Self-consistency and covariance of light-front quark models: testing via $f_{P,V,A}$ and $F_{P\to P\,,P\to V\,,V\to V}$ ......

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Outline

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6. Example 3: $\mathcal{F}^{P\to V}$

1. Motivation

- **Standard light-front quark model** (SLF QM);
  M. V. Terentev, SJNP 24 (1976) 106; P. L. Chung *et al.*, PLB 205 (1988) 545.

  The SLF QM is a relativistic constituent QM based on the LF formalism, which provides a conceptually simple and phenomenologically feasible framework for calculating the non-perturbative quantities of hadrons.

  two problems: non-manifestation of covariance; zero-mode issue

- **Covariant light-front quark model** (CLF QM);
  H. Y. Cheng *et al.*, Phys. Rev. D 57 (1998) 5598; W. Jaus, Phys. Rev. D 60 (1999) 054026;
  It provides a systematic way to explore the zero-mode effects; the results are guaranteed to be covariant after the spurious contribution proportional to $\omega = (0, 2, 0_\perp)$ is canceled by the inclusion of zero-mode contributions.

"Exclusive $D_s \to K$, K^* , ϕ decays " W. Wang and Y. L. Shen, PRD 78 (2008) 054002;

"J/ψ weak decays" Y. L. Shen and Y. M. Wang PRD 78, 074012 (2008);

" Ξ_{cc}^{++} ,Ξ_{cc}^{+} ,Ω_{cc}^{+} weak decays ($1/2 \to 3/2$ case)" Z. X. Zhao, EPJC 78 (2018) no.9, 756;

" $\eta_c \to \gamma^*\gamma$" H. Y. Ryu, H. M. Choi and C. R. Ji, Phys.Rev. D98 (2018) no.3, 034018;

" $D(D_s) \to (P, S, V, A)\ell\upsilon_\ell$ decays" H. Y. Cheng and X. W. Kang, EPJC 77 (2017) no.9, 587;

"Heavy pentaquark transition (Θ_c, Ξ_{5c})" H. Y. Cheng , C. K. Chua and C. W. Hwang, PRD 70 (2004) 034007

"Radiative decays of charmed vector mesons" H. M. Choi, PRD 75 (2007) 073016

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Two problems:

- **Self-consistency problem of CLF QM**

$$[f_V]^{\lambda=0}_{\rm CLF} \neq [f_V]^{\lambda=\pm}_{\rm CLF}$$

due to the additional contribution characterized by the $B_1^{(2)}$ function to $[f_V]^{\lambda=0}_{\rm CLF}$.

PHYSICAL REVIEW D **69**, 074025 (2004)

Covariant light-front approach for s-wave and p-wave mesons: Its application to decay constants and form factors

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[2]When $\mathcal{A}^V_\mu$ is contracted with the longitudinal polarization vector $\varepsilon^\mu(0)$, f_V will receive additional contributions characterized by the B functions defined in Appendix B [see Eq. (3.5) of [14]] which give about 10% corrections to f_V for the vertex function h'_V used in Eq. (2.11). It is not clear to us why the result of f_V depends on the polarization vector. Note that the new residual contributions are

Possible solution: H. M. Choi and C. R. Ji, Phys. Rev. D 89 (2014) no. 3, 033011.

$$\sqrt{2N_c}\frac{\chi(x,k_\perp)}{1-x} \to \frac{\psi(x,k_\perp)}{\sqrt{x(1-x)}\hat{M}_0}, \qquad D_{V,\rm con} \to D_{V,\rm LF}, \qquad (\textbf{type-I})$$

$\chi(x,k_\perp)$: CLF expressions $\longleftrightarrow$ SLF ones via Z.M. independent f_P or $f^+_{P\to P}$.
D: $D_{V,\rm con} = M + m_1 + m_2$ and $D_{V,\rm LF} = M_0 + m_1 + m_2$

$$\sqrt{2N_c}\frac{\chi(x,k_\perp)}{1-x} \to \frac{\psi(x,k_\perp)}{\sqrt{x(1-x)}\hat{M}_0}, \qquad M \to M_0. \qquad (\textbf{type-II})$$

$$\Longrightarrow \qquad [f_V]^{\lambda=0}_{\rm CLF} = [f_V]^{\lambda=\pm}_{\rm CLF} = [f_V]_{\rm SLF}$$

Questions: (i) f_A, $F_{P\to V}$ ...... ?

(ii) $[f_V]^{\lambda=0}_{\rm SLF} = [f_V]^{\lambda=\pm}_{\rm SLF}$? self-consistency of SLF QM ?

(iii) zero-mode contribution ?

■ Covariance problem of CLF QM

The manifest covariance is a remarkable feature of CLF QM relative to SLF QM. However,
the covariance is in fact violated
when the LF vertex function and operator are used (especially for spin-1 system).

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Covariant analysis of the light-front quark model

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The formulas for coupling constants and form factors have been derived in a manifestly covariant framework. However, if these formulas are evaluated with the symmetric light-front vertex function (5.2), the covariance conditions (3.32) are violated, i.e., the integrals of Eq. (3.32) are nonzero. Consequently, some residual ω dependence is introduced into these expressions if Eqs. (5.2) and (5.3) are used for the vertex function. This remaining ω dependence is minimal in the sense that only the B coefficients $B_n^{(m)}$ in the

Taking $\mathcal{A} \equiv \langle 0|\bar{q}_2 \Gamma q_1|M(p)\rangle$ as an example

$$\hat{\mathcal{A}}^{\mu}_{V} = M_V(\epsilon^{\mu} f_V + \omega^{\mu} g_V)\,,$$

$\hat{\mathcal{A}}^{\mu}_{V}$ is obviously not covariant unless the unphysical decay constant $g_V = 0$ since ω^{μ} is a fixed vector.

(i) Is the covariance violation minimal?

(ii) Can the strict covariance be recovered ?

Self-consistency, Covariance, Zero-mode contribution

2. Brief review of theoretical framework

The main task:

$$\mathcal{A} \equiv \langle 0|\bar{q}_2 \Gamma q_1|M(p)\rangle; \qquad \mathcal{B} \equiv \langle M''(p'')|\bar{q}_1''\Gamma q_1'|M'(p')\rangle$$

2.1 The SLF QM

$$|M\rangle = \sum_{h_1,h_2}\int \frac{\mathrm{d}k^+ \mathrm{d}^2 k_\perp}{(2\pi)^3 2\sqrt{k_1^+ k_2^+}} \Psi_{h_1,h_2}(k^+,k_\perp)|q_1 : k_1^+, k_{1\perp}, h_1\rangle|\bar{q}_2 : k_2^+, k_{2\perp}, h_2\rangle\,,$$

one-particle states: $|q_1\rangle = \sqrt{2k_1^+} b^\dagger|0\rangle$ with $\{b_h^\dagger(k), b_{h'}(k')\} = (2\pi)^3\delta(k^+ - k'^+)\delta^2(k_\perp - k'_\perp)\delta_{hh'}$.

Wavefunction:

$$\Psi_{h_1,h_2}(x,k_\perp) = S_{h_1,h_2}(x,k_\perp)\,\psi(x,k_\perp)\,,$$

Radial WF $\quad \psi_s(x,k_\perp) = 4\frac{\pi^{\frac{3}{4}}}{\beta^{\frac{3}{2}}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left[-\frac{k_z^2+k_\perp^2}{2\beta^2}\right]\,, \qquad$ s-wave

$$\psi_p(x,k_\perp) = \frac{\sqrt{2}}{\beta}\psi_s(x,k_\perp)\,. \qquad \text{p-wave}$$

Spin-orbital WF $\quad S_{h_1,h_2} = \frac{\bar{u}_{h_1}(k_1)\,\Gamma'\,v_{h_2}(k_2)}{\sqrt{2}\hat{M}_0}\,,$

obtained by the interaction-independent Melosh transformation, where

$$\Gamma'_{P,V,{}^1A,{}^3A} = \gamma_5\,,\ -\hat{\not\epsilon} + \frac{\hat{\epsilon}\cdot(k_1-k_2)}{D_{V,\rm LF}}\,,\ -\frac{\hat{\epsilon}\cdot(k_1-k_2)}{D_{{}^1A,\rm LF}}\gamma_5\,,\ -\frac{\hat{M}_0^2}{2\sqrt{2}M_0}\left[\hat{\not\epsilon} + \frac{\hat{\epsilon}\cdot(k_1-k_2)}{D_{{}^3A,\rm LF}}\right]\gamma_5$$

Equipped with the formulae given above, one can obtain

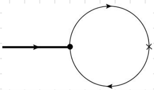
$$\mathcal{A} = \sqrt{N_c}\sum_{h_1,h_2}\int\frac{\mathrm{d}x\mathrm{d}^2k_\perp}{(2\pi)^3 2\sqrt{x\bar{x}}}\psi(x,k_\perp)\,S_{h_1,h_2}(x,k_\perp)\,C_{h_1,h_2}(x,k_\perp)\,,$$

