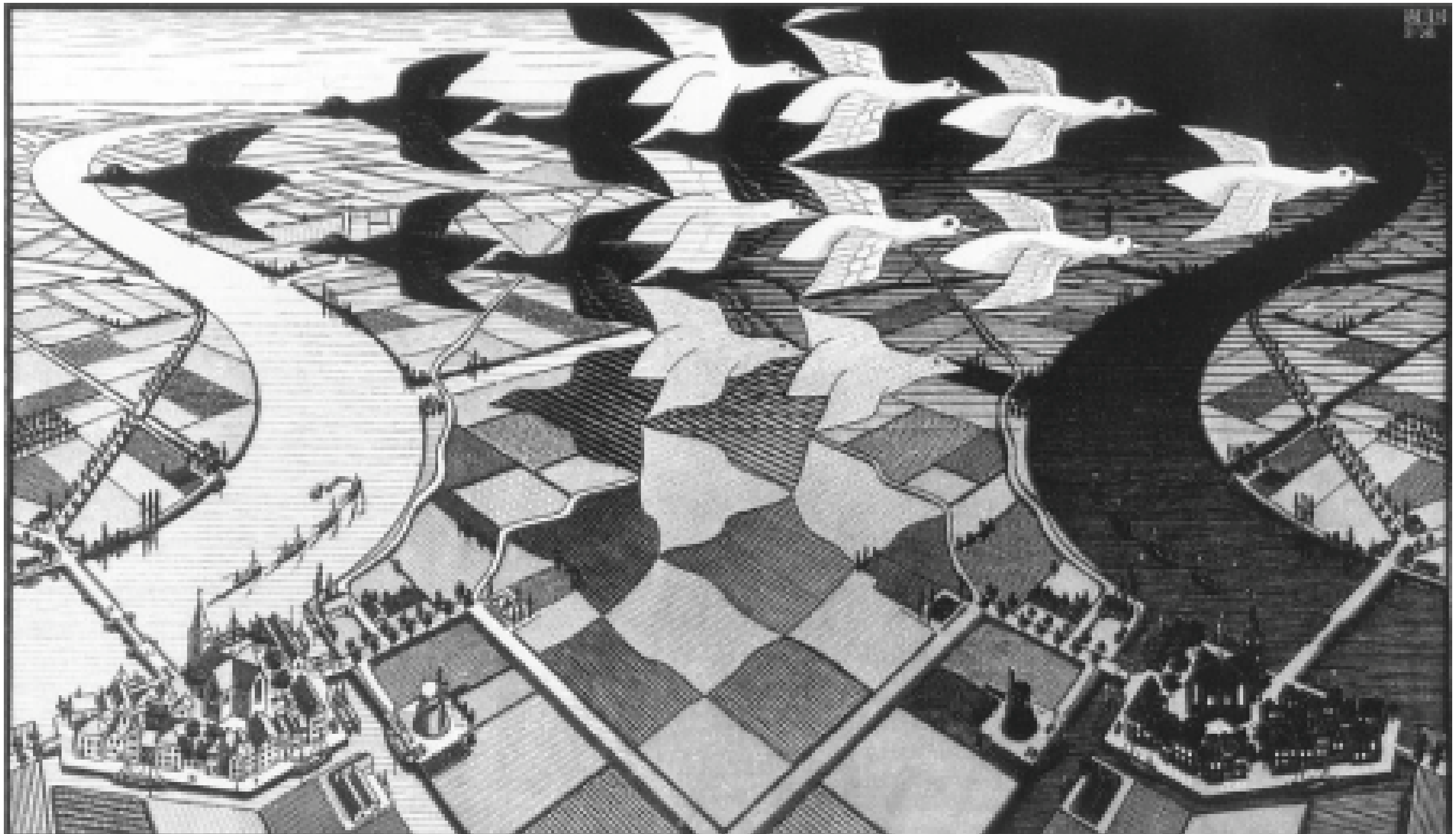
CP Violation

-- Lecture 4 --



M.C. Escher

Stephen Lars Olsen

中国科学院大学

University of Chinese Academy of Sciences

July-August, 2018

Summary of Lecture 3

- K^0 and $\bar{K}^0$ mesons are not mass eigenstates; K_S and K_L are mass eigenstates
- K_L mesons discovered at Brookhaven Laboratory
- a K^0 will oscillate into a $\bar{K}^0$ and vice versa
- neutral K mesons produced in ϕ decay are ~100% "quantum correlated"
- K_S mesons are "regenerated" when a K_L passes through matter

• Discovery of $K_L \rightarrow \pi^+\pi^-$ decays proved that CP symmetry is violated

• Measurements consistent with $K_L = K_2 + \boldsymbol{\varepsilon} K_1$; $\varepsilon = 2.2\text{x}10^{-3}$ $\varepsilon = \dfrac{i\,\mathrm{Im}\,M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S + \Gamma_L)}$

• Phase of $\boldsymbol{\varepsilon}$ consistent with CPV originating from short-distance Mass-Matrix terms

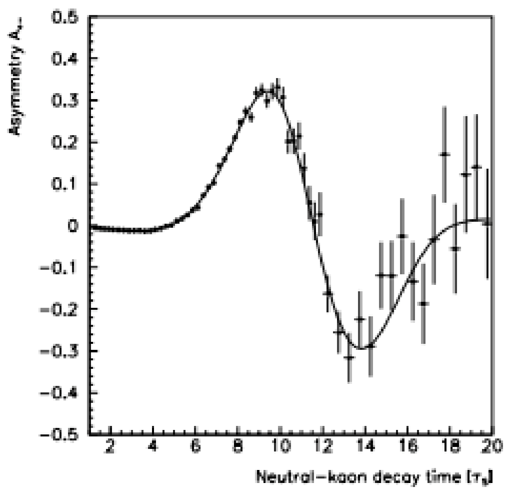
$$\varepsilon = \frac{i\,\mathrm{Im}\,M_{12} + \cancel{\tfrac{1}{2}\mathrm{Im}\Gamma_{12}}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S + \Gamma_L)}$$

• Many beautiful, high-statistics measurements of CPV interference effects reported

Question from lecture 3

how do we measure time?

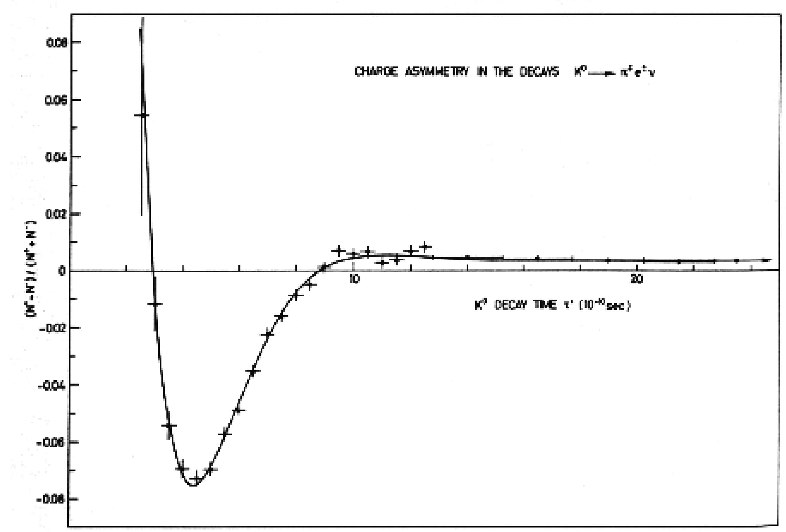
CPLEAR



$$A_{+-}(\tau) = \frac{\overline{N}(\tau) - \alpha N(\tau)}{\overline{N}(\tau) + \alpha N(\tau)}$$

$$= -2\frac{|\eta_{+-}|e^{\frac{1}{2}(\tau/\tau_S - \tau/\tau_L)}\cos(\Delta m\tau - \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\tau/\tau_S - \tau/\tau_L)}}.$$

NA31

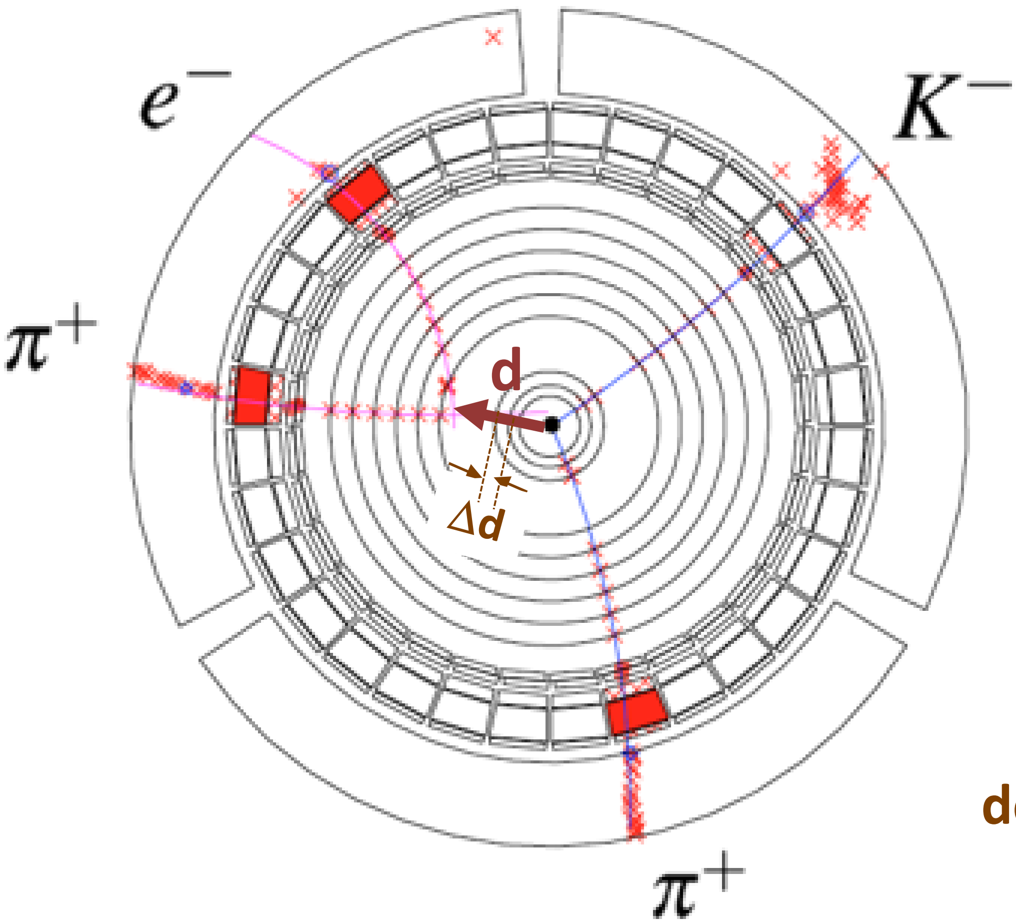


$$\frac{N^+ - N^-}{N^+ + N^-} = 2\left(\mathrm{Re}(\varepsilon) + \frac{e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}\cos\Delta M_K t}{e^{-\Gamma_L t} + e^{-\Gamma_L t}}\right)$$

$$I(K^0 \to K^0; t) = I_0 \left|\langle K^0 | K^0(t) \rangle\right|^2 = I_0 |f_+(t)|^2$$

$$I(K^0 \to \overline{K}^0; t) = I_0 \left|\langle \overline{K}^0 | K^0(t) \rangle\right|^2 = I_0 |f_-(t)|^2.$$

infer time from distance



$d=vt=\beta ct, \rightarrow t=d/\beta c$

this is the time in the lab frame

time in K^0 rest frame is $\tau=t/\gamma$

"proper time"

$\tau=d/\gamma\beta c=m_K d/p_K$

$(\gamma\beta c=p_K/m_K)$

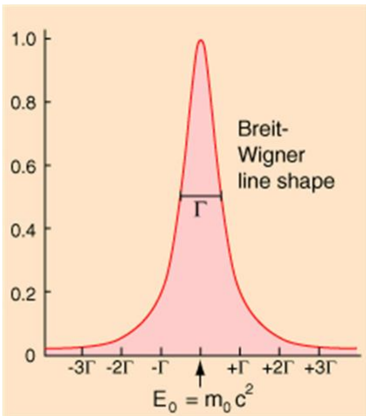
proper time interval

distance interval

$\Delta\tau=(m_K/p_K)\Delta d$

decay probabilty in Δd = $\Delta\tau/\tau_K$

comment on widths, lifetimes, branching factions, partial widths



Γ=total width $\Gamma = \dfrac{\hbar}{\tau} \rightarrow \dfrac{1}{\tau}$

τ=lifetime

we use units where $\hbar$=c=1

branching fraction

$$Bf(P \rightarrow xyz) = \frac{\#\text{ of } P \rightarrow xyz \text{ decays}}{\text{all } P \text{ decays}}$$

$$\sum_{i=\text{all modes}} Bf(P \rightarrow i) = 1$$

partial width

$$\Gamma_{xyz} = Bf(P \rightarrow xyz) \times \Gamma = \frac{Bf(P \rightarrow xyz)}{\tau}$$

these are what theories calculate

$$\Gamma = \sum_{i=\text{all modes}} \Gamma_i$$

example: # K_S & K_L mesons

K_S^0 DECAY MODES		Fraction (Γ_i/Γ)
Hadronic modes		
$\pi^0\pi^0$		(30.69 ± 0.05) %
$\pi^+\pi^-$		(69.20 ± 0.05) %
Semileptonic modes		
$\pi^\pm e^\mp \nu_e$	[p]	$(7.04\pm0.08)\times10^{-4}$

Mean life $\tau = (0.8954 \pm 0.0004) \times 10^{-10}$ s

K_L^0 DECAY MODES		Fraction (Γ_i/Γ)
Semileptonic modes		
$\pi^\pm e^\mp \nu_e$	[p]	(40.55 ± 0.11) %
Called K^0_{e3}.		
$\pi^\pm \mu^\mp \nu_\mu$	[p]	(27.04 ± 0.07) %
Hadronic modes		
$3\pi^0$		(19.52 ± 0.12) %
$\pi^+\pi^-\pi^0$		(12.54 ± 0.05) %

Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8}$ s

total widths

$$\Gamma_{Ks} = \frac{\hbar}{\tau_{Ks}} = \frac{6.58\times10^{-22}\,\mathrm{MeVs}}{0.895\times10^{-10}\,\mathrm{s}} = 7.35\times10^{-12}\,\mathrm{MeV}$$

$$\Gamma_{KL} = \frac{\hbar}{\tau_{KL}} = \frac{6.58\times10^{-22}\,\mathrm{MeVs}}{5.12\times10^{-8}\,\mathrm{s}} = 1.28\times10^{-14}\,\mathrm{MeV}$$

partial widths

$$\Gamma_{Ks\to\pi e\nu} = Bf(K_s \to \pi e\nu)\times\Gamma_{Ks} = 0.52\times10^{-14}\,\mathrm{MeV}$$

$$\Gamma_{KL\to\pi e\nu} = Bf(K_L \to \pi e\nu)\times\Gamma_{KL} = 0.52\times10^{-14}\,\mathrm{MeV}$$

although *Bf*($K_S \to \pi e \nu$)<<*Bf*($K_L \to \pi e \nu$), the partial widths are equal

units when *ħ=c=1*

mass, energy & momentum units are MeV   (sometimes GeV)

length and time units are MeV^{-1}

in "normal" units:

a 1 MeV^{-1} length unit = $\hbar c/1\ \mathrm{MeV} = 197\ \mathrm{fm} = 197\times10^{-13}\mathrm{cm}$

& a 1 MeV^{-1} time unit = $\hbar/1\ \mathrm{MeV} = 6.58\times10^{-22}\mathrm{s}$

Comments on choice of $\mathcal{C}$

Input: $\mathcal{C}^2$=1 and $\mathcal{C}|\pi^0\rangle = +|\pi^0\rangle$

-- my choice for $\mathcal{C}$ values --

$\mathcal{C}$, $\mathcal{P}$ & $\mathcal{CP}$ for π and K mesons

Particle	$\mathcal{P}$	$\mathcal{C}$	$\mathcal{CP}$
$\|\pi^+\rangle$	-1	$+\|\pi^-\rangle$	$-\|\pi^-\rangle$
$\|\pi^0\rangle$	-1	$+\|\pi^0\rangle$	$-\|\pi^0\rangle$
$\|\pi^-\rangle$	-1	$+\|\pi^+\rangle$	$-\|\pi^+\rangle$
$\|K^+\rangle$	-1	$+\|K^-\rangle$	$-\|K^-\rangle$
$\|K^0\rangle$	-1	$+\|\overline{K^0}\rangle$	$-\|\overline{K^0}\rangle$
$\|\overline{K^0}\rangle$	-1	$+\|K^0\rangle$	$-\|K^0\rangle$
$\|K^-\rangle$	-1	$+\|K^+\rangle$	$-\|K^+\rangle$

$$|K_1\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \Leftarrow \mathcal{CP}\,\text{even}$$

$$|K_2\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \Leftarrow \mathcal{CP}\,\text{odd}$$

$$\mathcal{CP}\langle\pi\pi|\mathcal{A}|K^0\rangle = +\langle\pi\pi|\mathcal{A}^*|\overline{K}^0\rangle$$

-- others (mostly theorist's) choices --

$\mathcal{C}$, $\mathcal{P}$ & $\mathcal{CP}$ for π and K mesons

Particle	$\mathcal{P}$	$\mathcal{C}$	$\mathcal{CP}$
$\|\pi^+\rangle$	-1	$+\|\pi^-\rangle$	$-\|\pi^-\rangle$
$\|\pi^0\rangle$	-1	$+\|\pi^0\rangle$	$-\|\pi^0\rangle$
$\|\pi^-\rangle$	-1	$+\|\pi^+\rangle$	$-\|\pi^+\rangle$
$\|K^+\rangle$	-1	$+\|K^-\rangle$	$-\|K^-\rangle$
$\|K^0\rangle$	-1	$-\|\overline{K^0}\rangle$	$+\|\overline{K^0}\rangle$
$\|\overline{K^0}\rangle$	-1	$-\|K^0\rangle$	$+\|K^0\rangle$
$\|K^-\rangle$	-1	$+\|K^+\rangle$	$-\|K^+\rangle$

$$|K_1\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \Leftarrow \mathcal{CP}\,\text{even}$$

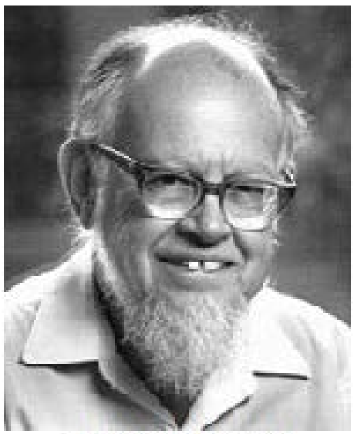
$$|K_2\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \Leftarrow \mathcal{CP}\,\text{odd}$$

$$\mathcal{CP}\langle\pi\pi|\mathcal{A}|K^0\rangle = -\langle\pi\pi|\mathcal{A}^*|\overline{K}^0\rangle$$

Physics stays the same (of course)

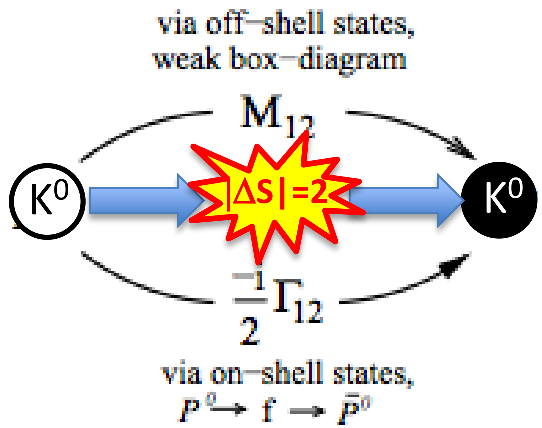
Superweak model for CPV?

$$\left|K_L\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2\right\rangle + \varepsilon\left|K_1\right\rangle\right) \qquad \varepsilon = \frac{i\,\mathrm{Im}\,M_{12} + \cancel{\tfrac{1}{2}\mathrm{Im}\,\Gamma_{12}}}{(M_L - M_S) - \frac{i}{2}(\Gamma_S + \Gamma_L)}$$



Lincoln Wolfenstein

CPV is very small, $\sim 10^{-3} G_F^2$: is there a new “superweak,” very short-ranged $\Delta S=2$ interaction that directly couples K^0 to $\bar{K}^0$?



This interaction has a coupling strength $\sim 10^{-10}\, G_F$ and is only strong enough to produce $\mathrm{Im}M_{12}$, i.e. $\boldsymbol{\varepsilon}$, and nothing else. It is seen in K_L decay because ΔM_K ($<10^{-12}$ MeV) is so small.

Decay of the K_L

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\overset{\mathcal{CP}\text{ odd}}{|K_2\rangle} + \varepsilon\overset{\mathcal{CP}\text{ even}}{|K_1\rangle}\right)$$

OK

direct CPV?

OK

indirect CPV

$\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$

$\mathcal{CP}$ odd

$\pi^+\pi^-$ or $\pi^0\pi^0$

$\mathcal{CP}$ even

Is there a direct decay $K_{\text{CP-odd}}$ to $\pi^+\pi^-$ or $\pi^0\pi^0$?

The Superweak model says no!

η_{+-} and η_{00} with direct decays

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle} = \frac{\langle \pi^+\pi^- | H | K_2 \rangle + \varepsilon \langle \pi^+\pi^- | H | K_1 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} = \varepsilon + \frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle}$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H | K_L \rangle}{\langle \pi^0\pi^0 | H | K_S \rangle} = \frac{\langle \pi^0\pi^0 | H | K_2 \rangle + \varepsilon \langle \pi^0\pi^0 | H | K_1 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} = \varepsilon + \frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle}$$

If Superweak is correct, and there are no direct CPV terms:

$$\frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} = 0 \quad \text{and} \quad \frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} = 0$$

Then $\eta_{+-} = \varepsilon \;\;\&\;\; \eta_{00} = \varepsilon$ $\quad \dfrac{\eta_{+-}}{\eta_{00}} = \dfrac{\langle \pi^+\pi^- | H | K_L \rangle / \langle \pi^+\pi^- | H | K_S \rangle}{\langle \pi^0\pi^0 | H | K_L \rangle / \langle \pi^0\pi^0 | H | K_S \rangle} = 1$

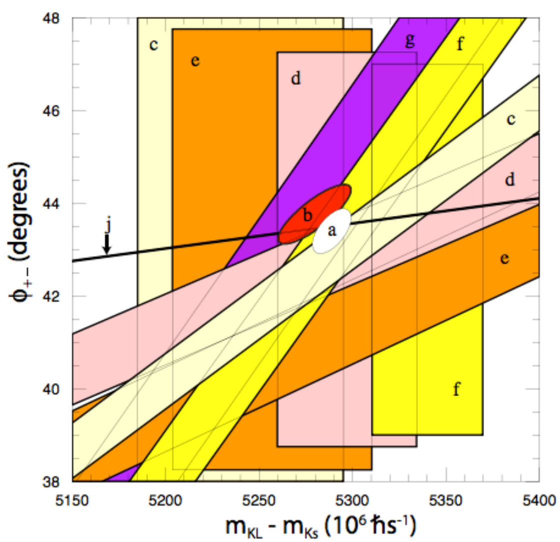
Predictions of Superweak model

All CPV is due to influence of $\boldsymbol{\varepsilon}$, no other measureable results

$$\eta_{+-} = \eta_{00} = \varepsilon; \quad \Re \equiv \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 = \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)} = 1$$

$$\text{phases of } \eta_{+-} \text{ and } \eta_{00},\ \phi_{+-} \text{ and } \phi_{00} = \arctan\left(\frac{2\Delta M_K}{\Delta\Gamma_K}\right) \approx 45^{\circ} \Leftarrow \text{"SW phase"}$$

different ϕ_{+-} measurements



SW phase

direct $K_2 \to \pi\pi$ effects $\pi^+\pi^-$ & $\pi^0\pi^0$ differently

since we use CP(K^0)=-$\overline{K}^0$

$$\langle \pi\pi; I=0 | H | K^0 \rangle = A_0 e^{i\delta_0} \qquad \langle \pi\pi; I=0 | H | \overline{K}^0 \rangle = -A_0^* e^{i\delta_0}$$

$$\langle \pi\pi; I=2 | H | K^0 \rangle = A_2 e^{i\delta_2} \qquad \langle \pi\pi; I=2 | H | \overline{K}^0 \rangle = -A_2^* e^{i\delta_2}$$

Clebsch-Gordon coefficients

$$|\pi^+\pi^-\rangle = \sqrt{\tfrac{2}{3}} |\pi\pi; I=0\rangle + \sqrt{\tfrac{1}{3}} |\pi\pi; I=2\rangle$$

$$\langle \pi^+\pi^- | H | K_2 \rangle = \tfrac{1}{\sqrt{2}} \left[\langle \pi^+\pi^- | H | K^0 \rangle + \langle \pi^+\pi^- | H | \overline{K}^0 \rangle \right]$$
$$= \tfrac{1}{\sqrt{2}} \left(\sqrt{\tfrac{2}{3}} (A_0 - A_0^*) e^{i\delta_0} + \sqrt{\tfrac{1}{3}} (A_2 - A_2^*) e^{i\delta_2} \right)$$
$$= \mathrm{i} \sqrt{\tfrac{2}{3}} \left(\sqrt{2}\, \mathrm{Im}\, A_0 e^{i\delta_0} + \mathrm{Im}\, A_2 e^{i\delta_2} \right)$$

$$\langle \pi^+\pi^- | H | K_1 \rangle = \sqrt{\tfrac{2}{3}} \left(\sqrt{2}\, \mathrm{Re}\, A_0 e^{i\delta_0} + \mathrm{Re}\, A_2 e^{i\delta_2} \right)$$

$$|\pi^0\pi^0\rangle = -\sqrt{\tfrac{1}{3}} |\pi\pi; I=0\rangle + \sqrt{\tfrac{2}{3}} |\pi\pi; I=2\rangle$$

$$\langle \pi^0\pi^0 | H | K_2 \rangle = \tfrac{1}{\sqrt{2}} \left[\langle \pi^0\pi^0 | H | K^0 \rangle - \langle \pi^0\pi^0 | H | \overline{K}^0 \rangle \right]$$
$$= \tfrac{1}{\sqrt{2}} \left(-\sqrt{\tfrac{1}{3}} (A_0 - A_0^*) e^{i\delta_0} + \sqrt{\tfrac{2}{3}} (A_2 - A_2^*) e^{i\delta_2} \right)$$
$$= \mathrm{i} \sqrt{\tfrac{2}{3}} \left(-\mathrm{Im}\, A_0 e^{i\delta_0} + \sqrt{2}\, \mathrm{Im}\, A_2 e^{i\delta_2} \right)$$

$$\langle \pi^0\pi^0 | H | K_1 \rangle = \sqrt{\tfrac{2}{3}} \left(-\mathrm{Re}\, A_0 e^{i\delta_0} + \sqrt{2}\, \mathrm{Re}\, A_2 e^{i\delta_2} \right)$$

