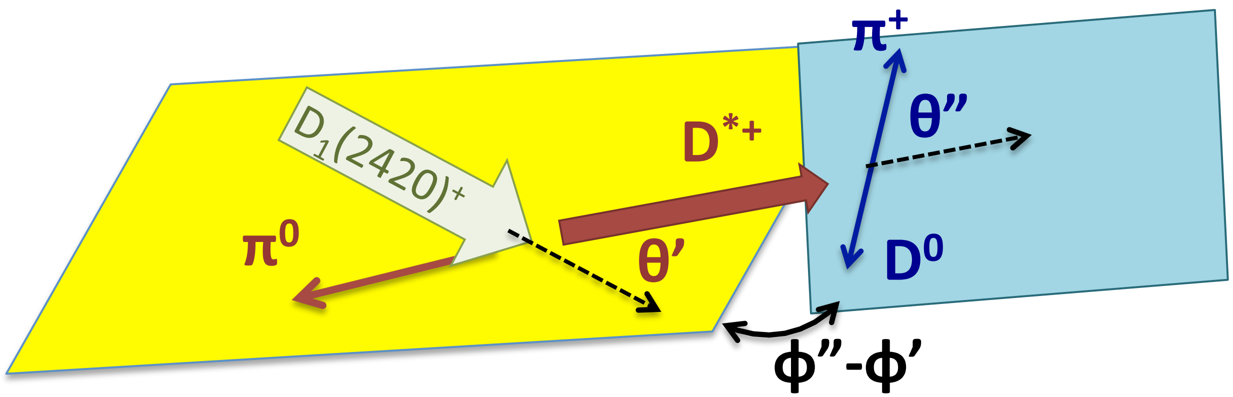
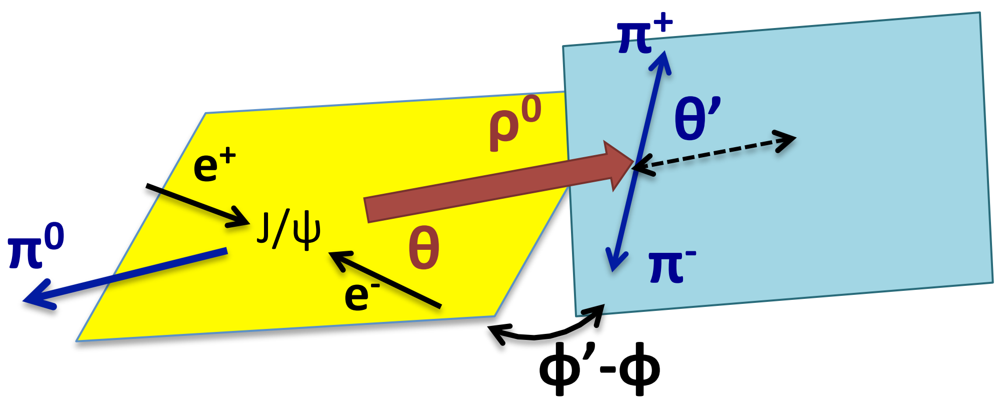
Beyond Dalitz plots II



Stephen Lars Olsen

Last time: $e^+e^- \rightarrow J/\psi \rightarrow \rho\pi;\ \rho \rightarrow \pi\pi$

$$1^{--} \rightarrow 1^{--}0^{-+};\ 1^{--} \rightarrow 0^{-+}0^{-+}$$



$$A(\theta,\phi,\theta',\phi',m_1,0,0,0\ ) = \sqrt{\frac{(2J_{J/\psi}+1)}{4\pi}\frac{(2J_\rho+1)}{4\pi}}B_{00}\sum_{\lambda_2=\pm1} A_{\lambda_2 0} D^{1*}_{1\lambda_2}(\theta,\phi) D^{1*}_{\lambda_2 0}(\theta',\phi')$$

parity conservation:

