Amplitude analyses of charmed meson hadronic decays at BESIII

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Outline

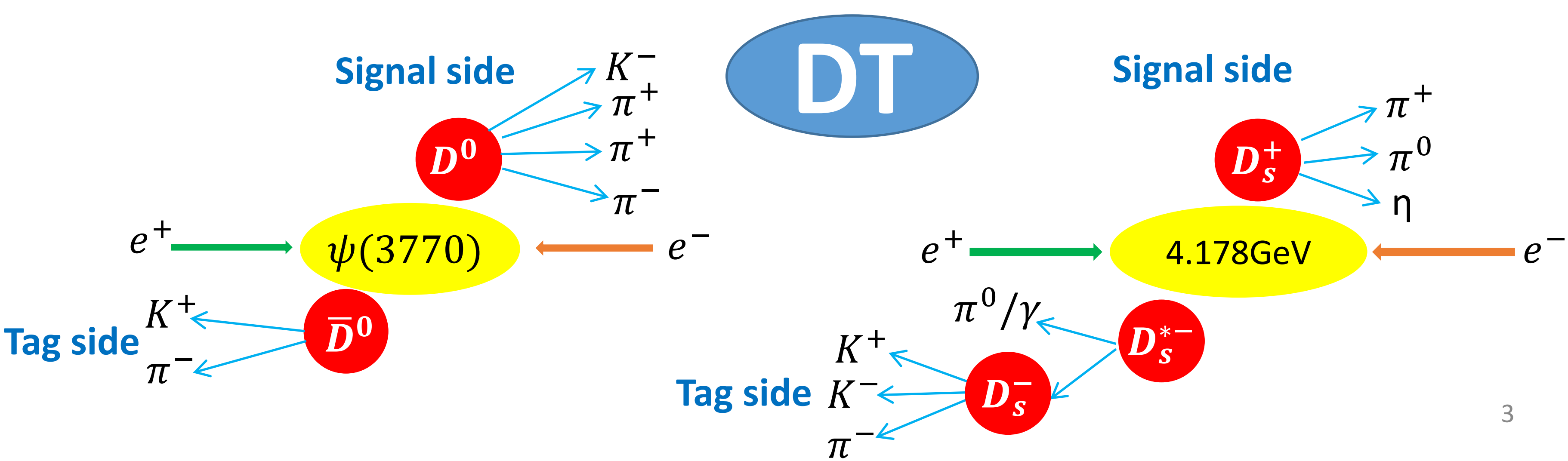
- Used dataset.
- Amplitude analysis.
- Recent results for $D_{(s)}$ hadronic decays
  1. Amplitude analysis of $D^0/D^+ \rightarrow K\pi\pi\pi$.
     - Amplitude analysis on strong phase measurement
  2. Amplitude analysis of $D_s^+ \rightarrow \pi^+\pi^0\eta$ and the first observation of $D_s^+ \rightarrow a_0(980)\pi$
- Summary

Used Dataset

- 2.93 fb^{-1} at Ecm = 3.773 GeV :
  ten millions of $D\overline{D}$ pairs (totally 21 M D^0 and 16 M D^+).
- 3.19 fb^{-1} at Ecm = 4.178 GeV :
  hundred thousands of $D_s\overline{D}_s^*$ pairs (totally 400K D_s).

Paired production allows two different ways:

- **Single Tag (ST):** reconstruct only one of the charmed meson.
- **Double Tag (DT):** reconstruct both of two charmed meson.
  - ➢ provides access to absolute branching fractions (BFs).
  - ➢ provides clean samples for amplitude analysis.



Amplitude analysis

- Based on RooFit:
  - Use RooFit framework to **construct the PDF**
  - Use RooMinuit to perform the unbinned likelihood fit
  - Use RooFitResult to deal with the fit results

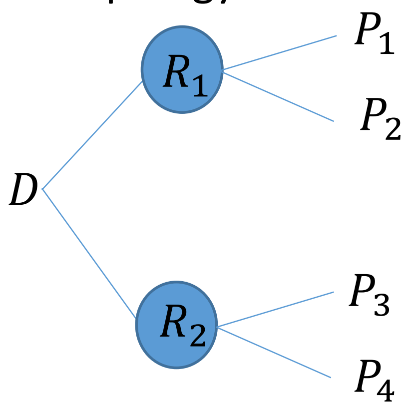
- Using GPU to accelerate the analysis:
  - C99 standard for programs in kernel
  - Connect the RooFit and GPU kernel with OpenCL and C++
  - Much faster than CPU version. (One or two weeks → less than three hours for a fit to data in Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$)

PDF construction

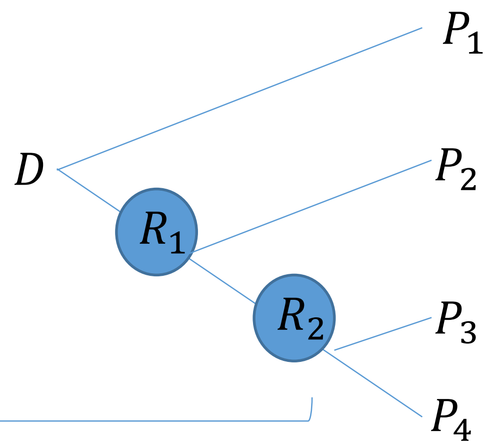
Isobar model is used in amplitude analysis:
the total amplitude $M(p_j)$ is the coherent sum of the sub-amplitudes $A_n(p_j)$,

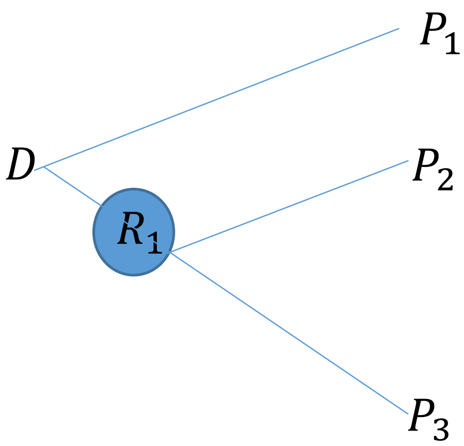
$$M(p_j) = \sum_n \rho_n e^{i\phi_n} A_n(p_j)$$

Topology A



Topology B





Four-body hadronic decays

$$A_n(p_j) = P_n^1(m_1) P_n^2(m_2) S_n(p_j) F_n^1(p_j) F_n^2(p_j) F_n^D(p_j)$$

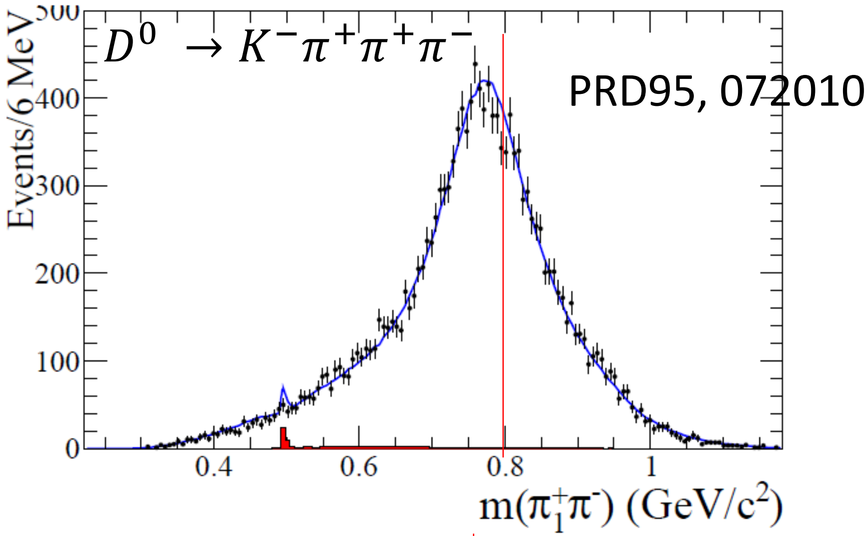
Three-body hadronic decays

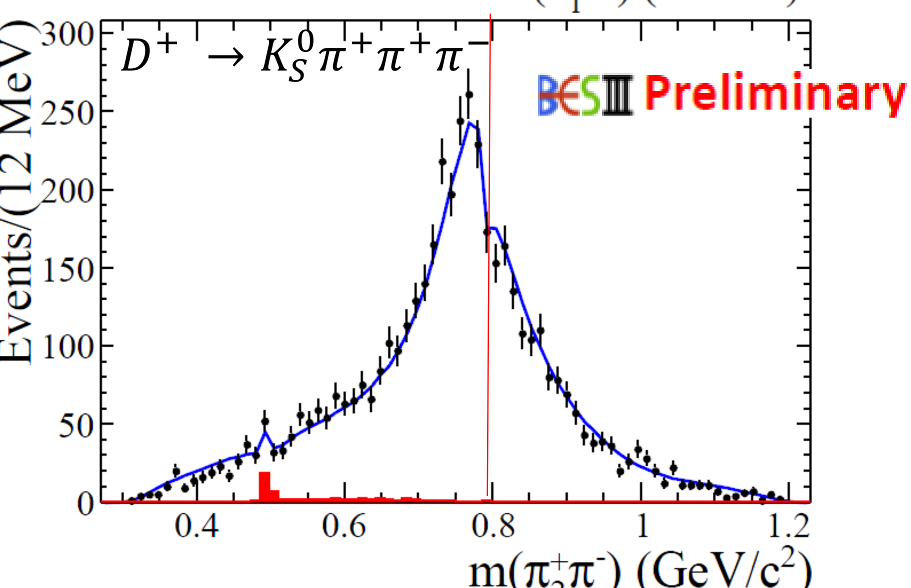
$$A_n(p_j) = P_n(m_R) S_n(p_j) F_n^R(p_j) F_n^D(p_j)$$