Physics is in the difference between A_0 & A_2 phases. It is customary* to chose A_0 to be real

$$\langle \pi^+\pi^- | H | K_2 \rangle = i\sqrt{\tfrac{2}{3}}\, \mathrm{Im}\, A_2 e^{i\delta_2}$$

$$\langle \pi^+\pi^- | H | K_1 \rangle = \sqrt{\tfrac{4}{3}} A_0 e^{i\delta_0} \left(1 + \frac{\mathrm{Re}\, A_2}{\sqrt{2} A_0} e^{i(\delta_2 - \delta_0)} \right)$$

$$\langle \pi^0\pi^0 | H | K_2 \rangle = i\sqrt{\tfrac{4}{3}}\, \mathrm{Im}\, A_2 e^{i\delta_2}$$

$$\langle \pi^0\pi^0 | H | K_1 \rangle = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0} \left(-1 + \frac{\sqrt{2}\, \mathrm{Re}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)} \right)$$

*T. T. Wu and C. N. Yang, *Phys. Rev. Lett.* **13**, 380 (1964) from previous slide

$$\langle \pi^+\pi^- | H | K_2 \rangle = i\sqrt{\tfrac{2}{3}}\,\mathrm{Im}\,A_2 e^{i\delta_2}$$

$$\langle \pi^+\pi^- | H | K_1 \rangle = \sqrt{\tfrac{4}{3}} A_0 e^{i\delta_0}\left(1+\frac{\mathrm{Re}\,A_2}{\sqrt{2}A_0}e^{i(\delta_2-\delta_0)}\right)$$

$$\langle \pi^0\pi^0 | H | K_2 \rangle = i\sqrt{\tfrac{4}{3}}\,\mathrm{Im}\,A_2 e^{i\delta_2}$$

$$\langle \pi^0\pi^0 | H | K_1 \rangle = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0}\left(-1+\frac{\sqrt{2}\,\mathrm{Re}\,A_2}{A_0}e^{i(\delta_2-\delta_0)}\right)$$

define:

$$\boxed{\varepsilon' = \frac{i}{\sqrt{2}}\frac{\mathrm{Im}\,A_2}{A_0}e^{i(\delta_2-\delta_0)} \qquad \omega = \frac{\mathrm{Re}\,A_2}{A_0}e^{i(\delta_2-\delta_0)}}$$

$$\frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} = \frac{\varepsilon'}{1+\frac{1}{\sqrt{2}}\omega}$$

$$\frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} = -\frac{2\varepsilon'}{1-\sqrt{2}\omega}$$

ΔI=1/2 rule →

$$\boxed{|\omega| = \frac{\mathrm{Re}\,A_2}{A_0} \approx \frac{\sqrt{Bf(K^+\to\pi^+\pi^0)\tau_{KS}}}{\sqrt{Bf(K_S\to\pi^+\pi^-)\tau_{K+}}} = \sqrt{\frac{0.21\times 0.1\text{ns}}{0.69\times 12\text{ns}}} \Box \frac{1}{22}}$$

$$\frac{\langle \pi^+\pi^- | H | K_2 \rangle}{\langle \pi^+\pi^- | H | K_1 \rangle} \Box\ \varepsilon'$$

$$\frac{\langle \pi^0\pi^0 | H | K_2 \rangle}{\langle \pi^0\pi^0 | H | K_1 \rangle} \Box\ -2\varepsilon'$$

$$\eta_{+-} = \varepsilon + \varepsilon'$$

← **different** →

$$\eta_{00} = \varepsilon - 2\varepsilon'$$

ΔI=1/2 rule

K_S^0 DECAY MODES	Fraction (Γ_i/Γ)
Hadronic modes	
$\pi^0\pi^0$	(30.69 ± 0.05) %
$\pi^+\pi^-$	(69.20 ± 0.05) %
Semileptonic modes	
$\pi^\pm e^\mp \nu_e$	[p] $(7.04\pm0.08)\times10^{-4}$

Mean life $\tau = (0.8954 \pm 0.0004)\times10^{-10}$ s

K^+ DECAY MODES	Fraction (Γ_i/Γ)
Hadronic modes	
$\pi^+\pi^0$	(20.67 ± 0.08) %
$\pi^+\pi^0\pi^0$	(1.760 ± 0.023) %
$\pi^+\pi^+\pi^-$	(5.583 ± 0.024) %

Mean life $\tau = (1.2380 \pm 0.0020)\times10^{-8}$ s

$$\Gamma_{KS\to\pi^+\pi^-} = \frac{Bf(K_S\to\pi^+\pi^-)}{\tau_{Ks}} = 5.1\times10^{-12}\,\mathrm{MeV}$$

$$\Gamma_{K^+\to\pi^+\pi^0} = \frac{Bf(K^+\to\pi^+\pi^0)}{\tau_{K^+}} = 2.4\times10^{-9}\,\mathrm{MeV}$$

$$\boxed{\Gamma_{Ks\to\pi^+\pi^-} \;\square\; 450\times\Gamma_{K^+\to\pi^+\pi^0}}$$

phase space is same for both modes

$$\Gamma_{Ks\to\pi^+\pi^-} \propto |A_0 + A_2|^2 \text{ phase space}$$

Ispin=0 $\pi^+\pi^-$ Ispin=2 $\pi^+\pi^-$

$$\Gamma_{K^+\to\pi^+\pi^0} \propto |A_2|^2 \text{ phase space}$$

Ispin=2 $\pi^+\pi^-$

$$\Rightarrow A_0 \approx \sqrt{450}\,A_2 \approx 22\,A_2$$

$$K_S \to (\pi^+\pi^-)_{I=0} \Leftarrow \Delta I = \tfrac{1}{2} \quad \text{favored}$$

$$K^+ \to (\pi^+\pi^0)_{I=2} \Leftarrow \Delta I = \tfrac{3}{2} \quad \text{disfavored}$$

direct CP violation double asymmetry

If there are direct CP violations:

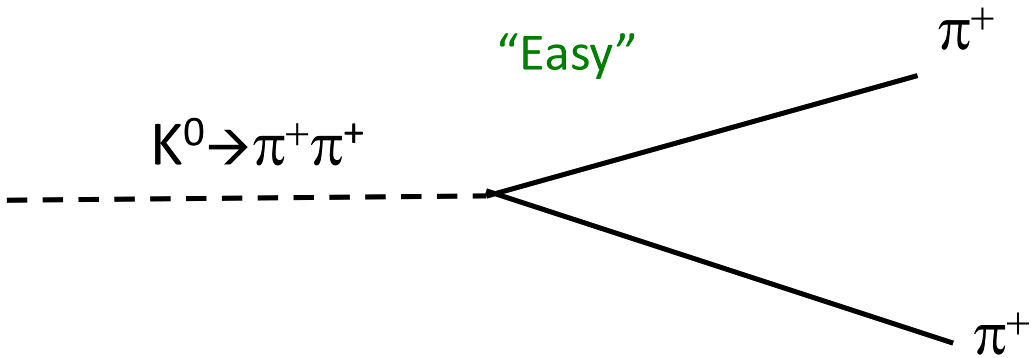
$$\frac{\Gamma\left(K_L \to \pi^+\pi^-\right)}{\Gamma\left(K_S \to \pi^+\pi^-\right)} = \left|\eta_{+-}\right|^2 = \left|\varepsilon + \varepsilon'\right|^2 = \varepsilon^2\left|1 + \varepsilon'/\varepsilon\right|^2 \approx \varepsilon^2\left(1 + 2\,\mathrm{Re}\,\varepsilon'/\varepsilon\right)$$

$$\frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} = \left|\eta_{00}\right|^2 = \left|\varepsilon - 2\varepsilon'\right|^2 = \varepsilon^2\left|1 - 2\varepsilon'/\varepsilon\right|^2 \approx \varepsilon^2\left(1 - 4\,\mathrm{Re}\,\varepsilon'/\varepsilon\right)$$

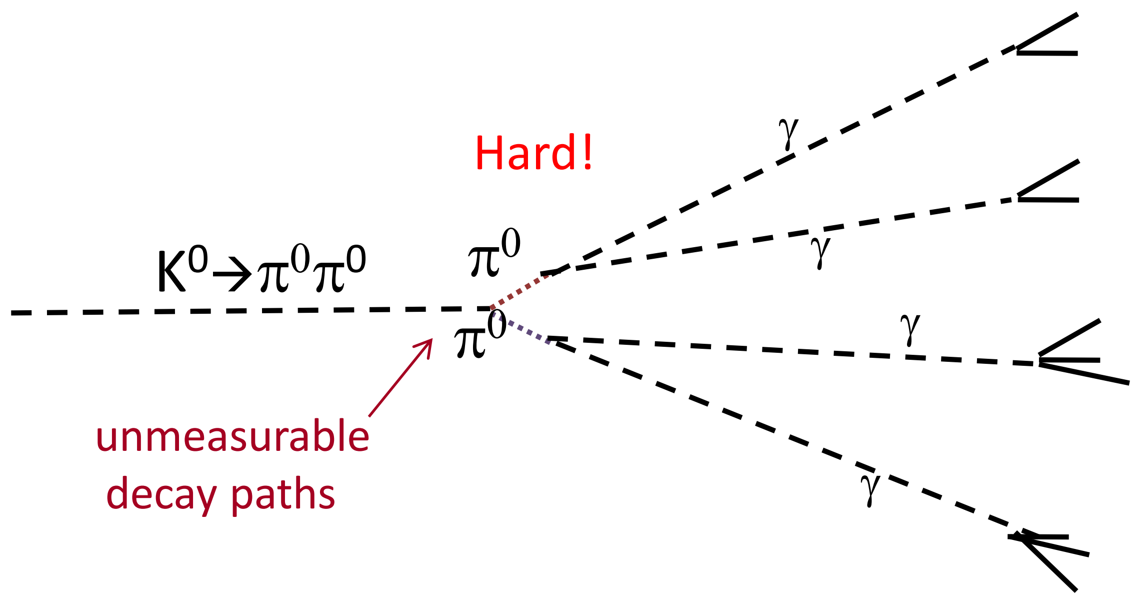
$$\Re = \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)} \approx \frac{1 + 2\,\mathrm{Re}\,\varepsilon'/\varepsilon}{1 - 4\,\mathrm{Re}\,\varepsilon'/\varepsilon} \approx 1 + 6\,\mathrm{Re}\,\varepsilon'/\varepsilon$$

precision measurements of $K_L \rightarrow \pi^0\pi^0$ are very important

Quest for $K_L \to \pi^0\pi^0$



distinguishable tracks
clean vertex
$\delta p \sim 0.2\% p$

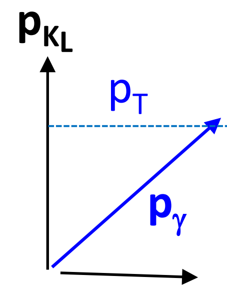


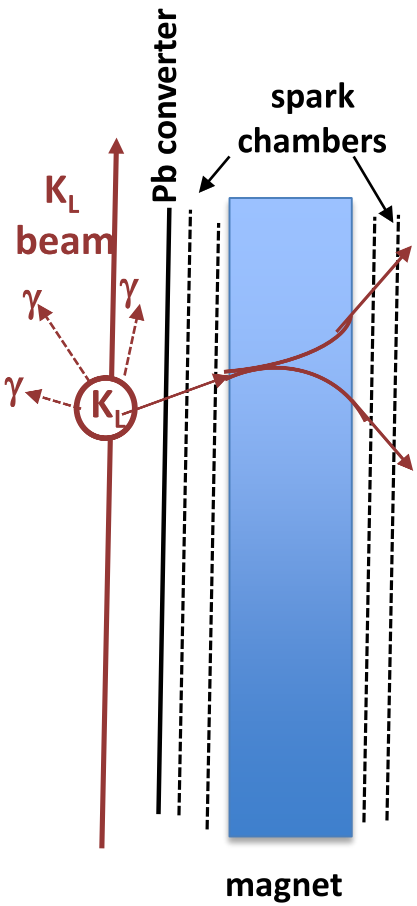
γ matching ambiguities
no well defined vertex
$\delta E \sim 5\%/\sqrt{E}$

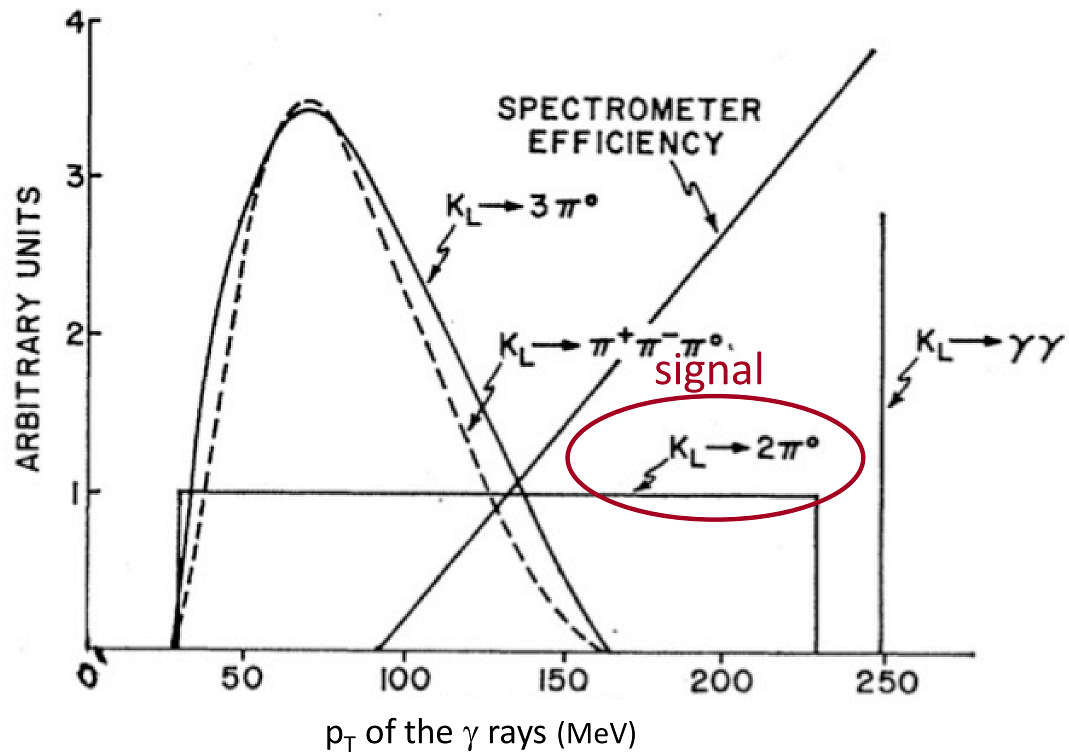
gets better at
high energy

1^{st} attempt by Cronin

Exploit kinematics for γ rays: $\dfrac{dN_\gamma}{dp_T} = \dfrac{dN_\gamma}{dE_{CM}}$, where p_T is the momentum component perpendicular to the beam direction

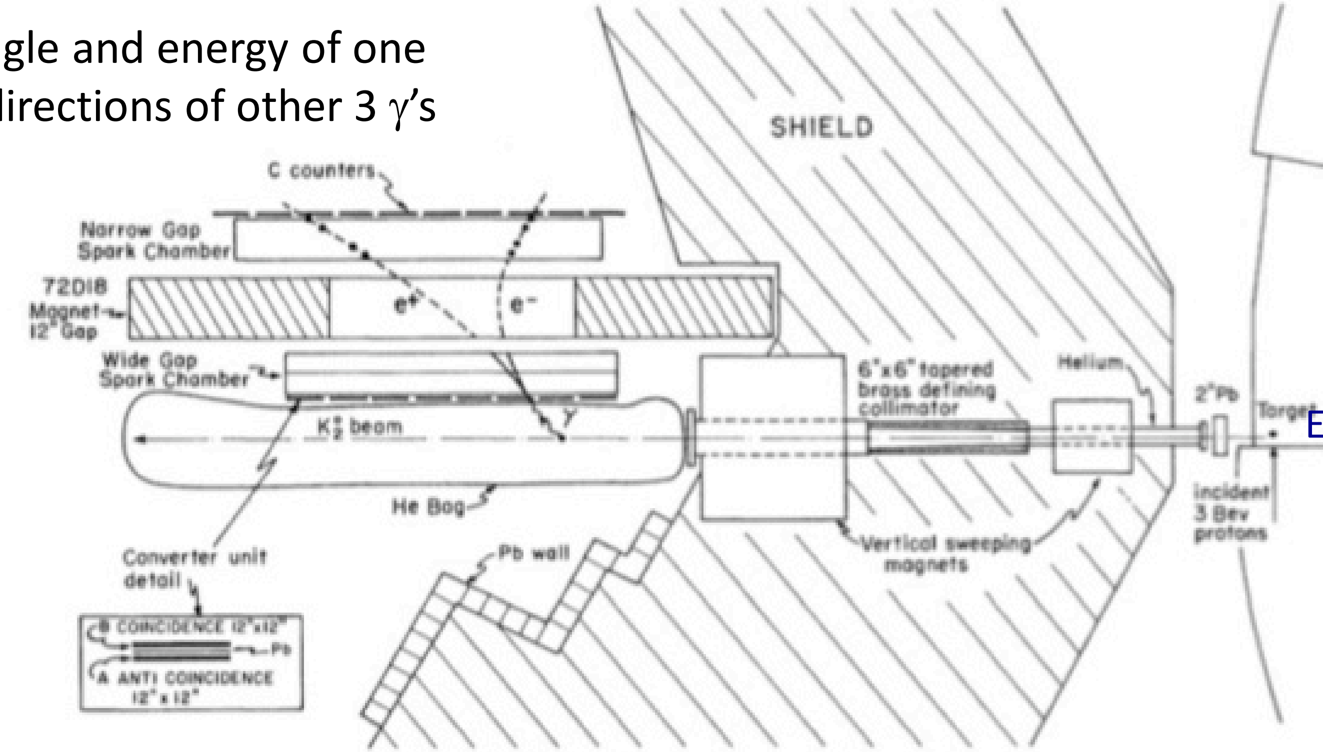






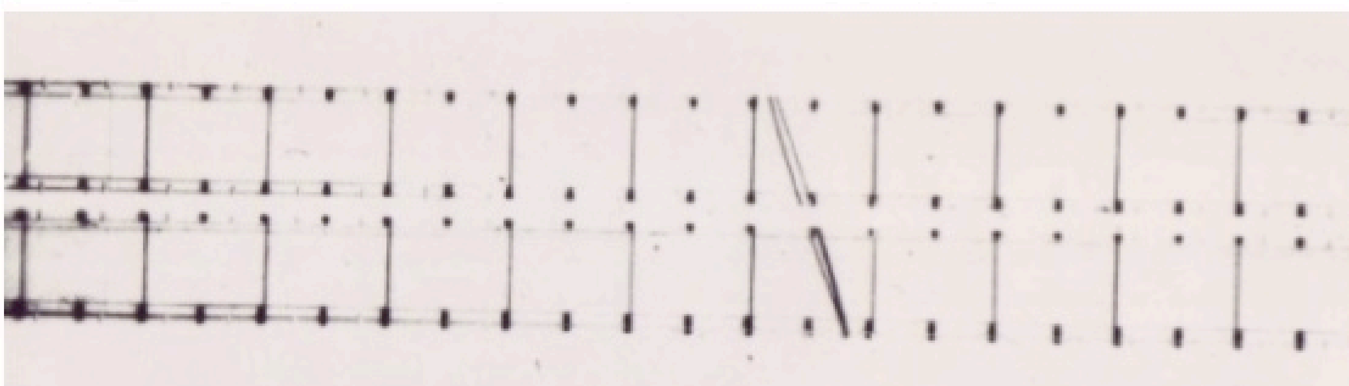
Trigger on one γ-ray with P_T>160 MeV

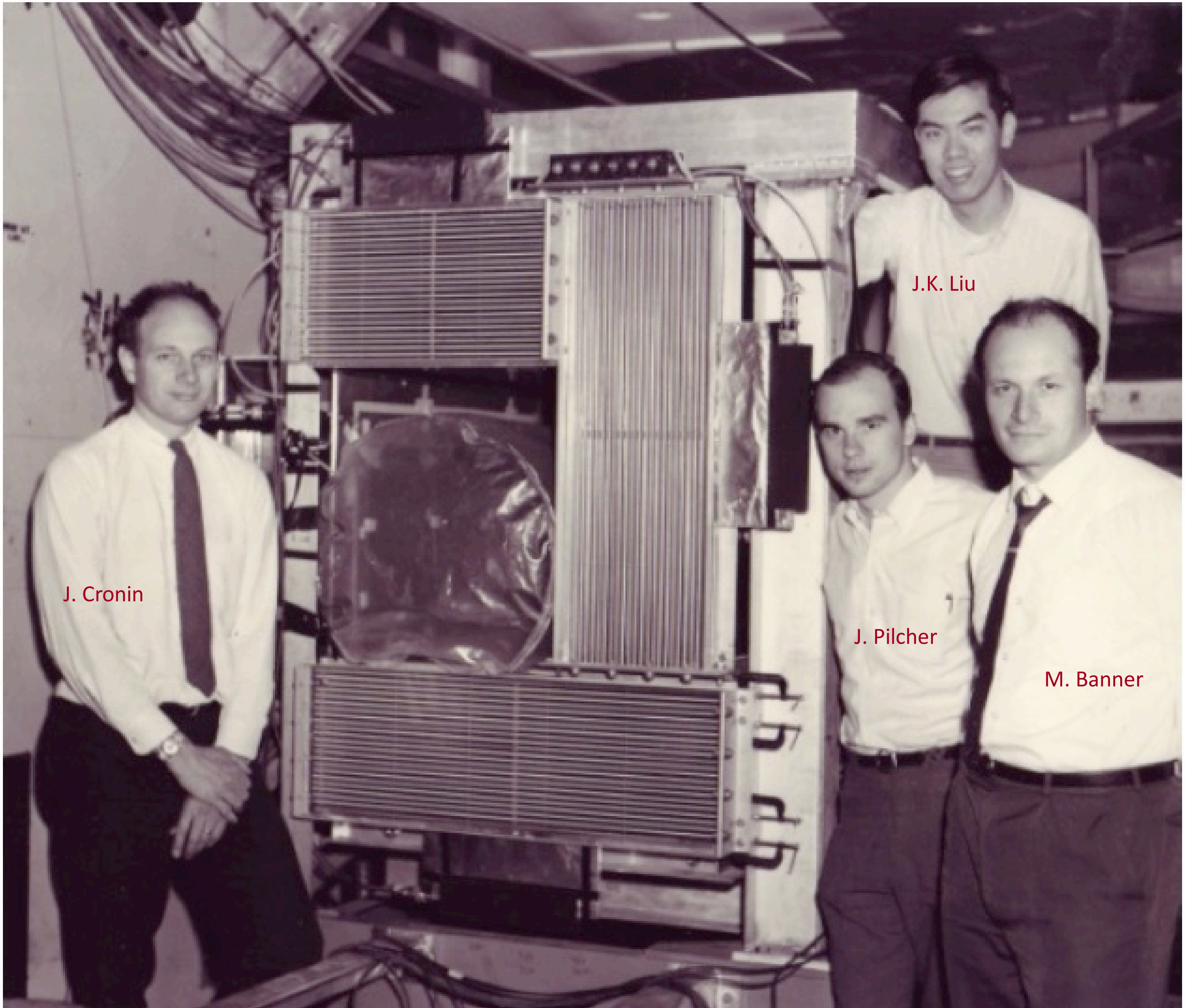
Measure angle and energy of one γ-ray well; directions of other 3 γ's



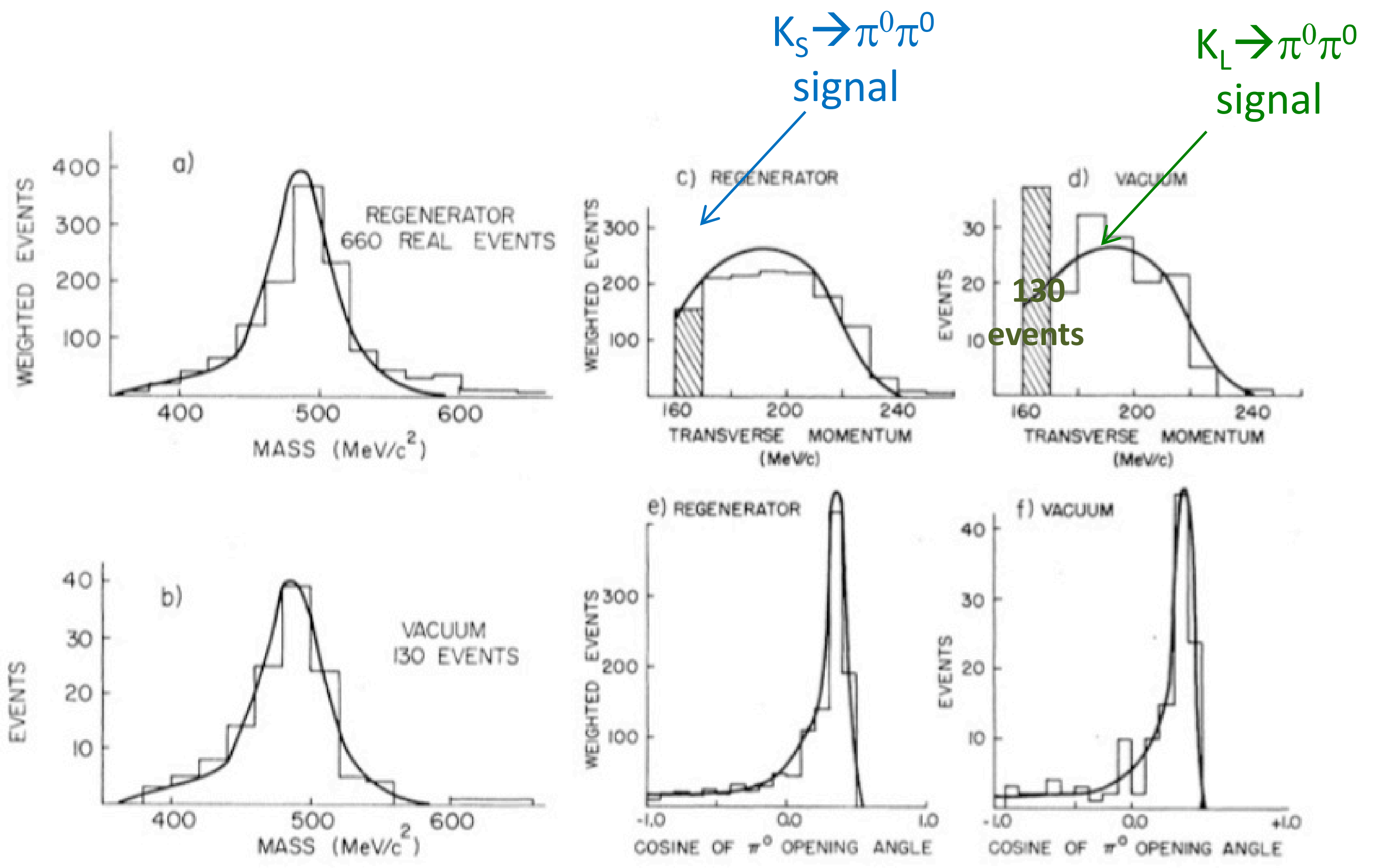
E<1 GeV beam

Figure 13: Spectrometer at PPA for the measurement of the γ-ray spectrum from K_L decay