$A_{-10} = -A_{10};\ A_{00} = 0$

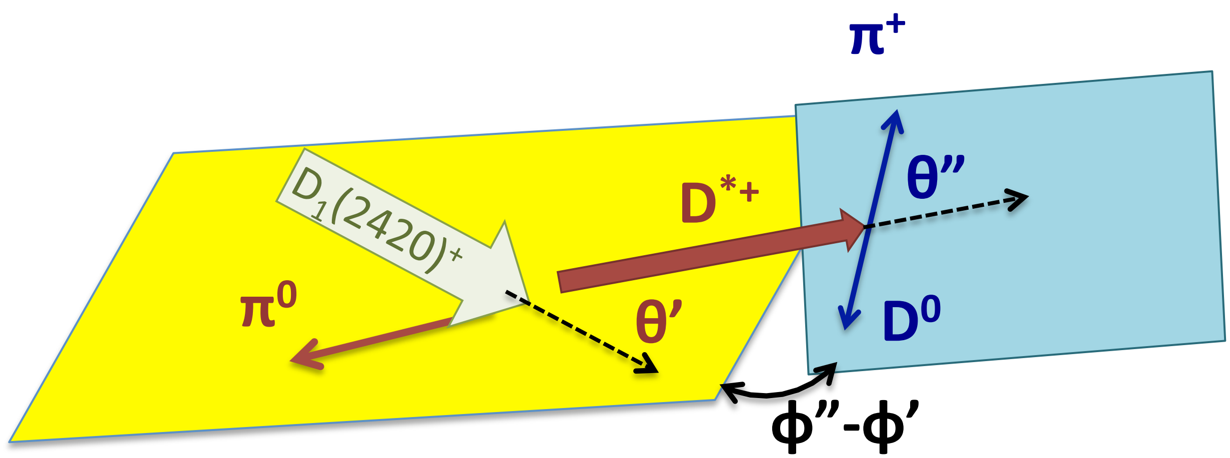
$$= \frac{-1}{\sqrt{2}} A_{10} B_{00} e^{i\phi} \sin\theta' \left(\cos(\phi'-\phi) + i\cos\theta \sin(\phi'-\phi)\right)$$

$$\left.\frac{d\Gamma}{d(\theta,\theta',(\phi'-\phi))}\right|_{m_{J/\psi}=+1} \propto \left|A(\theta,\theta',(\phi'-\phi),m_1=1)\right|^2 = \tfrac{1}{2}\left|A_{10}B_{00}\right|^2 \sin^2\theta' \left|\cos(\phi'-\phi) + i\cos\theta\sin(\phi'-\phi)\right|^2$$

