9=-2L_{10}=\frac{N_c}{48\pi^2}$

J. Balog, Phys.Lett.149B,197 1984; A.A.Andrianov and L.Bonora, Nucl.Phys.B233,232 1984; A.A.Andrianov,Phys.Lett.B157,425 1985;
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R.F.Alvarez-Estrada and A.Gomez Nicola,Phys.Lett.B355,288 1995; D.Espriu,E.De Rafael, and J.Taron, Nucl.Phys.B345,22 1990

- **Difficult to obtain SCSB** $F_0^2\propto 0,\ \Lambda^2$
- **Non trivial LECs** $_{\text{CL}\neq 0}$ **in absence of color interaction** $\alpha_s=0$
- **Some p^4 order coefficients have wrong signs**
- **No or divergent higher order terms**

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专　　业：理论物理
研 究 生：王　青
指导教师：张　礼　教授
　　　　　邝宇平　教授

一九八九年五月

TUTP-89/29

DERIVATION OF LOW ENERGY EFFECTIVE ACTION FOR MESONS FROM QCD *

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3. Institute of Theoretical Physics, Academia Sinica, Beijing, 100080, China

ABSTRACT

Low energy effective action for mesons is derived from the fundamental theory of QCD in different approaches. The obtained effective action is of the form of that in the chiral σ-model with vector and axial-vector mesons included but with fewer free parameters. Some of its consequences are discussed.

* Talk presented by Q.Wang at the Workshop on Weak Interactions and CP Violation, Aug.1989, Inst. High Energy Phys., Academia Sinica, Beijing, China. This work is supported by the National Natural Science Foundation of China

** Mailing Address

1

TUIMP-TH-92/48

AN ATTEMPT TO CALCULATE THE EFFECTIVE LAGRANGIAN FOR LOW LYING PSEUDOSCALAR MESONS FROM QCD STRONG COUPLING EXPANSION[1]

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CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China

and

Institute of Morden Physics, Tsinghua University, Beijing 100084, China[2]

and

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ABSTRACT

An attempt to derive the effective Lagrangian for low lying pseudoscalar mesons is given in QCD strong coupling expansion based on the idea of the effective field theory with a physical cut-off Λ.This theory provides the information about chiral symmetry breaking, and the quark condensates are calculable. The obtained effective Lagrangian contains the exact Wess-Zumino-Witten term and the complete Gasser-Leutwyler chiral Lagrangian with all the coefficients $F_0,B_0,L_1,L_2,\cdots,L_{10},H_1$ and H_2 given analytically as functions of the two fundamental parameters Λ and g_s (effective coupling constant in the cut-off QCD theory). Λ and g_s are then determined by taking the data of m_π and m_k as inputs. Up to order-$1/g_s^2$ contributions, the calculated m_η,F_π,F_k,F_η, quark condensates, pion- pion scattering lengths,and pseudoscalar-meson form factors are all in reasonable agreement with experiments.

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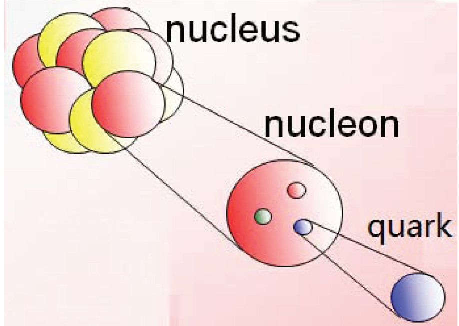
Derivation with approximations ?

清華大學 Tsinghua University　　中国科学院大学 University of Chinese Academy of Sciences

History of our first principle derivation

- Above PS meson, vector, scalar, baryon,....



- After long various prehistorical explorations

  anomaly approach, massive gluon...

- Exact derivation only succeed in PS mesons PRD61,54011(2000)
- Soon generalize DERIVATION to vector meson CTP34,519(2000); η' CTP34,683(2000); Large Nc
- Free dimensional parameter appear, leading arbitrary ρ mass
- Unsuccessful try for many years to solve problem, searching help
- by 2015 Find the key, original paper right! except the understandings

PHYSICAL REVIEW D, VOLUME 61, 054011

Derivation of the effective chiral Lagrangian for pseudoscalar mesons from QCD

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We formally derive the chiral Lagrangian for low lying pseudoscalar mesons from the first principles of QCD considering the contributions from the normal part of the theory without taking an approximation. The derivation is based on the standard generating functional of QCD in the path integral formalism. The gluon-field integration is formally carried out by expressing the result in terms of the physical Green's functions of the gluon. To integrate over the quark field, we introduce a bilocal auxiliary field $\Phi(x,y)$ representing the mesons. We then develop a consistent way of extracting the local pseudoscalar degree of freedom $U(x)$ in $\Phi(x,y)$ and integrating out the rest degrees of freedom such that the complete pseudoscalar degree of freedom resides in $U(x)$. With certain techniques, we work out the explicit $U(x)$ dependence of the effective action up to the p^4 terms in the momentum expansion, which leads to the desired chiral Lagrangian in which all the coefficients contributed from the normal part of the theory are expressed in terms of certain quark Green's functions in QCD. Together with the exsisting QCD formulas for the anomaly contributions, the present results lead to the complete effective chiral Lagrangian for pseudoscalar mesons. The final result can be regarded as the fundamental QCD definition of the coefficients in the chiral Lagrangian. The relation between the present QCD definition of the p^2-order coefficient F_0^2 and the well-known appoximate result given by Pagels and Stokar is discussed.