- $P_n(m_R)$: Propagator of intermediate resonance.
- $S_n(p_j)$: Angular terms, constructed with covariant tensor formalism (Zemach tensor).
- $F_n(p_j)$: Blatte-Weisskopf barriers

Propagator of intermediate resonance

- For the resonance ρ, the GS formula (PRL 21, 244) is considered. In the decay of $D^+ \to K_S^0\pi^+\pi^+\pi^-$, the obviously $\rho - \omega$ interference effect should be considered.





$$P_{\rho-\omega}(m) = P_\rho^{\mathrm{GS}}(m)(1 + a_r e^{i\phi_r} P_\omega^{\mathrm{RBW}}(m))$$

GS **RBW**

Relative magnitude and phase.

Propagator of intermediate resonance

For resonance σ ($f_0(500)$), the Bugg formula is used (PLB 572, 1):

$$P_{f_0(500)}(m) = \frac{1}{m_0^2 - m^2 - i m_0 \Gamma_{\rm tot}(m)},$$

where, $\Gamma_{tot}(m)$ is decomposed into two parts:

$$\Gamma_{\rm tot}(m) = g_1 \frac{\rho_{\pi\pi}(m)}{\rho_{\pi\pi}(m_0)} + g_2 \frac{\rho_{4\pi}(m)}{\rho_{4\pi}(m_0)},$$

and

$$g_1 = (b_1 + b_2 m^2) \frac{m^2 - m_\pi^2/2}{m_0^2 - m_\pi^2/2} e^{(m_0^2 - m^2)/a}.$$

$$\rho_{4\pi} = \sqrt{\left(1 - \frac{16 m_\pi^2}{m^2}\right)} / (1 + e^{\frac{2.8 - m^2}{3.5}})$$

All the parameters are fixed to Ref .PLB 598, 149.

For resonance $a_0(980)$ and $f_0(980)$, the flatte formula is used,

$$P_{a_0(f_0)}(m) = \frac{1}{m_0^2 - m^2 - i\left(\sum_j g_j^2 \rho_j\right)}.$$

Here, only two channels coupling ($\pi\eta$, KK for $a_0(980)$ and $\pi\pi$, KK for $f_0(980)$) is considered.

The parameters for $a_0(980)$ are fixed to Ref. PRD 95, 032002.

The parameters for $f_0(980)$ are fixed to Ref. PLB 607, 243.

- The $\rho_{\pi\pi}$ (ρ_j) above is the two-body phase space factor, $2q/m$, q is the magnitude of the daughter momentum in the resonance rest frame.
- For other resonances, the RBW is used.

Propagator of intermediate resonance

The $(K\pi)_{S-wave}$ parameterization:

The LASS model used in the analysis of $D^0 \to K_S^0\pi^+\pi^-$ of BABAR **(PRD 78, 034023)**,

$$A(m) = F sin\delta_F e^{i\delta_F} + R sin\delta_R e^{i\delta_R} e^{i2\delta_F},$$

Scattering term **$K_0^*(1430)$ term**

$$\delta_F = \phi_F + cot^{-1}\left(\frac{1}{aq} + \frac{rq}{2}\right),$$

$$\delta_R = \phi_R + tan^{-1}\left(\frac{M\Gamma(m_{K\pi})}{M^2 - m_{K\pi}^2}\right);$$

$M(\mathrm{GeV}/c^2)$	1.463 ± 0.002
$\Gamma(\mathrm{GeV}/c^2)$	0.233 ± 0.005
F	0.80 ± 0.09
ϕ_F	2.33 ± 0.13
R	$1(\mathrm{fixed})$
ϕ_R	-5.31 ± 0.04
a	1.07 ± 0.11
r	-1.8 ± 0.3

The parameters are fixed to BABAR results.

Angular terms

- The decays of $D^0 \to K^-\pi^+\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^0\pi^0$ allow the decay chains shown as topologies A and B.
- The decay of $D^+ \to K_S^0\pi^+\pi^+\pi^-$ only allows the decay chain of topology B since the contributions from doubly Cabibbo suppressed decays are negligible with our data statistic.

Decay mode	$S(p)$
$D[S] \to \mathrm{V_1 V_2},\ \mathrm{V_1 \to P_1 P_2},\ \mathrm{V_2 \to P_3 P_4}$	$\tilde{t}^{(1)\mu}(\mathrm{V_1})\tilde{t}^{(1)}_{\mu}(\mathrm{V_2})$
$D[P] \to \mathrm{V_1 V_2},\ \mathrm{V_1 \to P_1 P_2},\ \mathrm{V_2 \to P_3 P_4}$	$\epsilon_{\mu\nu\lambda\sigma}p^{\mu}(D)\tilde{T}^{(1)\nu}(D)\tilde{t}^{(1)\lambda}(\mathrm{V_1})\tilde{t}^{(1)\sigma}(\mathrm{V_2})$
$D[D] \to \mathrm{V_1 V_2},\ \mathrm{V_1 \to P_1 P_2},\ \mathrm{V_2 \to P_3 P_4}$	$\tilde{T}^{(2)\mu\nu}(D)\tilde{t}^{(1)}_{\mu}(\mathrm{V_1})\tilde{t}^{(1)}_{\nu}(\mathrm{V_2})$
$D \to \mathrm{AP_1},\ \mathrm{A}[S] \to \mathrm{VP_2},\ \mathrm{V \to P_3 P_4}$	$\tilde{T}^{\mu}_{1}(D)P^{(1)}_{\mu\nu}(\mathrm{A})\tilde{t}^{(1)\nu}(\mathrm{V})$
$D \to \mathrm{AP_1},\ \mathrm{A}[D] \to \mathrm{VP_2},\ \mathrm{V \to P_3 P_4}$	$\tilde{T}^{(1)\mu}(D)\tilde{t}^{(2)}_{\mu\nu}(\mathrm{A})\tilde{t}^{(1)\nu}(\mathrm{V})$
$D \to \mathrm{AP_1},\ \mathrm{A \to SP_2},\ \mathrm{S \to P_3 P_4}$	$\tilde{T}^{(1)\mu}(D)\tilde{t}^{(1)}_{\mu}(\mathrm{A})$
$D \to \mathrm{VS},\ \mathrm{V \to P_1 P_2},\ \mathrm{S \to P_3 P_4}$	$\tilde{T}^{(1)\mu}(D)\tilde{t}^{(1)}_{\mu}(\mathrm{V})$
$D \to \mathrm{V_1 P_1},\ \mathrm{V_1 \to V_2 P_2},\ \mathrm{V_2 \to P_3 P_4}$	$\epsilon_{\mu\nu\lambda\sigma}p^{\mu}_{\mathrm{V_1}}q^{\nu}_{\mathrm{V_1}}p^{\lambda}_{\mathrm{P_1}}q^{\sigma}_{\mathrm{V_2}}$
$D \to \mathrm{PP_1},\ \mathrm{P \to VP_2},\ \mathrm{V \to P_3 P_4}$	$p^{\mu}(\mathrm{P_2})\tilde{t}^{(1)}_{\mu}(\mathrm{V})$
$D \to \mathrm{TS},\ \mathrm{T \to P_1 P_2},\ \mathrm{S \to P_3 P_4}$	$\tilde{T}^{(2)\mu\nu}(D)\tilde{t}^{(2)}_{\mu\nu}(\mathrm{T})$

Four body

Three body

$$S_n = 1 \qquad (S\ wave),$$

$$S_n = \tilde{T}^{(1)\mu}(D_s)\tilde{t}^{(1)}_{\mu}(R) \quad (P\ wave),$$

$$S_n = \tilde{T}^{(2)\mu\nu}(D_s)\tilde{t}^{(2)}_{\mu\nu}(R) \quad (D\ wave),$$

- The determination of $\tilde{T}$ and $\tilde{t}$ are the same with **ref. EPJA 16, 537.**
- The amplitudes with angular momentum > 2 are not considered.