Today: $D_1(2420) \rightarrow D^*\pi$; $D^* \rightarrow D\pi$

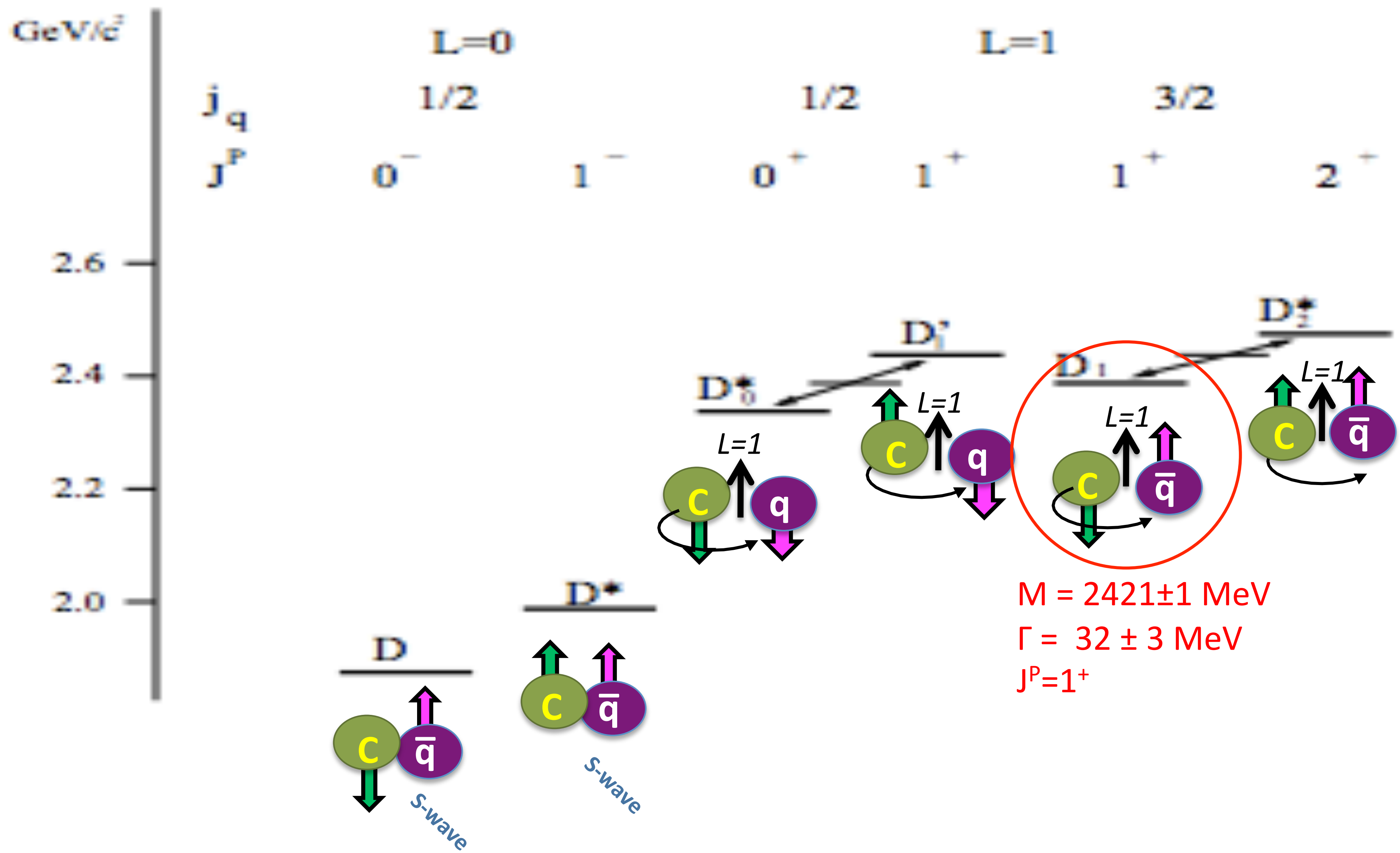
$$1^+ \rightarrow 1^- 0^-; \; 1^- \rightarrow 0^- 0^-$$

1^+ instead of 1^{--}



What is the $D_1(2420)$?