Commun. Theor. Phys. (Beijing, China) **34** (2000) pp 519–530

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Derivation of Effective Chiral Lagrangian Involving PSGB and Vector Bosons from QCD*

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Abstract *The effective chiral Lagrangian for a matter field content consisting of pseudo-scalar Goldstone bosons and vector bosons (with hidden symmetry) is derived from the underlying QCD theory. No approximations are made. All the free parameters of the effective chiral Lagrangian are expressed in terms of QCD-based Green's functions. These may be regarded as the QCD definitions of these Lagrangian coefficients.*

PACS numbers: 11.30.Rd, 12.38.Aw, 12.38.Lg, 12.39.Fe
Key words: effective chiral Lagrangian, vector boson, QCD

I. Introduction

Due to its nonperturbative nature, studying low energy hadron physics from the perspective of QCD has long been a difficult undertaking. Thus, in place of QCD, research into this low energy realm is often based on more empirical effective chiral Lagrangian (ECL)

Commun. Theor. Phys. (Beijing, China) **34** (2000) pp 683–704

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Derivation of Effective Chiral Lagrangian for the Whole Nonet Pseudo-Scalar Goldstone Bosons from QCD*

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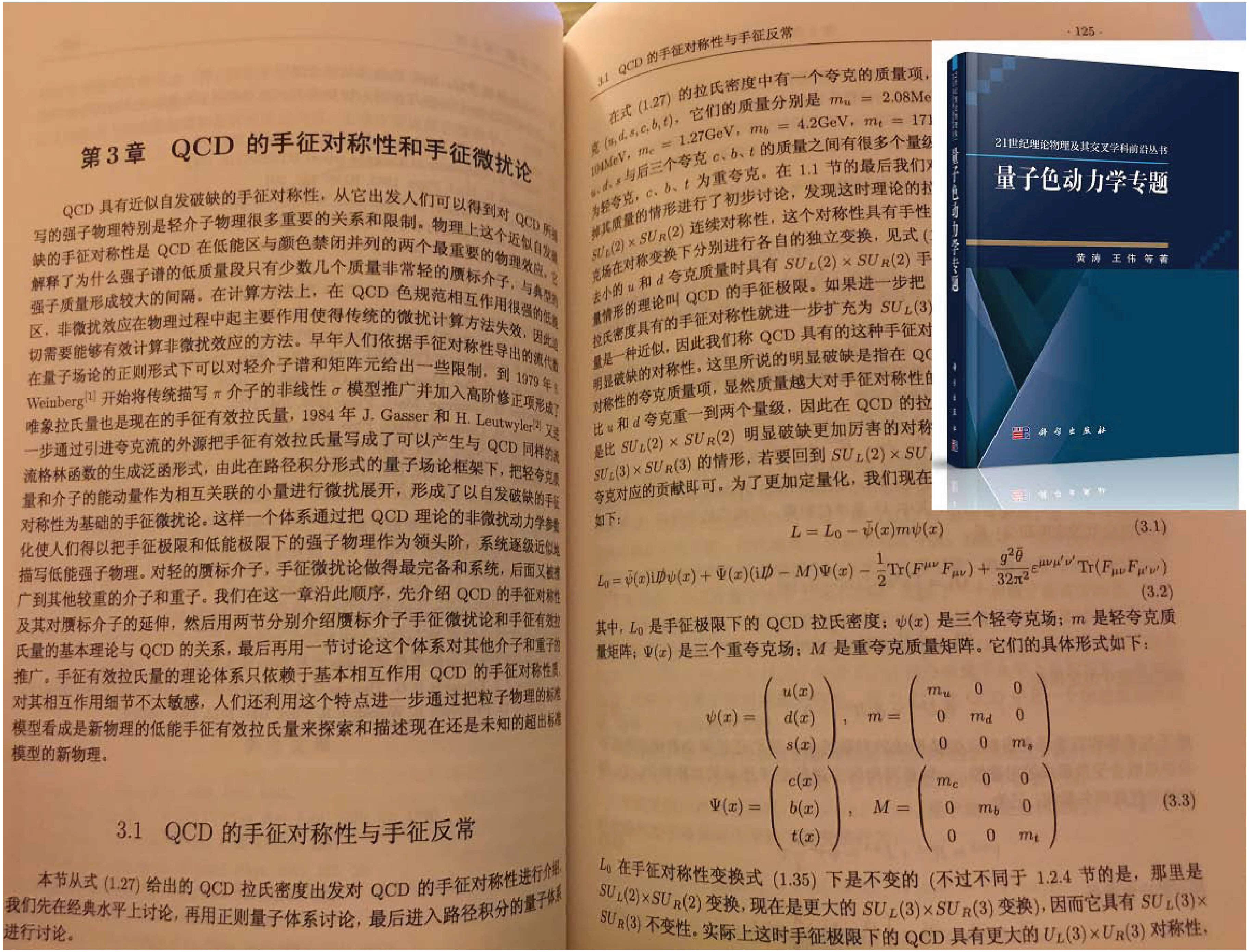
(Received March 15, 2000)

Abstract *The effective chiral Lagrangian derived from underlying QCD for pseudo-scalar Goldstone bosons has been generalized to involve the whole nonet pseudo-Goldstone bosons, no approximation is made in the derivation. The formulation offers general QCD definitions for the coefficients in effective chiral Lagrangian.*

PACS numbers: 11.30.Rd, 12.38.Aw, 12.38.Lg, 12.39.Fe
Key words: chiral Lagrangian, nonet Goldstone bosons, QCD

I. Introduction

At low energies, the effective interaction among pseudo-scalar Goldstone bosons (PSGB) induced from fundamental QCD can be described by an effective chiral Lagrangian (ECL).[1,2] This Lagrangian depends on a number of coupling coefficients which are not determined by symmetry requirements and must be taken as experimental inputs at level of phenomenology.