J.K. Liu
J. Cronin
J. Pilcher
M. Banner

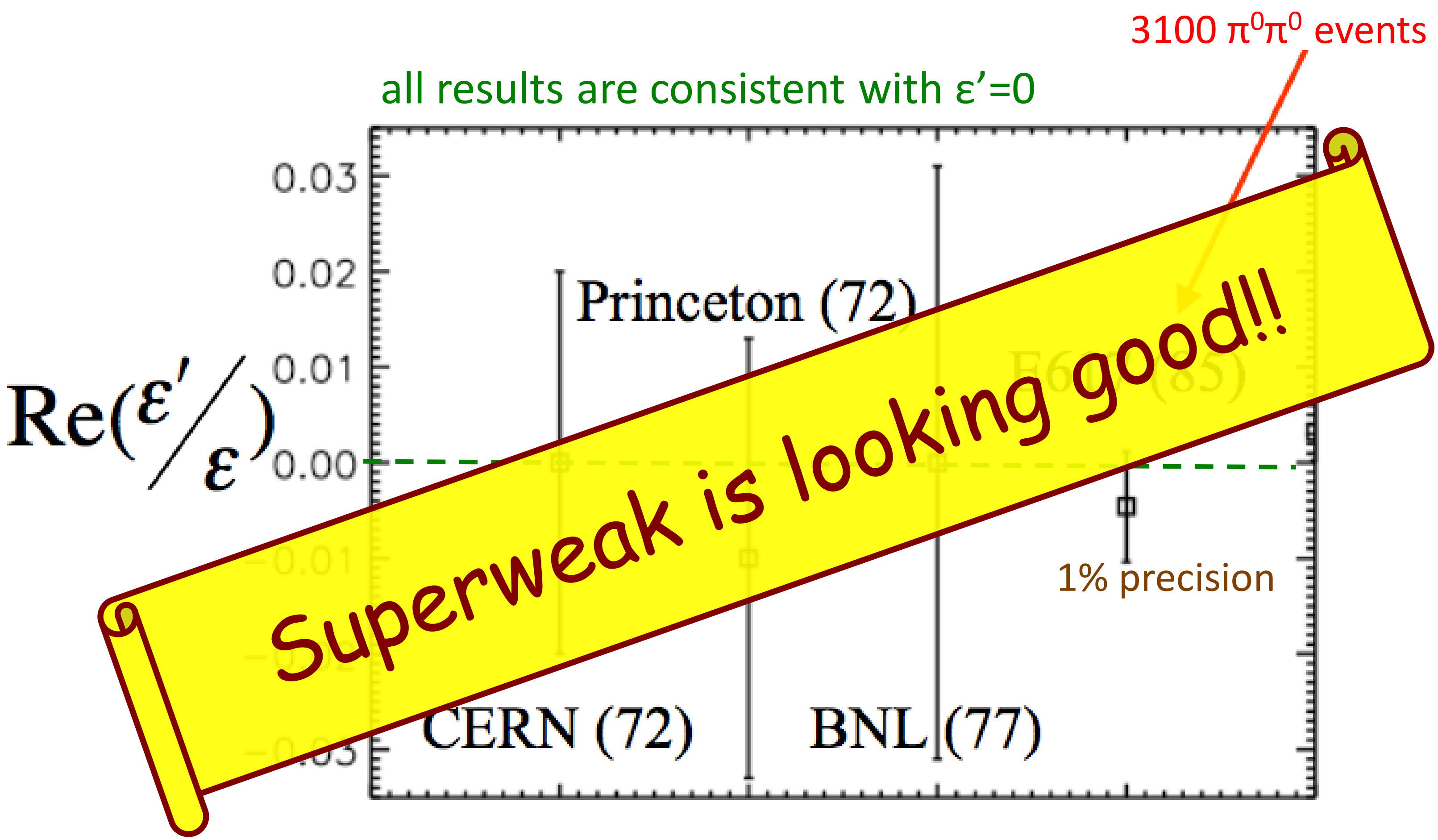
results



$|\eta_{+-}/\eta_{00}|^2 = 1.03 \pm 0.07$ $\Leftarrow$ Phys. Rev. 188, 2033 (1969)

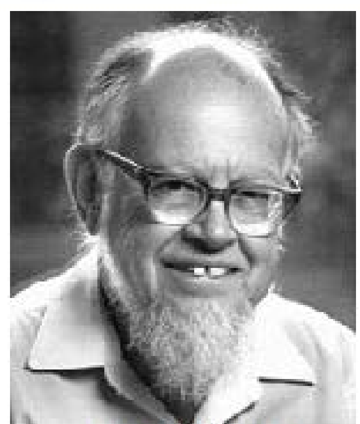
7% precision

The early measurements



1989 on non-zero measurement of ε'

After I submitted my paper to Physical Review Letters I received a reluctant acceptance from the referee who objected that my paper made no predictions. What he really meant was that the superweak theory predicted nothing; that is, nothing else would be found beyond the parameter ε in the K° system. Unfortunately, this prediction has proven all too true.



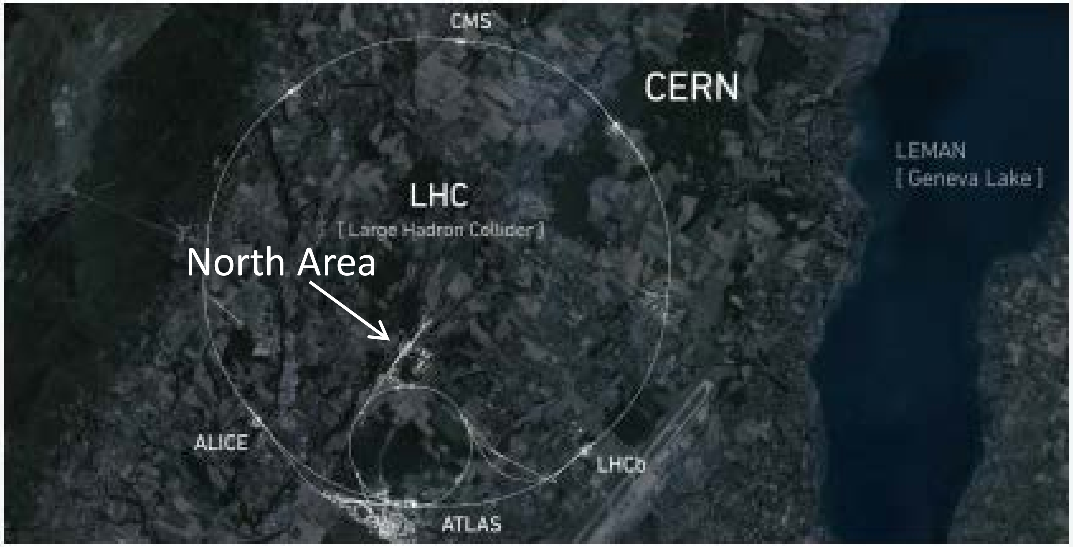
Lincoln Wolfenstein

L. Wolfenstein (1989)

Go to higher energy

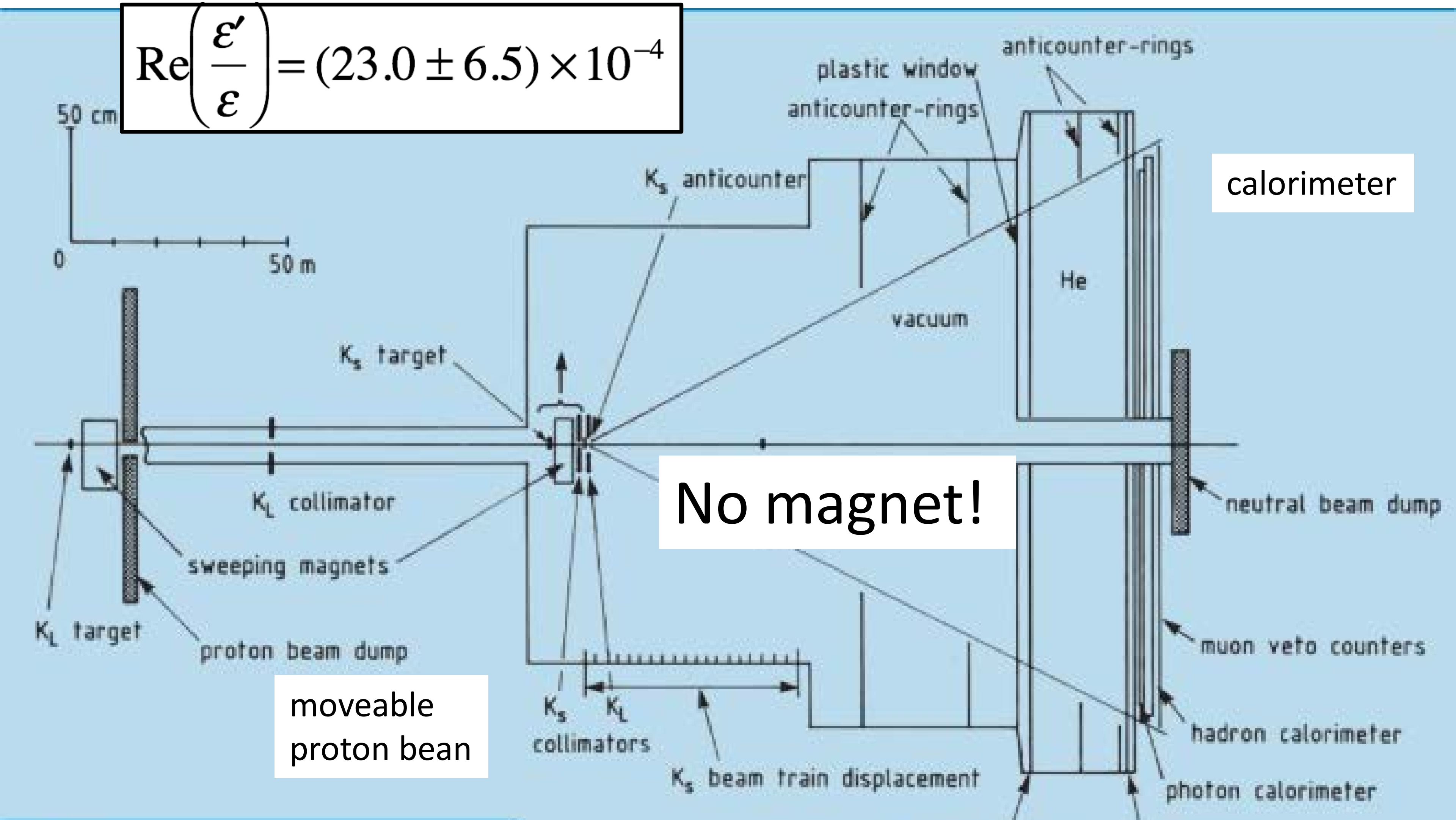
US: E731 at Fermilab

Europe: NA-31 at CERN



NA31 (CERN) 1st "evidence" for ε'≠0

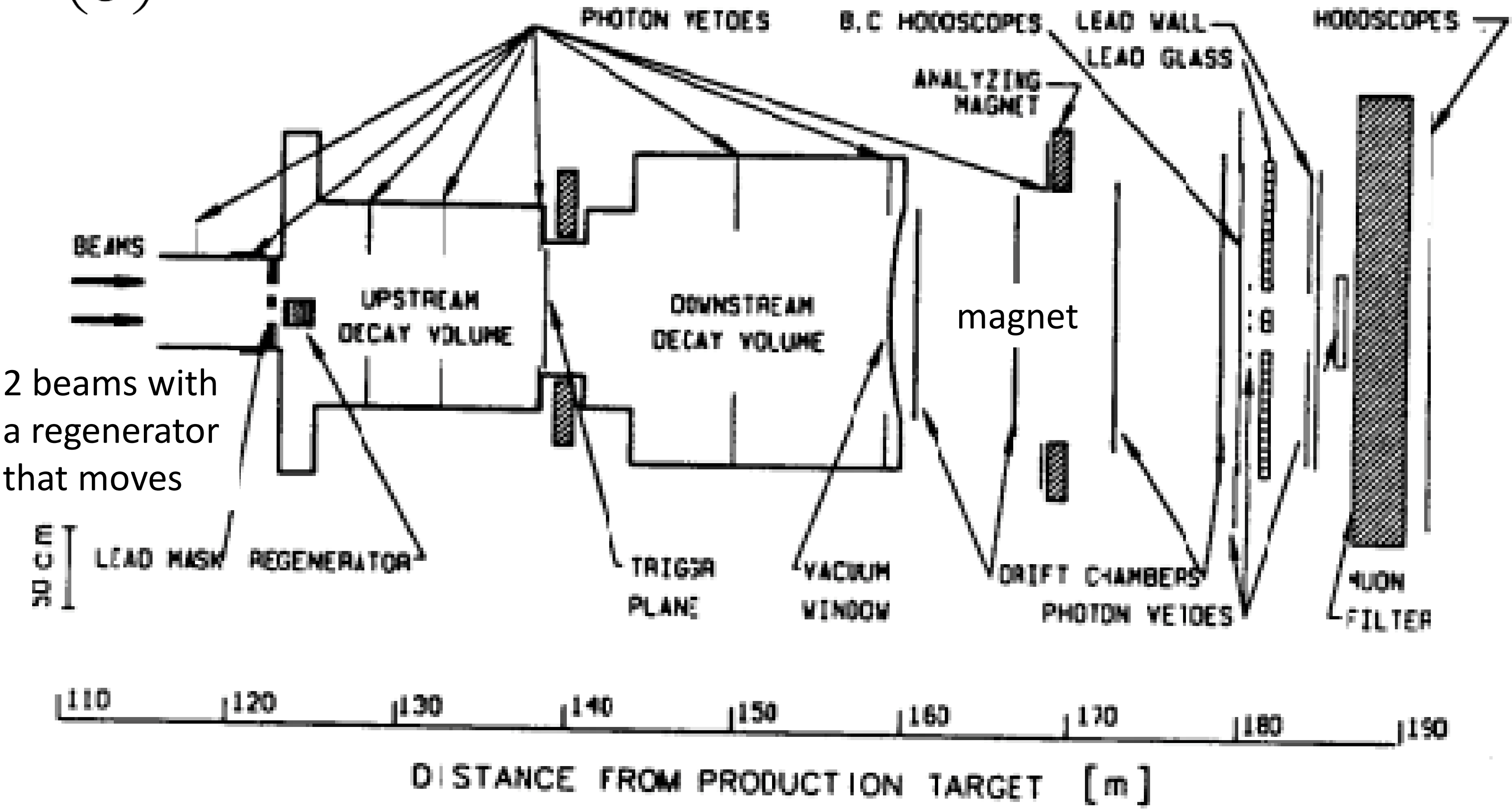
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23.0 \pm 6.5)\times 10^{-4}$$



▲ Fig.1: Schematic layout of the NA31 apparatus

E731 (Fermilab) finds ε′=consistent with 0

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 6.0) \times 10^{-4}$$



2 beams with a regenerator that moves

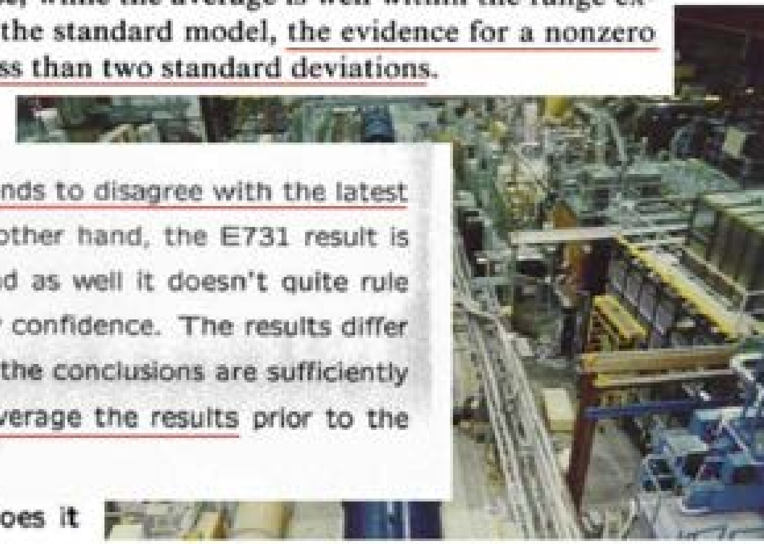
Big Controversy

E731:

$Re(\varepsilon'/\varepsilon) = (7.4 \pm 6.0) \cdot 10^{-4}$

Not dispro

In any case, while the average is well within the range expected in the standard model, the evidence for a nonzero effect is less than two standard deviations.

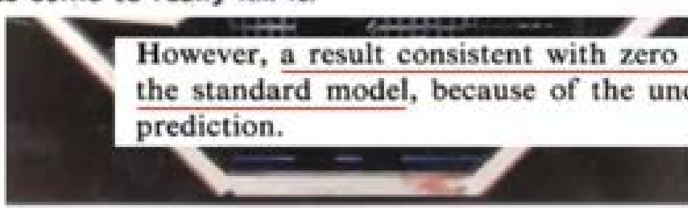


The NA31 result is more interesting in that it tends to disagree with the latest predictions from the Standard Model. On the other hand, the E731 result is in the range favored by the Standard Model and as well it doesn't quite rule out the Superweak Model ($Re\,\varepsilon'/\varepsilon = 0$) with any confidence. The results differ by about two standard deviations; nevertheless, the conclusions are sufficiently different that it would not be appropriate to average the results prior to the establishment of a non-zero effect.

The E731 result does not confirm the non-zero result of NA31 nor does it significantly disagree with it.

so that we cannot as yet claim that direct CP violation is established.

What are we to conclude from these experiments? The most important conclusion is that they must be continued to still higher accuracy. The point is not to find the exact value of ε'; the point is to make absolutely sure that ε' is non-zero. The NA31 experiment has wounded the superweak theory. The time has come to really kill it.



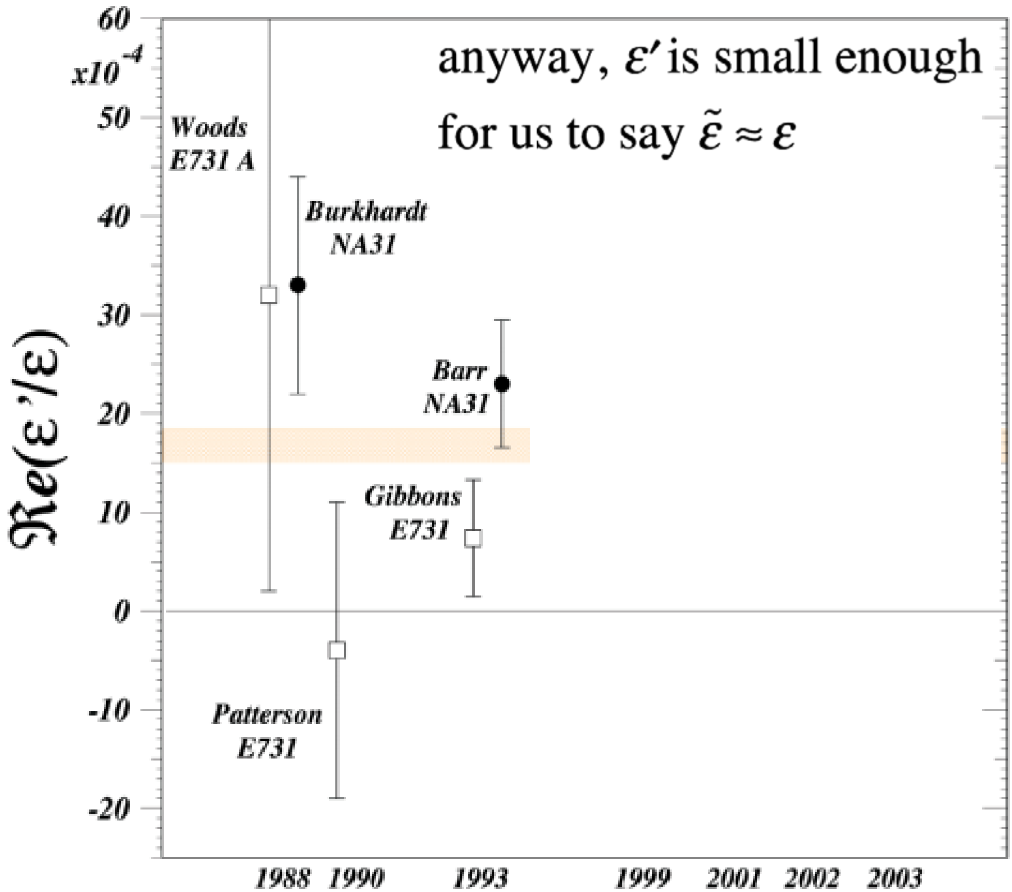
However, a result consistent with zero will not rule out the standard model, because of the uncertainties in the prediction.

$Re(\varepsilon'/\varepsilon) = (23.0 \pm 6.5) \cdot 10^{-4}$

Inconsistent with superweak

NA31

ε’=0? or ε’≠ 0?, that is the question



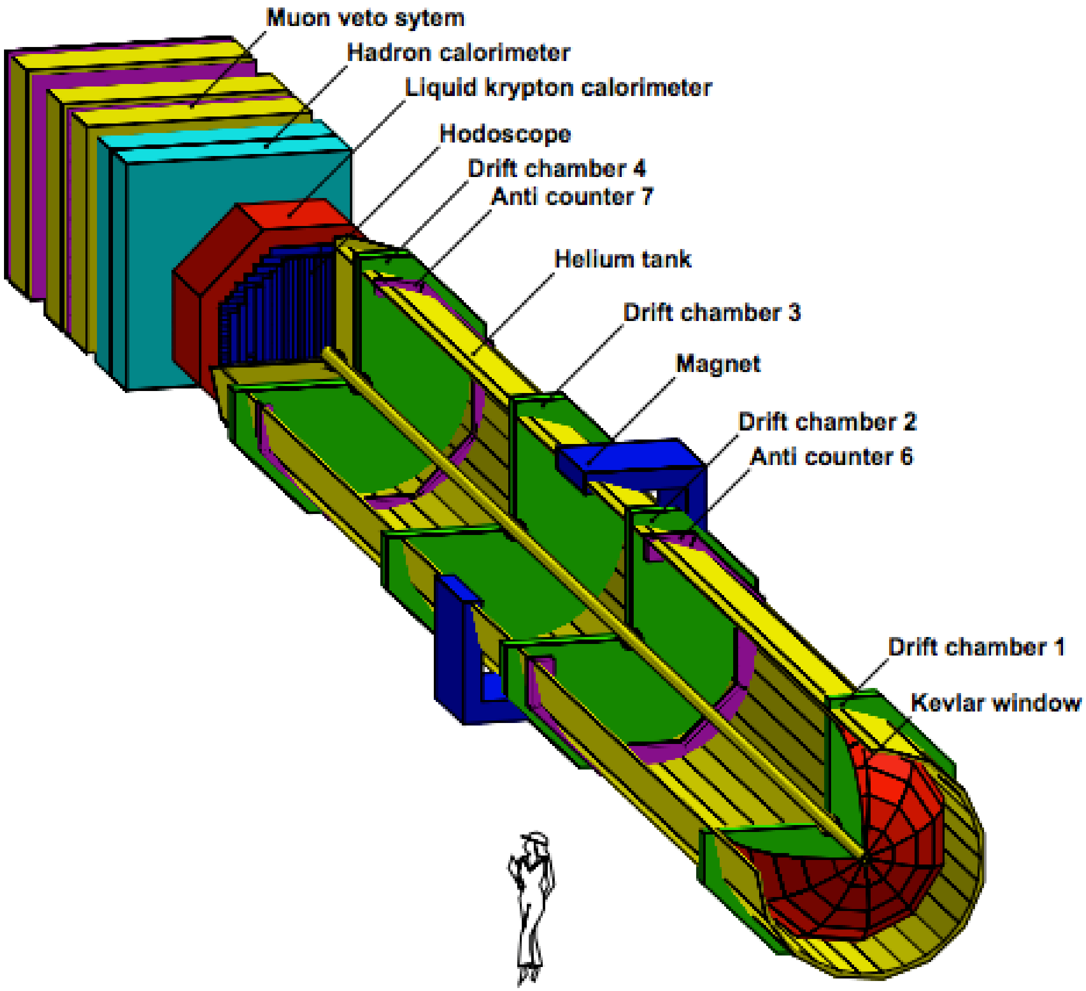
Experimental goal

Measure:

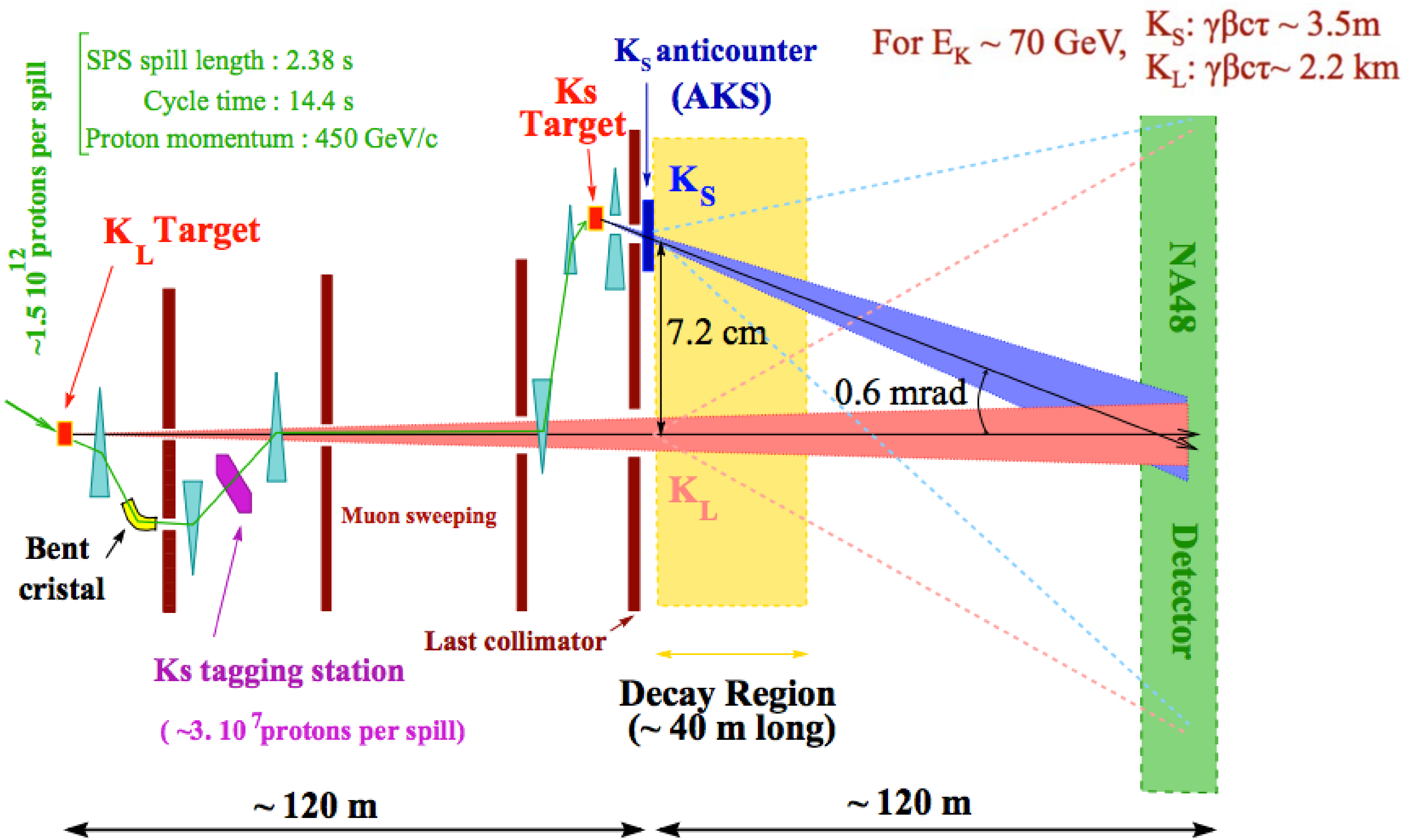
$$\Re = \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)}$$

with ≈± 0.1% precision

The NA-48 experiment at CERN

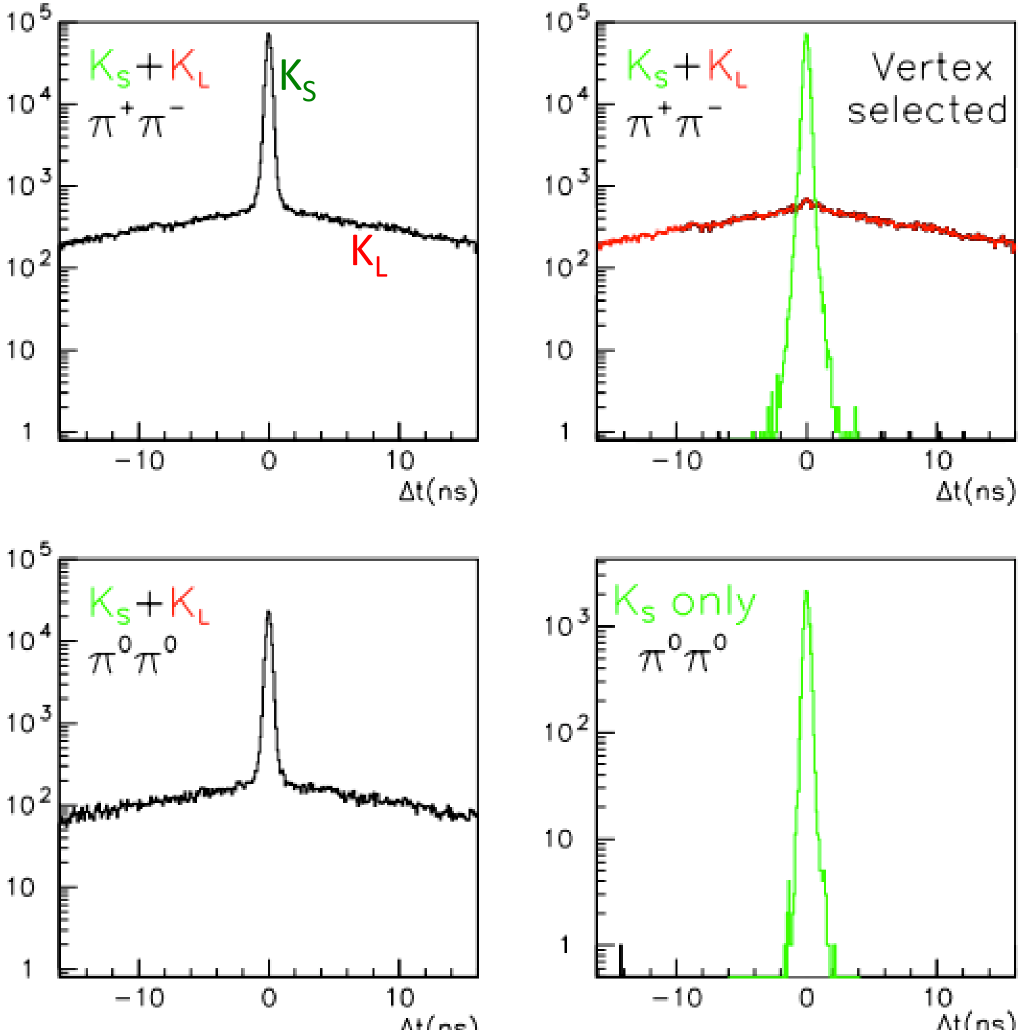


NA48: K_S and K_L beams simultaneously



K_S are distinguished from K_L by tagging the protons upstream of their production target.

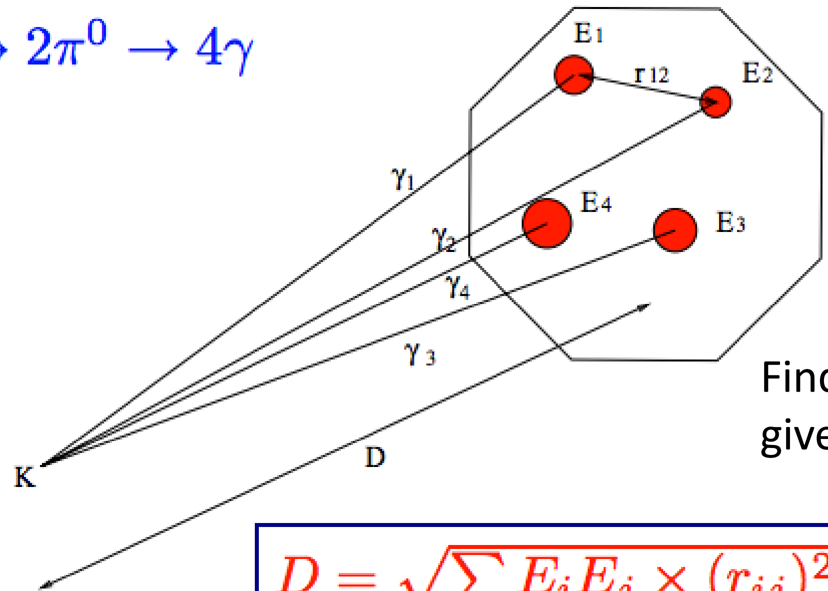
K_S and K_L signatures



Event time – proton beam counter time

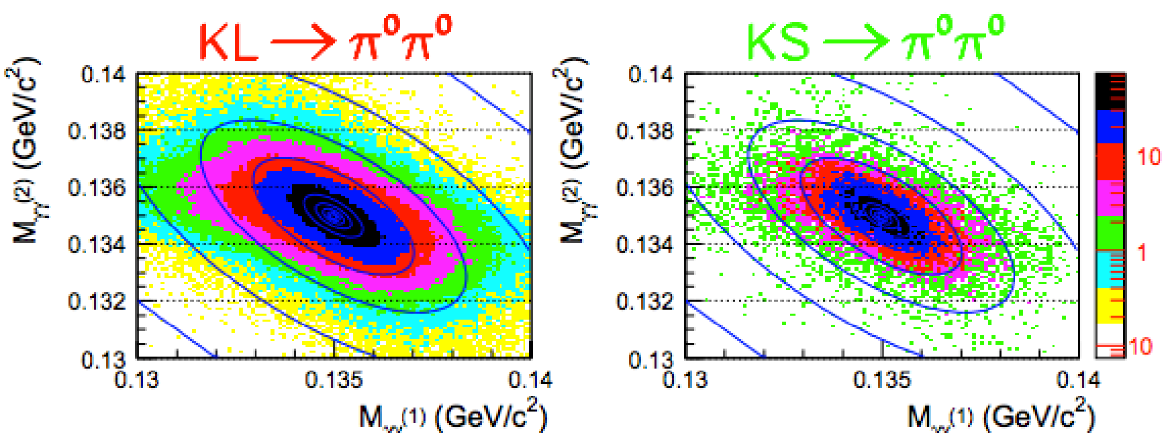
K→$\pi^0\pi^0$ events

$K_{L,S} \rightarrow 2\pi^0 \rightarrow 4\gamma$



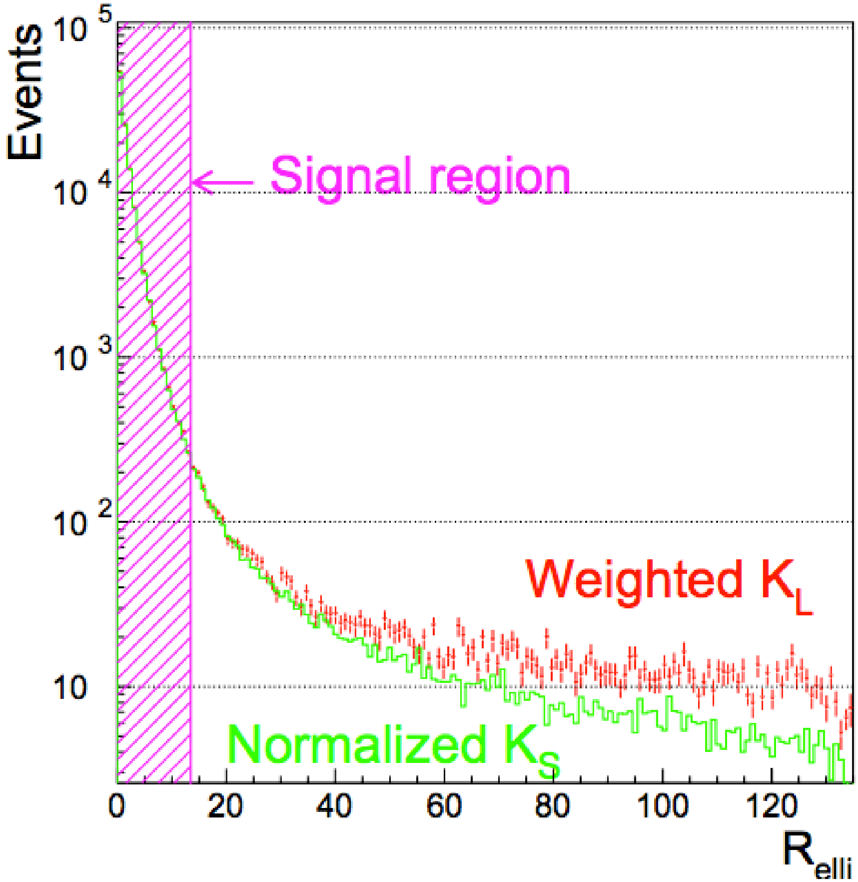
Find the value of D that gives the correct M_K value

$$D = \sqrt{\sum E_i E_j \times (r_{ij})^2} \;/M_K$$

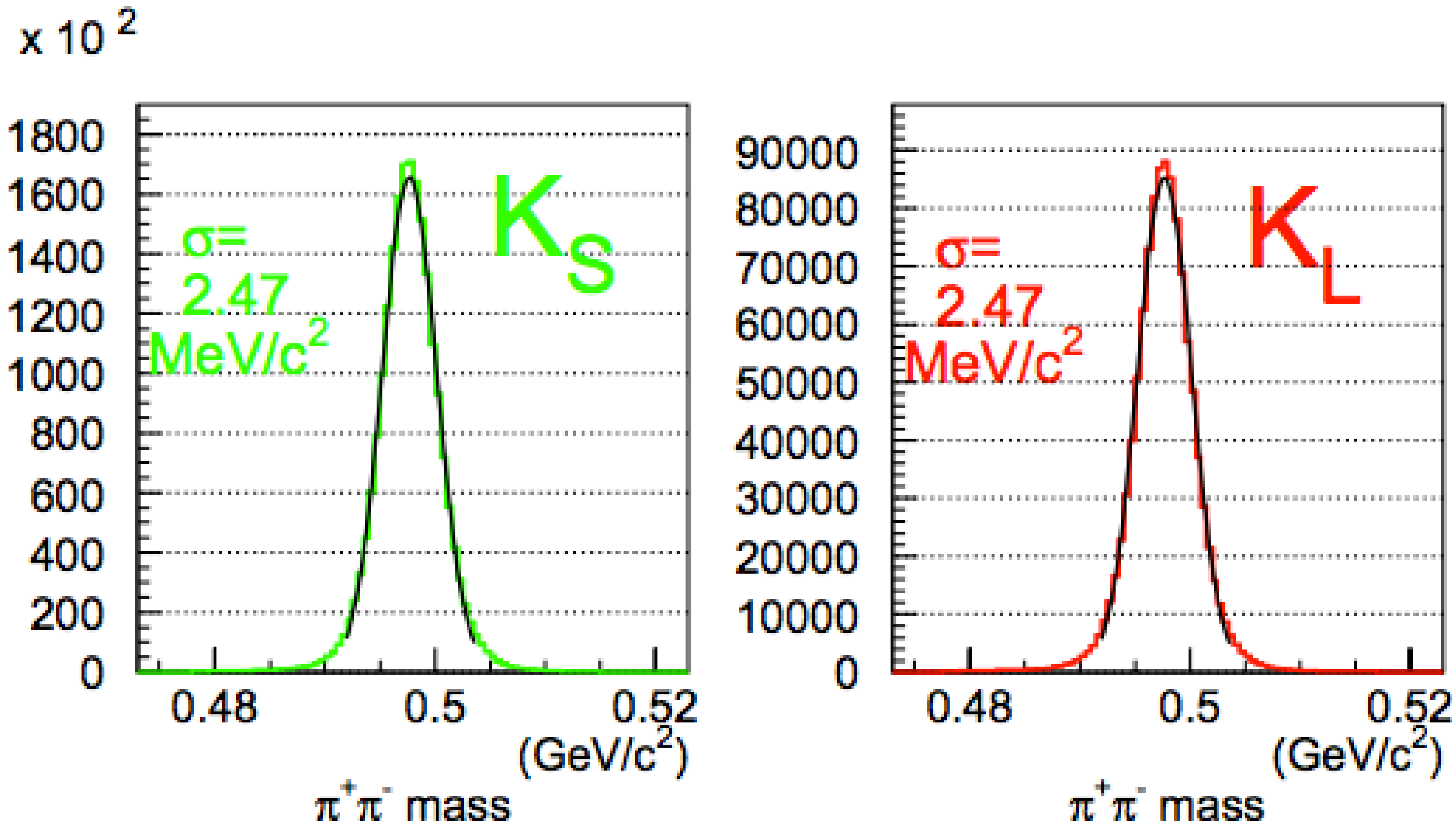


K→$\pi^0\pi^0$ event selection

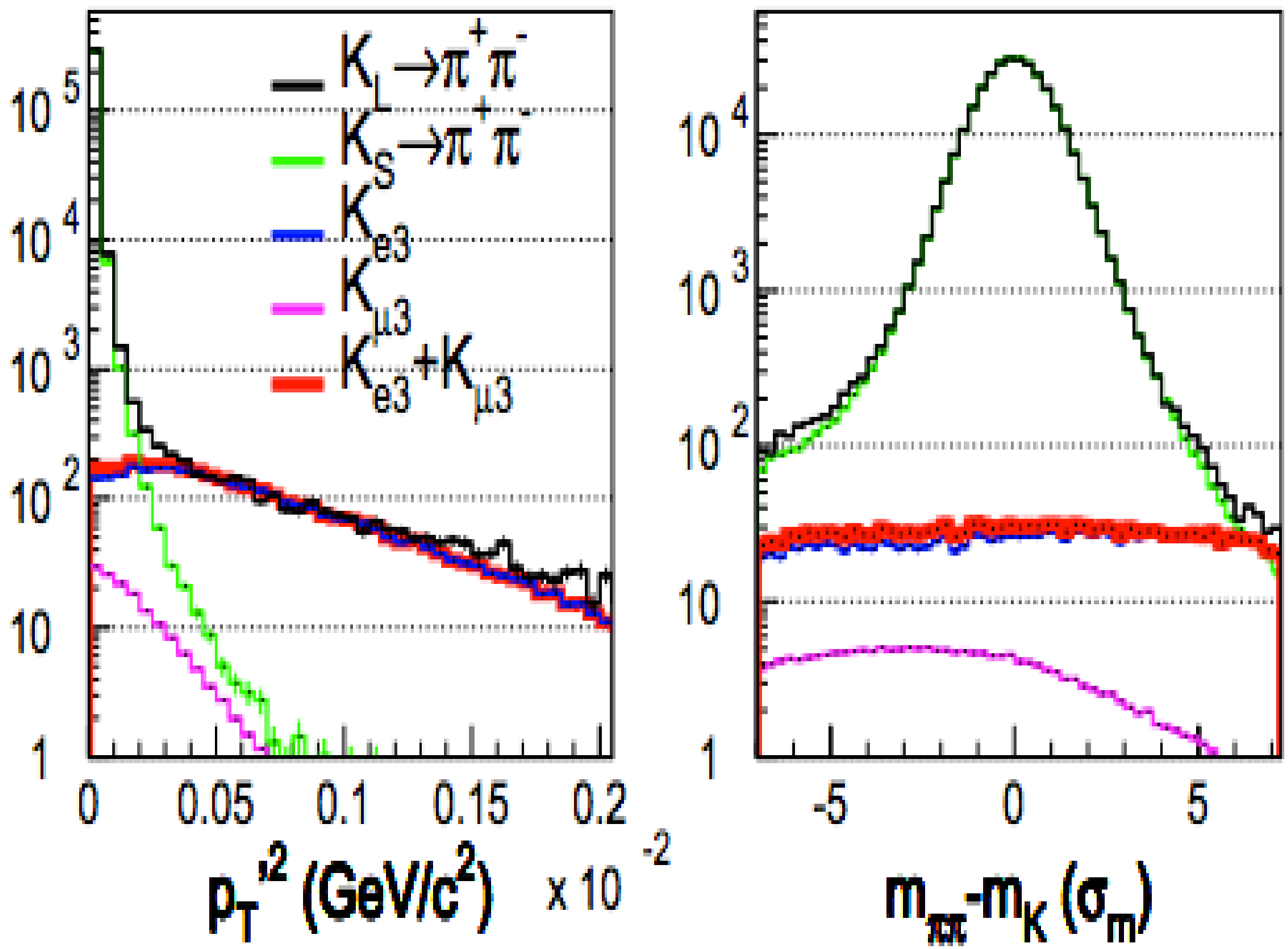
$$R_{ell} \equiv \left\{ \left(\frac{(m_1+m_2)-2m_{\pi^0}}{\sigma_{(m_1+m_2)}} \right)^2 + \left(\frac{m_1-m_2}{\sigma_{(m_1-m_2)}} \right)^2 \right\}$$



$K \rightarrow \pi^+\pi^-$ detection

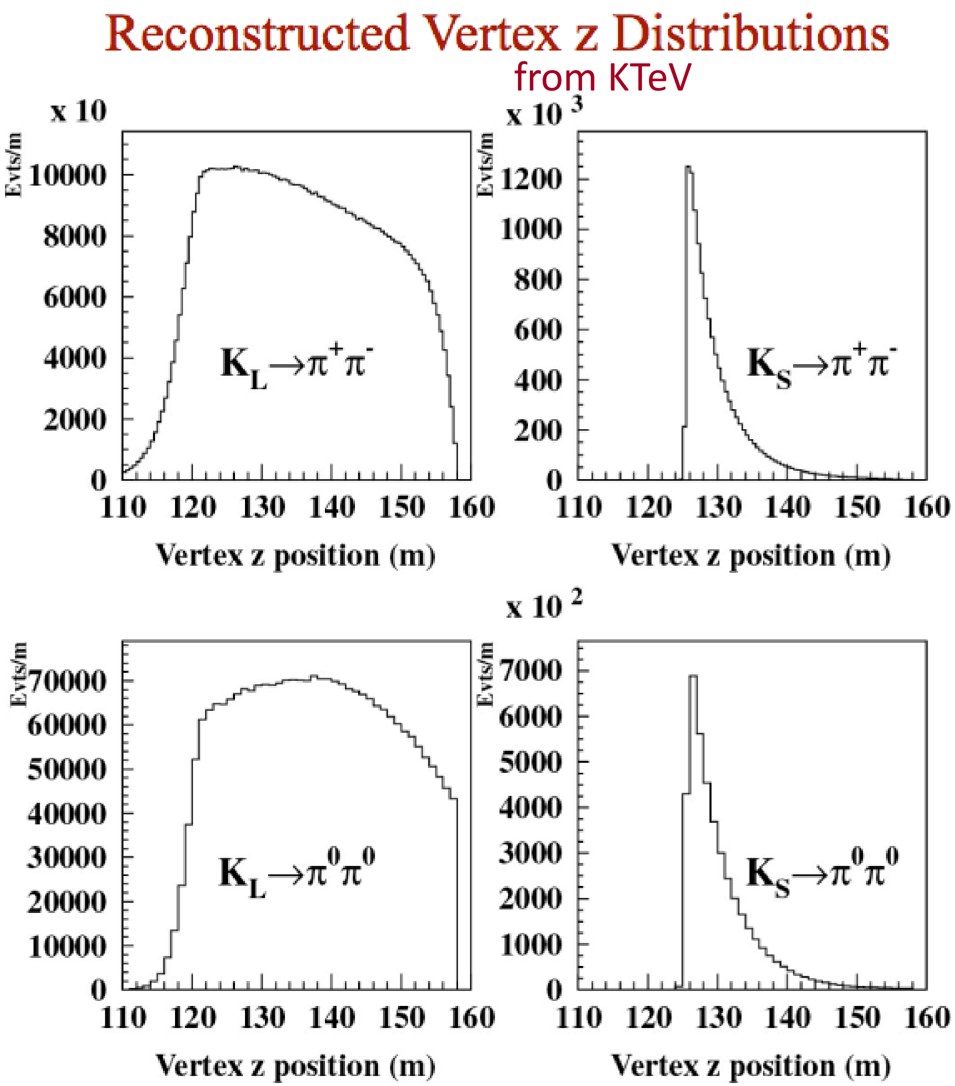


K→$\pi^+\pi^-$ selection



Problem: different decay-time distributions

from KTeV



Ideally, efficiency corrections cancel out in the ratios:

$$\Re = \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)}$$

However, because of the differences in the decay vertex distributions, these cancellations are not so effective.

Determining the R (double ratio)

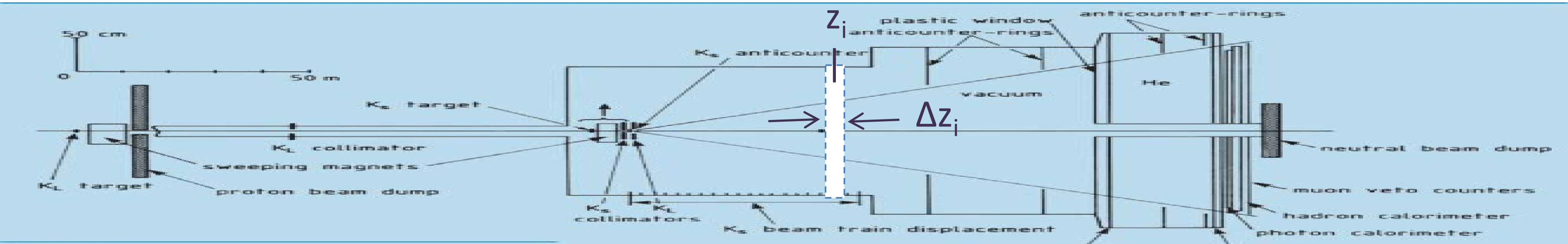


Fig.1: Schematic layout of the NA31 apparatus

Consider a slice of decay volume, Δz_i at z_i. and a range of K momentum Δp_j at p_j :

numbers of detected events

efficiencies

numbers of $K_{L(S)}$ in (Δz_i ,Δp_j) during the experiment

this is $\Delta\tau/\tau_{L(S)}$ probability for decay in Δz_i

decay branching fraction to this channel

$$N_{L(S)}^{00(+-)}(z_i,p_j) = \varepsilon^{00(+-)}(z_i,p_j)\,\frac{d^2 n_{L(S)}}{dzdp}(z_i,p_j)\Delta z_i \Delta p_j\,\frac{M_K \Delta z_i}{p_j c\tau_{L(S)}}\,\frac{\Gamma(K_{L(S)} \to (\pi\pi)^{00(+-)})}{\Gamma(K_{L(S)} \to all)}$$

this is 00(+-) specific; same for K_L & K_S

this is K_L & K_S specific same for 00(+-)

$$\varepsilon_{ij}^{00(+-)} = \varepsilon^{00(+-)}(z_i,p_j)$$

$$F_{ij}^{L(S)} \equiv \frac{d^2 n_{L(S)}}{dzdp}(z_i,p_j)\Delta z_i \Delta p_j\,\frac{M_K \Delta z_i}{p_j c\tau_{L(S)}\Gamma_{\mathrm{tot}}^{L(S)}}$$

$$\boxed{\Gamma(K_{L(S)} \to (\pi\pi)^{00(+-)}) = \frac{N_{L(S)}^{00(+-)}(z_i,p_j)}{\varepsilon_{ij}^{00(+-)} F_{ij}^{L(S)}}}$$

Low statistics experiments

Compute $\varepsilon^{00(+-)}(z,p)$ with Montecarlo;

Determine $F_L(z,p)$ from $K_L \rightarrow \pi^+\pi^-\pi^0$ decays + Monte Carlo, etc