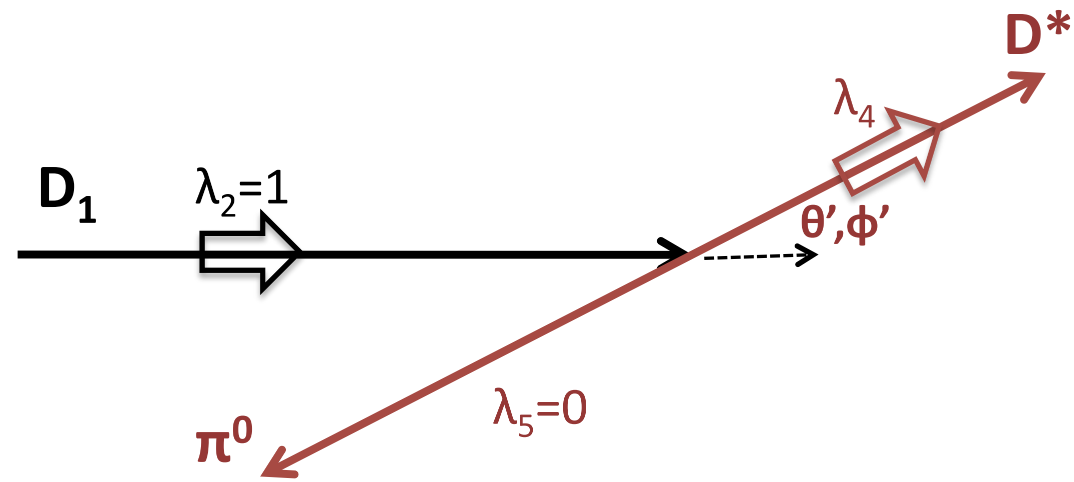
-- the low-mass D meson spectrum --



D1(2420)→D*π in the helicity basis

Let's do m_{D1}=+1

$$\left|J_1(=1)\, m_1=+1\right\rangle$$



$$\lambda_4=\begin{cases}+1\\0\\-1\end{cases}$$

$$D^j_{m'm}(\theta,\phi)=e^{-i(m'-m)\phi}d^j_{m'm}(\theta)$$

$$d^1_{1,1}=\frac{1+\cos\theta}{2}$$

$$d^1_{1,0}=-\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1}=\frac{1-\cos\theta}{2}$$

$$A_H(\theta',\phi';1,\lambda_4,0)=\sqrt{\frac{3}{4\pi}}B_{\lambda_4 0}D^{1*}_{1\lambda_4-0}(\theta',\phi')$$

$$=\sqrt{\frac{3}{4\pi}}\begin{Bmatrix}d^1_{11}(\theta')B_{10}\\ e^{i\phi'}d^1_{10}(\theta')B_{00}\\ e^{2i\phi'}d^1_{1-1}(\theta')B_{-10}\end{Bmatrix}=e^{i\phi'}\sqrt{\frac{3}{4\pi}}\begin{Bmatrix}B_{10}e^{-i\phi'}(1+\cos\theta')/2\\ -B_{00}(\sin\theta')/\sqrt{2}\\ B_{-10}e^{i\phi'}(1-\cos\theta')/2\end{Bmatrix}$$

Parity is conserved in $D_1 \rightarrow D^*\pi$

$$B_{m'm} = \eta_{D_1}\eta_{D^*}\eta_\pi(-1)^{S_{D_1}-s_{D^*}-s_\pi}B_{-m'-m}$$

$$B_{10} = (+1)_{D_1}(-1)_{D^*}(-1)_\pi(-1)^{1-1-0}B_{-10}$$

$$\Rightarrow B_{10} = +B_{-10}$$

-- 2 independent amplitudes --

$$B_{00} = (+1)_{D_1}(-1)_{D^*}(-1)_\pi(-1)^{1-1-0}B_{00}$$

$$\Rightarrow B_{00} = B_{00} \Rightarrow B_{00} \neq 0$$

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-- 2 independent amplitudes --

$$A_H(\theta,\phi;1,\lambda_4,0) = \sqrt{\frac{3}{4\pi}} B_{\lambda_4 0} D^{1*}_{1\lambda_4-0}(\theta',\phi')$$

$$= \sqrt{\frac{3}{4\pi}} \left\{ \begin{array}{c} d^1_{11}(\theta')B_{10} \\ e^{i\phi'}d^1_{10}(\theta')B_{00} \\ e^{2i\phi'}d^1_{1-1}(\theta')B_{-10} \end{array} \right\} = e^{i\phi'}\sqrt{\frac{3}{4\pi}} \left\{ \begin{array}{c} B_{10}e^{-i\phi'}(1+\cos\theta')/2 \\ -B_{00}(\sin\theta')/\sqrt{2} \\ B_{10}e^{i\phi'}(1-\cos\theta')/2 \end{array} \right\}$$

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$$B_{m'm} = \eta_{D_1}\eta_{D^*}\eta_\pi(-1)^{s_{D_1}-s_{D^*}-s_\pi} B_{-m'-m}$$