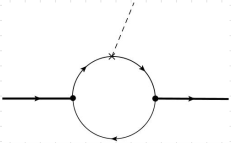
$$\mathcal{B} = \sum_{h'_1,h''_1,h_2}\int\frac{\mathrm{d}k'^+\mathrm{d}^2k'_\perp}{(2\pi)^3\,2\sqrt{k'^+\,k''^+}}\psi''^*(k''^+,\bar{k}''_\perp)\,\psi'(k'^+,k'_\perp)$$
$$\times\,S''^\dagger_{h''_1,h_2}(k''^+,k''_\perp)\,C_{h''_1,h'_1}(k''^+,k''_\perp,k'^+,k'_\perp)\,S'_{h'_1,h_2}(k'^+,k'_\perp)\,,$$

where $C_{h_1,h_2}\equiv\bar{v}_{h_2}\Gamma u_{h_1}$ and $C_{h''_1,h'_1}\equiv\bar{u}_{h''_1}\Gamma u_{h'_1}$

2.2 The CLF QM

Manifestly covariant one-loop integrals:





$$\mathcal{A} = N_c\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{H_M}{N_1N_2}S_{\mathcal{A}}\,,\qquad \mathcal{B} = N_c\int\frac{\mathrm{d}^4k'}{(2\pi)^4}\frac{H_{M'}H_{M''}}{N'_1\,N''_1\,N_2}iS_{\mathcal{B}}\,,$$

where, $\quad S_{\mathcal{A}} = \mathrm{Tr}\left[\Gamma\,(\not{k}_1+m_1)\,(i\Gamma_M)\,(-\not{k}_2+m_2)\right]$

$$S_{\mathcal{B}} = \mathrm{Tr}\left[\Gamma\,(\not{k}'_1+m'_1)\,(i\Gamma'_M)\,(-\not{k}_2+m_2)\,(i\gamma^0\Gamma''^\dagger_M\gamma^0)(\not{k}''_1+m''_1)\right]$$

Manifestly covariant expression $\xrightarrow{\text{integrating out } k^-}$ LF expression

Assumption: $H_{M,M',M''}$ are analytic in the upper complex k^- (k'^-) plane.

Consequently, q_2 is on mass-shell, and

$$N_1 \to \hat{N}_1\,,\qquad N_1^{\prime(\prime\prime)} \to \hat{N}_1^{\prime(\prime\prime)}\,,\qquad \mathcal{S} \to \hat{\mathcal{S}}\,,$$
$$\chi_M = H_M/N_1 \to h_M/\hat{N}_1\,,\qquad D_{M,\text{con}} \to D_{M,\text{LF}}\,.$$

Then,

$$\hat{\mathcal{A}} = N_c \int \frac{\mathrm{d}k^+ \mathrm{d}^2 k_\perp}{2(2\pi)^3} \frac{-i h_M}{\bar{x} p^+ \hat{N}_1} \hat{S}_{\mathcal{A}}\,,\qquad \hat{\mathcal{B}} = N_c \int \frac{\mathrm{d}k'^+ \mathrm{d}^2 k'_\perp}{2(2\pi)^3} \frac{h_{M'} h_{M''}}{\bar{x} p'^+ \hat{N}_1' \hat{N}_1''} \hat{S}_{\mathcal{B}}\,. \qquad (1)$$

In order to restore the zero-mode contribution and eliminate ω dependence, we need the following decomposition and replacements

Jaus, Phys. Rev. D 60 (1999) 054026. Phys. Rev. D 69 (2004) 074025

for $\hat{\mathcal{A}}$: $\quad \hat{k}_1^\mu \to x p^\mu + \boldsymbol{...(\omega, C_i^{(j)})}\,,$

$$\hat{k}_1^\mu \hat{k}_1^\nu \to -g^{\mu\nu} \frac{k_\perp^2}{2} + p^\mu p^\nu x^2 + \frac{p^\mu \omega^\nu + p^\nu \omega^\mu}{\omega \cdot p} \boldsymbol{B_1^{(2)}} + \boldsymbol{...(\omega, C_i^{(j)})}\,,$$
$$\hat{N}_2 \to Z_2 = \hat{N}_1 + m_1^2 - m_2^2 + (\bar{x} - x) M^2\,,$$

for $\hat{\mathcal{B}}$:
$$\hat{k}_1'^{\mu} \to P^{\mu} A_1^{(1)} + q^{\mu} A_2^{(1)} + \ldots(\omega, \mathbf{C_i^{(j)}}) ,$$
$$k_1'^{\mu}\hat{N}_2 \to q^{\mu}\left[A_2^{(1)} Z_2 + \frac{q\cdot P}{q^2} A_1^{(2)}\right] ,$$
$$\hat{k}_1'^{\mu}\hat{k}_1'^{\nu} \to g^{\mu\nu} A_1^{(2)} + P^{\mu}P^{\nu} A_2^{(2)} + (P^{\mu}q^{\nu} + q^{\mu}P^{\nu}) A_3^{(2)} + q^{\mu}q^{\nu} A_4^{(2)} + \frac{P^{\mu}\omega^{\nu} + \omega^{\mu}P^{\nu}}{\omega\cdot P} B_1^{(2)} + \ldots(\omega, \mathbf{C_i^{(j)}}) ,$$
$$\hat{k}_1'^{\mu}\hat{k}_1'^{\nu}\hat{N}_2 \to g^{\mu\nu} A_1^{(2)} Z_2 + q^{\mu}q^{\nu}\left(A_4^{(2)} Z_2 + 2\frac{q\cdot P}{q^2} A_2^{(1)} A_1^{(2)}\right) + \frac{P^{\mu}\omega^{\nu} + \omega^{\mu}P^{\nu}}{\omega\cdot P} B_3^{(3)} + \ldots(\omega, \mathbf{C_i^{(j)}}) ,$$
$$\ldots\ldots$$

where $P = p' + p''$, $q = p' - p''$ and

$$A_1^{(1)} = \frac{x}{2} , \qquad A_2^{(1)} = \frac{x}{2} - \frac{k'_{1\perp}\cdot q_\perp}{q^2} , \qquad A_1^{(2)} = -k_{1\perp}'^2 - \frac{(k'_{1\perp}\cdot q_\perp)^2}{q^2} ,$$
$$B_1^{(2)} = \frac{x}{2} Z_2 + \frac{k_\perp^2}{2} , \qquad B_3^{(3)} = B_1^{(2)} Z_2 + (P^2 - \frac{(q\cdot P)^2}{q^2}) A_1^{(1)} A_1^{(2)} .$$
$$Z_2 = \hat{N}_1' + m_1'^2 - m_2^2 + (\bar{x} - x) M'^2 + (q^2 + q\cdot P)\frac{k'_{1\perp}\cdot q_\perp}{q^2} ,$$

For a given quantity, in order to clearly show the zero-mode effect, we have

$$\mathcal{Q}^{\rm full} = \mathcal{Q}^{\rm val.} + \mathcal{Q}^{\rm z.m.}$$

$\mathcal{Q}^{\rm val.}$: assuming $k_2^+ \neq 0$ and $k_1^+ \neq 0 \Longrightarrow$ poles of N_2 and N_1 are safely located inside and outside, respectively, the contour of k^- (k'^-) integral; zero-mode contributions are absent.

decomposition and replacements $\longrightarrow k_2^2 = m_2^2$ and four-momentum conservation at each vertex.

It is expected that $\mathcal{Q}^{\rm full}$ $(\mathcal{Q}^{\rm val.}) = \mathcal{Q}^{\rm SLF}$ if we believe that zero-mode contribution has (not) been included in $\mathcal{Q}^{\rm SLF}$,

3. Example 1: f_P and f_V

Definition: $\langle 0|\bar{q}_2\gamma^\mu\gamma_5 q_1|P(p)\rangle = i f_P p^\mu\,, \qquad \langle 0|\bar{q}_2\gamma^\mu q_1|V(p,\lambda)\rangle \quad = f_V M_V \epsilon^\mu\,.$

3.1 f_P

$$[f_P]_{\rm SLF} = \sqrt{N_c}\int \frac{{\rm d}x\,{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_s(x,k_\perp)}{\sqrt{x\bar{x}}}\frac{2}{\sqrt{2}\hat{M}_0}(\bar{x}m_1+xm_2)\,,$$

$$[f_P]_{\rm full} = [f_P]_{\rm val.} = N_c\int \frac{{\rm d}x\,{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_P}{\bar{x}}2(\bar{x}m_1+xm_2)\,,$$

- no residual ω dependence
- $[f_P]_{\rm full} = [f_P]_{\rm val.}$: f_P is free of the Z.M. contribution
- $[f_P]_{\rm SLF} = [f_P]_{\rm val.} = [f_P]_{\rm full}$ within both type-I and -II schemes.