Likelihood construction

Independent with the floated parameters

The signal PDF $f(p_j)$ is given by:

$$f(p_j) = \frac{\epsilon(p_j)|M(p_j)|^2 d\phi_n(p_j)}{\int \epsilon(p_j)|M(p_j)|^2 d\phi_n(p_j)},$$

where, $\epsilon(p_j)$ is the efficiency function; $d\phi_n(p_j)$ is the n-body phase space. Then the likelihood is:

$$lnL = \sum_i^{N_{data}} w_i^{data} lnf(p_j) + \sum_i^{N_{bkg}} (-w_i^{bkg}) lnf(p_j),$$

Signal part

Background part

- Background sample can be simulated events or sideband events in data.
- The normalized integration is performed with MC integration method.

Fit Fraction and the statistical uncertainty

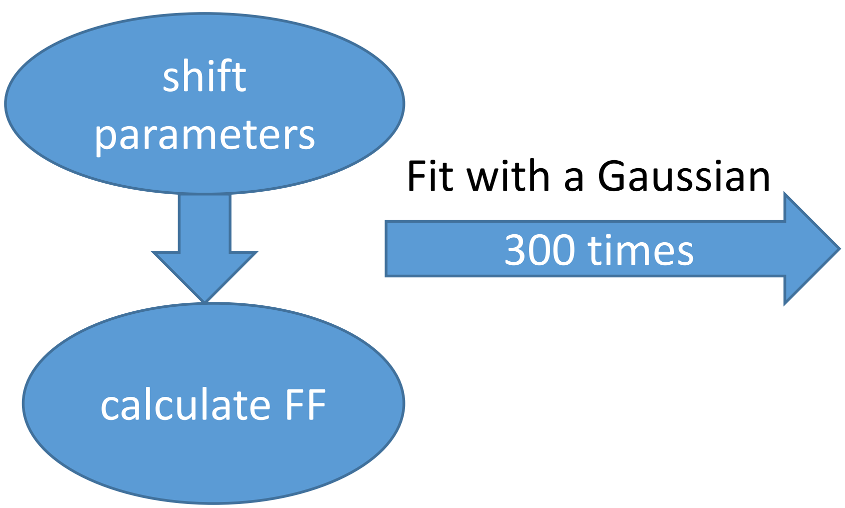
The fit fraction is calculated with MC integration method:

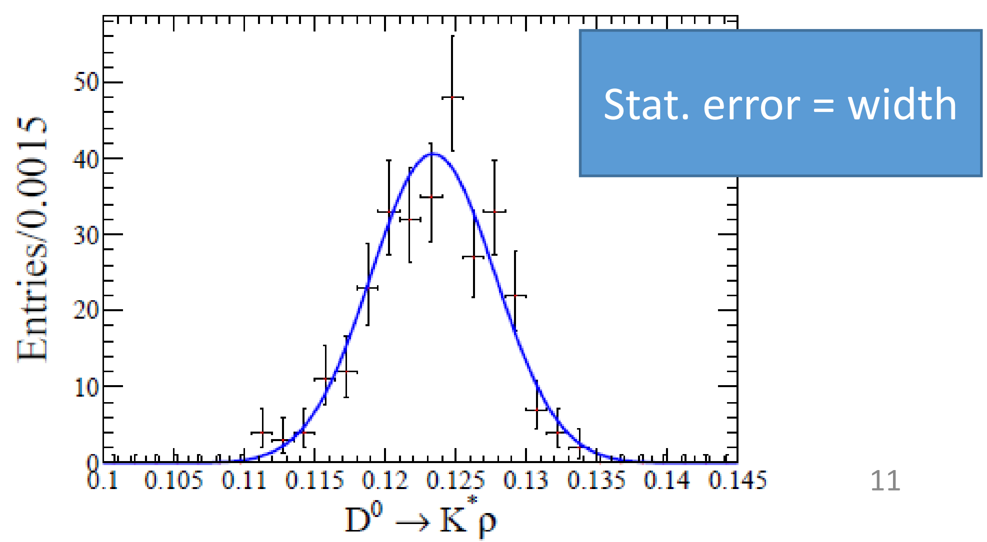
$$FF(n) = \frac{\sum_k^{N_{MC}} |\tilde{A}_{\mathbf{n}}(p_j)|^2}{\sum_k^{N_{MC}} |M(p_j)|^2},$$

- N_{MC} is the number of MC sample events.
- $\tilde{A}_{\mathbf{n}}(p_j)$ is either the $\mathbf{n}^{th}$ amplitude ($\tilde{A}_{\mathbf{n}}(p_j) = \rho_n e^{i\phi_n} A_n(p_j)$) or the $\mathbf{n}^{th}$ subset (component) of coherent sum of amplitudes ($\tilde{A}_{\mathbf{n}}(p_j) = \sum_{n_l} \rho_{n_l} e^{i\phi_{n_l}} A_{n_l}(p_j)$).

To obtain the statistical uncertainty, the fit results are randomly modified according to the covariance matrix of the fit result. Under the RooFit framework:

the function: randomizePars() returns the randomly shifted values.

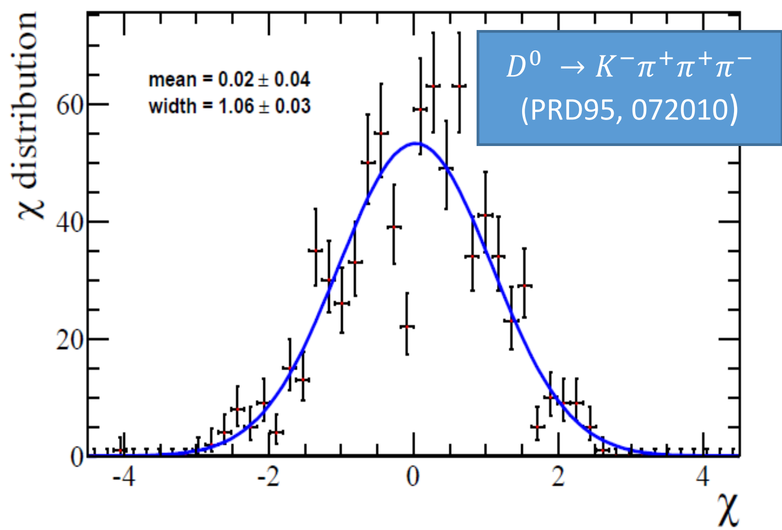




Fit quality

Binned method:

- The global χ^2 is calculated in a five-dimensional (two-dimensional) phase space for four-body (three-body) decays: $\chi^2 = \sum_p \chi_p$ and $\chi_p = \frac{(N_P - N_p^{\text{exp}})}{\sqrt{N_p^{\text{exp}}}}$.
- N_p and N_p^{exp} is the number of data and simulated events in p^{th} cell.
- Cells with expected events less than 20 will be merged with next cell until they satisfy the minimum number of events criterion.



Unbinned method:

A mixed-sample method is used to determine the fit quality of the $D^0 \to K^-\pi^+\pi^0\pi^0$ amplitude analysis results (M. Williams, Journal of Instrumentation 5, P09004 (2010)).

Recent results for $D_{(s)}$ hadronic decays

Amplitude analysis of $D^0/D^+ \to K\pi\pi\pi$:

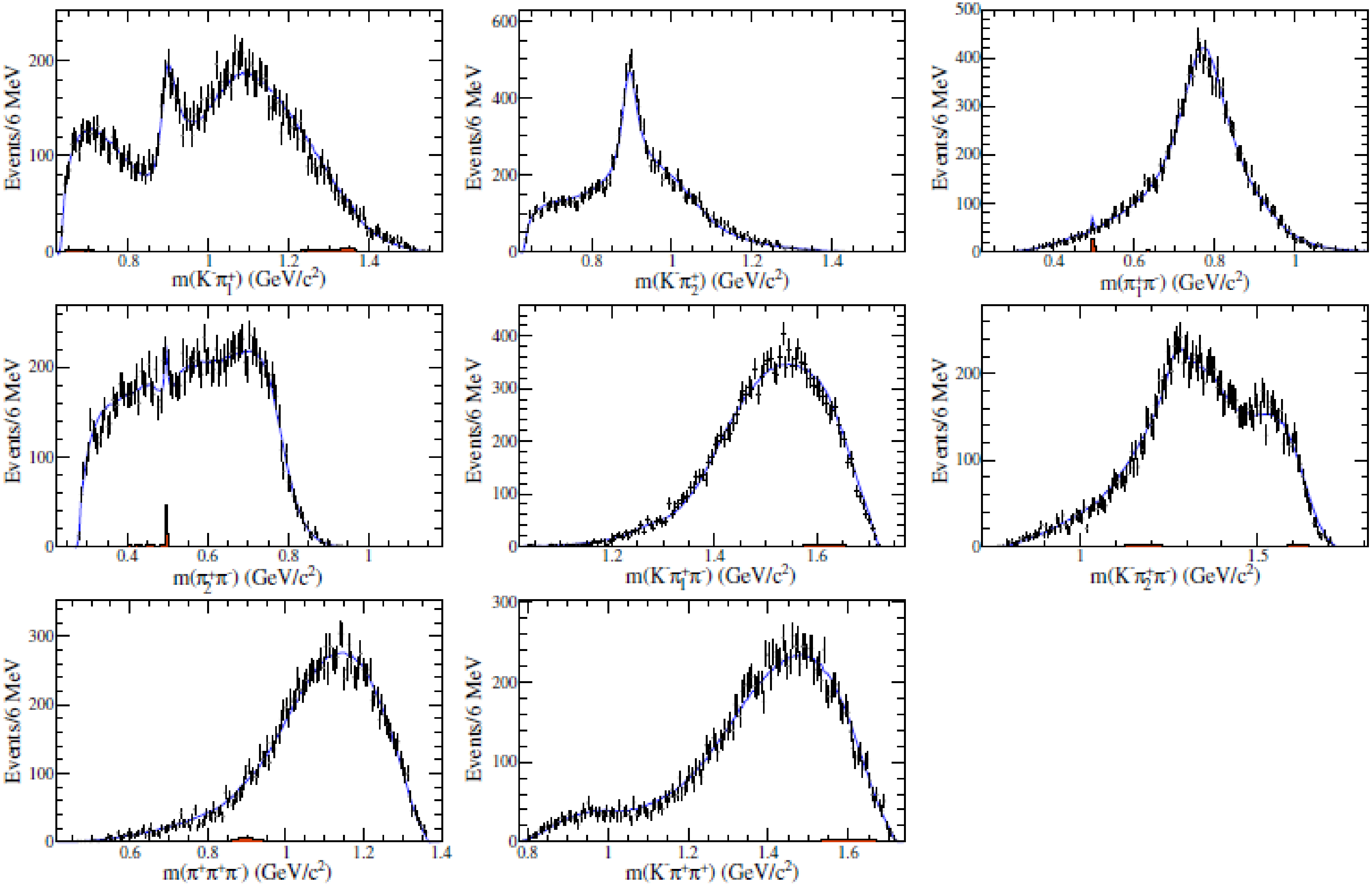
- These decays provide windows to investigate the decay modes $D \to VV$ and $D \to AP$ (P: pseudo-scalar, V: vector, A: axial-vector), which are important in searching for CPV and learning $D^0\overline{D}^0$mixing.
- The amplitude analysis results can also be used in many other experimental measurements:
  - Branching fraction measurement.
  - Strong phase determination (Only for D^0)
  - γ angle determination (Only for D^0)
- There are seven decays of $D^0/D^+ \to K\pi\pi\pi$, previous amplitude analyses for $D^0 \to K^-\pi^+\pi^+\pi^-$, $K_S^0\pi^+\pi^-\pi^0$, and $D^+ \to K_S^0\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^+\pi^0$ have been performed by Mark III and E691. Both measurements are affected by low statistics.
- The results of $D^0 \to K^-\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^0\pi^0$, and $D^+ \to K_S^0\pi^+\pi^+\pi^-$ are presented here.

Result of $D^0 \to K^-\pi^+\pi^+\pi^-$ (PRD95, 072010)

Double tag: $D^0 \to K^-\pi^+\pi^+\pi^-$ vs $\bar{D}^0 \to K^+\pi^-$ are reconstructed. About 16K events with a purity ~99.4% are selected for amplitude analysis. Model is constructed with 23 amplitudes.

Amplitude	ϕ_i	Fit fraction (%)
$D^0[S] \to \bar{K}^*\rho^0$	$2.35 \pm 0.06 \pm 0.18$	$6.5 \pm 0.5 \pm 0.8$
$D^0[P] \to \bar{K}^*\rho^0$	$-2.25 \pm 0.08 \pm 0.15$	$2.3 \pm 0.2 \pm 0.1$
$D^0[D] \to \bar{K}^*\rho^0$	$2.49 \pm 0.06 \pm 0.11$	$7.9 \pm 0.4 \pm 0.7$
$D^0 \to K^- a_1^+(1260),\ a_1^+(1260)[S] \to \rho^0\pi^+$	0(fixed)	$53.2 \pm 2.8 \pm 4.0$
$D^0 \to K^- a_1^+(1260),\ a_1^+(1260)[D] \to \rho^0\pi^+$	$-2.11 \pm 0.15 \pm 0.21$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270)[S] \to \bar{K}^{*0}\pi^-$	$1.48 \pm 0.21 \pm 0.24$	$0.1 \pm 0.1 \pm 0.1$
$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270)[D] \to \bar{K}^{*0}\pi^-$	$3.00 \pm 0.09 \pm 0.15$	$0.7 \pm 0.2 \pm 0.2$
$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270) \to K^-\rho^0$	$-2.46 \pm 0.06 \pm 0.21$	$3.4 \pm 0.3 \pm 0.5$
$D^0 \to (\rho^0 K^-)_{\rm A}\pi^+,\ (\rho^0 K^-)_{\rm A}[D] \to K^-\rho^0$	$-0.43 \pm 0.09 \pm 0.12$	$1.1 \pm 0.2 \pm 0.3$
$D^0 \to (K^-\rho^0)_{\rm P}\pi^+$	$-0.14 \pm 0.11 \pm 0.10$	$7.4 \pm 1.6 \pm 5.7$
$D^0 \to (K^-\pi^+)_{\text{S-wave}}\rho^0$	$-2.45 \pm 0.19 \pm 0.47$	$2.0 \pm 0.7 \pm 1.9$
$D^0 \to (K^-\rho^0)_{\rm V}\pi^+$	$-1.34 \pm 0.12 \pm 0.09$	$0.4 \pm 0.1 \pm 0.1$
$D^0 \to (\bar{K}^{*0}\pi^-)_{\rm P}\pi^+$	$-2.09 \pm 0.12 \pm 0.22$	$2.4 \pm 0.5 \pm 0.5$
$D^0 \to \bar{K}^{*0}(\pi^+\pi^-)_{\rm S}$	$-0.17 \pm 0.11 \pm 0.12$	$2.6 \pm 0.6 \pm 0.6$
$D^0 \to (\bar{K}^{*0}\pi^-)_{\rm V}\pi^+$	$-2.13 \pm 0.10 \pm 0.11$	$0.8 \pm 0.1 \pm 0.1$
$D^0 \to ((K^-\pi^+)_{\text{S-wave}}\pi^-)_{\rm A}\pi^+$	$-1.36 \pm 0.08 \pm 0.37$	$5.6 \pm 0.9 \pm 2.7$
$D^0 \to K^-((\pi^+\pi^-)_{\rm S}\pi^+)_{\rm A}$	$-2.23 \pm 0.08 \pm 0.22$	$13.1 \pm 1.9 \pm 2.2$
$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_{\rm S}$	$-1.40 \pm 0.04 \pm 0.22$	$16.3 \pm 0.5 \pm 0.6$
$D^0[S] \to (K^-\pi^+)_{\rm V}(\pi^+\pi^-)_{\rm V}$	$1.59 \pm 0.13 \pm 0.41$	$5.4 \pm 1.2 \pm 1.9$
$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_{\rm V}$	$-0.16 \pm 0.17 \pm 0.43$	$1.9 \pm 0.6 \pm 1.2$
$D^0 \to (K^-\pi^+)_{\rm V}(\pi^+\pi^-)_{\rm S}$	$2.58 \pm 0.08 \pm 0.25$	$2.9 \pm 0.5 \pm 1.7$
$D^0 \to (K^-\pi^+)_{\rm T}(\pi^+\pi^-)_{\rm S}$	$-2.92 \pm 0.14 \pm 0.12$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_{\rm T}$	$2.45 \pm 0.12 \pm 0.37$	$0.5 \pm 0.1 \pm 0.1$

Projections of $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ (PRD95, 072010)



Points with error bars: data, curves: fit, red histograms: background.

The two identical π^+ are required with: $m(\pi_1^+\pi^-) > m(\pi_2^+\pi^-)$

Branching fractions for different components (PRD95, 072010)

Component	Branching fraction (%)	PDG value (%)
$D^0 \to \bar{K}^{*0}\rho^0$	$0.99 \pm 0.04 \pm 0.04 \pm 0.03$	1.05 ± 0.23
$D^0 \to K^- a_1^+(1260)(\rho^0\pi^+)$	$4.41 \pm 0.22 \pm 0.30 \pm 0.13$	3.6 ± 0.6
$D^0 \to K_1^-(1270)(\bar{K}^{*0}\pi^-)\pi^+$	$0.07 \pm 0.01 \pm 0.02 \pm 0.00$	0.29 ± 0.03
$D^0 \to K_1^-(1270)(K^-\rho^0)\pi^+$	$0.27 \pm 0.02 \pm 0.04 \pm 0.01$	
$D^0 \to K^-\pi^+\rho^0$	$0.68 \pm 0.09 \pm 0.20 \pm 0.02$	0.51 ± 0.23
$D^0 \to \bar{K}^{*0}\pi^+\pi^-$	$0.57 \pm 0.03 \pm 0.04 \pm 0.02$	0.99 ± 0.23
$D^0 \to K^-\pi^+\pi^+\pi^-$	$1.77 \pm 0.05 \pm 0.04 \pm 0.05$	1.88 ± 0.26

Stat. uncertainty from FF

Sys. uncertainty from FF

uncertainties related to $B(D^0 \to K^-\pi^+\pi^+\pi^-)$ in PDG

Significantly improved than previous PDG values.