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$$A_H(\theta,\phi;1,\lambda_4,0) = \sqrt{\frac{3}{4\pi}} B_{\lambda_4 0} D^{1*}_{1\lambda_4-0}(\theta',\phi')$$

$$= \sqrt{\frac{3}{4\pi}}\left\{\begin{matrix} d^1_{11}(\theta')B_{10} \\ e^{i\phi'}d^1_{10}(\theta')B_{00} \\ e^{2i\phi'}d^1_{1-1}(\theta')B_{-10} \end{matrix}\right\} = e^{i\phi'}\sqrt{\frac{3}{4\pi}}\left\{\begin{matrix} B_{10}e^{-i\phi'}(1+\cos\theta')/2 \\ -B_{00}(\sin\theta')/\sqrt{2} \\ B_{10}e^{i\phi'}(1-\cos\theta')/2 \end{matrix}\right\}$$

$$\frac{d\Gamma}{d(\theta',\varphi',1,1,0)} = \frac{3|B_{10}|^2}{4\pi}\left|e^{-i\phi'}(1+\cos\theta')/2\right|^2 = \frac{3|B_{10}|^2}{16\pi}(1+2\cos\theta'+\cos^2\theta')$$

incoherent

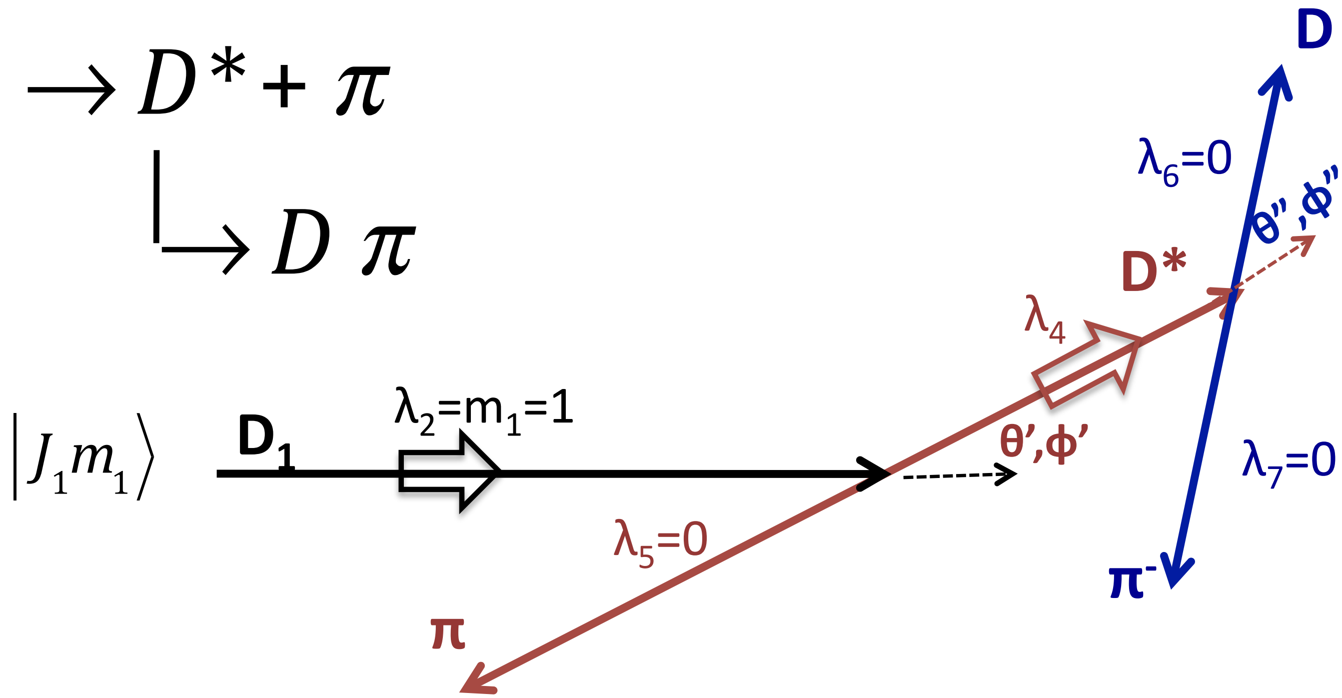
$$\frac{d\Gamma}{d(\theta',\varphi',1,0,0)} = \frac{3|B_{00}|^2}{4\pi}\left|-\frac{\sin\theta'}{\sqrt{2}}\right|^2 = \frac{3|B_{00}|^2}{8\pi}\sin^2\theta'$$