Fitting to the data of f_P

	$\beta_{q\bar{q}}$	$\beta_{s\bar{q}}$	$\beta_{s\bar{s}}$	$\beta_{c\bar{q}}$	$\beta_{c\bar{s}}$
this work	$314.1^{+0.5}_{-0.5}$	$342.8^{+1.3}_{-1.4}$	$365.8^{+1.2}_{-1.8}$	$464.1^{+11.2}_{-10.8}$	$537.5^{+9.0}_{-8.7}$
PLB 460 (1999) 461	365.9	388.6	412.8	467.9	501.6
	$\beta_{c\bar{c}}$	$\beta_{b\bar{q}}$	$\beta_{b\bar{s}}$	$\beta_{b\bar{c}}$	$\beta_{b\bar{b}}$
this work	$654.5^{+143.3}_{-132.4}$	$547.9^{+9.9}_{-10.2}$	$601.4^{+7.3}_{-7.3}$	$947.0^{+11.2}_{-10.9}$	$1391.2^{+51.6}_{-48.2}$
PLB 460 (1999) 461	650.9	526.6	571.2	936.9	1145.2

3.2 f_V

Theoretical results:

$$[f_V]^{\lambda=0}_{\rm SLF} = \sqrt{N_c}\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_s(x,k_\perp)}{\sqrt{x\,\bar{x}}}\frac{2}{\sqrt{2}\hat{M}_0}\left(\bar{x}m_1+xm_2+\frac{2k_\perp^2}{D_{V,\rm LF}}\right),$$

$$[f_V]^{\lambda=\pm}_{\rm SLF} = \sqrt{N_c}\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_s(x,k_\perp)}{\sqrt{x\,\bar{x}}}\frac{2}{\sqrt{2}\hat{M}_0}\left(\frac{\hat{M}_0^2}{2M_V}-\frac{k_\perp^2}{D_{V,\rm LF}}\frac{M_0}{M_V}\right),$$

$$[f_V]^{\lambda=0}_{\rm full} = N_c\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_V}{\bar{x}}\frac{2}{M_V}\left[xM_0^2-m_1(m_1-m_2)-\left(1-\frac{m_1+m_2}{D_{V,\rm con}}\right)(k_\perp^2\mathbf{-2B_1^{(2)}})\right],$$

$$[f_V]^{\lambda=\pm}_{\rm full} = N_c\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_V}{\bar{x}}\frac{2}{M_V}\left[xM_0^2-m_1(m_1-m_2)-\left(1-\frac{m_1+m_2}{D_{V,\rm con}}\right)k_\perp^2\right],$$

$[f_V]^{\lambda=\pm}_{\rm SLF}$ is usually ignored in previous works due to the traditional bias.

In order to clearly show their self-consistence we define:

$$\Delta^M_{\rm full}(x)\equiv\frac{{\rm d}[f_M]^{\lambda=0}_{\rm full}}{{\rm d}x}-\frac{{\rm d}[f_M]^{\lambda=\pm}_{\rm full}}{{\rm d}x},\qquad \Delta^M_{\rm SLF}(x)\equiv\frac{{\rm d}[f_M]^{\lambda=0}_{\rm SLF}}{{\rm d}x}-\frac{{\rm d}[f_M]^{\lambda=\pm}_{\rm SLF}}{{\rm d}x}.$$

The valence contributions:

$$[f_V]^{\lambda=0}_{\rm val.} = N_c\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_V}{\bar{x}}\frac{2}{M_V}\left[k_\perp^2+x\bar{x}M_V^2+m_1m_2+\frac{\bar{x}^2M_V^2-m_2^2-k_\perp^2}{\bar{x}D_{V,\rm con}}(\bar{x}m_1-xm_2)\right],$$

$$[f_V]^{\lambda=\pm}_{\rm val.} = N_c\int \frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_V}{\bar{x}}\frac{2}{M_V}\left[\frac{\bar{x}M_V^2+xM_0^2-(m_1-m_2)^2}{2}-\left(1-\frac{m_1+m_2}{D_{V,\rm con}}\right)k_\perp^2\right].$$

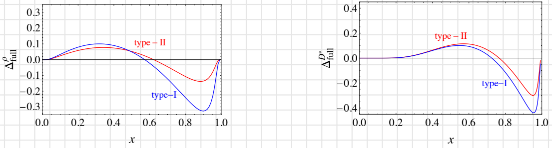
We do not find any relation in type-I scheme except for $[f_V]^{\lambda=0}_{\rm full}=[f_V]^{\lambda=\pm}_{\rm full}+\ldots\ldots B_1^{(2)}$

Numerical results: taking ρ and D^* as examples

	$[f_\rho]^{\lambda=0}_{\rm SLF}$	$[f_\rho]^{\lambda=\pm}_{\rm SLF}$	$[f_\rho]^{\lambda=0}_{\rm full}$	$[f_\rho]^{\lambda=\pm}_{\rm full}$	$[f_\rho]^{\lambda=0}_{\rm val.}$	$[f_\rho]^{\lambda=\pm}_{\rm val.}$
type-I	211.1	226.9	248.7	288.9	229.1	212.1
type-II	211.1	211.1	211.1	211.1	211.1	211.1
	$[f_{D^*}]^{\lambda=0}_{\rm SLF}$	$[f_{D^*}]^{\lambda=\pm}_{\rm SLF}$	$[f_{D^*}]^{\lambda=0}_{\rm full}$	$[f_{D^*}]^{\lambda=\pm}_{\rm full}$	$[f_{D^*}]^{\lambda=0}_{\rm val.}$	$[f_{D^*}]^{\lambda=\pm}_{\rm val.}$
type-I	252.6	273.5	275.3	305.6	244.6	258.9
type-II	252.6	252.6	252.6	252.6	252.6	252.6

Findings:

- Self-consistence of CLF QM: $\Delta^V_{\rm full}(x) = N_c \int \frac{{\rm d}^2 k_\perp}{(2\pi)^3} \frac{\chi_V}{\bar{x}} \frac{2}{M_V} \frac{D_{V,\rm con} - m_1 - m_2}{D_{V,\rm con}} 2B_1^{(2)}$

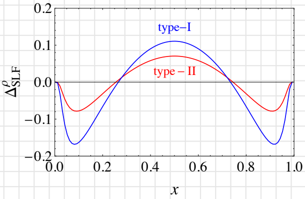


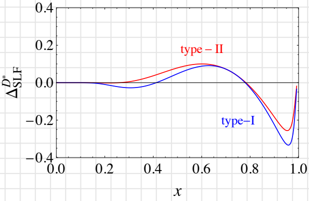
(i) $[f_V]^{\lambda=0}_{\rm full} \neq [f_V]^{\lambda=\pm}_{\rm full}$ (type-I) $\longrightarrow$ self-consistency problem of the CLF QM

(ii) Interestingly, we find: $[f_V]^{\lambda=0}_{\rm full} \doteq [f_V]^{\lambda=\pm}_{\rm full}$ (type-II) due to $\int {\rm d}x \Delta^V_{\rm full} = 0$

Type-II scheme provides a self-consistent correspondence between manifest covariant and LF approaches for f_V.

- Self-consistence of SLF QM: $\Delta^M_{\rm SLF}(x)$





(i) $[f_V]^{\lambda=0}_{\rm SLF} < [f_V]^{\lambda=\pm}_{\rm SLF}$ (type-I) $\longrightarrow$ self-consistency problem exists also in the traditional SLF QM

(ii) $[f_V]^{\lambda=0}_{\rm SLF} \doteq [f_V]^{\lambda=\pm}_{\rm SLF}$ (type-II) due to $\int {\rm d}x \Delta^V_{\rm SLF} = 0$

Type-II scheme is also favored by the self-consistency of the SLF QM.

- Relation between $[f_V]^{\lambda=0,\pm}_{\rm SLF}$ and $[f_V]^{\lambda=0,\pm}_{\rm val.}$:

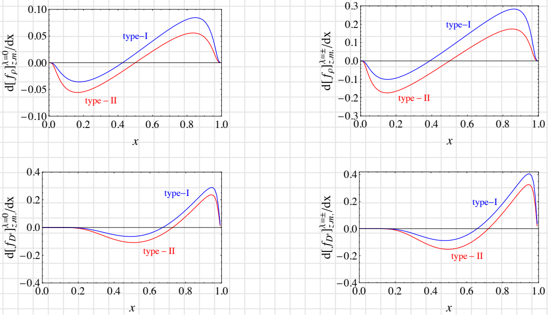
(i) No relation can be found (traditional type-I scheme).

(ii) Taking type-II scheme and making some simplifications, we find surprisingly

$$[f_V]^{\lambda=0}_{\rm SLF} = [f_V]^{\lambda=0}_{\rm val.} \text{ and } [f_V]^{\lambda=\pm}_{\rm SLF} = [f_V]^{\lambda=\pm}_{\rm val.} \quad (\text{type-II})$$

which are exactly ones expected.

- Zero-mode effects: $[f_V]_{\rm z.m.}$



(i) $0 < [f_V]^{\lambda=0}_{\rm z.m.} < [f_V]^{\lambda=\pm}_{\rm z.m.}$ **(type-I)** $\longrightarrow [f_V]_{\rm z.m.}$ are non-zero and dependent on λ.