Amplitude analysis benefits the strong phase measurement

$$D^0 \rightarrow K_S^0 \pi^+ \pi^-$$

Modeled with the amplitude analysis result (by BABAR)

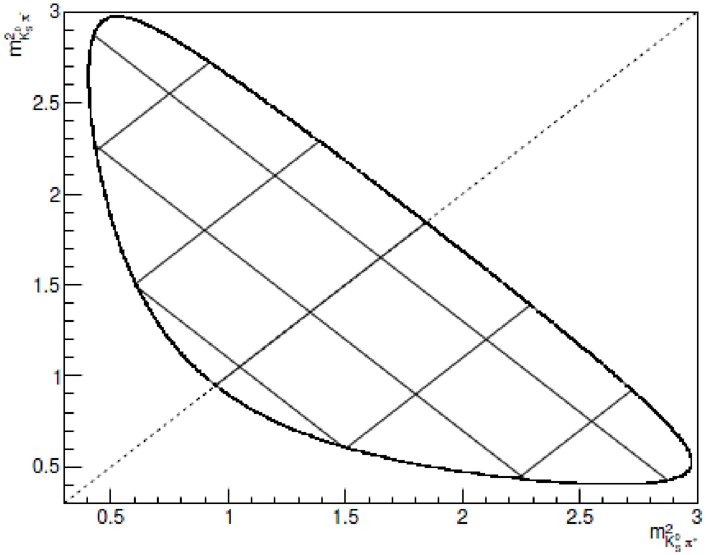


FIG. 2: Square binned Dalitz plot with symmetric bins over an exchange $m^2_{K^0_S\pi^+}$ and $m^2_{K^0_S\pi^-}$.

- **For instance, if we take the Belle's Dalitz result (PRD85, 112014 (2012)),** **γ/ϕ_3 (in degrees) = $77.3^{+15.1}_{-14.9}$ (stat.) ± 4.2 (syst.) ± 4.3 (c_i/s_i)** **With our preliminary result from the $D^0 \rightarrow K_S\pi^+\pi^-$,** **this would be → ± 2.4 (c_i/s_i)** **Very important inputs for the future analyses by LHCb and Belle II, where the statistical sensitivity starts to reach ~1~2 degrees.**

From Hajime's report on 2016 BESIII summer collaboration meeting

Amplitude analysis benefits the strong phase measurement

$D^0 \to K^-\pi^+\pi^+\pi^-$

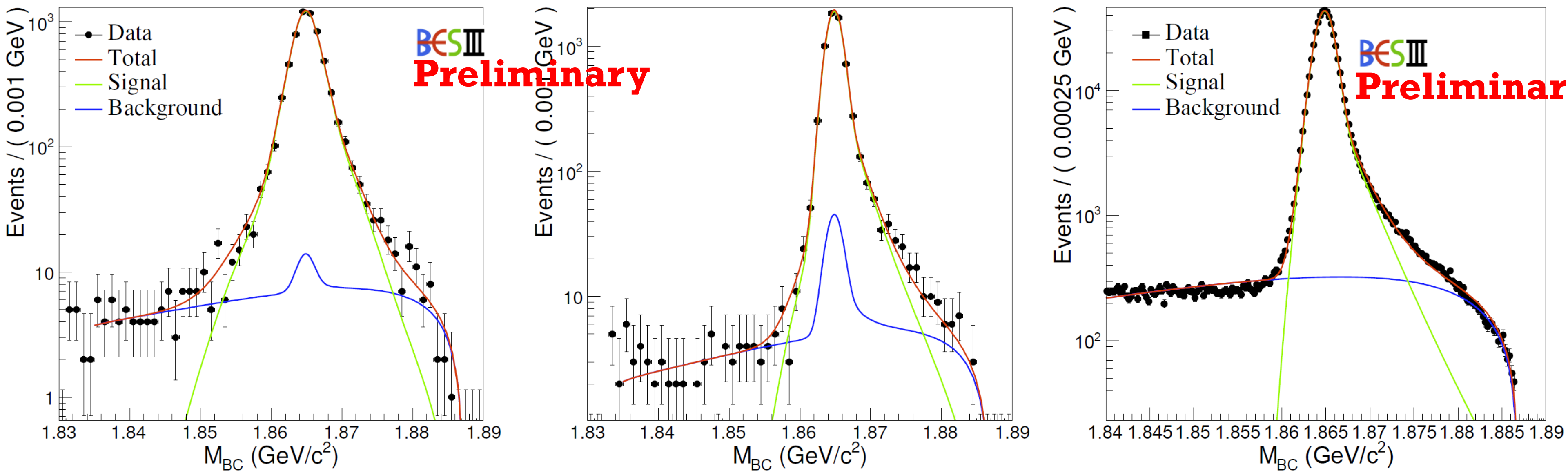
Only with previous CLEOc data:
$R_{K3\pi} = 0.32^{+0.20}_{-0.28}$ and $\delta_{K3\pi} = (255^{+21}_{-78})^\circ$ (PLB. 731, 197 (2014))
Combined with LHCb data (provide the time-dependent mixing rate $R(t)$):
$R_{K3\pi} = 0.43^{+0.17}_{-0.13}$ and $\delta_{K3\pi} = (128^{+28}_{-17})^\circ$ (PLB. 757, 520 (2016))

With the BESIII data, the statistic uncertainty can be suppressed by ~40%. With the amplitude analysis, the sys. uncertainties due to efficiency, BF, cross-feed, mis-combination, ... could be significantly suppressed.

Branching fractions for $D^0 \rightarrow K^-\pi^+\pi^0\pi^0$ (BESIII Preliminary)

Fits to $M_{\rm BC}$ distributions of DT and ST data:

Beam-Constrained Mass: $M_{\rm BC} = \sqrt{E_{\rm beam}^2 - |\vec{p}_D|^2}$ Signal: $M_{\rm BC}$ peaks at D mass



DT $(K^-\pi^+\pi^0\pi^0)$ DT $(K^+\pi^-)$ ST

Yields: DT, 6101 ± 83; ST, 534581 ± 769.

The amplitude analysis results are used to determine the reconstructed efficiency.

First measurement!

$$\mathcal{B}(D^0 \rightarrow K^-\pi^+\pi^0\pi^0) = (8.98 \pm 0.13(\text{stat}) \pm 0.40(\text{syst}))\%$$

BESIIIPreliminary

Result of $D^0 \to K^-\pi^+\pi^0\pi^0$ (BESIII Preliminary)

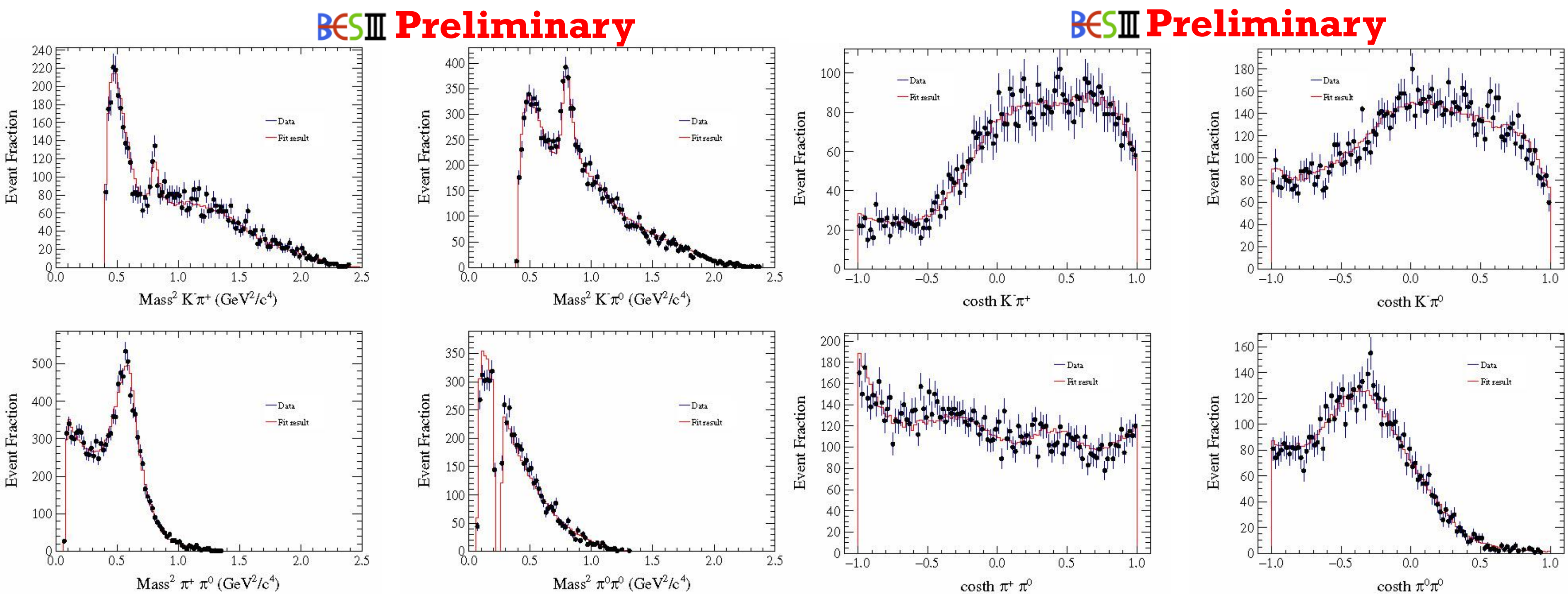
Double tag: $D^0 \to K^-\pi^+\pi^0\pi^0$ vs $\bar{D}^0 \to K^+\pi^-$ are reconstructed. About 5.95K events with a purity ~99% are selected for amplitude analysis. Model is constructed with 26 amplitudes.