Weight events

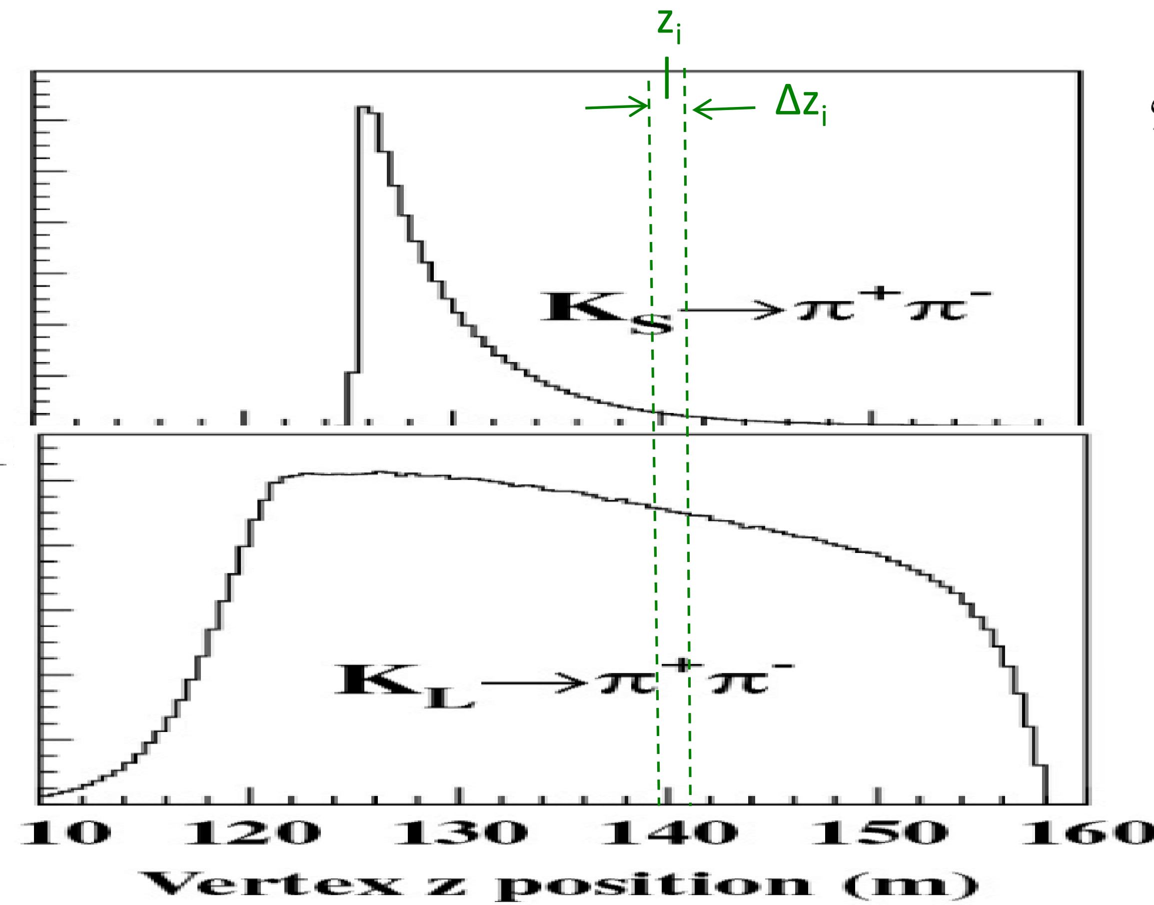
Determine: $$\Gamma\left(K_{L(S)} \rightarrow (\pi\pi)^{00(+-)}\right) = \sum_{ij} \frac{N_{L(S)}^{00(+-)}(z_i, p_j)}{\varepsilon_{ij}^{00(+-)} F_{ij}^{00(+-)}}$$

Make ratios

Systematic errors are dominated by MC, but are smaller than statistical errors

High statistics experiments (+-)

measure $\Re i_{ij}^{+-} = \dfrac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}$ for events with vertices in the small region $\Delta z_i, \Delta p_j$



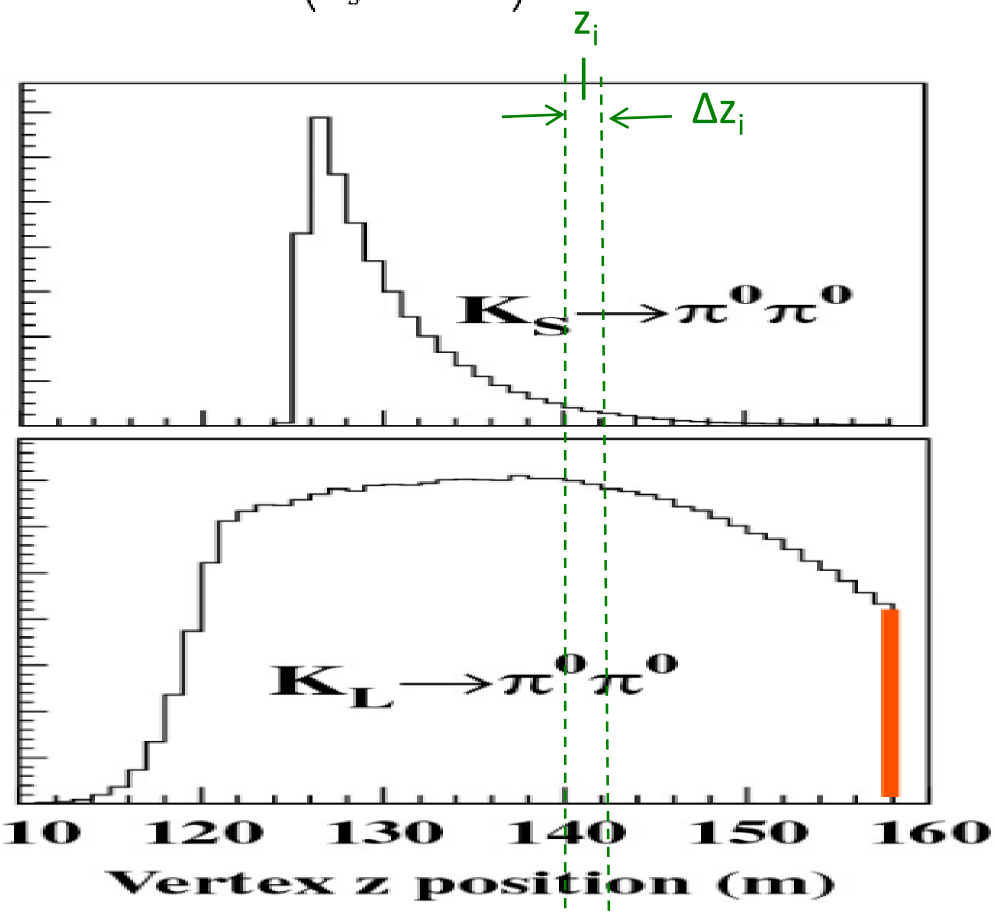
$$\Re_{ij}^{+-} = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{\dfrac{N_{ij}(K_L \to \pi^+\pi^-)}{\varepsilon_{ij}^{+-} F_{ij}^{L}}}{\dfrac{N_{ij}(K_S \to \pi^+\pi^-)}{\varepsilon_{ij}^{+-} F_{ij}^{S}}}$$

$$\Re_{ij}^{+-} = \frac{F_{ij}^{S} N_{ij}(K_L \to \pi^+\pi^-)}{F_{ij}^{L} N_{ij}(K_S \to \pi^+\pi^-)}$$

efficiencies cancel

Efficiency weighting (00)

measure $\Re_{ij}^{00} = \dfrac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)}$ for events with vertices in the same region Δz_i



$$\Re_{ij}^{00} = \frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} = \frac{\dfrac{N_{ij}\left(K_L \to \pi^0\pi^0\right)}{\varepsilon_{ij}^{00} F_{ij}^{L}}}{\dfrac{N_{ij}\left(K_S \to \pi^0\pi^0\right)}{\varepsilon_{ij}^{00} F_{ij}^{S}}}$$

$$\Re_{ij}^{00} = \frac{F_{ij}^{S} N_{ij}\left(K_L \to \pi^0\pi^0\right)}{F_{ij}^{L} N_{ij}\left(K_S \to \pi^0\pi^0\right)}$$

efficiencies cancel

High statistics experiments

make a double ratio:

$$\Re_{ij}^{tot} = \frac{\Re_{ij}^{+-}}{\Re_{ij}^{00}} = \frac{F_{ij}^{S} \cdot N_{ij}\left(K_L \to \pi^+\pi^-\right) / F_{ij}^{L} \cdot N_{ij}\left(K_S \to \pi^+\pi^-\right)}{F_{ij}^{S} \cdot N_{ij}\left(K_L \to \pi^0\pi^0\right) / F_{ij}^{L} \cdot N_{ij}\left(K_S \to \pi^0\pi^0\right)}$$

-- beam flux factors cancel --

no dependence on efficiency
or beam intensity

just count events

$$\boxed{\Re_{ij}^{tot} = \frac{N_{ij}\left(K_L \to \pi^+\pi^-\right) / N_{ij}\left(K_S \to \pi^+\pi^-\right)}{N_{ij}\left(K_L \to \pi^0\pi^0\right) / N_{ij}\left(K_S \to \pi^0\pi^0\right)}}$$

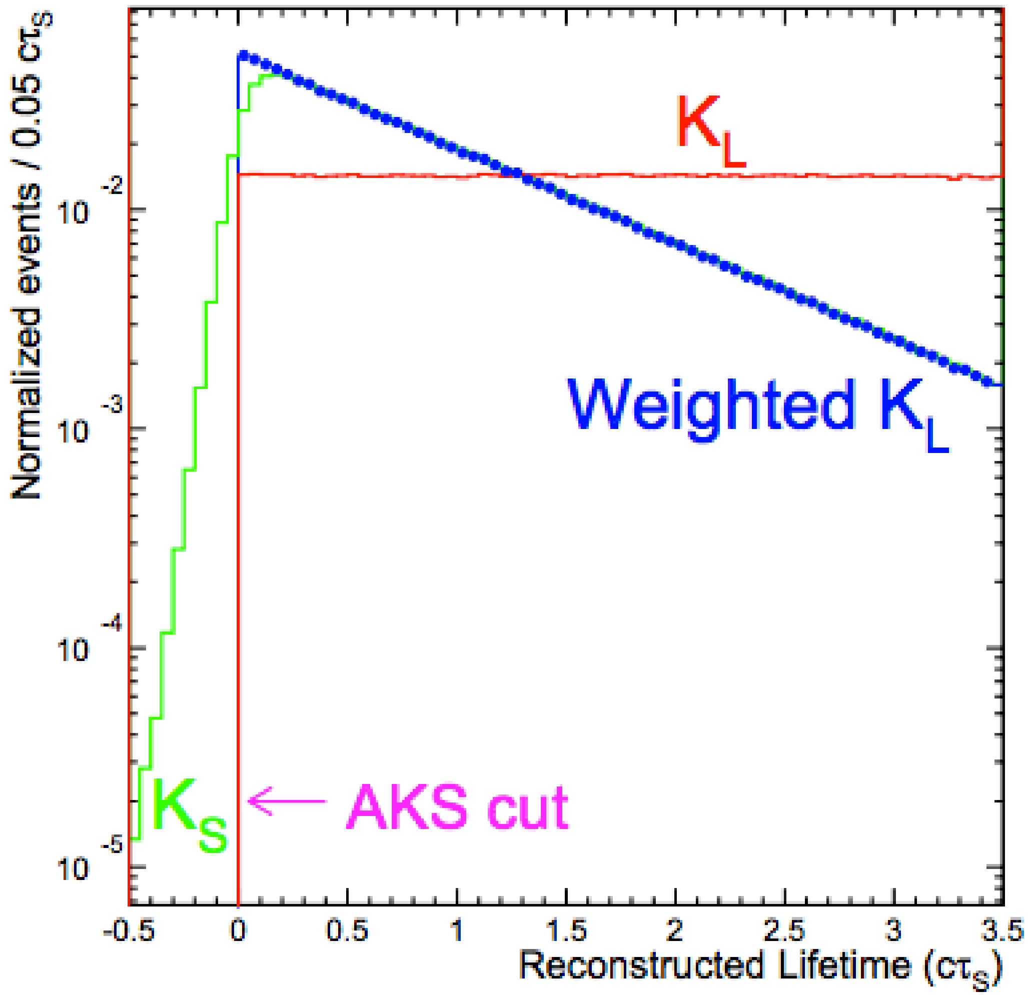
This results in many ($N=Z_{tot}/\Delta z \times P_{tot}/\Delta p$?) independent measurements with no MC dependence & small systematic errors

combine these to get the final result:

$$\Re_{\text{final}} = \frac{\sum_{ij} \frac{\Re_{ij}^{tot}}{\sigma_{ij}^2}}{\sum_{i} \frac{1}{\sigma_{ij}^2}} \pm \frac{1}{\sqrt{\sum_{i} \frac{1}{\sigma_{ij}^2}}}$$

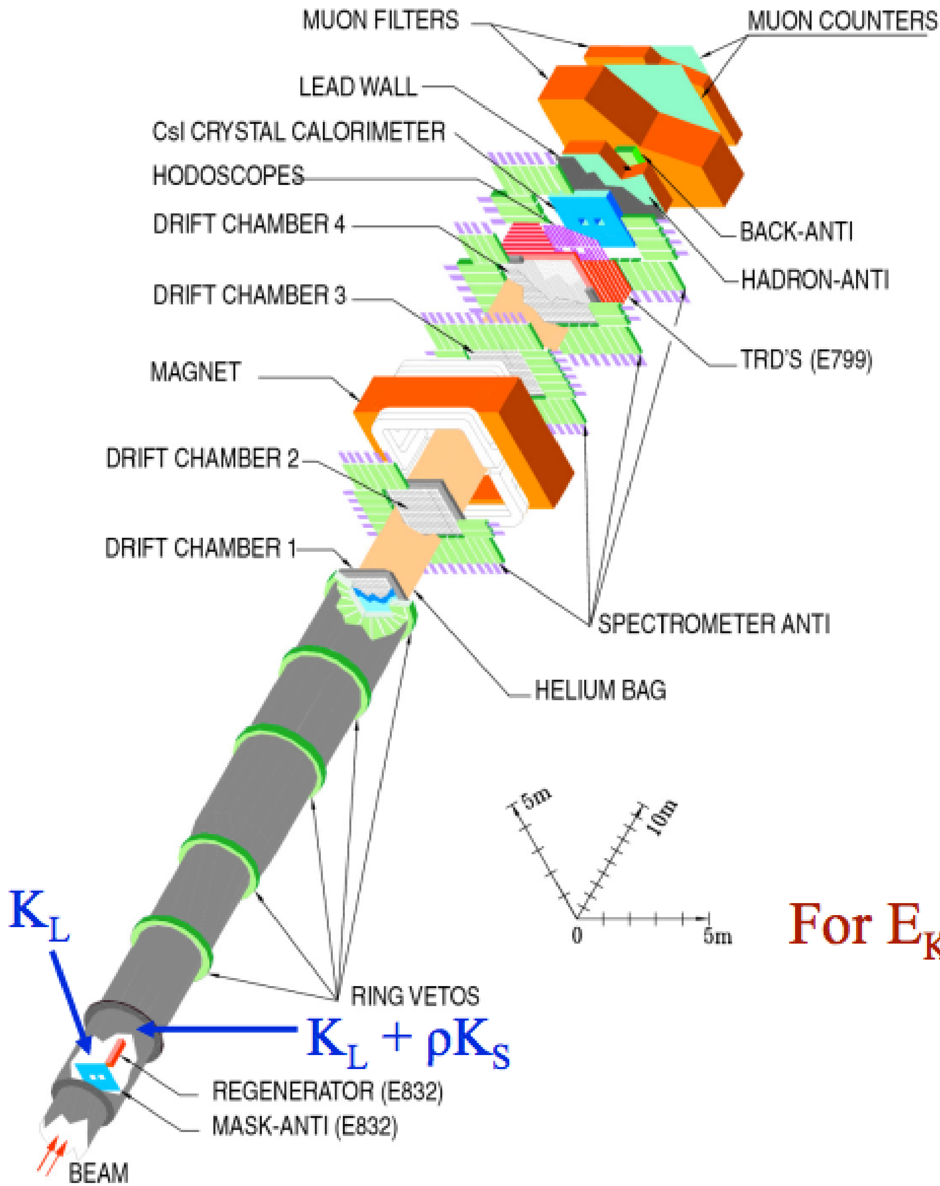
The final statistical error is larger, but the systematic error is smaller

Use “life-time weighting”



Important to correct for different decay vertex distributions

The KTeV Detector

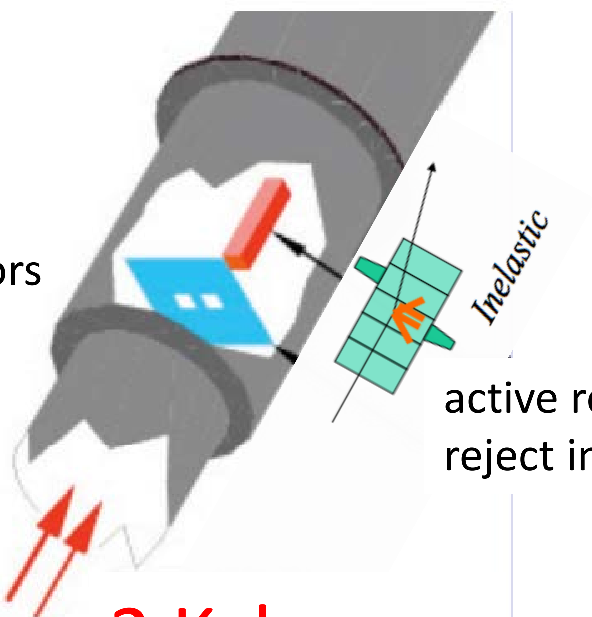


- Charged particle momentum resolution < 1% for p>8 GeV/c; Momentum scale known to 0.01% from $K \to \pi^+\pi^-$
- CsI energy resolution < 1% for E_γ > 3 GeV; energy scale known to 0.05% from $K \to \pi e \nu$.

For $E_K \sim 70$ GeV, K_S: $\gamma\beta c\tau \sim 3.5$m
K_L: $\gamma\beta c\tau \sim 2.2$ km

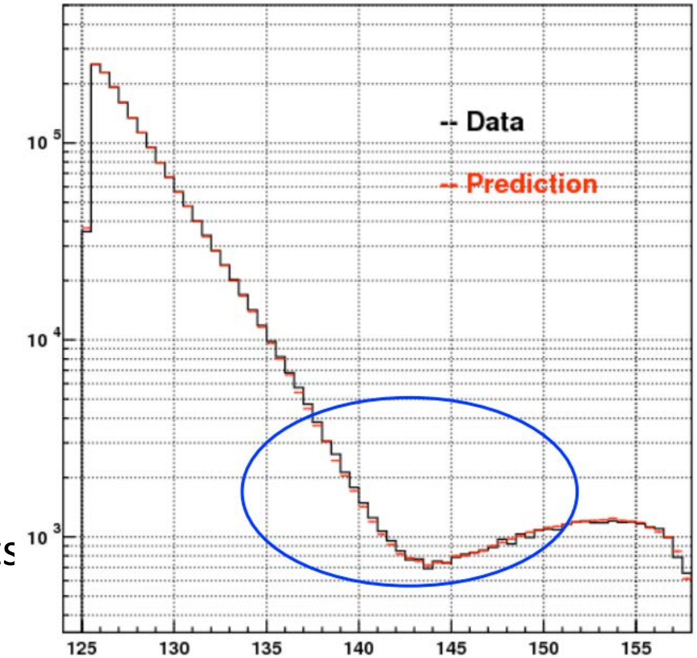
Double-beam with active regenerator

Beam collimators



active regenerator:
reject incoherent events

2 K_L beams



use $K_L \rightarrow \pi\pi$ – $K_S \rightarrow \pi\pi$ interference
to measure ρ

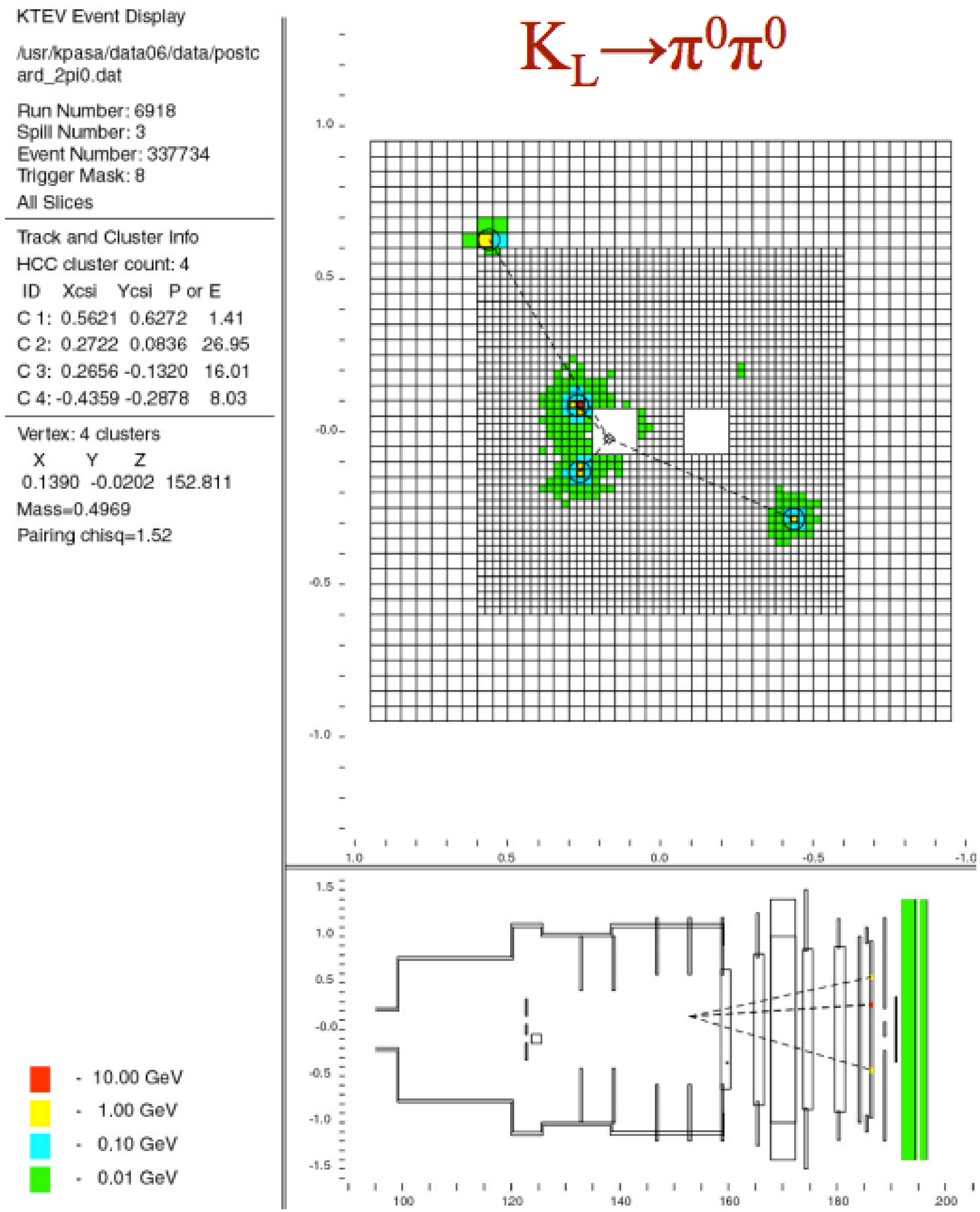
$K_L \longrightarrow$ $\qquad$ K_L

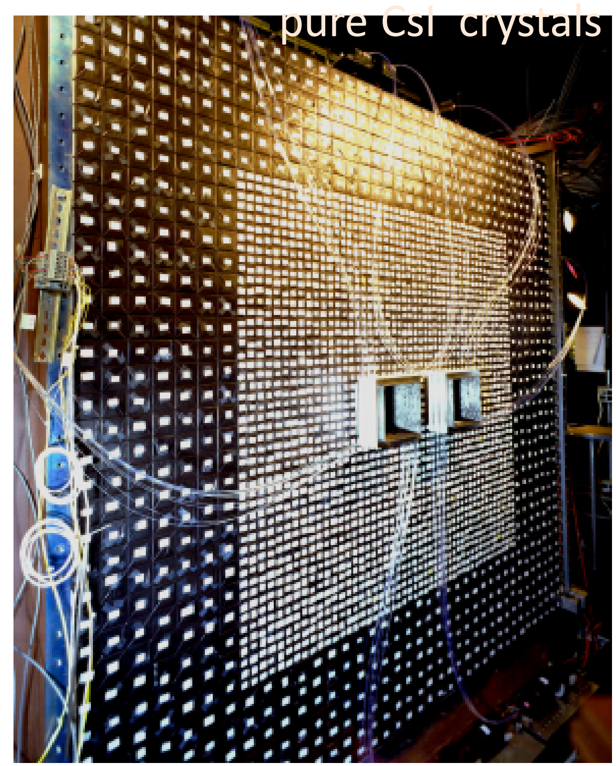
$K_L \longrightarrow \boxed{}$ $\qquad$ $K_L + \rho K_S$

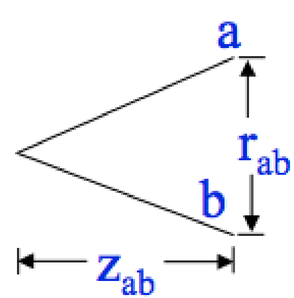
$\left.\begin{array}{l} K_L \\ K_L + \rho K_S \end{array}\right\} \pi^+\pi^- \text{ or } \pi^0\pi^0$