(ii) $[f_V]^{\lambda=0,\pm}_{\rm z.m.} \doteq 0$ **(type-II)** $\longrightarrow [f_V]^{\lambda=0}_{\rm full} \doteq [f_V]^{\lambda=0}_{\rm val.}$ and $[f_V]^{\lambda=\pm}_{\rm full} \doteq [f_V]^{\lambda=\pm}_{\rm val.}$

Summarizing the findings above:

$$[f_V]^{\lambda=0}_{\rm SLF} = [f_V]^{\lambda=0}_{\rm val.} \doteq [f_V]^{\lambda=0}_{\rm full} \doteq [f_V]^{\lambda=\pm}_{\rm full} \doteq [f_V]^{\lambda=\pm}_{\rm val.} = [f_V]^{\lambda=\pm}_{\rm SLF} \quad \text{(type-II)}$$

Our Updated predictions for f_V (in unit of MeV):

	data	LQCD	QCD SR	this work
f_ρ	210 ± 4	199 ± 4	206 ± 7	211 ± 1
f_{K^*}	204 ± 7	$-$	222 ± 8	223 ± 1
f_ϕ	228.5 ± 3.6	238 ± 3	215 ± 5	236 ± 1
f_{D^*}	$-$	223.5 ± 8.4	250 ± 8	253 ± 7
$f_{D_s^*}$	301 ± 13	268.8 ± 6.6	290 ± 11	314 ± 6
$f_{J/\psi}$	411 ± 5	418 ± 9	401 ± 46	382 ± 96
f_{B^*}	$-$	185.9 ± 7.2	210 ± 6	205 ± 5
$f_{B_s^*}$	$-$	223.1 ± 5.4	221 ± 7	246 ± 4
$f_{B_c^*}$	$-$	422 ± 13	453 ± 20	465 ± 7
$f_{\Upsilon(1S)}$	708 ± 8	$-$	$-$	713 ± 34

LQCD: Nucl. Phys. B 883 (2014) 306; Phys. Rev. D 96 (2017) no. 7, 074502; JHEP 1704 (2017) 082; PoS LATTICE 2016 (2017) 291; Phys. Rev. D 91 (2015) no.11, 114509.

QCD SR: Nucl. Phys. B 883 (2014) 306; Phys. Rev. D 75 (2007) 054004; Part. Phys. Proc. 270-272 (2016) 143.

4. Example 2: f_A

Definition:

$$\langle 0|\bar q_2\gamma^\mu\gamma_5 q_1|A(p,\lambda)\rangle = f_A\, M_A \epsilon^\mu_\lambda$$

^3A: $^{2S+1}L_J = {}^3P_1$; ^1A: $^{2S+1}L_J = {}^1P_1$.

Theoretical results for ^1A:

$$[f_{1_A}]^{\lambda=0}_{\rm SLF} = -\sqrt{N_c}\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_p(x,k_\perp)}{\sqrt{x\,\bar x}}\frac{1}{\sqrt{2}\hat M_0}\frac{2}{M_0}\frac{(\bar x m_1+xm_2)[(\bar x-x)k_\perp^2+\bar x^2m_1^2-x^2m_2^2]}{x\bar x D_{1_A,{\rm LF}}}\,,$$

$$[f_{1_A}]^{\lambda=\pm}_{\rm SLF} = -\sqrt{N_c}\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_p(x,k_\perp)}{\sqrt{x\,\bar x}}\frac{1}{\sqrt{2}\hat M_0}\frac{2}{M_{1_A}}\frac{m_1-m_2}{D_{1_A,{\rm LF}}}k_\perp^2\,;$$

$$[f_{1_A}]^{\lambda=0}_{\rm full} = -N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{1_A}}{\bar x}\frac{2}{M_{1_A}}\frac{m_1-m_2}{D_{1_A,{\rm con}}}\left(k_\perp^2-\mathbf{2B_1^{(2)}}\right)\,,$$

$$[f_{1_A}]^{\lambda=\pm}_{\rm full} = -N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{1_A}}{\bar x}\frac{2}{M_{1_A}}\frac{m_1-m_2}{D_{1_A,{\rm con}}}k_\perp^2\,;$$

$$[f_{1_A}]^{\lambda=0}_{\rm val.} = -N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{1_A}}{\bar x}\frac{2}{M_{1_A}}\frac{M_{1_A}^2\bar x^2-m_2^2-k_\perp^2}{\bar x D_{1_A,{\rm con}}}(\bar x m_1+xm_2)\,,$$

$$[f_{1_A}]^{\lambda=\pm}_{\rm val.} = -N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{1_A}}{\bar x}\frac{2}{M_{1_A}}\frac{m_1-m_2}{D_{1_A,{\rm con}}}k_\perp^2\,.$$

Theoretical results for ${}^3A$:

$$[f_{^3A}]^{\lambda=0}_{\rm SLF} = \sqrt{N_c}\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_p(x,k_\perp)}{\sqrt{x\,\bar{x}}}\frac{1}{\sqrt{2}\hat{M}_0}\frac{\hat{M}_0^2}{2\sqrt{2}M_0}\frac{2}{M_0}\Big\{2k_\perp^2+(m_1-m_2)(\bar{x}m_1-xm_2) - \frac{(\bar{x}m_1+xm_2)[(\bar{x}-x)k_\perp^2+\bar{x}^2m_1^2-x^2m_2^2]}{x\bar{x}\,D_{^3A,{\rm LF}}}\Big\},$$

$$[f_{^3A}]^{\lambda=\pm}_{\rm SLF} = \sqrt{N_c}\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi_p(x,k_\perp)}{\sqrt{x\,\bar{x}}}\frac{1}{\sqrt{2}\hat{M}_0}\frac{\hat{M}_0^2}{2\sqrt{2}M_0}\frac{2}{M_{^3A}}\left[\frac{k_\perp^2-2\bar{x}xk_\perp^2+(\bar{x}m_1-xm_2)^2}{2\bar{x}x} - \frac{k_\perp^2(m_1-m_2)}{D_{^3A,{\rm LF}}}\right];$$

$$[f_{^3A}]^{\lambda=0}_{\rm full} = N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^3A}}{\bar{x}}\frac{2}{M_{^3A}}\left\{xM_0^2-m_1(m_1+m_2)-\left(1+\frac{m_1-m_2}{D_{^3A,{\rm con}}}\right)(k_\perp^2-{\color{red}\mathbf{2B_1^{(2)}}})\right\},$$

$$[f_{^3A}]^{\lambda=\pm}_{\rm full} = N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^3A}}{\bar{x}}\frac{2}{M_{^3A}}\left[xM_0^2-m_1(m_1+m_2)-\left(1+\frac{m_1-m_2}{D_{^3A,{\rm con}}}\right)k_\perp^2\right];$$

$$[f_{^3A}]^{\lambda=0}_{\rm val.} = N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^3A}}{\bar{x}}\frac{2}{M_{^3A}}\left[k_\perp^2+x\bar{x}M_{^3A}^2-m_1m_2-\frac{M_{^3A}^2\bar{x}^2-m_2^2-k_\perp^2}{\bar{x}D_{^3A,{\rm con}}}(\bar{x}m_1+xm_2)\right],$$

$$[f_{^3A}]^{\lambda=\pm}_{\rm val.} = N_c\int\frac{{\rm d}x{\rm d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^3A}}{\bar{x}}\frac{2}{M_{^3A}}\left[\frac{\bar{x}M_{^3A}^2+xM_0^2-(m_1+m_2)^2}{2}-\left(1+\frac{m_1-m_2}{D_{^3A,{\rm con}}}\right)k_\perp^2\right].$$

Numerical results: taking $^1A_{(q\bar q)}$, $^3A_{(q\bar q)}$, $^1A_{(c\bar q)}$ and $^3A_{(c\bar q)}$ ($b_1(1235)$, $a_1(1260)$, $D_1(2420)$ and $D_1(2430)$) as examples

	$[f_{^1A_{(q\bar q)}}]^{\lambda=0}_{\rm SLF}$	$[f_{^1A_{(q\bar q)}}]^{\lambda=\pm}_{\rm SLF}$	$[f_{^1A_{(q\bar q)}}]^{\lambda=0}_{\rm full}$	$[f_{^1A_{(q\bar q)}}]^{\lambda=\pm}_{\rm full}$	$[f_{^1A_{(q\bar q)}}]^{\lambda=0}_{\rm val.}$	$[f_{^1A_{(q\bar q)}}]^{\lambda=\pm}_{\rm val.}$
type-I	0	0	0	0	-47.4	0
type-II	0	0	0	0	0	0
	$[f_{^1A_{(c\bar q)}}]^{\lambda=0}_{\rm SLF}$	$[f_{^1A_{(c\bar q)}}]^{\lambda=\pm}_{\rm SLF}$	$[f_{^1A_{(c\bar q)}}]^{\lambda=0}_{\rm full}$	$[f_{^1A_{(c\bar q)}}]^{\lambda=\pm}_{\rm full}$	$[f_{^1A_{(c\bar q)}}]^{\lambda=0}_{\rm val.}$	$[f_{^1A_{(c\bar q)}}]^{\lambda=\pm}_{\rm val.}$
type-I	-78.5	-84.6	-78.4	-84.6	-65.2	-84.6
type-II	-78.5	-78.5	-78.5	-78.5	-78.5	-78.5