第 3 章 QCD 的手征对称性和手征微扰论

QCD 具有近似自发破缺的手征对称性，从它出发人们可以得到对 QCD 所描写的强子物理特别是轻介子物理很多重要的关系和限制。物理上这个近似自发破缺的手征对称性是 QCD 在低能区与颜色禁闭并列的两个最重要的物理效应，它解释了为什么强子谱的低质量段只有少数几个质量非常轻的赝标介子，与典型的强子质量形成较大的间隔。在计算方法上，在 QCD 色规范相互作用很强的低能区，非微扰效应在物理过程中起主要作用使得传统的微扰计算方法失效，因此迫切需要能够有效计算非微扰效应的方法。早年人们依据手征对称性导出的流代数在量子场论的正则形式下可以对轻介子谱和矩阵元给出一些限制，到 1979 年 Weinberg[1] 开始将传统描写 π 介子的非线性 σ 模型推广并加入高阶修正项形成了唯象拉氏量也是现在的手征有效拉氏量，1984 年 J. Gasser 和 H. Leutwyler[2] 又进一步通过引进夸克流的外源把手征有效拉氏量写成了可以产生与 QCD 同样的流格林函数的生成泛函形式，由此在路径积分形式的量子场论框架下，把轻夸克质量和介子的能动量作为相互关联的小量进行微扰展开，形成了以自发破缺的手征对称性为基础的手征微扰论。这样一个体系通过把 QCD 理论的非微扰动力学参量化使人们得以把手征极限和低能极限下的强子物理作为领头阶，系统逐级近似地描写低能强子物理。对轻的赝标介子，手征微扰论做得最完备和系统，后面又被推广到其他较重的介子和重子。我们在这一章沿此顺序，先介绍 QCD 的手征对称性及其对赝标介子的延伸，然后用两节分别介绍赝标介子手征微扰论和手征有效拉氏量的基本理论与 QCD 的关系，最后再用一节讨论这个体系对其他介子和重子的推广。手征有效拉氏量的理论体系只依赖于基本相互作用 QCD 的手征对称性而对其相互作用细节不太敏感，人们还利用这个特点进一步通过把粒子物理的标准模型看成是新物理的低能手征有效拉氏量来探索和描述现在还是未知的超出标准模型的新物理。

3.1 QCD 的手征对称性与手征反常

本节从式 (1.27) 给出的 QCD 拉氏密度出发对 QCD 的手征对称性进行介绍。我们先在经典水平上讨论，再用正则量子体系讨论，最后进入路径积分的量子体系进行讨论。

在式 (1.27) 的拉氏密度中有一个夸克的质量项，
克 (u,d,s,c,b,t)，它们的质量分别是 $m_u = 2.08$Me
104MeV，$m_c = 1.27$GeV，$m_b = 4.2$GeV，$m_t = 17$
u、d、s 与后三个夸克 c、b、t 的质量之间有很多个量级
为轻夸克，c、b、t 为重夸克。在 1.1 节的最后我们对
掉其质量的情形进行了初步讨论，发现这时理论的拉
$SU_L(2) \times SU_R(2)$ 连续对称性，这个对称性具有手性
克场在对称变换下分别进行各自的独立变换，见式 (
去小的 u 和 d 夸克质量时具有 $SU_L(2) \times SU_R(2)$ 手
量情形的理论叫 QCD 的手征极限。如果进一步把
拉氏密度具有的手征对称性就进一步扩充为 $SU_L(3)$
量是一种近似，因此我们称 QCD 具有的这种手征对
明显破缺的对称性。这里所说的明显破缺是指在 QC
对称性的夸克质量项，显然质量越大对手征对称性的
比 u 和 d 夸克重一到两个量级，因此在 QCD 的拉
是比 $SU_L(2) \times SU_R(2)$ 明显破缺更加厉害的对称
$SU_L(3) \times SU_R(3)$ 的情形，若要回到 $SU_L(2) \times SU$
夸克对应的贡献即可。为了更加定量化，我们现在
如下：

$$L = L_0 - \bar{\psi}(x) m \psi(x) \tag{3.1}$$

$$L_0 = \bar{\psi}(x) i \not{D} \psi(x) + \bar{\Psi}(x)(i\not{D} - M)\Psi(x) - \frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{g^2\bar{\theta}}{32\pi^2}\varepsilon^{\mu\nu\mu'\nu'}\mathrm{Tr}(F_{\mu\nu}F_{\mu'\nu'}) \tag{3.2}$$

其中，L_0 是手征极限下的 QCD 拉氏密度；$\psi(x)$ 是三个轻夸克场；m 是轻夸克质量矩阵；$\Psi(x)$ 是三个重夸克场；M 是重夸克质量矩阵。它们的具体形式如下：

$$\psi(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}, \quad m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$
$$\Psi(x) = \begin{pmatrix} c(x) \\ b(x) \\ t(x) \end{pmatrix}, \quad M = \begin{pmatrix} m_c & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{3.3}$$

L_0 在手征对称性变换式 (1.35) 下是不变的 (不过不同于 1.2.4 节的是，那里是 $SU_L(2)\times SU_R(2)$ 变换，现在是更大的 $SU_L(3)\times SU_R(3)$ 变换)，因而它具有 $SU_L(3)\times SU_R(3)$ 不变性。实际上这时手征极限下的 QCD 具有更大的 $U_L(3)\times U_R(3)$ 对称性，

Key Difference between Pseudoscalar & Other Mesons

- Pseudoscalar mesons are Pseudo Goldstone Bosons of SCSB
- Take χPT, PS mesons become exact Goldstone bosons of SCSB
- They are massless, their fields are taken as chiral rotation angles
- EFT of PS mesons has no mass and self-interaction terms
- Other mesons are all massive with some mass gap with PS mesons
- Their EFTs have mass and self-interaction terms
- They especially mass are determined by BSE or Faddeev Eq special feature to PS
- How can derive a EFT with right mass and self-interaction terms ?

without knowing how to solve BSE

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Derivation of the effective chiral Lagrangian for pseudoscalar, scalar, vector, and axial-vector mesons from QCD