$\rho=0.03$

$K_L \to \pi^0\pi^0$

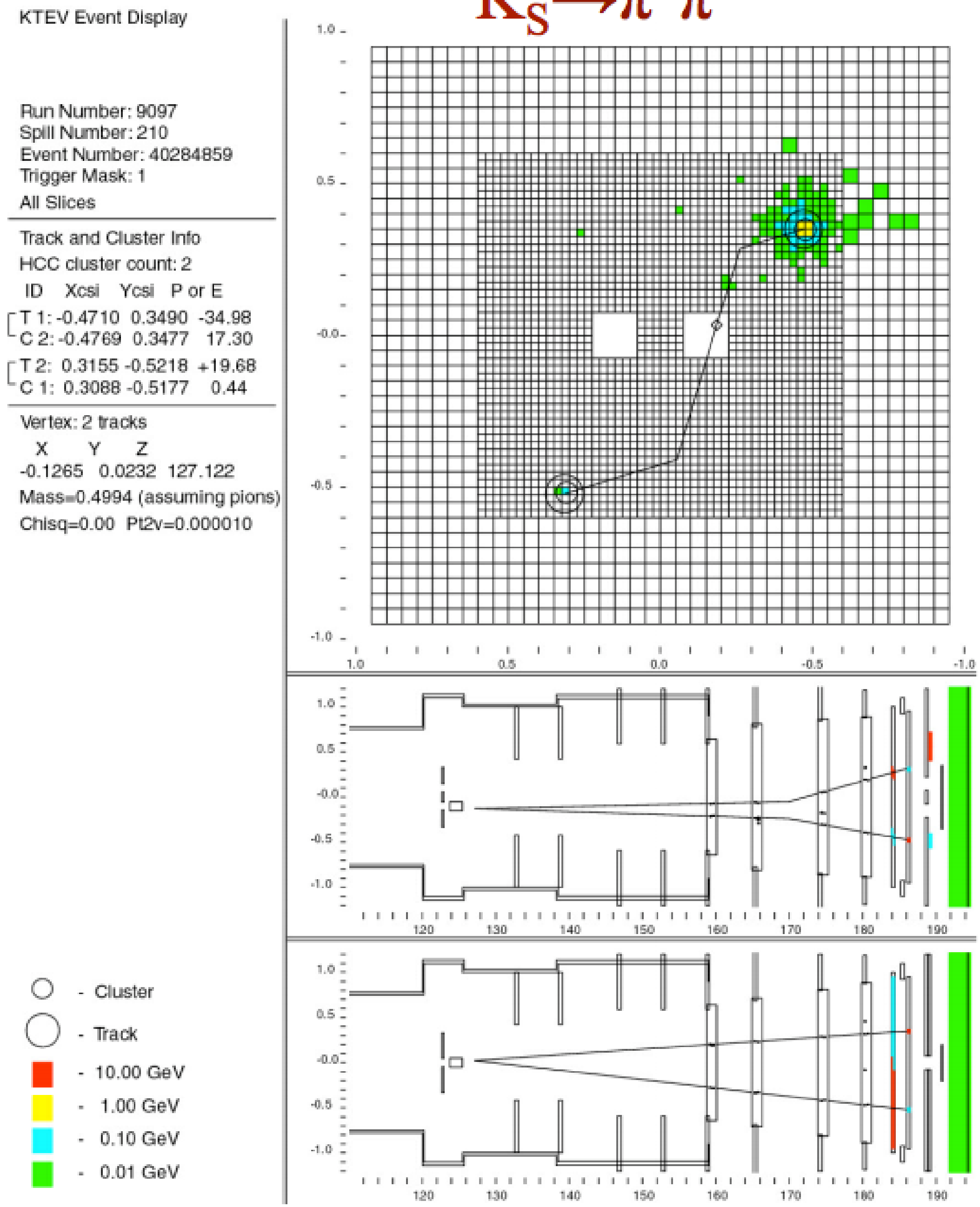




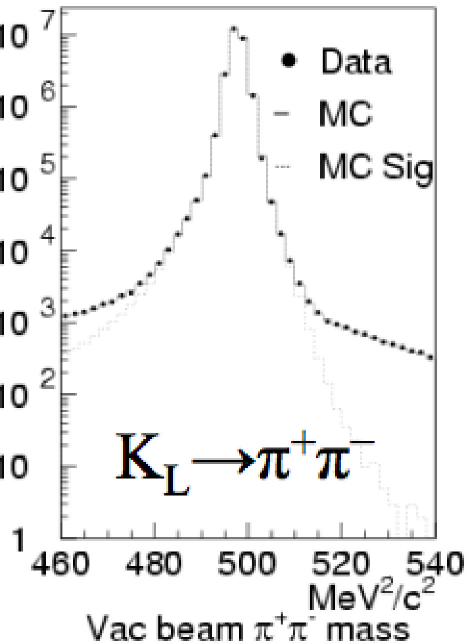


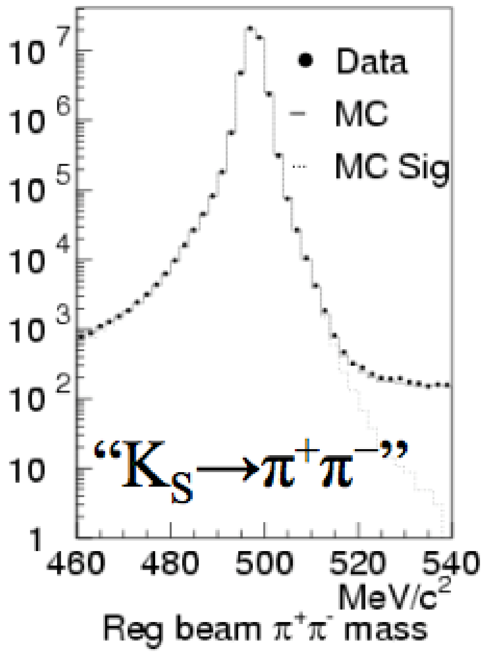
$$z_{ab}^2 \approx \frac{E_a E_b r_{ab}^2}{m_{\pi^0}^2}$$

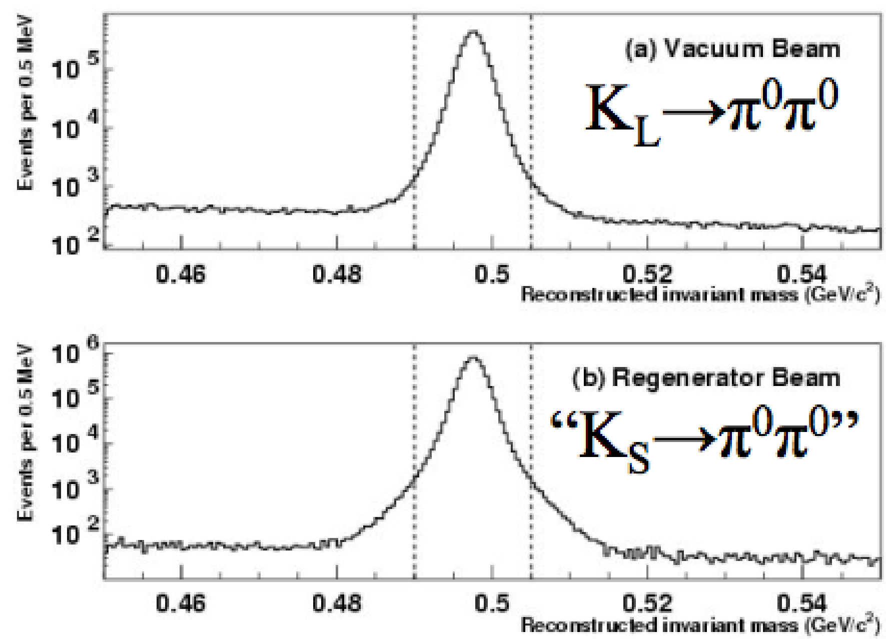
$K_S \to \pi^+\pi^-$


KTEV Event Display
Run Number: 9097
Spill Number: 210
Event Number: 40284859
Trigger Mask: 1
All Slices
Track and Cluster Info
HCC cluster count: 2
ID Xcsi Ycsi P or E
T 1: -0.4710 0.3490 -34.98
C 2: -0.4769 0.3477 17.30
T 2: 0.3155 -0.5218 +19.68
C 1: 0.3088 -0.5177 0.44
Vertex: 2 tracks
X Y Z
-0.1265 0.0232 127.122
Mass=0.4994 (assuming pions)
Chisq=0.00 Pt2v=0.000010
- Cluster
- Track
- 10.00 GeV
- 1.00 GeV
- 0.10 GeV
- 0.01 GeV

Invariant Mass Plots

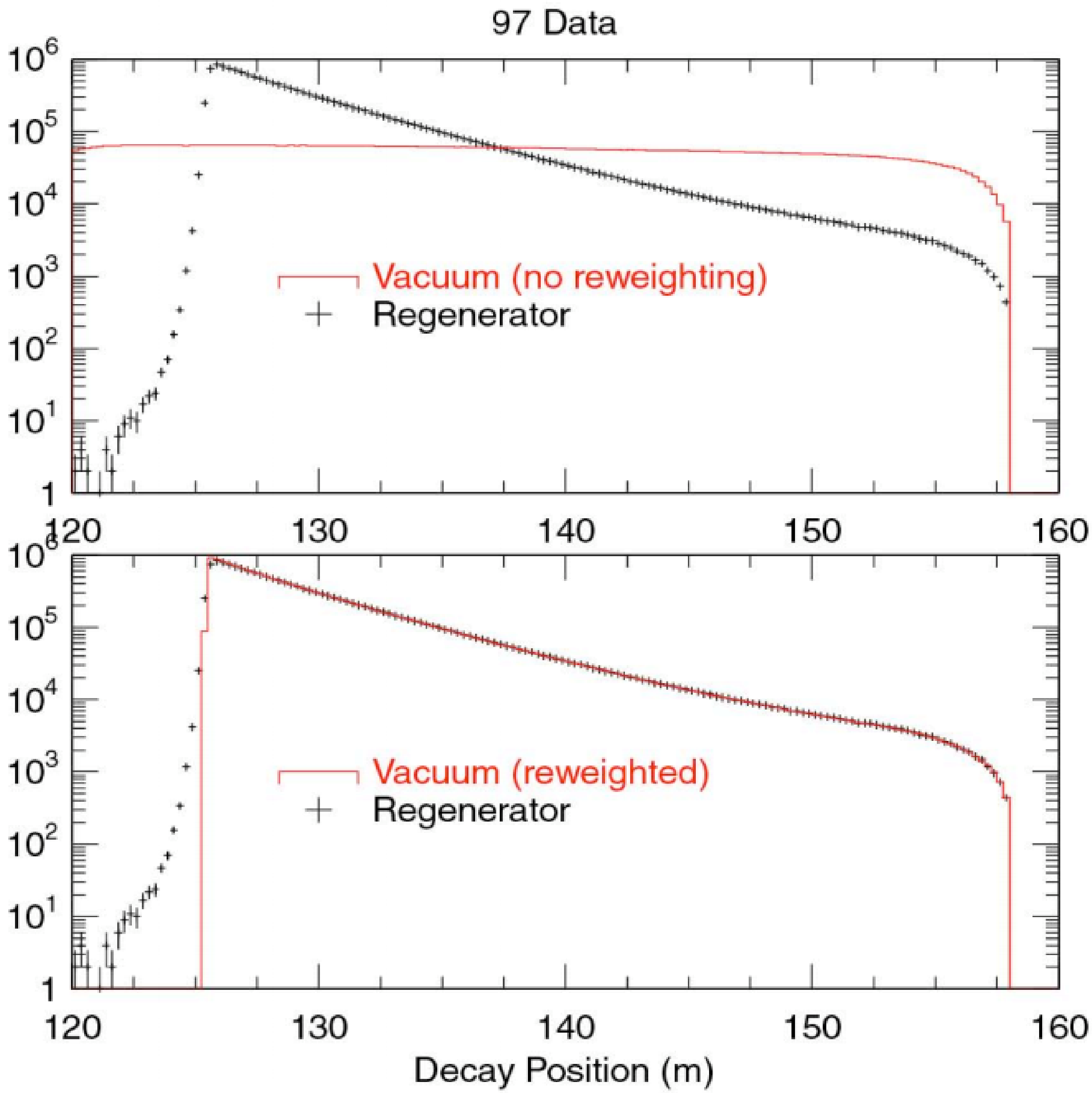






Mass resolution is ~1.5 MeV/c^2 for both decay modes.

Reweight K_L decays to reg. beam distribution.



Huge numbers of events

KTeV

	K_L Vacuum Beam	"K_S" Reg. Beam
$K \to \pi^+\pi^-$	8,593,988	14,903,532
$K \to \pi^0\pi^0$	$\boxed{2{,}489{,}537}$	4,130,392

NA-48

4.8 million $\pi^0\pi^0$ events total

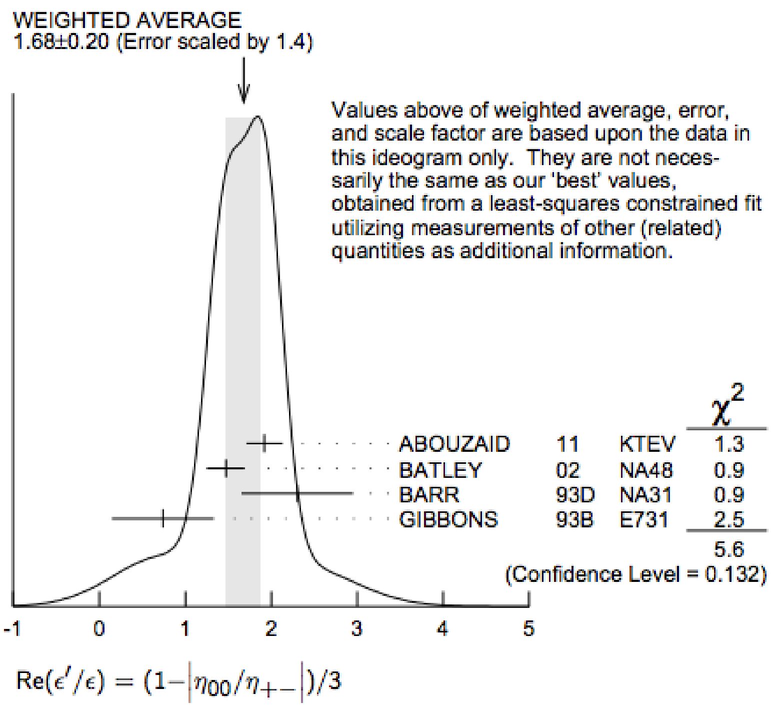
~1000x increase over the best pre-1990 experiments

Final ε' results

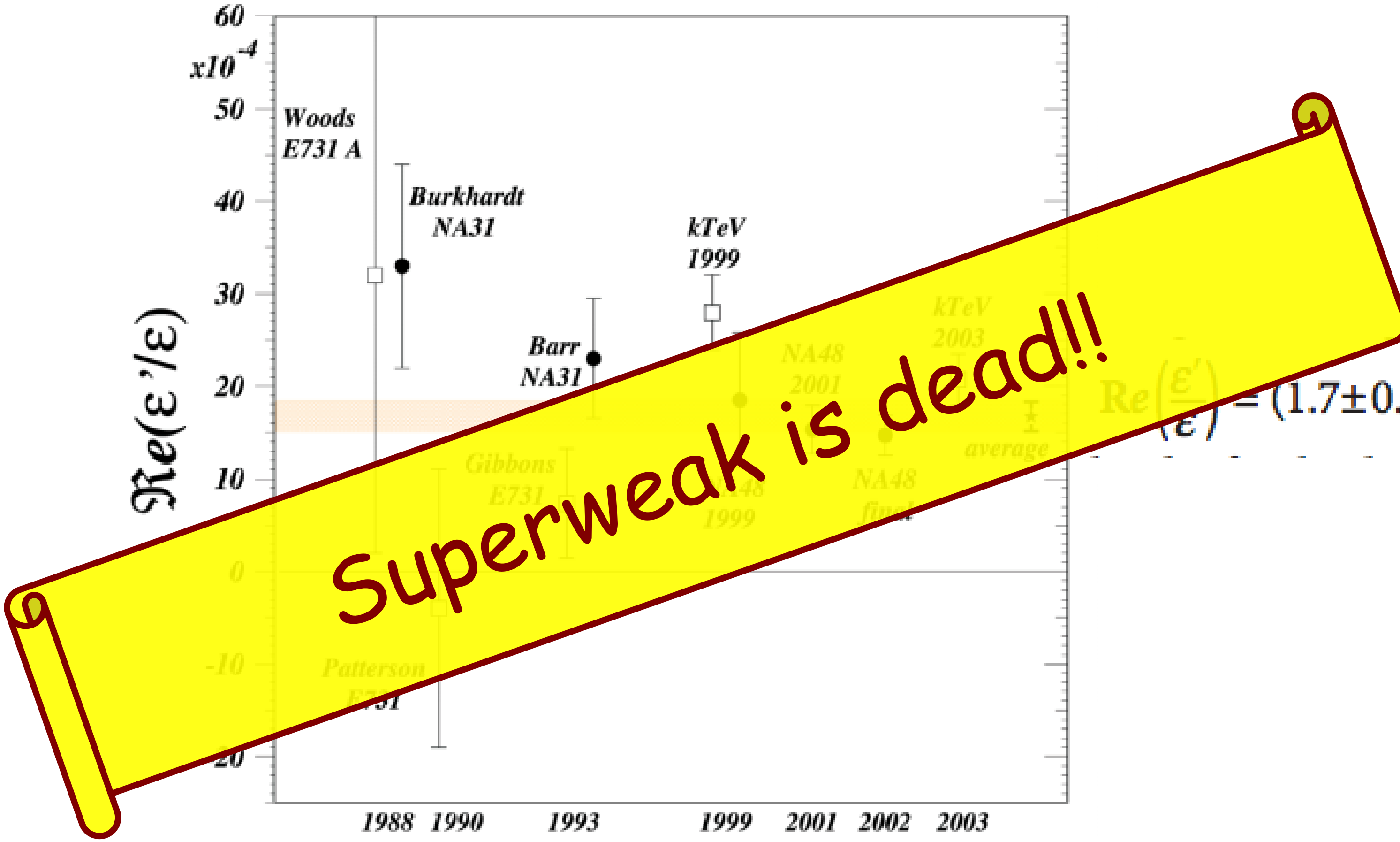
$\mathrm{Re}(\epsilon'/\epsilon) = (1-|\eta_{00}/\eta_{+-}|)/3$

We have neglected terms of order $\omega \cdot \mathrm{Re}(\epsilon'/\epsilon)$, where $\omega = \mathrm{Re}(A_2)/\mathrm{Re}(A_0) \simeq 1/22$. If included, this correction would lower $\mathrm{Re}(\epsilon'/\epsilon)$ by about 0.04×10^{-3}. See SOZZI 04.

VALUE (units 10^{-3})	*DOCUMENT ID*	*TECN*	*COMMENT*	
1.66 ±0.23 OUR FIT Error includes scale factor of 1.6.				
1.68 ±0.20 OUR AVERAGE Error includes scale factor of 1.4. See the ideogram below.				
1.92 ±0.21	[1] ABOUZAID 11	KTEV	Assuming *CPT*	**64M $\pi^+\pi^-$ evts & 16M $\pi^0\pi^0$ evts**
1.47 ±0.22	BATLEY 02	NA48		**17M $\pi^+\pi^-$ evts & 3.6M $\pi^0\pi^0$ evts**
0.74 ±0.52 ±0.29	GIBBONS 93B	E731		



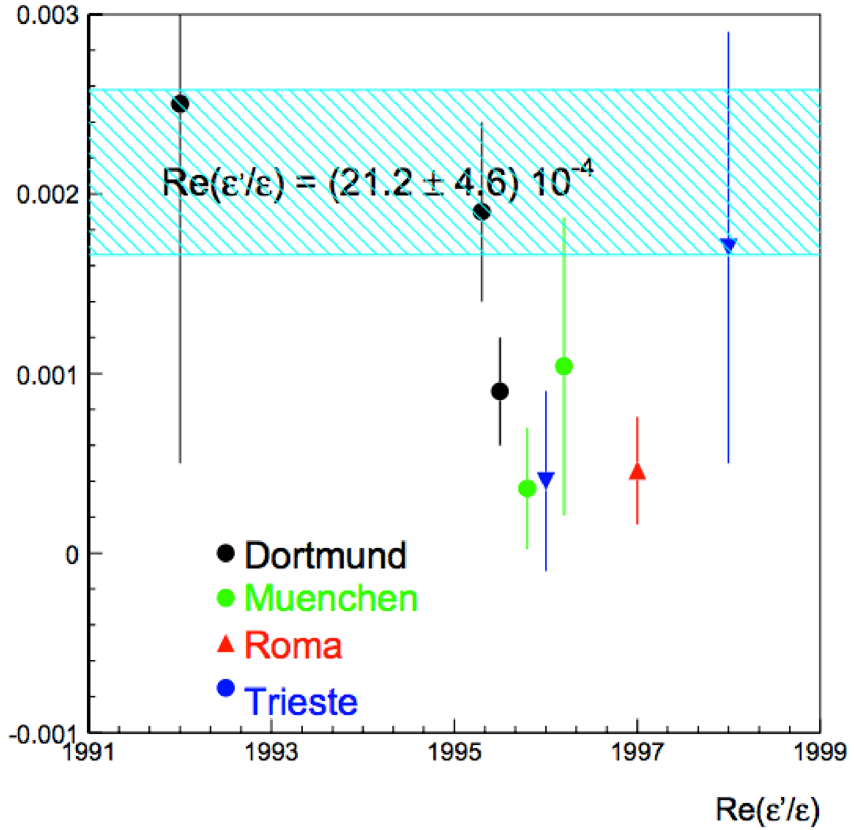
Final ε’ results



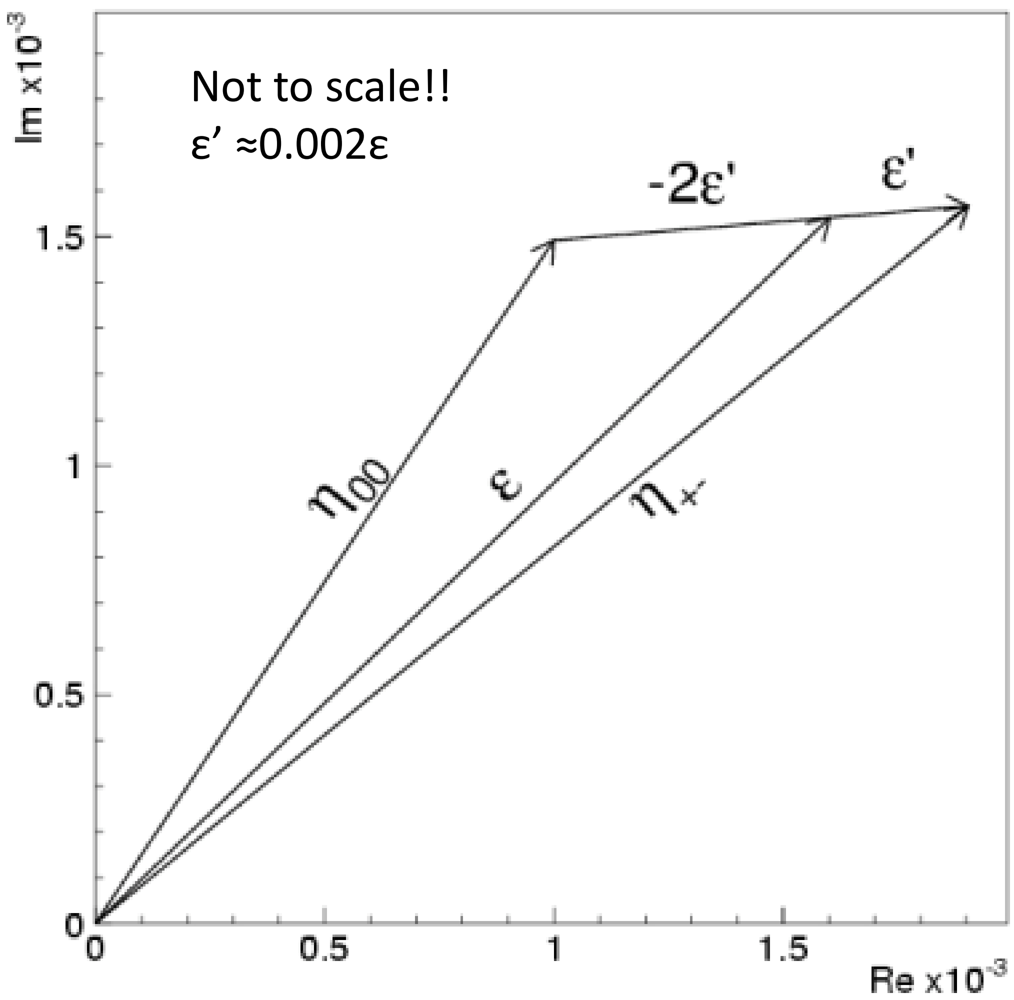
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.7\pm0.2)\,\mathrm{x}10^{-3}.$$

What do the theorists say?

- Theoretical predictions for $\mathrm{Re}(\varepsilon'/\varepsilon)$ generally below $1 \cdot 10^{-3}$



ε, η_{+-} and η_{00} today



CP violating asymmetries in QM

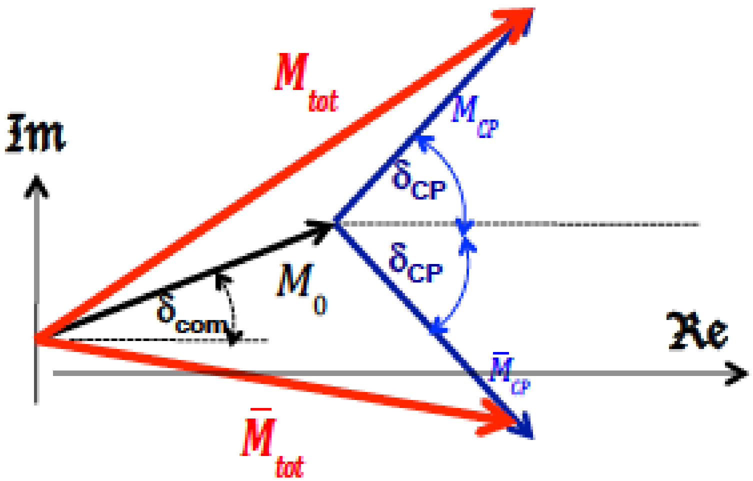
requirements for a CPV-generated particle-antiparticle asymmetry:

1) a process with a non-zero CP phase (δ_{CP})

2) a competing process with the same final state

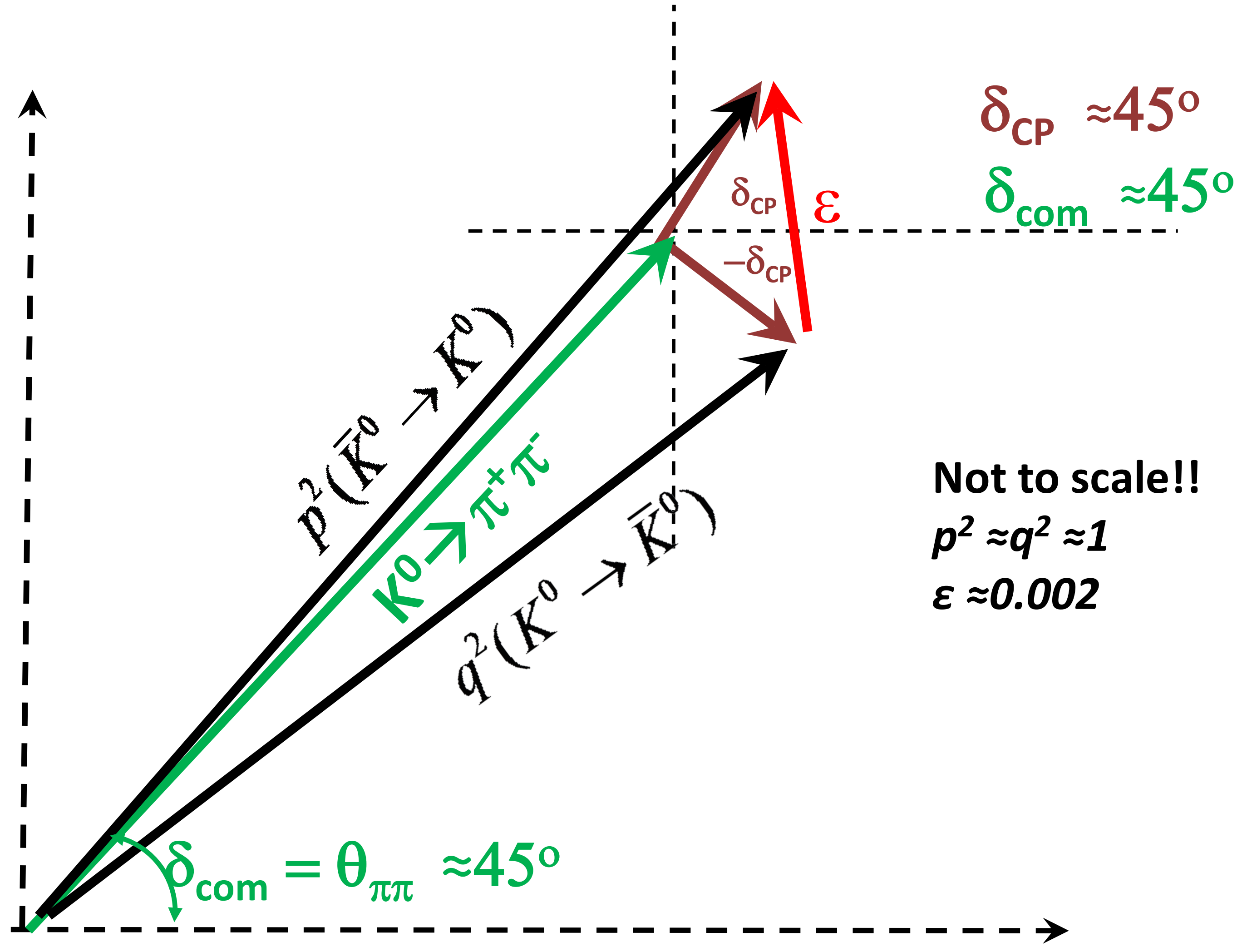
3) a non-zero common (or strong) phase (δ_{com})