	$[f_{^3A_{(q\bar q)}}]^{\lambda=0}_{\rm SLF}$	$[f_{^3A_{(q\bar q)}}]^{\lambda=\pm}_{\rm SLF}$	$[f_{^3A_{(q\bar q)}}]^{\lambda=0}_{\rm full}$	$[f_{^3A_{(q\bar q)}}]^{\lambda=\pm}_{\rm full}$	$[f_{^3A_{(q\bar q)}}]^{\lambda=0}_{\rm val.}$	$[f_{^3A_{(q\bar q)}}]^{\lambda=\pm}_{\rm val.}$
type-I	218.7	223.6	260.6	223.6	263.1	263.1
type-II	218.7	218.7	218.7	218.7	218.7	218.7
	$[f_{^3A_{(c\bar q)}}]^{\lambda=0}_{\rm SLF}$	$[f_{^3A_{(c\bar q)}}]^{\lambda=\pm}_{\rm SLF}$	$[f_{^3A_{(c\bar q)}}]^{\lambda=0}_{\rm full}$	$[f_{^3A_{(c\bar q)}}]^{\lambda=\pm}_{\rm full}$	$[f_{^3A_{(c\bar q)}}]^{\lambda=0}_{\rm val.}$	$[f_{^3A_{(c\bar q)}}]^{\lambda=\pm}_{\rm val.}$
type-I	231.7	256.7	244.7	256.7	228.5	228.5
type-II	231.7	231.7	231.7	231.7	231.7	231.7

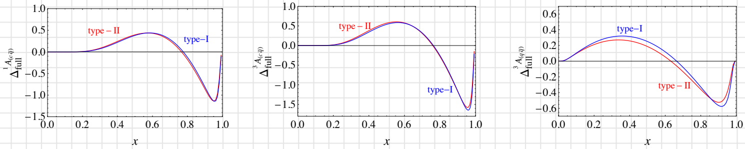
(i). Self-consistency problem also exists in ^1A and ^3A systems

(ii). $^1A_{(q\bar q)}$ meson is not ideal for testing the self-consistency due to $m_1 = m_2$.

$[f_{^1A_{(q\bar q)}}]^{\lambda=\pm}_{\rm val.,SLF,full}$, $[f_{^1A_{(q\bar q)}}]^{\lambda=0}_{\rm full}$ $:\propto m_1 - m_2$

$[f_{^1A_{(q\bar q)}}]^{\lambda=0}_{\rm SLF}$: anti-symmetry under $x \leftrightarrow \bar x$.

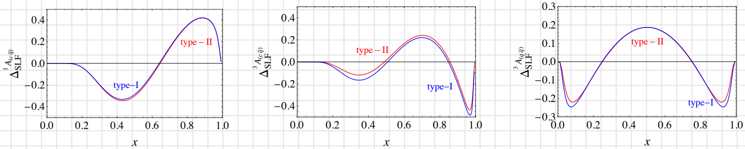
- Self-consistence of CLF QM:



(i) The violation of self-consistence is very small but non-zero in traditional type-I scheme.

(ii) $[f_A]^{\lambda=0}_{\rm full} \doteq [f_A]^{\lambda=\pm}_{\rm full}$ **(type-II)** due to $\int {\rm d}x\Delta^{1(3)A}_{\rm full} = 0$

- Self-consistence of SLF QM:



Self-consistency holds only in type-II scheme: $[f_A]^{\lambda=0}_{\rm SLF} \doteq [f_A]^{\lambda=\pm}_{\rm SLF}$ **(type-II)**

Above findings are similar to the case of V meson.

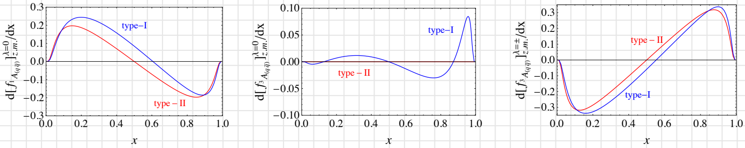
- Relation between $[f_A]^{\lambda=0,\pm}_{\rm SLF}$ and $[f_A]^{\lambda=0,\pm}_{\rm val.}$:

  Taking type-II scheme and making some simplifications, we find that:

  $[f_{1(3)A}]^{\lambda=0}_{\rm SLF} = [f_{1(3)A}]^{\lambda=0}_{\rm val.}\,,\quad [f_{1(3)A}]^{\lambda=\pm}_{\rm SLF} = [f_{1(3)A}]^{\lambda=\pm}_{\rm val.}\,,$ (type-II)

  in which, only $[f_{1A}]^{\lambda=\pm}_{\rm SLF} = [f_{1A}]^{\lambda=\pm}_{\rm val.}$ holds in the type-I scheme.

- Zero-mode effects: $[f_A]_{\rm z.m.}$



(i) $[f_{1A}]^{\lambda=\pm}_{\rm z.m.} = 0$ (type-I and -II) , $[f_{1A}]^{\lambda=0}_{\rm z.m.} \neq 0$ (type-I) $\longrightarrow$ The existence or absence of $[f_{1A}]_{\rm z.m.}$ depends on the choice of λ in type-I scheme.

(ii) $[f_{3A}]^{\lambda=0,\pm}_{\rm z.m.} \neq 0$ (type-I) $\longrightarrow$ Its contribution depends on the choice of λ .

(iii) $[f_A]^{\lambda=0,\pm}_{\rm z.m.} \doteq 0$ (type-II) $\longrightarrow$ $[f_A]^{\lambda=0}_{\rm full} \doteq [f_A]^{\lambda=0}_{\rm val.}$ and $[f_A]^{\lambda=\pm}_{\rm full} \doteq [f_A]^{\lambda=\pm}_{\rm val.}$

Combining the findings for the V and A mesons,

$$[\mathcal{Q}]^{\lambda=0}_{\rm SLF} = [\mathcal{Q}]^{\lambda=0}_{\rm val.} \doteq [\mathcal{Q}]^{\lambda=0}_{\rm full} \doteq [\mathcal{Q}]^{\lambda=\pm}_{\rm full} \doteq [\mathcal{Q}]^{\lambda=\pm}_{\rm val.} = [\mathcal{Q}]^{\lambda=\pm}_{\rm SLF}\,, \qquad \text{(type-II)}$$

where $\mathcal{Q} = f_V$, $f_{^1A}$ and $f_{^3A}$, and the first and the last "$\doteq$" should be replaced by "$=$" for the $^3A_{(q\bar{q})}$ and ^1A mesons, respectively.

Updated predictions for $f_{^1A}$ and $f_{^3A}$ (in unit of MeV)

	$f_{q\bar{q}}$	$f_{s\bar{q}}$	$f_{s\bar{s}}$	$f_{c\bar{q}}$	$f_{c\bar{s}}$
^1A	0	-27 ± 1	0	-78 ± 2	-62 ± 2
^3A	220 ± 1	219 ± 2	203 ± 2	231 ± 8	257 ± 8
	$f_{c\bar{c}}$	$f_{b\bar{q}}$	$f_{b\bar{s}}$	$f_{b\bar{c}}$	$f_{b\bar{b}}$
^1A	0	-95 ± 3	-88 ± 2	-86 ± 3	0
^3A	250 ± 90	176 ± 6	180 ± 5	281 ± 7	353 ± 25

5. Covariance of CLF QM: $f_{V,A}$

Taking $\mathcal{A}_V \equiv \langle 0|\bar{q}_2\gamma^\mu q_1|V(p)\rangle$ as an example,

$$\hat{\mathcal{A}}_V^\mu = M_V(\epsilon^\mu f_V + \omega^\mu g_V)\,,$$

Note: covariance holds only when $g_V = 0$.

Origin of violation:
After integrating out the k^- component and taking into account the zero-mode contributions (most of ω dependences are eliminated), we can decompose $\hat{S}_\mathcal{A}$ (integrand) as

$$\hat{S}_V^\mu = 4\left\{2\left(1-\frac{m_1+m_2}{D_{V,\rm con}}\right)\frac{\omega\cdot\epsilon}{\omega\cdot p}p^\mu B_1^{(2)} + \epsilon^\mu\left[\cdots\right]\right\}\,,$$

- Second term: the physical contribution to f_V
  First term: the ω-dependent part
- Case of $\lambda = \pm$: the ω dependence vanishes due to $\omega\cdot\epsilon_\pm = 0 \longrightarrow$ Covariant

  Note that: $\lambda = \pm$ is not alway a "good choice" to avoid the covariance problem

- Case of $\lambda = 0$:
  In order to separate the physical and unphysical contributions, we have to use the identity

$$p^\mu \frac{\epsilon\cdot\omega}{\omega\cdot p} = \epsilon^\mu - \frac{\omega^\mu}{\omega\cdot p}\left(\epsilon\cdot p - \epsilon\cdot\omega\frac{p^2}{\omega\cdot p}\right) - \frac{i\lambda}{\omega\cdot p}\varepsilon^{\mu\nu\alpha\beta}\omega_\nu\epsilon_\alpha p_\beta \, .$$

The third term: $= 0$

The first term: gives an additional contribution to f_V that results in the self-consistency problem;

The second term: the residual ω-dependent part that contributes to g_V and may violate the Lorentz covariance.

The problems of self-consistency and covariance of the CLF quark model within the type-I scheme have the same origin!