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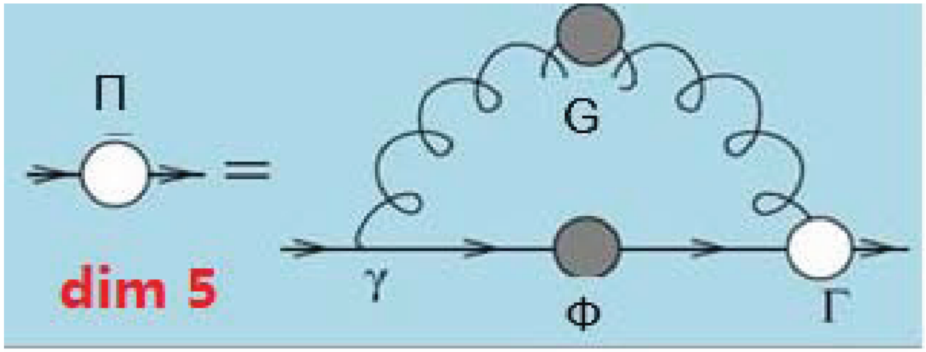
A previous formal derivation of the effective chiral Lagrangian for low-lying pseudoscalar mesons from first-principles QCD without approximations [Q. Wang, Y.-P. Kuang, X.-L. Wang, and M. Xiao, Phys. Rev. D **61**, 054011 (2000)] is generalized to further include scalar, vector, and axial-vector mesons. In the large N_c limit and with an Abelian approximation, we show that the properties of the newly added mesons in our formalism are determined by the corresponding underlying fundamental homogeneous Bethe-Salpeter equation in the ladder approximation, which yields the equations of motion for the scalar, vector, and axial-vector meson fields at the level of an effective chiral Lagrangian. The masses appearing in the equations of motion of the meson fields are those determined by the corresponding Bethe-Salpeter equation.

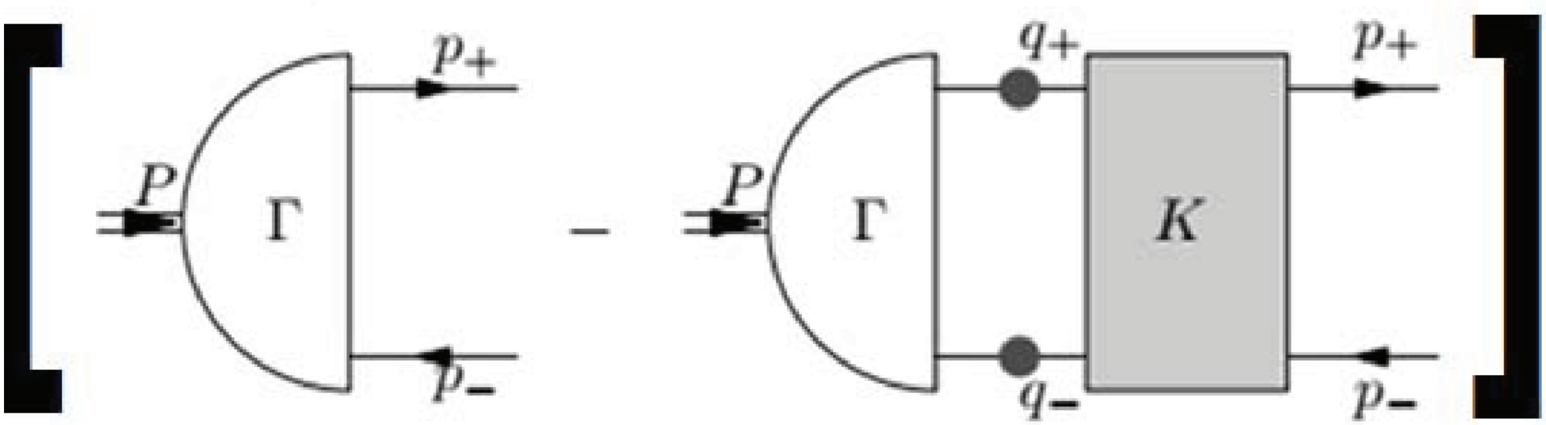
DOI: 10.1103/PhysRevD.95.074012



Extract out effective meson field from coincidence limit of quark self energy Π

$$\int \mathcal{D}\phi\,\delta\Big[\phi^{(a\xi)(b\zeta)}(x)+\frac{1}{\mu^4}\{[e^{-\frac{i\theta(x)}{2N_f}}\xi_L(x)P_R+e^{\frac{i\theta(x)}{2N_f}}\xi_R(x)P_L]$$
$$\times\Pi(x,x)[e^{-\frac{i\theta(x)}{2N_f}}\xi_R^\dagger(x)P_R+e^{\frac{i\theta(x)}{2N_f}}\xi_L^\dagger(x)P_L]\}^{(a\xi')(b\zeta')}P^{\xi\zeta}_{\xi\zeta}\Big].$$

free dimentional parameter $\times \dfrac{\partial S_{\mathrm{eff}}}{\partial\phi(\mathbf{x})}\delta(x-y)+$



$\times\,\phi = 0$

No further investigations on mesons; Similar result for baryons，preparing paper

Complexities in the QCD Derivation of Baryon EFT

- **Baryon apparently is more interesting and important than mesons**
- **But in QCD, baryon is consists of Nc quarks which depend on Nc**

  meson composed of quark and anti-quark, independent of Nc

- **Nc counting of baryon is highly nontrivial, needs special treatment**

  "Baryons in the 1/N Expansion" E.Witten，Nucl.Phys. B160,57(1979)

- **Nc counting of Faddeev Eq and EFT** itself **are different** due to Nc dep baryon fields