	$i \to f$	$\bar{i} \to \bar{f}$
M_{CP} phase	δ_{CP}	$-\delta_{CP}$
M_0 phase	δ_{com}	δ_{com}

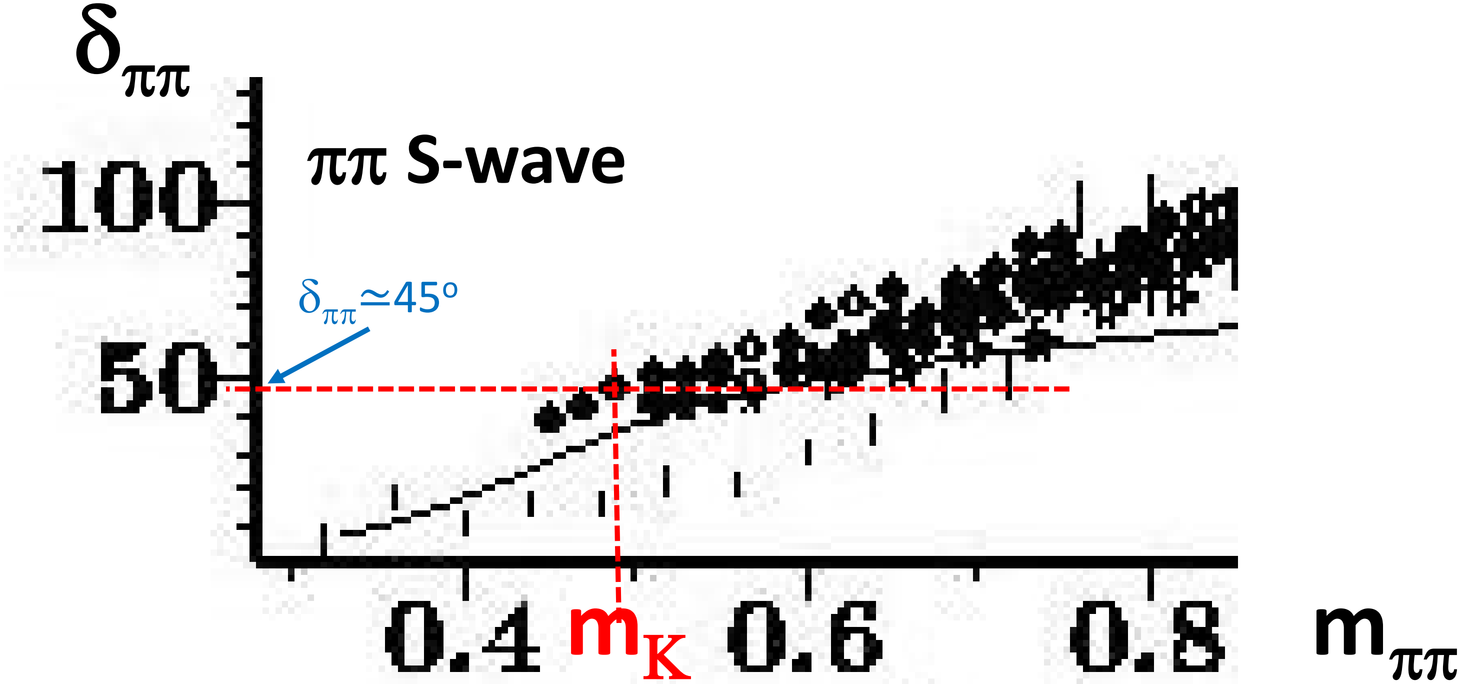


δ_{com}, the "common" or "strong" phase, of M_0, is the same for $i \to f$ and $\bar{i} \to \bar{f}$

CPV in $K_L \rightarrow \pi^+\pi^-$



S-wave $\pi\pi$ phase shifts

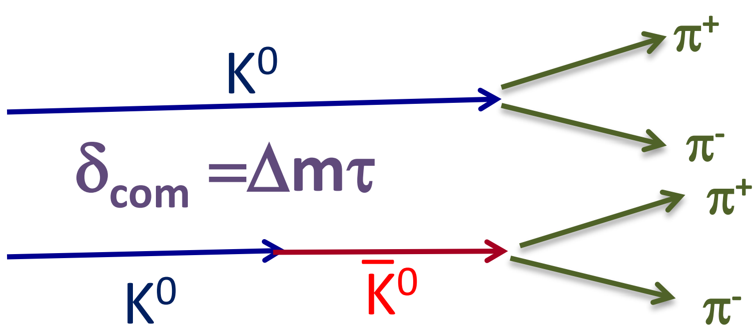


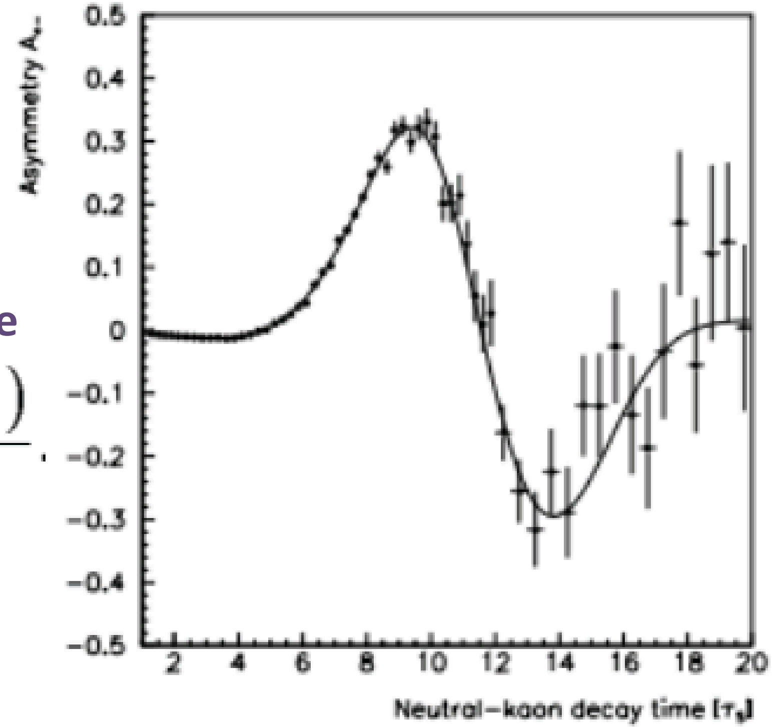
$K^0 \rightarrow \bar{K}^0 \rightarrow K^0$ in $\bar{p}p \rightarrow K^0 K^- \pi^+$ ($\bar{K}^0 K^+ \pi^-$) (CPLEAR)

$$A_{+-}(\tau) = \frac{\overline{N}(\tau) - \alpha N(\tau)}{\overline{N}(\tau) + \alpha N(\tau)}$$

common phase

$$= -2\frac{|\eta_{+-}|e^{\frac{1}{2}(\tau/\tau_S - \tau/\tau_L)}\cos(\Delta m\tau - \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\tau/\tau_S - \tau/\tau_L)}}.$$





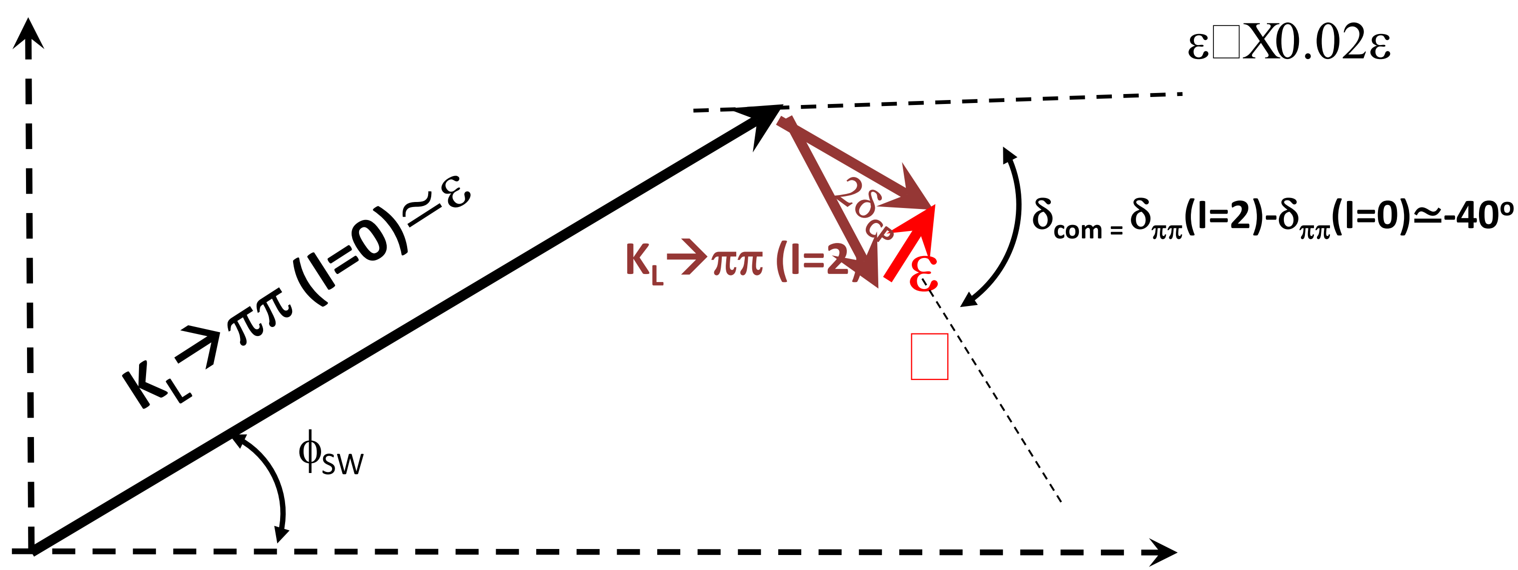
direct CP parameter ε’

$$\varepsilon' = \frac{i}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)}$$

common phase

Not to scale!!

ε□X0.02ε



If it’s not superweak, what is it?

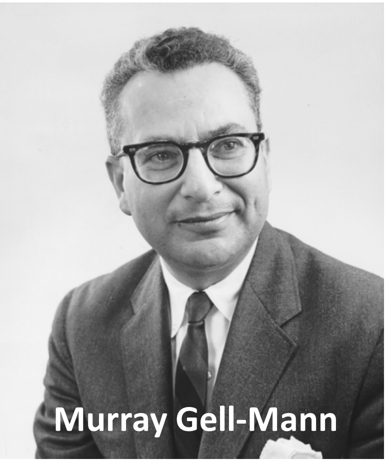
Can CPV fit into the Standard model?

Clue: CPV is seen in strangeness-changing weak decays.

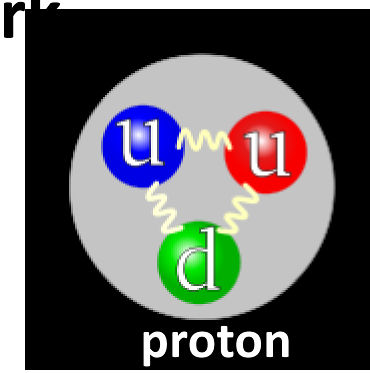
maybe CPV has something to do with flavor-changing Weak Interactions

Flavor mixing
&
CP Violation

Three Quarks for Müster Mark

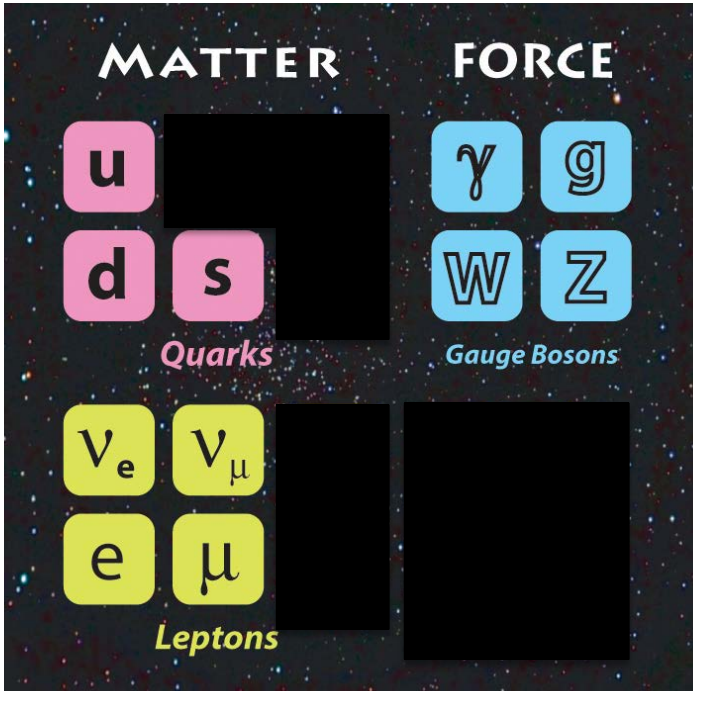


1963: all known strongly interacting particles are comprised of three basic constituents: fractionally charged quarks (and their three anti-quark partners).



$$\begin{array}{ll} q=+2/3 & \\ q=-1/3 & \end{array} \begin{pmatrix} u \\ d \end{pmatrix} \quad s$$

The elementary particles before 1974



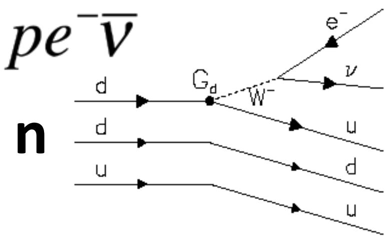
Weak Interactions in the 3-quark era

1964--1974

3 quarks:

q=+2/3

$n \to p e^- \bar{\nu}$



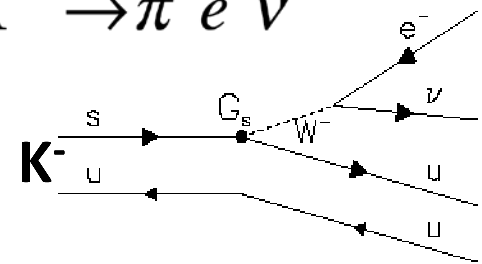
$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad s$$

|ΔS|=0

|ΔS|=1

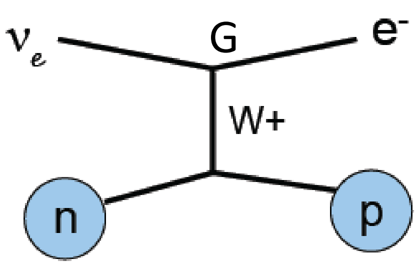
q=−1/3

$K^- \to \pi^0 e^- \bar{\nu}$



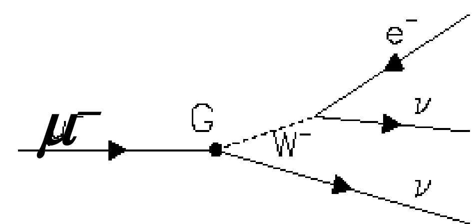
4 leptons:

$\nu_e n \to e^- p$



$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \qquad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$$

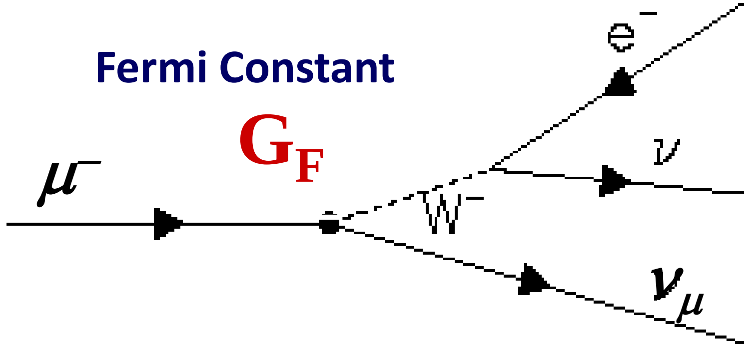
$\mu^- \to e^- \nu_\mu \bar{\nu}_e$



Problems with the Weak Interactions & the 3-quark model

1) anomalous quark W.I."charges"

Strength of the weak interaction, characterized by the Fermi constant, G_F, is well measured in muon decays



$$G_F = 1.1663787(6)\times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$n \to p e^- \bar{\nu}$$

d→u

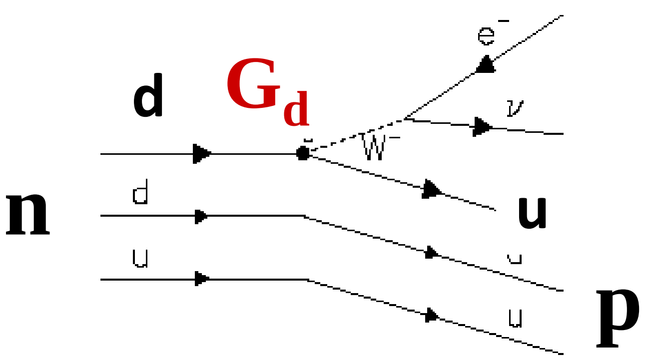
$G_d \approx 0.98 G_F$

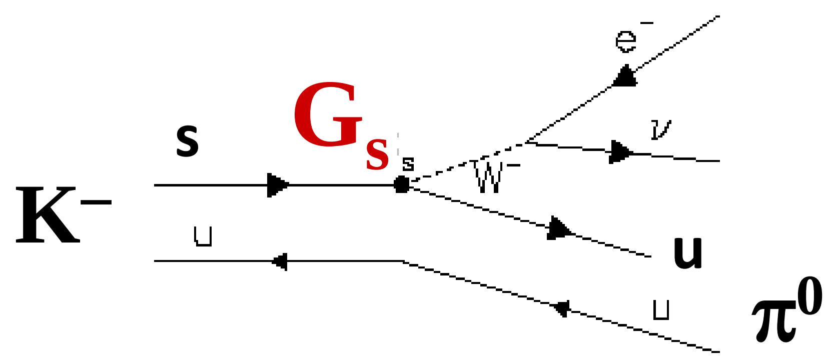
same quantity measured for
d- and s-quarks gives different results

$$K^- \to \pi^0 e^- \bar{\nu}$$

s→u

$G_s \approx 0.21 G_F$



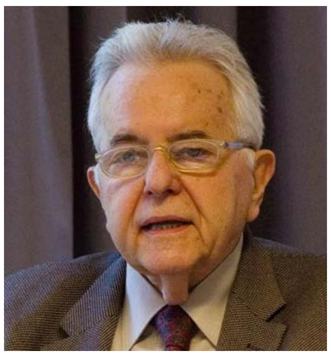


Cabbibo's solution: flavor mixing

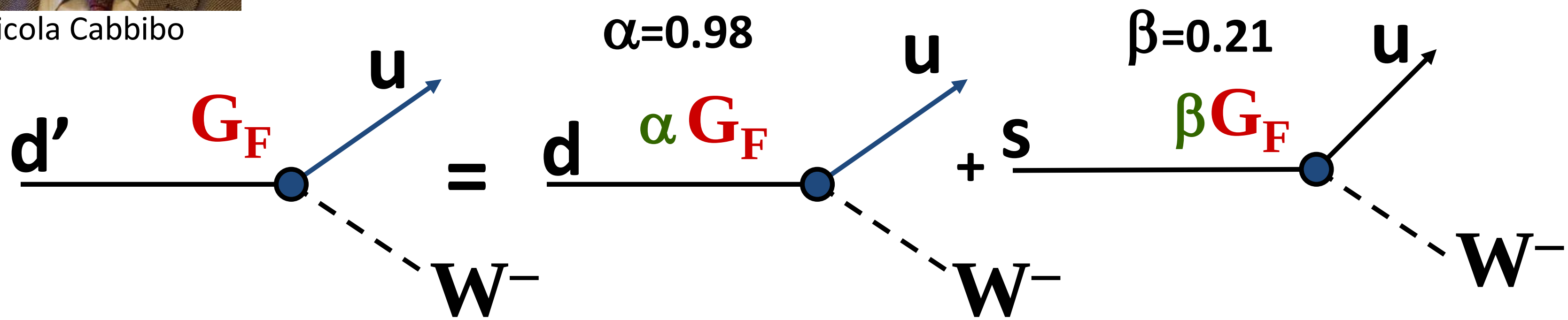
Weak Int flavor state

mass eigenstates

$$d' = \alpha\, d + \beta\, s$$



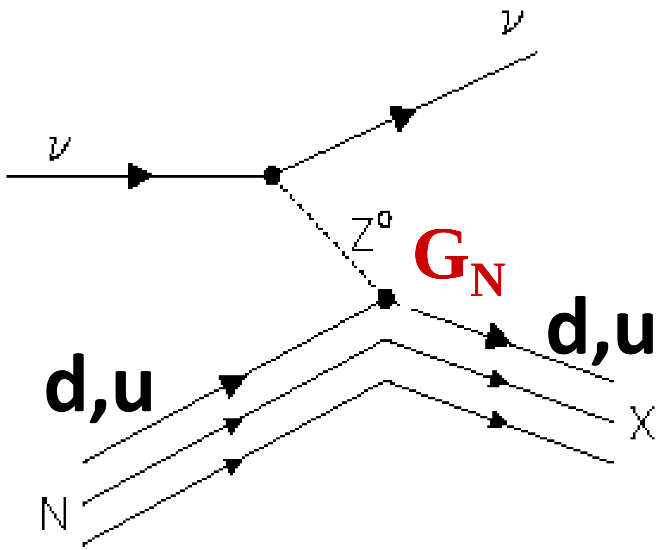
Nicola Cabbibo

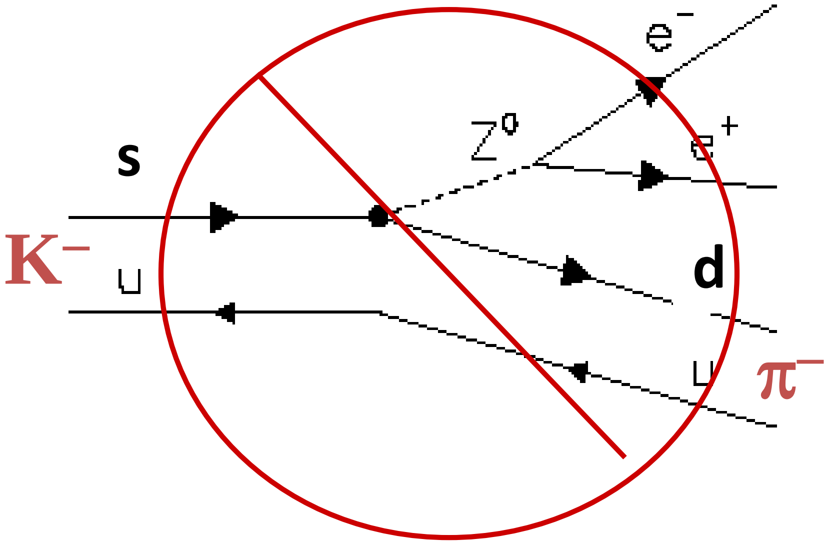


$\alpha^2+\beta^2$X1

Missing neutral currents

2: no <u>flavor-changing</u> "neutral currents" seen.





flavor-preserving neutral currents (e.g. $\nu N \rightarrow \nu X$) are seen

flavor-changing neutral currents (e.g. $K \rightarrow \pi\, l^+ l^-$) are strongly supressed

discovered at CERN

GIM sol'n: Introduce a 4th quark

2 quark doublets:

charmed quark

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \qquad\qquad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

weak eigenstates

mass eigenstates

GIM: Glashow, Iliopoulis and Maiani

d' & s' are mixed d & s

4-quark
flavor-mixing
matrix

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

weak
eigenstates

mass
eigenstates

Mixing matrix must be Unitary

$UU^{\dagger} = 1$

$$\begin{pmatrix} \alpha & \beta \\ -\beta & a \end{pmatrix} \begin{pmatrix} \alpha^{*} & -\beta^{*} \\ \beta^{*} & \alpha^{*} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1 \quad \& \quad \alpha^*\beta - \alpha\beta^* = 0$$

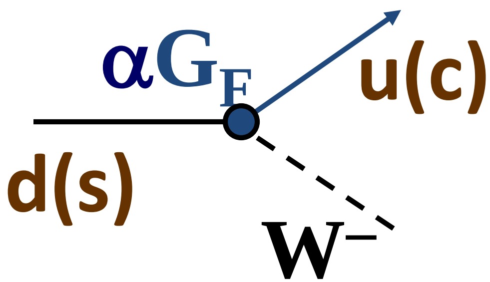
W.I. quark spinors

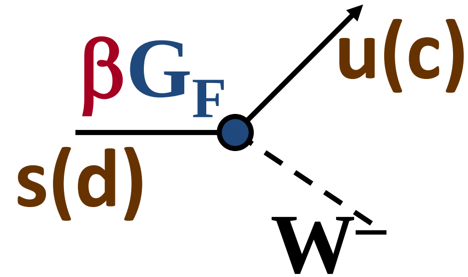
$$\begin{pmatrix} u^{+\frac{2}{3}} \\ d'^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} c^{+\frac{2}{3}} \\ s'^{-\frac{1}{3}} \end{pmatrix} = \begin{pmatrix} u^{+\frac{2}{3}} \\ \alpha d^{-\frac{1}{3}} + \beta s^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} c^{+\frac{2}{3}} \\ -\beta d^{-\frac{1}{3}} + \alpha s^{-\frac{1}{3}} \end{pmatrix}$$

Charged currents (u-quark)

$$\begin{pmatrix} u^{2/3} \\ \alpha d^{-1/3} + \beta s^{-1/3} \end{pmatrix}$$

|ΔS|=1





G_F modified by α

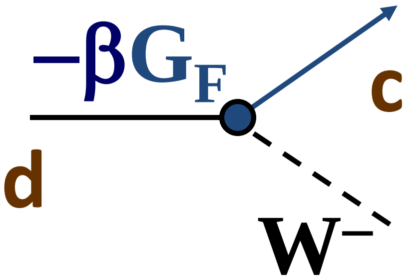
G_F modified by β

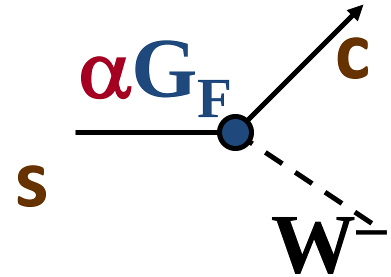
Charged currents (c-quark)

|ΔC|=1
|ΔS|=0

$$\begin{pmatrix} c^{2/3} \\ -\beta d^{-1/3} + \alpha s^{-1/3} \end{pmatrix}$$

|ΔC|=1
|ΔS|=1

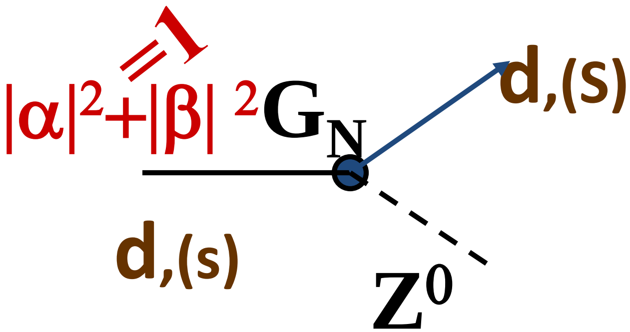




G_F modified by β

G_F modified by α

Flavor preserving Neutral Current



$$|d\rangle = \alpha|d'\rangle - \beta|s'\rangle$$

$$|s\rangle = \alpha|s'\rangle + \beta|d'\rangle$$

$$\langle d\|d\rangle = \left(\alpha^*\langle d'| - \beta^*\langle s'|\right)\left(\alpha|d'\rangle - \beta|s'\rangle\right)$$