Theoretical results

$$[g_V]^{\lambda=0} = \frac{N_c}{2}\int\frac{\mathrm{d}x\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\chi_V(x,k_\perp^2)}{\bar{x}}4\left(1-\frac{m_1+m_2}{D_{V,\mathrm{con}}}\right)\frac{2}{\omega\cdot p}B_1^{(2)} \, ,$$

$$[g_{^3A}]^{\lambda=0} = \frac{N_c}{2}\int\frac{\mathrm{d}x\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^3A}(x,k_\perp^2)}{\bar{x}}4\left(1+\frac{m_1-m_2}{D_{^3A,\mathrm{con}}}\right)\frac{2}{\omega\cdot p}B_1^{(2)} \, ,$$

$$[g_{^1A}]^{\lambda=0} = \frac{N_c}{2}\int\frac{\mathrm{d}x\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\chi_{^1A}(x,k_\perp^2)}{\bar{x}}4\frac{m_1-m_2}{D_{^1A,\mathrm{con}}}\frac{2}{\omega\cdot p}B_1^{(2)} \, ,$$

- Conditions for the covariance: $[g_V]=[g_A]=0 \Longrightarrow$

$$\int \mathrm{d}x\mathrm{d}^2k_\perp \frac{\chi_M(x,k_\perp^2)}{\bar{x}} B_1^{(2)} = 0\,, \qquad \int \mathrm{d}x\mathrm{d}^2k_\perp \frac{\chi_M(x,k_\perp^2)}{\bar{x}} \frac{B_1^{(2)}}{D_{M,\mathrm{con}}} = 0\,,$$

which is much stricter than the one given by Jaus.

- $[g_{V,A}] \propto 1/p^+$: the size of covariance violation within the type-I scheme is in fact out of control because p^+ is reference-frame dependent.
- Covariance is violated in the type-I scheme but can be recovered in the type-II scheme.
In the rest frame ($p^+=M$),

$$[g_{V,A}]^{\lambda=0} = [f_{V,A}]^{\lambda=0}_{\mathrm{full}} - [f_{V,A}]^{\lambda=\pm}_{\mathrm{full}} = \int \mathrm{d}x\, \Delta^{V,A}_{\mathrm{full}}(x)\,,$$

So,

$$[g_{\rho,\,D^*,\,{}^1A_{(c\bar{q})},\,{}^3A_{(q\bar{q})},{}^3A_{(c\bar{q})}}]^{\lambda=0} = (-40.2,\,-30.3,\,6.2,\,37.0,\,-12.0)\,\mathrm{MeV} \neq 0\,, \quad \text{(type-I)}$$

$$[g_{\rho,\,D^*,\,{}^1A_{(c\bar{q})},\,{}^3A_{(q\bar{q})},{}^3A_{(c\bar{q})}}]^{\lambda=0} = 0\,, \quad \text{(type-II)}$$

The problems of self-consistency and covariance of the CLF quark model can be "resolved" simultaneously within the type-II scheme.

- The type-I and -II schemes are consistent with each other in the heavy quark limit.
  $M \sim m_Q \gg m_{\bar{q}}$
  $x \sim m_Q/M$ and $\bar{x} \sim m_q/M$ $\Longrightarrow$ $M_0 \to M$
  $f(g)_{V,A}$ are dominated by $|k_\perp| \lesssim 1$ GeV

Brief summary:

- In the traditional SLF and CLF QMs (type-I scheme), $f_{V,{}^1A,{}^3A}$ suffer from the self-consistency and covariance problems.
- In the CLF QMs, the self-consistency and covariance problems can be resolved simultaneously by taking type-II correspondence.
- The zero-mode contributions exist only formally but vanish numerically (type-II).
- For the decay constants of spin-1 systems,

$$[\mathcal{Q}]^{\lambda=0}_{\rm SLF} = [\mathcal{Q}]^{\lambda=0}_{\rm val.} \doteq [\mathcal{Q}]^{\lambda=0}_{\rm full} \doteq [\mathcal{Q}]^{\lambda=\pm}_{\rm full} \doteq [\mathcal{Q}]^{\lambda=\pm}_{\rm val.} = [\mathcal{Q}]^{\lambda=\pm}_{\rm SLF}\,; \qquad \text{(type-II)}$$

- The two schemes are consistent with each other in the heavy-quark limit.

6. Example 3: $\mathcal{F}^{P\to V}$

Definition:

$$\langle V(p'',\lambda)|\bar{q}_1''\gamma_\mu q_1'|P(p')\rangle = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha q^\beta\, g(q^2)\,,$$

$$\langle V(p'',\lambda)|\bar{q}_1''\gamma_\mu\gamma^5 q_1'|P(p')\rangle = -f(q^2)\,\epsilon^*_\mu - \epsilon^*\cdot P\left[a_+(q^2)P_\mu + a_-(q^2)q_\mu\right]\,.$$

Bauer-Stech-Wirbel (BSW) form factors:

$$V(q^2) = -(M'+M'')g(q^2)\,,\quad A_1(q^2) = -\frac{f(q^2)}{M'+M''}\,,\quad A_2(q^2) = (M'+M'')a_+(q^2)\,,$$

$$A_0(q^2) = -\frac{1}{2M''}\left[q^2 a_-(q^2) + f(q^2) + (M'^2 - M''^2)a_+(q^2)\right]$$

Theoretical results: $g(q^2)$ and $a_+(q^2)$

$$[g(q^2)]_{\rm SLF} = -\int \frac{{\rm d}x\,{\rm d}^2\mathbf{k}'_\perp}{(2\pi)^3\,2x}\frac{\psi''^*(x,\mathbf{k}''_\perp)\psi'(x,\mathbf{k}'_\perp)}{2\hat{M}'_0\hat{M}''_0}$$
$$2\left\{\bar{x}m_1' + xm_2 + (m_1'-m_1'')\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{q^2} + \frac{2}{D''_{\rm V,LF}}\left[\mathbf{k}'^2_\perp + \frac{(\mathbf{k}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right]\right\}\,,$$

$$[g(q^2)]_{\rm full} = [g(q^2)]_{\rm val.} = -N_c\int\frac{{\rm d}x\,{\rm d}^2\mathbf{k}'_\perp}{2(2\pi)^3}\frac{\chi'_P\chi''_V}{\bar{x}}$$
$$2\left\{\bar{x}m_1' + xm_2 + (m_1'-m_1'')\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{q^2} + \frac{2}{D''_{\rm V,con}}\left[\mathbf{k}'^2_\perp + \frac{(\mathbf{k}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right]\right\}\,,$$

$$[a_+(q^2)]_{\rm SLF} = \int \frac{{\rm d}x\, {\rm d}^2\mathbf{k}'_\perp}{(2\pi)^3\, 2x} \frac{\psi''^*(x,\mathbf{k}''_\perp)\psi'(x,\mathbf{k}'_\perp)}{2\hat{M}'_0\hat{M}''_0}$$
$$2\Big\{(m''_1 - 2xm'_1 + m'_1 + 2xm_2)\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{\mathbf{q}^2_\perp} + (x-\bar{x})(\bar{x}m'_1 + xm_2)$$
$$+ \frac{2}{D''_{\rm V,LF}}\frac{\mathbf{k}''_\perp\cdot\mathbf{q}_\perp}{\bar{x}\mathbf{q}^2_\perp}\Big[\mathbf{k}'_\perp\cdot\mathbf{k}''_\perp + (xm_2 - \bar{x}m''_1)(xm_2 + \bar{x}m'_1)\Big]\Big\},$$
$$[a_+(q^2)]_{\rm full} = [a_+(q^2)]_{\rm val.} = N_c \int \frac{{\rm d}x\, {\rm d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_P\chi''_V}{\bar{x}}$$
$$2\Big\{(m''_1 - 2xm'_1 + m'_1 + 2xm_2)\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{\mathbf{q}^2_\perp} + (x-\bar{x})(\bar{x}m'_1 + xm_2)$$
$$+ \frac{2}{D''_{\rm V,con}}\frac{\mathbf{k}''_\perp\cdot\mathbf{q}_\perp}{\bar{x}\mathbf{q}^2_\perp}\Big[\mathbf{k}'_\perp\cdot\mathbf{k}''_\perp + (xm_2 - \bar{x}m''_1)(xm_2 + \bar{x}m'_1)\Big]\Big\},$$

For $\mathcal{F} = g(q^2)$ and $a_+(q^2)$:

- $[\mathcal{F}]^{\lambda=0}_{\rm full} = [\mathcal{F}]^{\lambda=\pm}_{\rm full}$: no self-consistency problem.
- $[\mathcal{F}]_{\rm z.m.} = 0$: no z.m. contribution.
- $[\mathcal{F}]_{\rm SLF} = [\mathcal{F}]_{\rm full} = [\mathcal{F}]_{\rm val.}$