  $\phi\partial^2\phi+\frac{1}{N_c}\phi^{N_c}$

- **Choose to keep SDE/BSE & Faddeev Eq at the same**

  EFT is no longer all at the same Nc orders

Steps for First Principle Derivation for Hadron EFT

- Integrate out gluon fields, obtain nonlocal multi-quark theory
- Integrate in bilocal meson fields, obtain bilocal meson fields theory

an alternative path integral expression of QCD

DSE, BSE, Faddeev Eq all can be derived from it

****

- Integrate in hadron fields, obtain EFT for hadrons $\Leftarrow$ Formal Derivation
- Obtain hadron EFT$\&$ corresponding large $\mathrm{N_c}$ BSE or Faddeev Eq

this now achieved successfully for all low energy hadrons

****

- Solve BSE or Faddeev Eq to fix hadron EFT order by orders or nonlocally

only achieved for p^2 order PS mesons; more simplified ansatz are used to compute all LECs up to p^6 order of PS mesons

$$e^{iZ[J,\bar\theta]}=\int\mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar\psi\mathcal{D}\psi\ e^{i\int d^4x\ [\mathcal{L}_{QCD}+\bar\psi J\psi]}=\int\mathcal{D}\bar\psi\mathcal{D}\psi\ e^{i\int d^4x\bar\psi(i\partial\!\!\!/+J)\psi}e^{iZ'[\bar\psi\gamma^\mu\frac{\lambda^C_i}{2}\psi,\bar\theta]}$$

$$e^{iZ'[I^\mu_i,\bar\theta]}=\int\mathcal{D}\mathcal{A}_{\mu,i}\ e^{i\int d^4x\ [-gA_{\mu,i}I^\mu_i+\mathcal{L}']}\qquad \mathcal{L}'=-\frac{1}{4}\mathcal{G}_{\mu\nu,i}\mathcal{G}^{\mu\nu}_i+\frac{g^2\bar\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}$$

$$Z'[I^\mu_i,\bar\theta]=\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n\frac{(-i)^ng^n}{n!}\underbrace{G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)I^{\mu_1}_{i_1}(x_1)\cdots I^{\mu_n}_{i_n}(x_n)}_{\to GCM\to NJL}$$

$G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)$ has well defined large N_c limit, except necessary renormalization constant

$$G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n\left[\bar\psi_{f_1\alpha_1}(x_1)(\frac{\lambda^C_{i_1}}{2})_{\alpha_1\beta_1}\gamma^{\mu_1}\psi_{f_1\beta_1}(x_1)\right]\cdots\left[\bar\psi_{f_n\alpha_n}(x_n)(\frac{\lambda^C_{i_n}}{2})_{\alpha_n\beta_n}\gamma^{\mu_n}\psi_{f_n\beta_n}(x_n)\right]$$

$$=\int d^4x_1'\cdots d^4x_n'g^{n-2}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\bar\psi^{\sigma_1}_{\alpha_1}(x_1)\psi^{\rho_1}_{\alpha_1}(x_1')\cdots\bar\psi^{\sigma_n}_{\alpha_n}(x_n)\psi^{\rho_n}_{\alpha_n}(x_n')$$

$$e^{iZ[J,\bar\theta]}=\int\mathcal{D}\bar\psi\mathcal{D}\psi\ \exp\Big\{i\int d^4x\bar\psi(i\partial\!\!\!/+J)\psi+\sum_{n=2}^{\infty}\int d^4x_1d^4x_1'\cdots d^4x_nd^4x_n'$$
$$\times\frac{(-i)^n(g^2)^{n-1}}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\bar\psi^{\sigma_1}_{\alpha_1}(x_1)\psi^{\rho_1}_{\alpha_1}(x_1')\cdots\bar\psi^{\sigma_n}_{\alpha_n}(x_n)\psi^{\rho_n}_{\alpha_n}(x_n')\Big\}$$

$$G^{i_1i_2}_{\mu_1\mu_2}(x_1,x_2)\left[\bar\psi_{f_1\alpha_1}(x_1)(\frac{\lambda^C_{i_1}}{2})_{\alpha_1\beta_1}\gamma_{\mu_1}\psi_{f_1\beta_1}(x_1)\right]\left[\bar\psi_{f_2\alpha_2}(x_2)(\frac{\lambda^C_{i_2}}{2})_{\alpha_2\beta_2}\gamma_{\mu_2}\psi_{f_2\beta_2}(x_2)\right]$$

$$G^{i_1i_2}_{\mu_1\mu_2}(x_1,x_2)=\delta_{i_1i_2}G_{\mu_1\mu_2}(x_1,x_2) \qquad (\lambda^C_{i_1})_{\alpha_1\beta_1}(\lambda^C_{i_1})_{\alpha_2\beta_2}=2\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}-\frac{2}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}$$

$$=\frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)\left[\bar q_{f_1\alpha_1}(x_1)\gamma^{\mu_1}q_{f_1\beta_1}(x_1)\right]\left[\bar q_{f_2\alpha_2}(x_2)\gamma^{\mu_2}q_{f_2\beta_2}(x_2)\right](\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}-\frac{1}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2})$$

$$=\int d^4x' d^4x'_2 G_{\mu_1\mu_2}(x_1,x_2)[-\frac{1}{2}\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)$$

Key is transforming λ^C matrices into $\delta_{\alpha\beta}$!