different D* helicity states

$$\frac{d\Gamma}{d\ \ \theta'\ \varphi'\ \ -} = \frac{|B\ \ |}{\pi}\left|e^{i\phi'}\ \ -\ \ \theta'\ \ \right| = \frac{|B\ \ |}{\pi}\ \ -\ \ \theta' + \ \ \theta'$$

add D*→Dπ decay

$$D_1 \to D^* + \pi$$
$$\quad\quad \lfloor\!\!\to D\,\pi$$



$$A(\theta',\phi',\theta'',\phi'',m_1,0,0,0\ )=$$

$$\tfrac{3}{4\pi}\sum_{\lambda_4=+1,0,-1} B_{\lambda_4 0} D^{1*}_{1\lambda_4}(\theta',\phi') C_{00} D^{1*}_{\lambda_4 0}(\theta'',\phi'')$$

Evaluate: $$C_{00}\sum_{\lambda_4=+1,0,-1} B_{\lambda_4 0} D^{1*}_{1\lambda_4}(\theta',\phi') D^{1*}_{\lambda_4 0}(\theta'',\phi'')$$

$$B_{-10}=B_{10};\ B_{00}\neq 0$$

$$D^{1*}_{1\pm1}(\theta',\phi')=e^{i\phi'}\left(e^{\mp i\phi'}(1\pm\cos\theta')/2\right)$$

$$D^{1*}_{10}(\theta',\phi')=-\tfrac{1}{\sqrt{2}}e^{i\phi'}\sin\theta'$$

$$D^{1*}_{\pm10}(\theta'',\phi'')=e^{\pm i\phi''}d^{1}_{\pm10}(\theta'')$$
$$=\mp\tfrac{1}{\sqrt{2}}e^{\pm i\phi''}\sin\theta''$$

$$D^{1}_{00}(\theta'')=d^{1}_{00}(\theta'')=\cos\theta''$$

$$C_{00}\sum_{\lambda_4=+1,0,-1} B_{\lambda_4 0} D^{1*}_{1\lambda_4}(\theta',\phi') D^{1*}_{\lambda_2 0}(\theta'',\phi'')$$
$$=\tfrac{-1}{2\sqrt{2}}C_{00}e^{i\phi'}\left(B_{10}\sin\theta''(e^{-i\phi'}(1+\cos\theta')e^{i\phi''}-e^{i\phi'}(1-\cos\theta')e^{-i\phi''})+2B_{00}\sin\theta'\cos\theta''\right)$$

$$\left.\frac{d\Gamma}{d(\theta',\theta'',(\phi''-\phi'))}\right|_{m_{D_1}=+1} \propto \left| \sum_{\lambda_4=+1,0,-1} B_{\lambda_4 0} D^{1*}_{1\lambda_4}(\theta',\phi') D^{1*}_{\lambda_4 0}(\theta'',\phi'') \right|^2$$

$$\propto \left|B_{10}\right|^2 \frac{\sin^2\theta''}{4}(1+\cos^2\theta'+\sin^2\theta'\cos 2(\phi''-\phi'))$$

$$+\left|B_{00}\right|^2 \frac{\cos^2\theta''\sin^2\theta'}{2}$$

$$+\mathcal{R}e(B^*_{10}B_{00})\sin\theta'\cos\theta'\sin\theta''\cos\theta''\cos(\phi''-\phi')$$