Theoretical results: $f(q^2)$ and $a_-(q^2)$

$$
\begin{aligned}
[f(q^2)]_{\rm SLF} =& -\int \frac{{\rm d}x\,{\rm d}^2\mathbf{k}'_\perp}{(2\pi)^3\,2x}\frac{\psi''^*(x,\mathbf{k}''_\perp)\psi'(x,\mathbf{k}'_\perp)}{2\hat{M}'_0\hat{M}''_0} \\
& \frac{4M''}{\bar{x}M''_0}\Big\{\Big[\mathbf{k}'^2_\perp(\bar{x}m'_1+m''_1-\bar{x}m_2)-\bar{x}\mathbf{k}'_\perp\cdot\mathbf{q}_\perp(m''_1+2\bar{x}m'_1+xm_2-\bar{x}m_2) \\
& +(\bar{x}m'_1+xm_2)(m''_1m_2+x\bar{x}M''^2_0+\bar{x}^2\mathbf{q}^2_\perp)\Big] \\
& +\frac{\mathbf{k}''^2_\perp+m_2^2-\bar{x}^2M''^2_0}{\bar{x}D''_{\rm V,LF}}\left[\mathbf{k}'_\perp\cdot\mathbf{k}''_\perp+(xm_2-\bar{x}m''_1)(xm_2+\bar{x}m'_1)\right]\Big\} \\
& -\left(M'^2-M''^2+\mathbf{q}^2_\perp\right)[a_+(q^2)]_{\rm SLF}\,, \\
[f(q^2)]_{\rm val.} =& -N_c\int\frac{{\rm d}x\,{\rm d}^2\mathbf{k}'_\perp}{2(2\pi)^3}\frac{\chi'_P\chi''_V}{\bar{x}} \\
& \frac{4}{\bar{x}}\Big\{\mathbf{k}'^2_\perp(\bar{x}m'_1+m''_1-\bar{x}m_2)-\bar{x}\mathbf{k}'_\perp\cdot\mathbf{q}_\perp(m''_1+2\bar{x}m'_1+xm_2-\bar{x}m_2) \\
& +(\bar{x}m'_1+xm_2)\left[m''_1m_2+x\bar{x}M''^2+\bar{x}^2\mathbf{q}^2_\perp\right] \\
& +\frac{\mathbf{k}''^2_\perp+m_2^2-\bar{x}^2M''^2}{\bar{x}D''_{\rm V,con}}\left[\mathbf{k}'_\perp\cdot\mathbf{k}''_\perp+(xm_2+\bar{x}m'_1)(xm_2-\bar{x}m''_1)\right]\Big\} \\
& -\left(M'^2-M''^2+\mathbf{q}^2_\perp\right)[a_+(q^2)]_{\rm val.}
\end{aligned}
$$

$$
\begin{aligned}
[f(q^2)]^{\lambda=0}_{\rm full} =& - N_c \int \frac{dx\, d^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_P \chi''_V}{\bar{x}} \\
& 2\Big\{ - (m'_1 + m''_1)^2 (m'_1 - m_2) + (x m_2 - \bar{x} m'_1) M'^2 + (x m_2 + \bar{x} m'_1) M''^2 \\
& - x(m_2 - m'_1)(M'^2_0 + M''^2_0) + 2x m''_1 M'^2_0 - 4\left(m'_1 - m_2\right)\left(\mathbf{k}'^2_\perp + \frac{(\mathbf{k}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right) \\
& - m_2 q^2 - (m'_1 + m''_1)(q^2 + q\cdot P)\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{q^2} + 4(m'_1 - m_2) B_1^{(2)} \\
& + \frac{2}{D''_{\rm V,con}}\Big[\left(\mathbf{k}'^2_\perp + \frac{(\mathbf{k}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right)\Big((x - \bar{x})M'^2 + M''^2 - 2(m'_1 - m''_1)(m'_1 - m_2) \\
& + 2x M'^2_0 - q^2 - 2(q^2 + q\cdot P)\frac{\mathbf{k}'_\perp\cdot\mathbf{q}_\perp}{q^2}\Big) \\
& - \Big(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2)\Big) B_1^{(2)} + 2B_3^{(3)}\Big]\Big\},
\end{aligned}
$$

$$
[f(q^2)]^{\lambda=\pm}_{\rm full} = [f(q^2)]^{\lambda=0}_{\rm full}\Big|_{B_1^{(2)}=B_3^{(3)}=0}
$$

$$
\begin{aligned}
[a_-(q^2)]_{\rm SLF} = & \int \frac{{\rm d}x\, {\rm d}^2\mathbf{k}'_\perp}{(2\pi)^3\, 2x} \frac{\psi''^*(x,\mathbf{k}''_\perp)\psi'(x,\mathbf{k}'_\perp)}{2\hat{M}'_0\hat{M}''_0} \\
& \frac{4}{\mathbf{q}_\perp^2}\Big\{ m''_1 M'^2_0 + m'_1 M''^2_0 - (m'_1+m''_1)(m'_1-m_2)(m''_1-m_2) \\
& - \bar{x}(m'_1-m_2)\mathbf{q}_\perp^2 + [m'_1 - m''_1 + 2\bar{x}(m'_1-m_2)]\mathbf{k}'_\perp\cdot\mathbf{q}_\perp - 2(m'_1-m_2)\mathbf{k}'^2_\perp \\
& + \frac{1}{D''_{\rm V,LF}}\Big\{ -\mathbf{k}''_\perp\cdot\mathbf{q}_\perp \left[M'^2_0 - (m'_1-m_2)^2\right] \\
& + \mathbf{k}'_\perp\cdot\mathbf{k}''_\perp \left[M''^2_0 + M'^2_0 + 2(m''_1+m_2)(m'_1-m_2) + \mathbf{q}_\perp^2\right] \Big\}\Big\} - \frac{2}{q^2} f(q^2) + a_+(q^2)
\end{aligned}
$$

$$
\begin{aligned}
[a_-(q^2)]_{\rm val.} = & N_c \int \frac{{\rm d}x\, {\rm d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_P\chi''_V}{\bar{x}} \\
& \frac{4}{\mathbf{q}_\perp^2}\Big\{ x(m''_1-m_2)M'^2_0 + x(m'_1-m_2)M''^2_0 + (\bar{x}m''_1 + xm_2)M'^2 \\
& + (\bar{x}m'_1 + xm_2)M''^2 - (m'_1+m''_1)(m'_1-m_2)(m''_1-m_2) - \bar{x}(m'_1-m_2)\mathbf{q}_\perp^2 \\
& + [m'_1 - m''_1 + 2\bar{x}(m'_1-m_2)]\mathbf{k}'_\perp\cdot\mathbf{q}_\perp - 2(m'_1-m_2)\mathbf{k}'^2_\perp \\
& + \frac{1}{D''_{\rm V,con}}\Big[-\mathbf{k}''_\perp\cdot\mathbf{q}_\perp \left(xM'^2_0 + \bar{x}M'^2 - (m'_1-m_2)^2\right) \\
& + \mathbf{k}'_\perp\cdot\mathbf{k}''_\perp \left(M''^2 + M'^2 + 2(m''_1+m_2)(m'_1-m_2) + \mathbf{q}_\perp^2\right) \Big]\Big\} \\
& - \frac{2}{q^2} f(q^2) + a_+(q^2)\,.
\end{aligned}
$$

$$
\begin{aligned}
[a_-(q^2)]^{\lambda=0}_{\text{full}} = &- N_c \int \frac{\mathrm{d}x\, \mathrm{d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_P \chi''_V}{\bar{x}} \\
& 2\Bigg\{ (3-2x)(\bar{x}m'_1 + x m_2) - \left[(6x-7)m'_1 + (4-6x)m_2 + m''_1\right] \frac{\mathbf{k}'_\perp \cdot \mathbf{q}_\perp}{q^2} \\
& + 4(m'_1 - m_2)\left[2\left(\frac{\mathbf{k}'_\perp \cdot \mathbf{q}_\perp}{q^2}\right)^2 + \frac{\mathbf{k}'^2_\perp}{q^2}\right] - 4\frac{(m'_1 - m_2)}{q^2} B^{(2)}_1 \\
& + \frac{1}{D''_{\text{V,con}}}\Bigg[- 2\left(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2)\right)(A^{(2)}_3 + A^{(2)}_4 - A^{(1)}_2 \\
& + \left(2M'^2 - q^2 - \hat{N}'_1 + \hat{N}''_1 - 2(m'_1 - m_2)^2 + (m'_1 + m''_1)^2\right)\left(A^{(1)}_1 + A^{(1)}_2 - 1\right) \\
& + 2Z_2\left(2A^{(2)}_4 - 3A^{(1)}_2 + 1\right) + 2\frac{q\cdot P}{q^2}\left(4A^{(1)}_2 A^{(2)}_1 - 3A^{(2)}_1\right) \\
& + \frac{2}{q^2}\left(\left(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2)\right) B^{(2)}_1 - 2B^{(3)}_3\right)\Bigg]\Bigg\}.
\end{aligned}
$$

$$
[a_-(q^2)]^{\lambda=\pm}_{\text{full}} = [a_-(q^2)]^{\lambda=0}_{\text{full}}\Big|_{B^{(2)}_1 = B^{(3)}_3 = 0}
$$

For $\mathcal{F} = f(q^2)$ and $a_-(q^2)$:

- $[\mathcal{F}]^{\lambda=0}_{\text{full}} \neq [\mathcal{F}]^{\lambda=\pm}_{\text{full}}$: self-consistency problem.
- $[\mathcal{F}]_{\text{SLF}} \neq [\mathcal{F}]_{\text{full}} \neq [\mathcal{F}]_{\text{val.}}$ within traditional type-I scheme

Taking $D \to K^*$ as example. "—": divergent at $q^2 = 0$.