$$-\frac{1}{2N_c}\gamma^{\mu_1}_{\sigma_1\rho_1}\gamma^{\mu_2}_{\sigma_2\rho_2}\delta(x'_1-x_1)\delta(x'_2-x_2)]\bar q^{\sigma_1}_{\alpha_1}(x_1)q^{\rho_1}_{\alpha_1}(x'_1)\bar q^{\sigma_2}_{\alpha_2}(x_1)q^{\rho_2}_{\alpha_1}(x'_2)$$

$$\overline{G}^{\sigma_1\sigma_2}_{\rho_1\rho_2}(x_1,x'_1,x_2,x'_2)=-\frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)[\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)+\frac{1}{N_c}\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)]$$

$$G^{i_1i_2i_3}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)=gf^{i_1i_2i_3}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)+gd^{i_1i_2i_3}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)$$

$$f^{i_1i_2i_3}=-2i\,\mathrm{tr}\left[[\frac{\lambda^C_{i_1}}{2},\frac{\lambda^C_{i_2}}{2}]\frac{\lambda^C_{i_3}}{2}\right]=\frac{i}{4}\lambda^{i_2}_{\alpha\beta}\lambda^{i_1}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha}-\frac{i}{4}\lambda^{i_1}_{\alpha\beta}\lambda^{i_2}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha} \qquad d^{i_1i_2i_3}=-2i\,\mathrm{tr}\left[\{\frac{\lambda^C_{i_1}}{2},\frac{\lambda^C_{i_2}}{2}\}\frac{\lambda^C_{i_3}}{2}\right]$$

$$f^{i_1i_2i_3}\lambda^{i_1}_{\alpha_1\beta_1}\lambda^{i_2}_{\alpha_2\beta_2}\lambda^{i_3}_{\alpha_3\beta_3}=2i(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3}-\delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_3})$$

$$d^{i_1i_2i_3}\lambda^{i_1}_{\alpha_1\beta_1}\lambda^{i_2}_{\alpha_2\beta_2}\lambda^{i_3}_{\alpha_3\beta_3}=2(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3}+\delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_3}-\frac{2}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_3}\delta_{\alpha_3\beta_2})$$

$$-\frac{2}{N_c}\delta_{\alpha_1\beta_3}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_1}-\frac{2}{N_c}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_3\beta_3}+\frac{4}{N_c^2}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_3})$$

$$\int \mathcal{D}\Phi\ \delta\Big(N_c\Phi^{\sigma\rho}(x,x') - \bar\psi^\sigma_\alpha(x)\psi^\rho_\alpha(x')\Big)$$

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}\Phi\delta\Big(N_c\Phi^{\sigma\rho}(x,x') - \bar\psi^\sigma_\alpha(x)\psi^\rho_\alpha(x')\Big)\exp\Big\{i\int d^4x\bar\psi(i\partial\!\!\!/ + J)\psi$$

$$+N_c\sum_{n=2}^{\infty}\int d^4x_1d^4x_1'\cdots d^4x_nd^4x_n'\frac{(-i)^n(N_cg^2)^{n-1}}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')$$

$$\times\Phi^{\sigma_1\rho_1}(x_1,x_1')\cdots\Phi^{\sigma_n\rho_n}(x_n,x_n')\Big\}$$

$$\delta\Big(N_c\Phi^{\sigma\rho}(x,x') - \bar q^\sigma_\alpha(x)q^\rho_\alpha(x')\Big) = C\int\mathcal{D}\Pi e^{i\int d^4xd^4x'\Pi^{\sigma\rho}(x,x')[N_c\Phi^{\sigma\rho}(x,x')-\bar\psi^\sigma_\alpha(x)\psi^\rho_\alpha(x')]}$$

$$e^{iZ[J,\bar\theta]} = \int\mathcal{D}\Phi\mathcal{D}\Pi\ e^{i\Gamma[J,\Phi,\Pi]}$$

$$\Gamma[J,\Phi,\Pi] = -iN_c\mathrm{Trln}[i\partial\!\!\!/ + J - \Pi] + N_c\int d^4xd^4x'\Phi^{\sigma\rho}(x,x')\Pi^{\sigma\rho}(x,x')$$

$$+N_c\sum_{n=2}^{\infty}\int d^4x_1d^4x_1'\cdots d^4x_nd^4x_n'\frac{(-i)^n(N_cg^2)^{n-1}}{n!}$$

$$\times\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\Phi^{\sigma_1\rho_1}(x_1,x_1')\cdots\Phi^{\sigma_n\rho_n}(x_n,x_n')$$

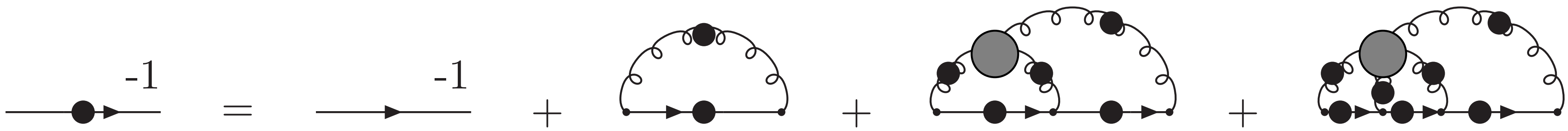
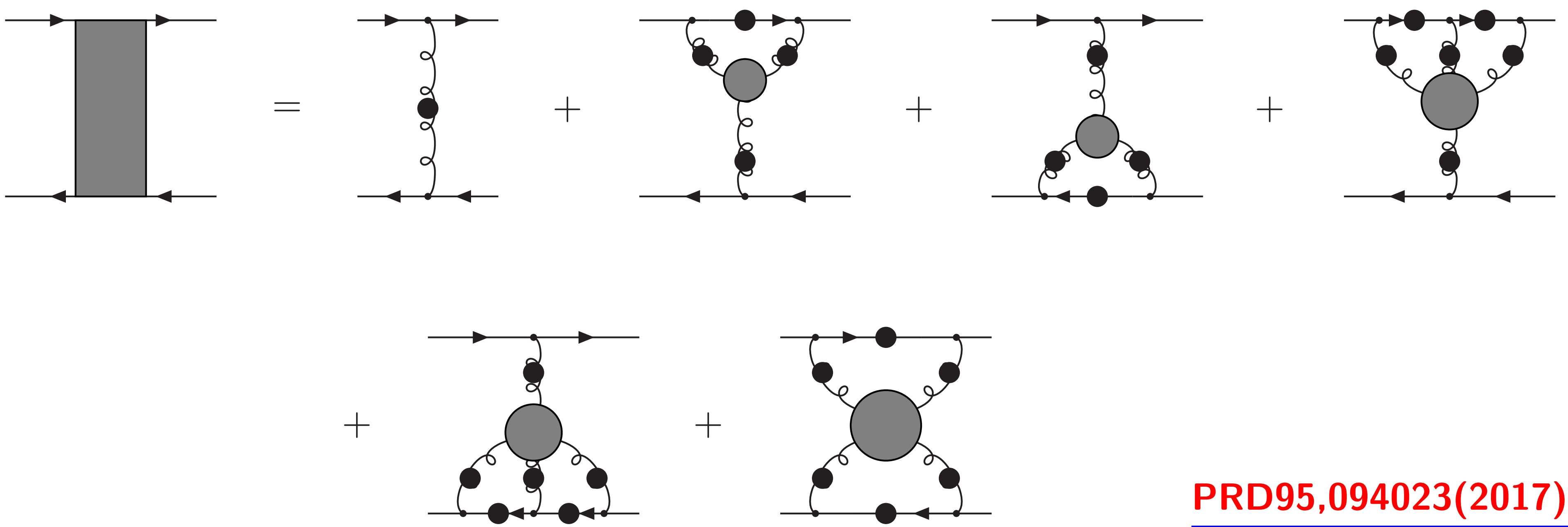