BESIII Preliminary

Amplitude mode	FF(%)	Phase (ϕ)
$D \to SS$		
$D \to (K^-\pi^+)_{S\text{-wave}}(\pi^0\pi^0)_S$	$6.92 \pm 1.44 \pm 2.86$	$-0.75 \pm 0.15 \pm 0.47$
$D \to (K^-\pi^0)_{S\text{-wave}}(\pi^+\pi^0)_S$	$4.18 \pm 1.02 \pm 1.77$	$-2.90 \pm 0.19 \pm 0.47$
$D \to AP, A \to VP$		
$D \to K^-a_1(1260)^+, \rho^+\pi^0[S]$	$28.36 \pm 2.50 \pm 3.53$	0 (fixed)
$D \to K^-a_1(1260)^+, \rho^+\pi^0[D]$	$0.68 \pm 0.29 \pm 0.30$	$-2.05 \pm 0.17 \pm 0.25$
$D \to K_1(1270)^-\pi^+, K^{*-}\pi^0[S]$	$0.15 \pm 0.09 \pm 0.18$	$1.84 \pm 0.34 \pm 0.43$
$D \to K_1(1270)^0\pi^0, K^{*0}\pi^0[S]$	$0.39 \pm 0.18 \pm 0.30$	$-1.55 \pm 0.20 \pm 0.26$
$D \to K_1(1270)^0\pi^0, K^{*0}\pi^0[D]$	$0.11 \pm 0.11 \pm 0.13$	$-1.35 \pm 0.43 \pm 0.48$
$D \to K_1(1270)^0\pi^0, K^-\rho^+[S]$	$2.71 \pm 0.38 \pm 0.29$	$-2.07 \pm 0.09 \pm 0.20$
$D \to (K^{*-}\pi^0)_A\pi^+, K^{*-}\pi^0[S]$	$1.85 \pm 0.62 \pm 1.11$	$1.93 \pm 0.10 \pm 0.15$
$D \to (K^{*0}\pi^0)_A\pi^0, K^{*0}\pi^0[S]$	$3.13 \pm 0.45 \pm 0.58$	$0.44 \pm 0.12 \pm 0.21$
$D \to (K^{*0}\pi^0)_A\pi^0, K^{*0}\pi^0[D]$	$0.46 \pm 0.17 \pm 0.29$	$-1.84 \pm 0.26 \pm 0.42$
$D \to (\rho^+K^-)_A\pi^0, K^-\rho^+[D]$	$0.75 \pm 0.40 \pm 0.60$	$0.64 \pm 0.36 \pm 0.53$
$D \to AP, A \to SP$		
$D \to ((K^-\pi^+)_{S\text{-wave}}\pi^0)_A\pi^0$	$1.99 \pm 1.08 \pm 1.55$	$-0.02 \pm 0.25 \pm 0.53$
$D \to VS$		
$D \to (K^-\pi^0)_{S\text{-wave}}\rho^+$	$14.63 \pm 1.70 \pm 2.41$	$-2.39 \pm 0.11 \pm 0.35$
$D \to K^{*-}(\pi^+\pi^0)_S$	$0.80 \pm 0.38 \pm 0.26$	$1.59 \pm 0.19 \pm 0.24$
$D \to K^{*0}(\pi^0\pi^0)_S$	$0.12 \pm 0.27 \pm 0.27$	$1.45 \pm 0.48 \pm 0.51$
$D \to VP, V \to VP$		
$D \to (K^{*-}\pi^+)_V\pi^0$	$2.25 \pm 0.43 \pm 0.45$	$0.52 \pm 0.12 \pm 0.17$
$D \to VV$		
$D[S] \to K^{*-}\rho^+$	$5.15 \pm 0.75 \pm 1.28$	$1.24 \pm 0.11 \pm 0.23$
$D[P] \to K^{*-}\rho^+$	$3.25 \pm 0.55 \pm 0.41$	$-2.89 \pm 0.10 \pm 0.18$
$D[D] \to K^{*-}\rho^+$	$10.90 \pm 1.53 \pm 2.36$	$2.41 \pm 0.08 \pm 0.16$
$D[P] \to (K^-\pi^0)_V\rho^+$	$0.36 \pm 0.19 \pm 0.27$	$-0.94 \pm 0.19 \pm 0.28$
$D[D] \to (K^-\pi^0)_V\rho^+$	$2.13 \pm 0.56 \pm 0.92$	$-1.93 \pm 0.22 \pm 0.25$
$D[D] \to K^{*-}(\pi^+\pi^0)_V$	$1.66 \pm 0.52 \pm 0.61$	$-1.17 \pm 0.20 \pm 0.39$
$D[S] \to (K^-\pi^0)_V(\pi^+\pi^0)_V$	$5.17 \pm 1.91 \pm 1.82$	$-1.74 \pm 0.20 \pm 0.31$
$D \to TS$		
$D \to (K^-\pi^+)_{S\text{-wave}}(\pi^0\pi^0)_T$	$0.30 \pm 0.21 \pm 0.32$	$-2.93 \pm 0.31 \pm 0.82$
$D \to (K^-\pi^0)_{S\text{-wave}}(\pi^+\pi^0)_T$	$0.14 \pm 0.12 \pm 0.10$	$2.23 \pm 0.38 \pm 0.65$

BESIII Preliminary

Projections of $D^0 \to K^-\pi^+\pi^0\pi^0$ (BESIII Preliminary)



Points with error bars: data, red histograms: fit.

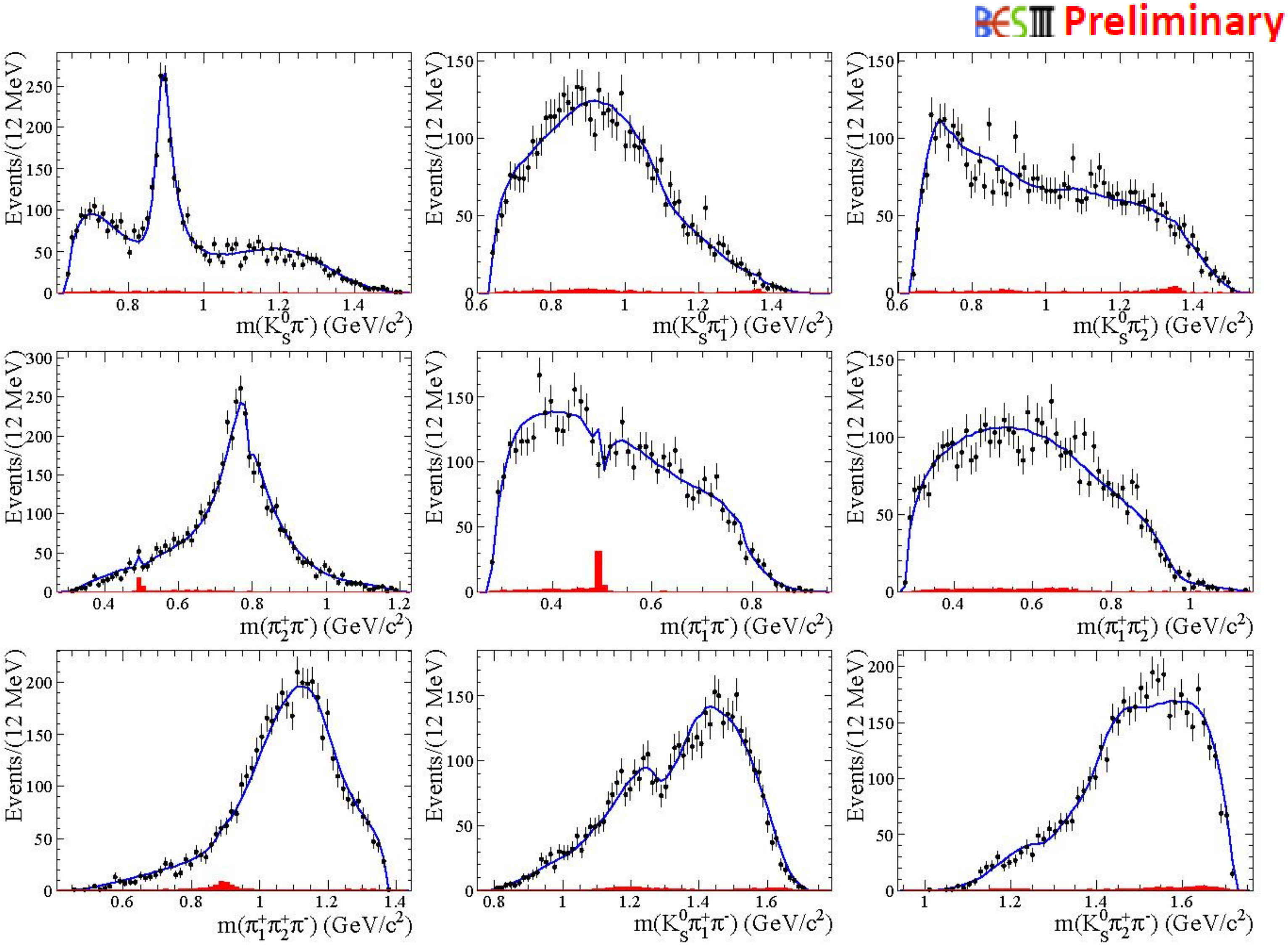
Result of $D^+ \to K_S^0\pi^+\pi^+\pi^-$ (BESIII Preliminary)