$D \to K^*$		$[f(\mathbf{q}_\perp^2)]_{\text{SLF}}$	$[f(\mathbf{q}_\perp^2)]_{\text{full}}^{\lambda=0}$	$[f(\mathbf{q}_\perp^2)]_{\text{full}}^{\lambda=\pm1}$	$[f(\mathbf{q}_\perp^2)]_{\text{val.}}$
$\mathbf{q}_\perp^2 = 0$	type-I	-1.93	-1.76	-2.17	-2.19
	type-II	-2.66	-2.66	-2.66	-2.66
$\mathbf{q}_\perp^2 = 1$	type-I	-1.75	-1.59	-1.89	-1.97
	type-II	-2.37	-2.37	-2.37	-2.37
$\mathbf{q}_\perp^2 = 2$	type-I	-1.61	-1.50	-1.69	-1.79
	type-II	-2.14	-2.14	-2.14	-2.14
$D \to K^*$		$[a_-(\mathbf{q}_\perp^2)]_{\text{SLF}}$	$[a_-(\mathbf{q}_\perp^2)]_{\text{full}}^{\lambda=0}$	$[a_-(\mathbf{q}_\perp^2)]_{\text{full}}^{\lambda=\pm1}$	$[a_-(\mathbf{q}_\perp^2)]_{\text{val.}}$
$\mathbf{q}_\perp^2 = 0$	type-I	—	—	-0.34	—
	type-II	-0.35	-0.35	-0.35	-0.35
$\mathbf{q}_\perp^2 = 1$	type-I	0.97	-0.94	-0.27	-0.49
	type-II	-0.27	-0.27	-0.27	-0.27
$\mathbf{q}_\perp^2 = 2$	type-I	0.32	-0.62	-0.22	-0.30
	type-II	-0.21	-0.21	-0.21	-0.21

$a_-(q^2)$, are in fact calculable in the SLF QM after taking $M \to M_0$

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Covariant analysis of the light-front quark model

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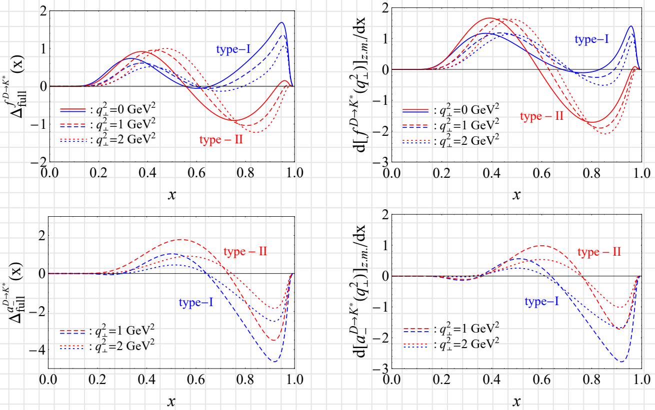
the standard approach are reproduced, except for those that de... covariant approach permits also the calculation of the scalar ... mesons, and the form factor $a_-(q^2)$ for transitions between pseudoscalar and vector mesons, which is not possible in the standard light-front formalism. The practical application of the covariant extension of the



- $[\mathcal{F}]^{\lambda=0}_{\rm full} \doteq [\mathcal{F}]^{\lambda=\pm}_{\rm full}$ (type-II): self-consistency problem is “resolved”.
- $[\mathcal{F}]_{\rm full} \doteq [\mathcal{F}]_{\rm val.}$ (type-II): Z.M contribution vanishes numerically
- $[\mathcal{F}]_{\rm SLF} = [\mathcal{F}]_{\rm val.}$ (type-II)

$$[\mathcal{Q}]_{\rm SLF} = [\mathcal{Q}]_{\rm val.} \doteq [\mathcal{Q}]_{\rm full}\,, \qquad \text{(type-II)}$$

✓

Covariance:

$$\hat{S}^{\mu}_{P\to V} = 4\,\frac{P^\mu \epsilon^*\cdot\omega + \omega^\mu \epsilon^*\cdot P}{\omega\cdot P}\Big\{2(m_1'-m_2)B_1^{(2)} - \frac{1}{D''_{\rm V,con}}\Big[(M'^2+M''^2-q^2+2(m_1'-m_2)(m_1''+m_2))B_1^{(2)} - 2B_3^{(3)}\Big]\Big\} + \dots,$$

"...": physical contribution to $f(q^2)$ and $a_\pm(q^2)$.

For the first term, we shall use the identity

$$P^\mu\frac{\epsilon\cdot\omega}{\omega\cdot P} = \epsilon^\mu - \frac{q^\mu}{q^2}\left(\epsilon\cdot q - q\cdot P\frac{\omega\cdot\epsilon}{\omega\cdot P}\right) - \frac{\omega^\mu}{\omega\cdot P}\left[\epsilon\cdot P - \epsilon\cdot q\frac{q\cdot P}{q^2} - \epsilon\cdot\omega\frac{P^2}{\omega\cdot P} + \epsilon\cdot\omega\frac{(q\cdot P)^2}{q^2\omega\cdot P}\right] - \frac{i\lambda}{\omega\cdot P}\frac{\epsilon\cdot q}{q^2}\varepsilon^{\mu\alpha\beta\nu}\omega_\alpha q_\beta P_\nu\,.$$

For $\lambda = 0$

ϵ^μ term: additional contribution to $f(q^2)$;
$q^\mu \epsilon\cdot q$ term : additional contribution to $a_-(q^2)$
ω^μ term: contribution to unphysical $\mathcal{F}$, which violates covariance. λ term: $=0$

$$\left.\frac{P^\mu\epsilon^*\cdot\omega+\omega^\mu\epsilon^*\cdot P}{\omega\cdot P}\right|_{\lambda=0} = \epsilon^{*\mu} - \frac{q^\mu}{q^2}\epsilon^*\cdot q + q^\mu\frac{q\cdot P}{q^2 M''} + \omega^\mu\frac{2M''}{\omega\cdot P}\,.$$

The self-consistency and the covariance problems have the same origin!

For $\lambda = \pm$

Instead of using such identity, we can directly write the pre-factor as, due to $\epsilon^*_{\lambda=\pm}\cdot\omega = 0$,

$$\frac{P^\mu \epsilon^*\cdot\omega + \omega^\mu \epsilon^*\cdot P}{\omega\cdot P}\Big|_{\lambda=\pm} = \omega^\mu \frac{\epsilon^*\cdot P}{\omega\cdot P}\,.$$

"no" addition contribution to $f(q^2)$ and $a_-(q^2)$.

Unphysical $\mathcal{F}$:

$$[h(q^2)]^{\lambda=0} = N_c \int \frac{dx\, d^2k'_\perp}{2(2\pi)^3}\frac{\chi'_P\,\chi''_V}{\bar{x}}\, 4\left[2(m'_1 - m_2)B_1^{(2)} - \frac{1}{D''_{V,\rm con}}\left([\hat{A}']B_1^{(2)} - 2B_3^{(3)}\right)\right]\frac{2M''}{\omega\cdot P}$$

$$[h(q^2)]^{\lambda=\pm} = N_c \int \frac{dx\, d^2k'_\perp}{2(2\pi)^3}\frac{\chi'_P\,\chi''_V}{\bar{x}}\, 4\left[2(m'_1 - m_2)B_1^{(2)} - \frac{1}{D''_{V,\rm con}}\left([\hat{A}']B_1^{(2)} - 2B_3^{(3)}\right)\right]\frac{\epsilon^*\cdot P}{\sqrt{2}\omega\cdot P}$$

- The decomposition for the case of $\lambda = \pm$ is in fact ambiguous.
  One can also decompose $\frac{P^\mu\epsilon^*\cdot\omega+\omega^\mu\epsilon^*\cdot P}{\omega\cdot P}\big|_{\lambda=\pm}$ in the same way of $\frac{P^\mu\epsilon^*\cdot\omega+\omega^\mu\epsilon^*\cdot P}{\omega\cdot P}\big|_{\lambda=0}$. At this moment, the self-consistency problem vanishes, which however is at the expense of introducing more unphysical form factors ($\varepsilon^{\mu\alpha\beta\nu}\omega_\alpha q_\beta P_\nu$).
  The self-consistency problem is not real, it is dependent on the way of decomposition.
  Such ambiguous decomposition is trivial only when the B function contributions are zero.

- **Covariance:**

$$h(q^2) \propto \int \mathrm{d}x \, \Delta^{f,a_-}_{\mathrm{full}}(x) \,,$$

type-I: $h(q^2) \neq 0$, covariance is violated;
type-II: $h(q^2) \doteq 0$, covariance is recovered.

- $h(q^2) \propto 1/P^+$: the violation of covariance is out of control (traditional type-I scheme).
- $\lambda = \pm$ is not alway a good choice to avoid the covariance problem

Thank you !

Why $M \to M_0$?

Manifest covariant formalism | LF vertex function | LF vertex operator ($\hat{\epsilon}$)

↓ problem is possibly caused by the SLF QM

Spin WF | "zero-binding-energy" limit with dressed "effective" quark

$\hat{\epsilon}$ v.s. ϵ (M v.s. M_0) | conservation of momentum | v.s. all are on mass-shell

momentum conservation → $p^2 = (k_1 + k_2)^2 = M_0^2$, meson mass is understood as

$$M^2 = \int \frac{\mathrm{d}x \mathrm{d}\mathbf{k}_\perp^2}{2(2\pi)^3} M_0^2 |\psi(x, \mathbf{k}_\perp)|^2$$

when a hadron mass appeared in a integral for a physical quantity expressed in terms of WF, M_0 should be used.

CLF QM is treated as a covariant expression for the SLF QM + z.m..