Figure 2: The gap equation up to the NNL-order truncation.



PRD95,094023(2017)

Figure 3: The meson BS kernel up to the NNL-order truncation.

Modern Chiral Effective Theories Industry

http://home.thep.lu.se/ bijnens/chpt/

- Built p^2 LO, p^4 NLO, p^6 NNLO theories for SU(3) PS mesons normal，abnormal；generalized
- Marching to higher energy region Hard pion ChPT...
- S.Z.Jiang,F.J.Ge,Q.Wang, *Full pseudoscalar mesonic chiral Lagrangian at p^6 order under the unitary group*, PRD89,074048(2014)
- With unitarity；Include in other hadrons：scalars、vectors、axial scalars...
- Include in baryons B-ChPT...；Nucleon Multipole moments force；Carbogenesis...；finite ρ, T, V...
- Combined with HQEFT, include in heavy mesons of c、b quarks Heavy Meson ChPT...
- Add in EM interaction include photons、various lepton processes、weak interactions；Lame shift；external B-field...
- Combined with lattice theory Quenched；Partially Quenched；Staggered；Unitarized；Wilson；...
- Apply to EW int...；New physics...；Non-fund rep of quarks Real、Pseudo-Real...；

Tsinghua University

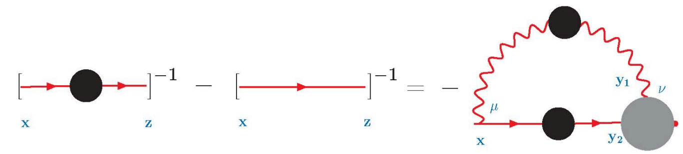
University of Chinese Academy of Sciences

Computation of LECs

- **Self energy of quarks:** $\Sigma(p^2)$

$$\Sigma(p^2) - \frac{3}{2}N_c\int\frac{d^4q}{4\pi^3}\frac{\alpha_s[(p-q)^2]}{(p-q)^2}\frac{\Sigma(q^2)}{q^2+\Sigma^2(q^2)} = 0$$

- $\alpha_s(p^2) = \begin{cases} 7\frac{12\pi}{(11N_c-2N_f)} & \ln\frac{p^2}{\Lambda^2_{\rm QCD}} \le -2 \\ \{7-\frac{4}{5}[2+\ln\frac{p^2}{\Lambda^2_{\rm QCD}}]^2\}\frac{12\pi}{(11N_c-2N_f)} & -2 \le \ln\frac{p^2}{\Lambda^2_{\rm QCD}} \le 0.5 \\ \frac{1}{\ln\frac{p^2}{\Lambda^2_{\rm QCD}}}\frac{12\pi}{(11N_c-2N_f)} & 0.5 \le \ln\frac{p^2}{\Lambda^2_{\rm QCD}} \end{cases}$



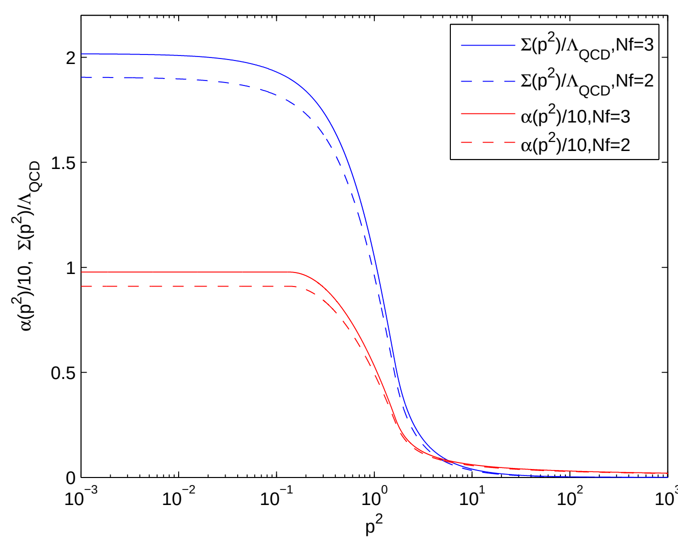
Input $N_c=3, N_f, \Lambda, F_0 = 87$**MeV fix** $\Lambda_{\rm QCD}$