Double tag: $D^+ \to K_S^0\pi^+\pi^+\pi^-$ vs $D^- \to K^+\pi^-\pi^-$ are reconstructed. About 4.56K events with a purity ~99% are selected for amplitude analysis. Model is constructed with 12 amplitudes.

BESIII **Preliminary**

Amplitude	ϕ	Fit fraction
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to \rho^0\pi^+[S]$	0.000(fixed)	$0.567 \pm 0.020 \pm 0.044$
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to f_0(500)\pi^+$	$-2.023 \pm 0.068 \pm 0.113$	$0.050 \pm 0.006 \pm 0.007$
$D^+ \to \bar{K}_1(1400)^0\pi^+, \bar{K}_1(1400)^0 \to K^{*-}\pi^+[S]$	$-2.714 \pm 0.038 \pm 0.051$	$0.380 \pm 0.013 \pm 0.014$
$D^+ \to \bar{K}_1(1400)^0\pi^+, \bar{K}_1(1400)^0 \to K^{*-}\pi^+[D]$	$3.431 \pm 0.137 \pm 0.117$	$0.015 \pm 0.004 \pm 0.005$
$D^+ \to \bar{K}_1(1270)^0\pi^+, \bar{K}_1(1270)^0 \to K_S^0\rho^0[S]$	$-0.418 \pm 0.070 \pm 0.087$	$0.036 \pm 0.004 \pm 0.002$
$D^+ \to \bar{K}(1460)^0\pi^+, \bar{K}(1460)^0 \to K_S^0\rho^0$	$-1.850 \pm 0.120 \pm 0.223$	$0.014 \pm 0.004 \pm 0.003$
$D^+ \to (K_S^0\rho^0)_A[D]\pi^+$	$2.328 \pm 0.097 \pm 0.068$	$0.011 \pm 0.003 \pm 0.002$
$D^+ \to K_S^0(\rho^0\pi^+)_P$	$1.656 \pm 0.083 \pm 0.056$	$0.031 \pm 0.004 \pm 0.010$
$D^+ \to (K^{*-}\pi^+)_A[S]\pi^+$	$1.962 \pm 0.047 \pm 0.073$	$0.132 \pm 0.011 \pm 0.011$
$D^+ \to (K^{*-}\pi^+)_A[D]\pi^+$	$0.989 \pm 0.158 \pm 0.229$	$0.013 \pm 0.004 \pm 0.004$
$D^+ \to (K_S^0(\pi^+\pi^-)_S)_A\pi^+$	$-2.935 \pm 0.060 \pm 0.125$	$0.051 \pm 0.004 \pm 0.003$
$D^+ \to ((K_S^0\pi^-)_S\pi^+)_P\pi^+$	$1.864 \pm 0.069 \pm 0.288$	$0.022 \pm 0.003 \pm 0.003$

Projections of $D^+ \to K_S^0\pi^+\pi^+\pi^-$ (BESIII Preliminary)



Points with error bars: data, red histograms: fit, green histograms: background estimated from MC.

The two identical π^+ are required with: $m(\pi_1^+\pi^-) < m(\pi_2^+\pi^-)$.

Branching fractions for different components (BESIII Preliminary)

BESIII Preliminary

Component	Branching fraction (%)
$D^+ \to K_S^0 a_1(1260)^+(\rho^0\pi^+)$	$1.684 \pm 0.059 \pm 0.131 \pm 0.062$
$D^+ \to K_S^0 a_1(1260)^+(f_0(500)\pi^+)$	$0.149 \pm 0.018 \pm 0.021 \pm 0.006$
$D^+ \to \bar{K}_1(1400)^0(K^{*-}\pi^+)\pi^+$	$1.105 \pm 0.045 \pm 0.048 \pm 0.041$
$D^+ \to \bar{K}_1(1270)^0(K_S^0\rho^0)\pi^+$	$0.107 \pm 0.012 \pm 0.006 \pm 0.004$
$D^+ \to \bar{K}(1460)^0(K_S^0\rho^0)\pi^+$	$0.042 \pm 0.012 \pm 0.009 \pm 0.002$
$D^+ \to K_S^0\pi^+\rho^0$	$0.131 \pm 0.015 \pm 0.015 \pm 0.005$
$D^+ \to K^{*-}\pi^+\pi^+$	$0.413 \pm 0.036 \pm 0.059 \pm 0.015$
$D^+ \to K_S^0\pi^+\pi^+\pi^-$	$0.220 \pm 0.015 \pm 0.024 \pm 0.008$

Stat. uncertainty from FF

Sys. uncertainty from FF

uncertainties related to $B(D^+ \to K_S^0\pi^+\pi^+\pi^-)$ in PDG

Amplitude analysis of $D_s^+ \to \pi^+\pi^0\eta$:

- Extract the branching fraction of the W-annihilation process involved decay $D_s^+ \to \rho^+\eta$.

- Improve the precise of $B(D_s^+ \to \pi^+\pi^0\eta)$.
  - A Cabibbo favored decay with large branching fraction but poor precision (PDG: $(9.2 \pm 1.2)\%$).

- Search for the pure W-annihilation decay $D_s^+ \to a_0(980)\pi$.

Result of $D_s^+ \to \pi^+\pi^0\eta$ (BESIII Preliminary)

Events are selected with double tag:

- Tag modes: $D_s^- \to K_S^0K^-$, $D_s^- \to K^+K^-\pi^-$, $D_s^- \to K_S^0K^-\pi^0$, $D_s^- \to K^+K^-\pi^-\pi^0$, $D_s^- \to K_S^0K^+\pi^-\pi^-$, $D_s^- \to \pi^-\eta_{\gamma\gamma}$, and $D_s^- \to \pi^-\eta'_{\pi^+\pi^-\eta}$.
- Signal mode: $D_s^+ \to \pi^+\pi^0\eta$.
- Multi-variate analysis is performed to suppress the background from fake η.

1239 events are selected with a purity of $(97.7 \pm 0.5)\%$ for amplitude analysis.

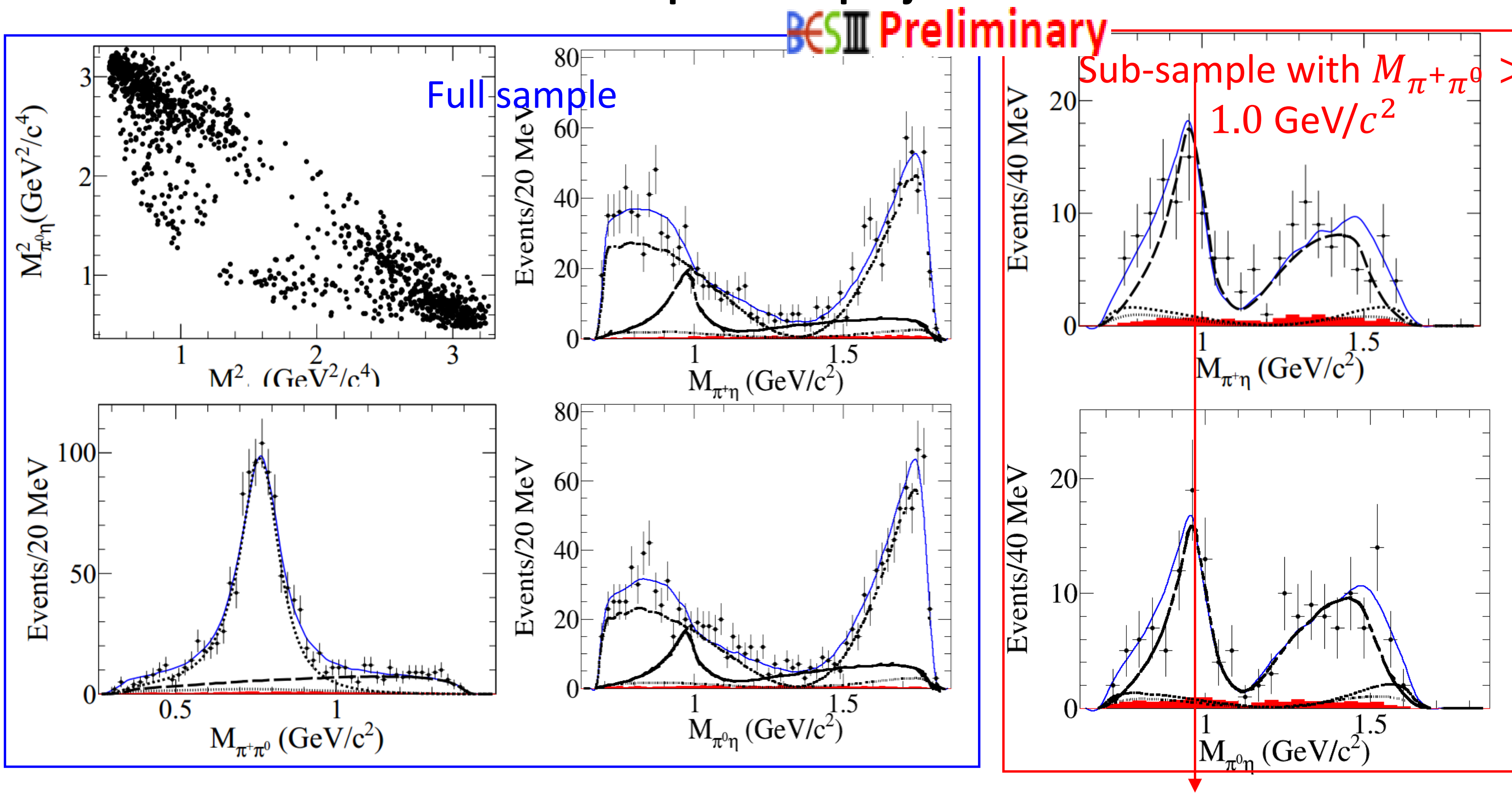
The significances, phases, and FFs for intermediate processes.