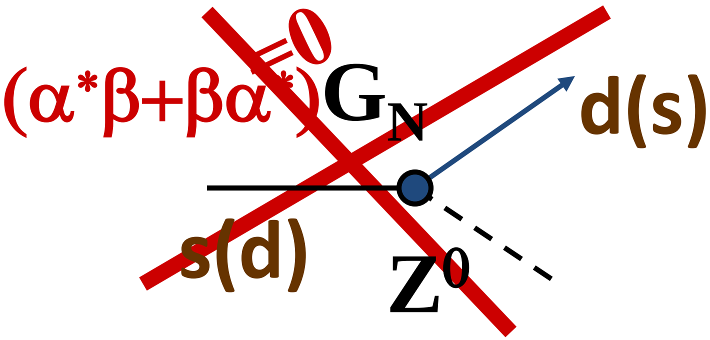
$$= |\alpha|^2 \underbrace{\langle d'|d'\rangle}_{=1} + \alpha^*\beta\underbrace{\langle d'|s'\rangle}_{=0} - \beta^*\alpha\underbrace{\langle s'|d'\rangle}_{=0} + |\beta|^2\underbrace{\langle s'|s'\rangle}_{=1}$$

$$= |\alpha|^2 + |\beta|^2 \quad = 1$$

From Unitarity

OK

Flavor changing Neutral Current



$$|d\rangle = \alpha|d'\rangle - \beta|s'\rangle$$
$$|s\rangle = \alpha|s'\rangle + \beta|d'\rangle$$

$$\langle s||d\rangle = \left(\alpha^*\langle s'| - \beta^*\langle d'|\right)\left(\alpha|d'\rangle + \beta|s'\rangle\right)$$

$$= |\alpha|^2 \underbrace{\langle s'|d'\rangle}_{=0} + \alpha^*\beta\underbrace{\langle s'|s'\rangle}_{=1} - \beta^*\alpha\underbrace{\langle d'|d'\rangle}_{=1} + |\beta|^2\underbrace{\langle d'|s'\rangle}_{=0}$$

$$= \left(\alpha^*\beta - \beta^*\alpha\right) = 0$$ From Unitarity

FCNC forbidden by Unitarity

GIM-Mechanism

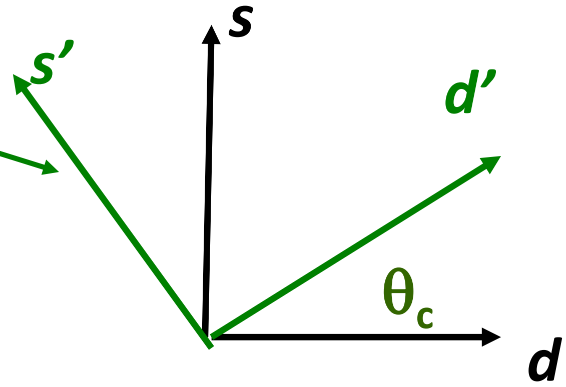
Flavor mixing with 4 quarks

-- 2-dimensional rotation --

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak Int flavor state

mass eigenstates

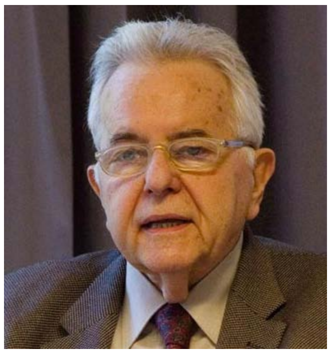


Cabibbo's flavor mixing revisitd

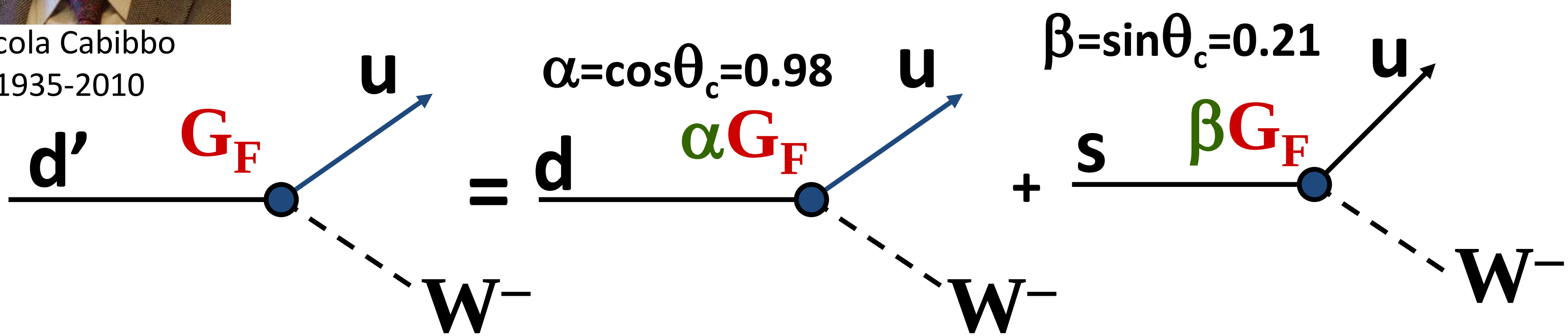
Weak Int flavor state

flavor mass eigenstates

$$d' = \alpha\, d + \beta\, s$$



Nicola Cabibbo
1935-2010

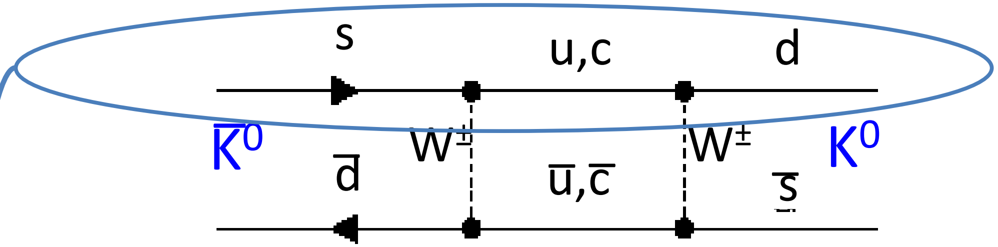


Unitarity: $|\alpha|^2 + |\beta|^2 = 1$

$\alpha = \cos\theta_c$; $\beta = \sin\theta_c$

θ_c="Cabibbo angle"$\approx 12^0$

short-distance $K^0 \leftrightarrow \bar{K}^0$ mixing



$\Delta S=2$ process

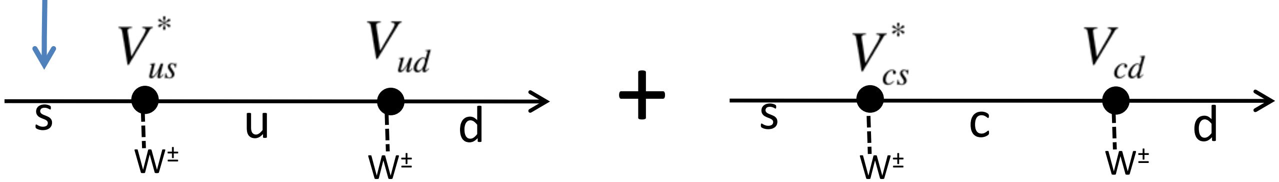
4-quark world:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

θ_C=Cabibbo angle

W.I. eigenstates

mass eigenstates



s→d FCNC

$$\boxed{\Delta m^{s-d}_{SM} \propto G_F^2 (\underbrace{V^*_{us} V_{ud}}_{\cos\theta_c \sin\theta_c} f(m_u) + \underbrace{V^*_{cs} V_{cd}}_{-\cos\theta_c \sin\theta_c} f(m_c))}$$

$$\Delta m^{s-d}_{SM} \propto G_F^2 \left(f(m_u) - f(m_c) \right) \cos\theta \sin\theta \approx G_F^2 \underline{m_c^2} \cos\theta \sin\theta$$

if m_u and m_c were equal, GIM mechanism would make $\Delta m^{s-d}_{SM} = 0$

$|\Delta m_s| = |m_{K_S} - m_{K_L}|$ constrained original prediction of c-quark mass

Original GIM paper:

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139

(Received 5 March 1970)

... from the observed K_1K_2 mass difference we now conclude that Δ must be not larger than 3–4 GeV.

m_c

Japan physicists knew about the *c* quark

1971 paper by Nagoya particle physicist Kiyoshi Niu and colleagues

Prog. Theor. Phys. Vol. 46 (1971), No. 5

A Possible Decay in Flight of a New Type Particle

Kiyoshi NIU, Eiko MIKUMO and Yasuko MAEDA*

Institute for Nuclear Study University of Tokyo

**Yokohama National University*

August 9, 1971

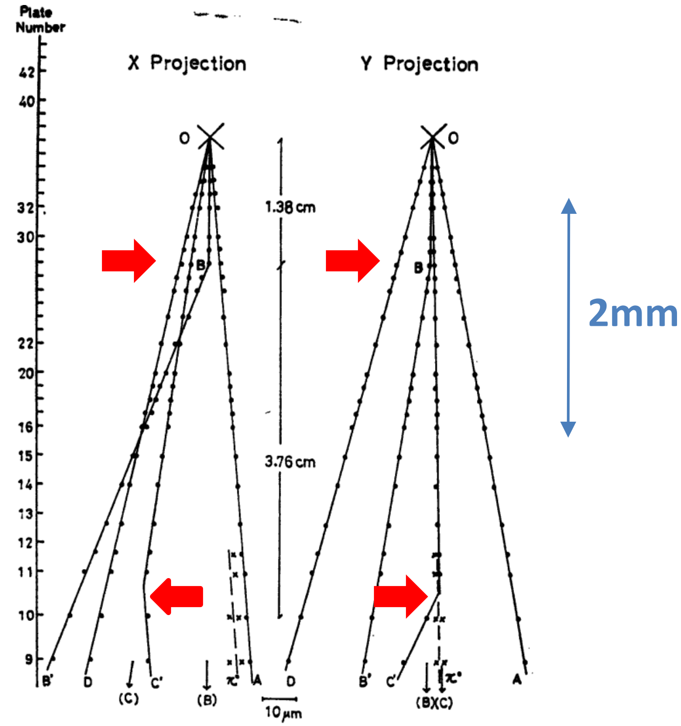


Fig. 3(a). Fig. 3(b).

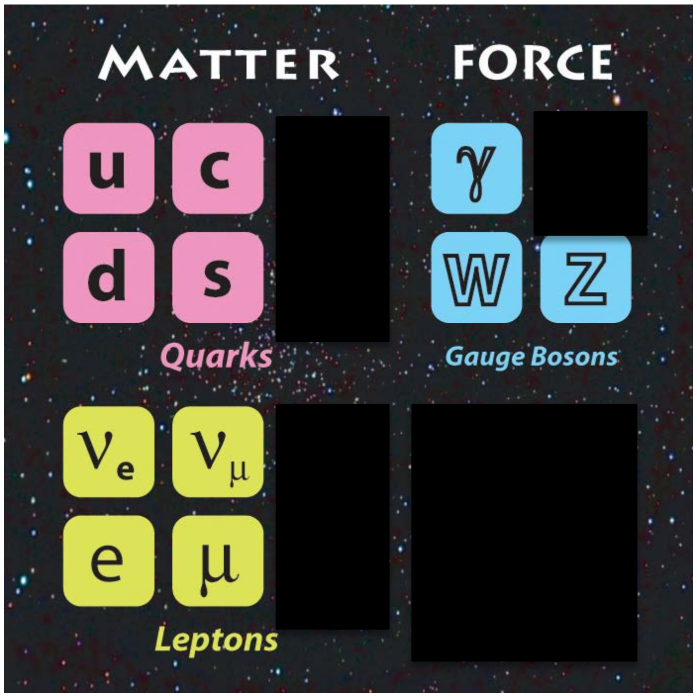
Assumed decay mode	M_x GeV	T_x sec
$X \to \pi^0 + \pi^\pm$	1.78	2.2×10^{-14}
$X \to \pi^0 + p$	2.95	3.6×10^{-14}

Shuzo Ogawa (Nagoya) interpreted this event as production of one particle with a c-quark ($X \to \pi^0 p$) and one with an anti-c-quark ($X \to \pi^0 \pi^\pm$).

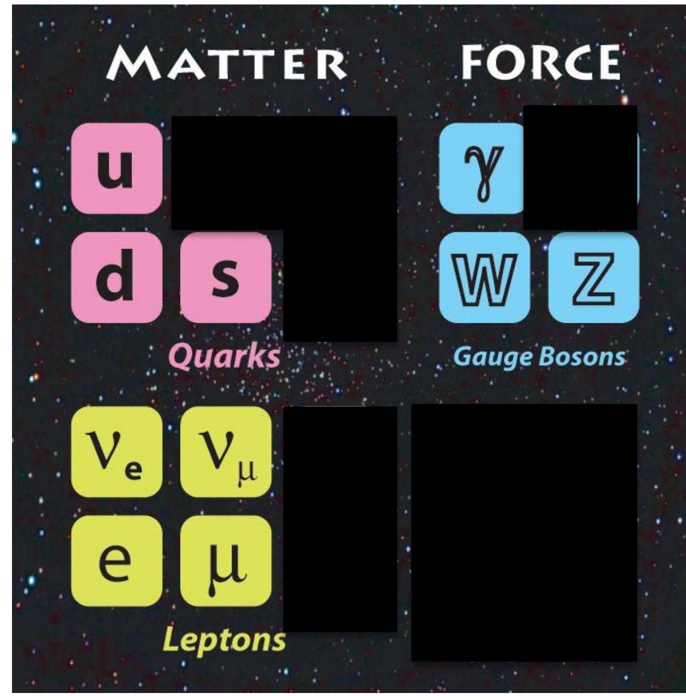
Introducing a CP-violating, complex amplitude into the Standard Model

The elementary particles in 1973

In Nagoya



In the rest of the world



1973 Kobayashi & Maskawa

Makoto Kobayashi

Toshihide Maskawa

PAGE 1

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are

"quartet scheme"
=4-quark model

Quark-flavor-mixing for 4 flavors

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak int eigenstates

Mass eigenstates

Can we add a complex CPV phase to one of these matrix elements?

Number of parameters:	8
Unitarity conditions:	
4 flavors; # of arb. phases:	
	___
# of free parameters:	

Only 1 free parameter: the Cabibbo angle, θ_C

$V^\dagger V = 1$

$$\Rightarrow 4 \text{ conditions}: \begin{cases} V_{ud}^* V_{ud} + V_{cd}^* V_{cd} = 1 \\ V_{us}^* V_{us} + V_{cs}^* V_{cs} = 1 \\ V_{ud}^* V_{us} + V_{cd}^* V_{cd} = 0 \\ V_{us}^* V_{ud} + V_{cs}^* V_{cd} = 0 \end{cases}$$

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

KM paper, page 12

"6 quark model"

Next we consider a 6-plet model, another interesting model of CP-violation.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3 Euler angles: θ_1 θ_2 & θ_3, plus 1 CP-violating phase: δ

Then, we have CP-violating effects through the interference among these different current components.

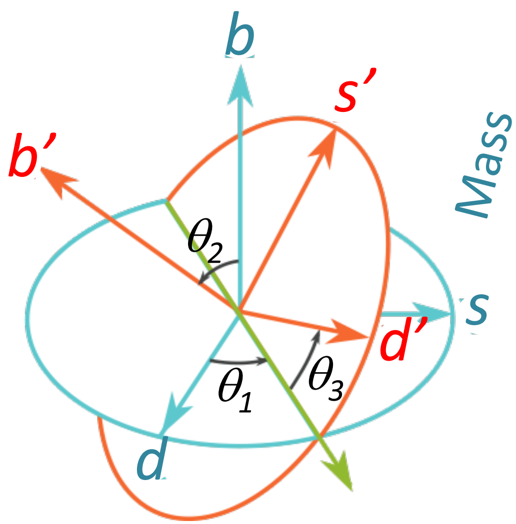
i.e., theory can accommodate CP violation, but only with 6 (or more) quarks

What about 6 quark flavors

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak int eigenstates

Mass eigenstates



Can we add a complex CPV phase to one of these matrix elements?

Number of parameters:	18
Unitarity conditions:	9
6 flavors; # of arb. phases:	5
# of free parameters:	4

Yes!

4 parameters, 3 or rotations (Euler angles: θ_1 θ_2 & θ_3) with one left over for a CPV phase

Quark field "re-phasing"

multiply each quark by an arbitrary phase factor:

$$q_i \to e^{i\phi_i} q_i$$

6 (2N) arbitrary phase factors: ϕ_i

simultaneously "rephase" the CKM matrix:

$$V \to \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & 0 & 0 \\ 0 & e^{-i\phi_s} & 0 \\ 0 & 0 & e^{-i\phi_b} \end{pmatrix}$$

or $V_{ij} \to e^{i(\phi_i - \phi_j)} V_{ij}$

$\langle \bar{u}_i | V_{ij} | d_j \rangle$ is unchanged: $\langle \bar{u}_i | V_{ij} | d_j \rangle \to \langle \bar{u}_i | e^{-i\phi_i} e^{i(\phi_i - \phi_j)} V_{ij} e^{i\phi_j} | d_j \rangle = \langle \bar{u}_i | V_{ij} | d_j \rangle$

1 overall phase can be factored out:

for example ϕ_u

$$q_i \to e^{i(\phi_i - \phi_u)} q_i$$

$$V \to e^{i\phi_u} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\phi_c - \phi_u)} & 0 \\ 0 & 0 & e^{i(\phi_t - \phi_u)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & 0 & 0 \\ 0 & e^{-i\phi_s} & 0 \\ 0 & 0 & e^{-i\phi_b} \end{pmatrix}$$

5 (2N-1) arbitrary phases in CKM matrix + 1 overall (trivial) phase

KM paper was in 1973, the 3-quark age

1964-1974

3x3 matrix ⇒3 generations, i.e. 6 quarks

predicted by GIM
discovered in Nagoya 1971
rest of the world: Nov 1974

In 1973, these were not in our dreams.

6 quarks:

q=+2/3

q=–1/3

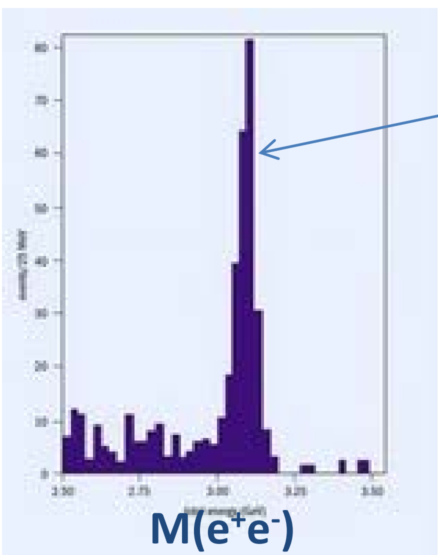
$$\begin{pmatrix} u^{2/3} \\ d^{-1/3} \end{pmatrix} \begin{pmatrix} c^{2/3} \\ s^{-1/3} \end{pmatrix} \begin{pmatrix} t^{2/3} \\ b^{-1/3} \end{pmatrix}$$

History

November 1974:

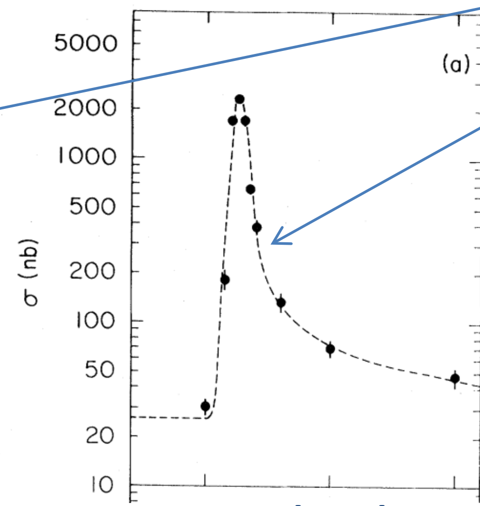
Charmed (4^{th}) quark "discovered"
@ Brookhaven & SLAC

$J/\psi = c\bar{c}$



$pp \rightarrow J/\psi + X;\ J/\psi \rightarrow e^+e^-$

Phys.Rev.Lett.33:1404-1406,1974.



$E_{cm}(e^+e^-)$

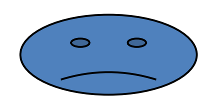
$e^+e^- \rightarrow$ hadrons

Phys.Rev.Lett.33:1406-1408,1974

1976 Nobel prize

Sam Ting

Burt Richter

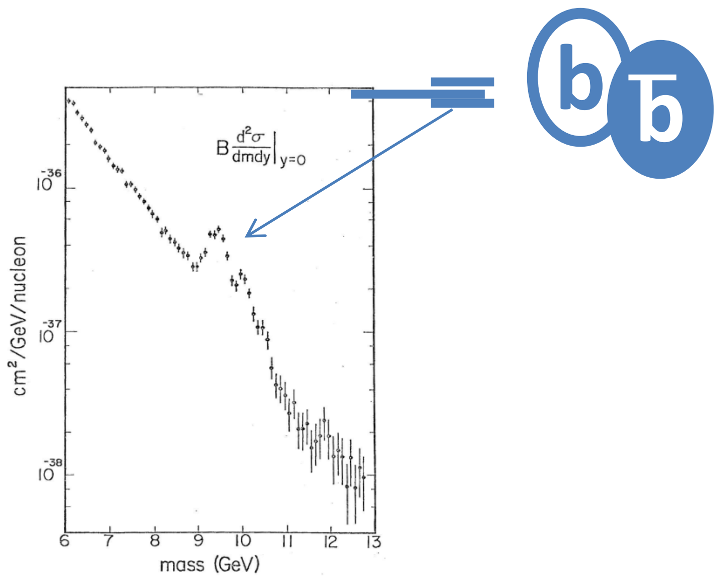


Kiyoshi Niu

More History

November 1977:

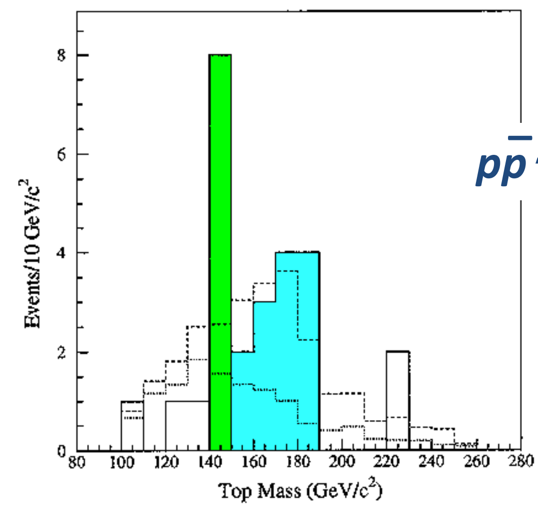
Bottom (5th) quark discovered
@ Fermilab



Phys.Rev.Lett.39:252-255,1977.

February 1995:

Top (6th) quark discovered
@ Fermilab



$\ell^+\nu$

$p\bar{p} \rightarrow t\,\bar{t}\,X$

$b\bar{c}$

CDF: Phys.Rev.Lett.74:2626-2631,1995

D0: Phys.Rev.Lett.74:2632-2637,1995

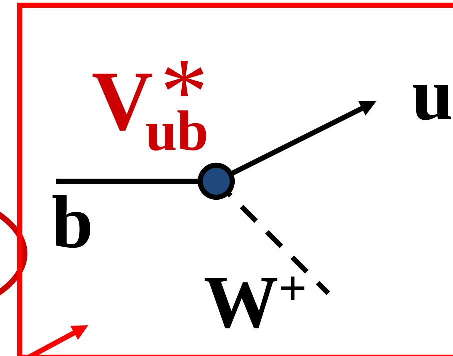
A little history

- 1963 CP violation seen in K^0 system
- 1973 KM 6-quark model proposed
- 1974 charm (4^{th}) quark discovered
- 1978 beauty/bottom (5^{th}) quark discovered
- 1995 truth/top (6^{th}) quark discovered

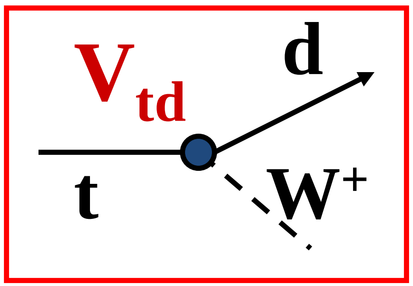
CKM matrix today

CPV phases are in the corners

ϕ_3 (γ)

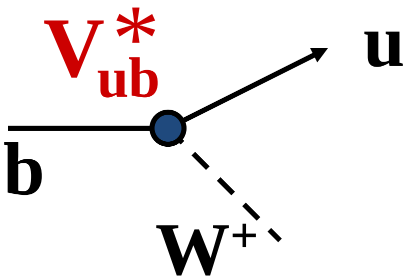


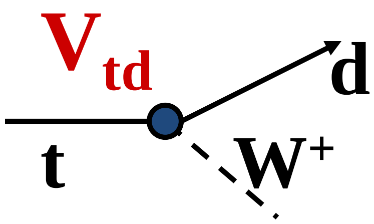
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



ϕ_1 (β)

The challenge





Measure a complex phase for b→u

or in t→d

or, even better, both

Summary of Lecture 4

- The Superweak model was a plausible explanation for the $K_L \rightarrow \pi^+\pi^-$ observation
  It predicted the phase of $\varepsilon = \phi_{SW} = \arctan(2\Delta M_K/\Delta\Gamma_K) \simeq 45^o$, in agreement with experiment, no other observable CPV processes, & a dull future for specialists in the field.

- Precise comparisons of the rates for $K_L \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0$ in high-statistics experiments exposed a direct CPV amplitude in $K_L \rightarrow \pi\pi$ decays, killing the Superweak model

- The measured Weak Interaction charge of the d-quark is 0.98 G_F, that for the s-quark is 0.21 G_F. These differences from G_F are due to quark-flavor mixing

- The non-existence of Flavor-Changing Neutral Currents was explained by the discovery of the charmed quark & Unitarity of the 4-quark flavor mixing matrix

- Kobayashi & Maskawa: a CP violating phase can be accommodated in the quark-flavor mixing matrix but only if there are 6 quark flavors (not 3, known in 1973)

- Three more quark-flavors are discovered: charm, bottom and top.