- quark two point function $\underbrace{\Pi^{\sigma\rho}_{\Omega c}(x,y) \approx [\Sigma(\overline{\nabla}^2_x)]^{\sigma\rho}\delta^4(x-y)}$ Σ quark self energy $\overline{\nabla}^\mu_x = \partial^\mu_x - iv^\mu_\Omega(x)$

GND/CQM **PRD66,014019(2002)；PLB532,240(2002)；PLB560,188(2003)**

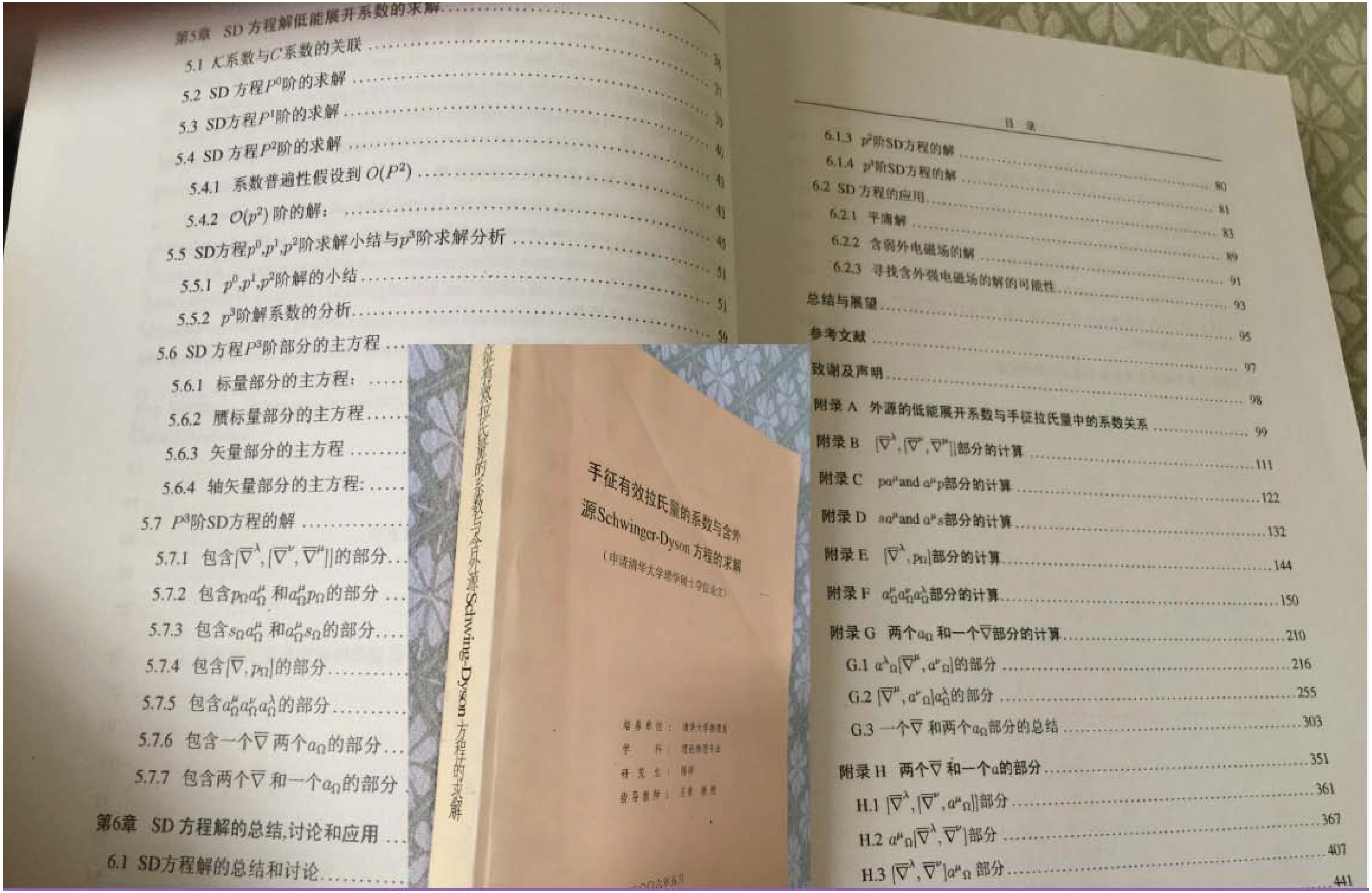
- **Simplest approx，help with computer，obtained all p^6 LECs**

1. **S.Z.Jiang, Y.Zhang, C.Li, Q.Wang PRD81,014001(2010)**
   Computation of the p^6 order chiral Lagrangian coefficients
2. **S.Z.Jiang, Q.Wang PRD81 ,094037(2010)**
   Computation of the p^6 order anomalous chiral Lagrangian



- **Beyond low energy EFT** C.Li, S.Z.Jiang, Q.Wang

Minimal Ward-Takahashi Vertices and Pion Light Cone Distribution Amplitudes from Gauge Invariant, Nonlocal, Dynamical Quark Model **CPL, 30, no8，081101(2013)**



手征有效拉氏量的系数与含外源Schwinger-Dyson 方程的求解
（申请清华大学理学博士学位论文）
手征有效拉氏量的系数与含外源Schwinger-Dyson方程的求解

Summary

- QCD gen func is of an alternative form in terms of bilocal meson fields
- automatically Reproduce non-pert SDE/BSE & lead many applications
- Further integrating in local hadron fields leads EFT for hadrons

  exact QCD first principle derivation is achieved !

- Bound State Eqs and Hadron EOMs live in the same equation

  quark-gluon level BS、Faddeev Meson、Baryon hadron level

- Simplified Ansatz used to compute PS EFT LECs up to p^6 orders
- even for PS mesons Still not solve SDE to achieve fully first principle cal
- Need to go beyond formal der of EFT to perform detail computations