Amplitude	Significance (σ)	Phase BESIII Preliminary	FF
$D_s^+ \to \rho^+\eta$	> 20	0.0 (fixed)	$0.783 \pm 0.050 \pm 0.021$
$D_s^+ \to (\pi^+\pi^0)_V\eta$	5.7	$0.612 \pm 0.172 \pm 0.342$	$0.054 \pm 0.021 \pm 0.026$
$\boldsymbol{D_s^+ \to a_0(980)\pi}$	$\mathbf{16.2}$	$\mathbf{2.794 \pm 0.087 \pm 0.041}$	$\mathbf{0.232 \pm 0.023 \pm 0.034}$

The amplitude analysis agrees with the isospin conserved expectation ($A(D_s^+ \to$

Projections of $D_s^+ \to \pi^+\pi^0\eta$ (BESIII Preliminary)

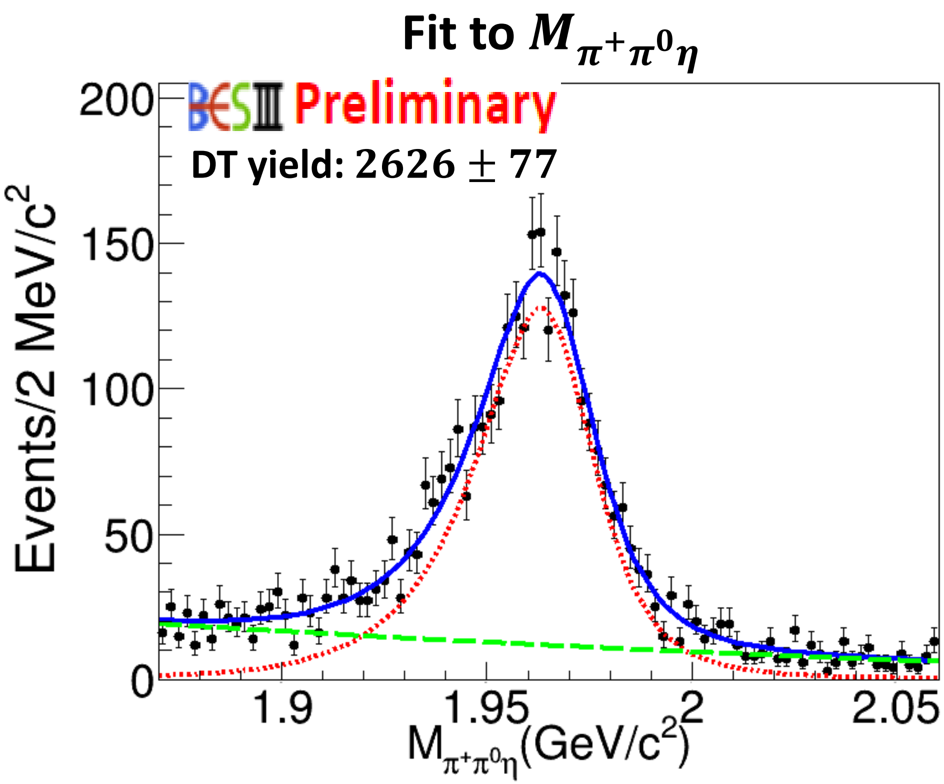
Dalitz plot and projections



Dots with error bar: data; solid: total fit; dashed: $D_s^+ \to \rho^+\eta$; dotted: $D_s^+ \to (\pi^+\pi^0)_V\eta$; long dashed: $D_s^+ \to a_0(980)\pi$.

Obvious peaks for two $a_0(980)$ mesons!

Branching fraction measurement (BESIII Preliminary)



- Dots with error bars: data.
- Total fit.
- Signal: MC shape convoluted with a Gaussian.
- Background: second-order Chebychev.

Efficiency is determined with the amplitude analysis result.

$BF(D_s^+ \to \pi^+\pi^0\eta) = (9.50 \pm 0.28_{stat.} \pm 0.41_{sys.})\%$ BESIII Preliminary

The systematic uncertainty on $BF(D_s^+ \to \pi^+\pi^0\eta)$ is dominated by π^0 and η reconstruction (4%).

Branching fraction measurement (BESIII Preliminary)

For the n^{th} amplitude, the BF can be calculated with:

$$BF(n) = BF(D_s^+ \to \pi^+\pi^0\eta)FF(n)$$

Branching fraction	This measurement (%)	PDG value (%)
$BF(D_s^+ \to \rho^+\eta)$	$7.44 \pm 0.48_{stat.} \pm 0.44_{sys.}$	8.9 ± 0.9
$BF(D_s^+ \to a_0(980)\pi)$ *	$2.20 \pm 0.22_{stat.} \pm 0.34_{sys.}$	-
$BF(D_s^+ \to a_0(980)^+\pi^0)$ * $BF(D_s^+ \to a_0(980)^0\pi^+)$ *	$1.46 \pm 0.15_{stat.} \pm 0.22_{sys.}$	-

***Here, $a_0(980) \to \pi\eta$.**

- The $BF(D_s^+ \to a_0(980)^{+/0}\pi^{0/+})$ is larger than the PDG values of $BF(D_s^+ \to \omega\pi^+)$ ($(2.4 \pm 0.6) \times 10^{-3}$) and $BF(D_s^+ \to p\bar{n})$ ($(1.3 \pm 0.4) \times 10^{-3}$) by one order of magnitude.
- The magnitude for the ratio of the W-annihilation amplitude over tree-emission amplitude $|A/T|$ is obtained to be 0.84$\pm$0.23 in the decay mode of $D \to SP$, which is much larger than the level of 0.1-0.2 in the decay mode of $D \to VP$ (PRD 93, 114010).
- The contribution from W-annihilation processes should be taken into considered in investigating the $D \to SP$ decays.
- This provides theoretical challenge in understanding such a large W-annihilation contribution in $D \to SP$ decays.

Summary

- BESIII provides large data samples close to charm related threshold to study the $D_{(s)}$ multi-body hadronic decays.
- Amplitude analysis is a powerful method to investigate the multi-body decays and to extract the physical information.
- Three amplitude analysis results for $D \to K\pi\pi\pi$ and one for D_s^+ decay are presented.
- Amplitude analyses for $D^0 \to K^-\pi^+\pi^0\pi^0$ and $D_s^+ \to \pi^+\pi^0\eta$ are firstly performed.
  - With the amplitude analysis result, the $BF(D^0 \to K^-\pi^+\pi^0\pi^0)$ is firstly measured to be $(8.98 \pm 0.13_{stat.} \pm 0.40_{sys.})\%$.
  - The precision of $BF(D_s^+ \to \pi^+\pi^0\eta)$ is improved with a factor of 2.5 to the PDG values.
  - The decays $BF(D_s^+ \to a_0(980)^+\pi^0)$ and $BF(D_s^+ \to a_0(980)^0\pi^+)$ via $a_0(980) \to \pi\eta$ are firstly observed and measured to be $(1.46 \pm 0.15_{stat.} \pm 0.22_{sys.})\%$
- More results for $D_{(s)}$ multi-body hadronic decays are coming.

Thank you!