Mixing and CP-Violation in Charm

Where We Are & Where We Are Going

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Why Study Charm Mixing and CPV

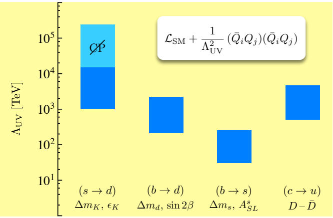
What should you remember tomorrow, or a year from now?

- Flavor physics, generically, allows searches for manifestations of **New Physics at the highest energy scales** by studying rare and forbidden decays and and searching for CP violation beyond that described by the Kobayashi-Maskawa phase of the CKM matrix.
  - ***CP* violation in** D^0, K^0, B_d and B_s **mixing** provide complementary sensitivities to BSM physics;
  - LHCb is collecting fully reconstructed charm samples **$100\times$ to $1000\times$** larger than previous experiments, and expects to collect another $10\times$ to $50\times$ more in Run 3;
  - We are already probing mass scales **higher** than can be searched for directly at the LHC.
- **Direct CPV** provides complementary insights related to new amplitudes.
- **Combining time-dependent amplitude analyses with measurements of correlated decays at $D\overline{D}$ threshold**, including amplitude analyses, will lead to significantly improved precision by 2025.

Flavor Constrains BSM Physics

Operator	Bounds on Λ in TeV ($c_{\rm NP}=1$)		Bounds on $c_{\rm NP}$ ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert_D$, ϕ_D
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	
$(\bar{b}_L\gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	Δm_{B_d}; $\sin(2\beta)$ from $B_d\to\psi K$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	
$(\bar{b}_L\gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $\sin(\phi_s)$ from $B_s\to\psi\phi$
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	

Flavor Structure in the SM and Beyond



Generic bounds without a flavor symmetry

$$\Delta\mathcal{L}^{\Delta F=2} = \sum_{i\neq j}\frac{c_{ij}}{\Lambda^2}(\overline{Q}_{Li}\gamma^\mu Q_{Lj})^2 \ ,$$

- Table above from Isidori and Teubert, Eur.Phys.J.Plus **129**, 40 (2014). Bounds on representative dimension-six $\Delta F = 2$ operators.
- Image to the left from M. Neubert, EPS-HEP-2011.

Direct CP Violation

adapted from Khodjamirian and Petrov, PLB 774 (2017) 235 - 242

Observables sensitive to CP-violation are most often written in terms of asymmetries

$$a_{\mathrm{CP}}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}, \qquad (1)$$

formed from the partial rates of a D-meson decay to a final state f and of its CP-conjugated counterpart. $\cdots$ the asymmetry in Eq. (1) could be a function of time, if $D^0\overline{D}^0$-mixing is taken into account. The measured time-integrated asymmetry contains a *direct* component, [which] occurs when the absolute values of the $D \to f$ decay amplitude, which we denote by $A_f \equiv A(D \to f)$, and of the corresponding CP-conjugated amplitude $\overline{A}_{\overline{f}} \equiv A(\overline{D} \to \overline{f})$ are different. This can be realized if the decay amplitude A_f can be separated into at least two different parts,

$$A_f = A_f^{(1)} e^{i\delta_1} e^{i\phi_1} + A_f^{(2)} e^{i\delta_2} e^{i\phi_2}, \qquad (2)$$

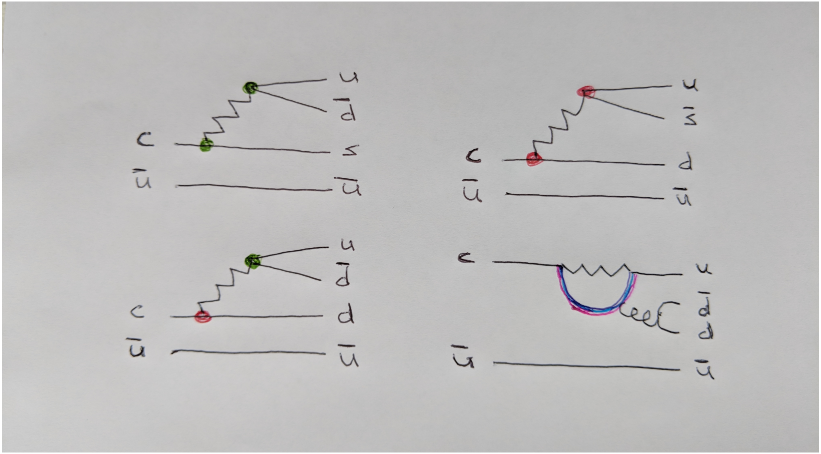
where $\phi_1 \neq \phi_2$ are the weak phases (odd under CP), and $\delta_1 \neq \delta_2$ are the strong phases (even under CP). The CP-violating asymmetry is then given by

$$a_{\mathrm{CP}}^{\mathrm{dir}}(f) \propto \frac{A_f^{(1)}}{A_f^{(2)}} \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2). \qquad (3)$$

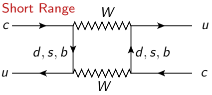
The amplitude pattern of Eq. (2) naturally emerges in SCS nonleptonic decays such as $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$. [as penguin amplitudes augment tree amplitudes]

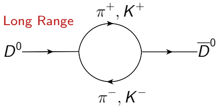
Tree Amplitudes and Penguin Amplitudes

Strong and Weak Phases



Neutral Meson Oscillation and *CP* Violation in Mixing

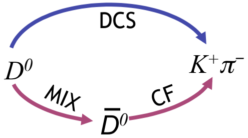




$$|P_{1,2}\rangle = p|P^0\rangle \pm q|\overline{P}^0\rangle \, ; \qquad p^2 + q^2 = 1$$

$$x \equiv \frac{\Delta m}{\Gamma} \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

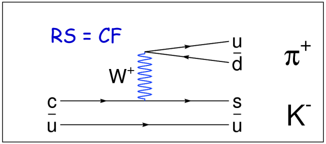
$$|\mathcal{M}|^2 \propto \frac{1}{2} e^{-\Gamma t} \Big\{ |\mathcal{A}_\alpha|^2 \left(\cosh y\,\Gamma t + \cos x\,\Gamma t \right)$$
$$+ |\overline{\mathcal{A}}_\alpha|^2 \left|\frac{q}{p}\right|^2 \left(\cosh y\,\Gamma t - \cos x\,\Gamma t \right)$$
$$+ 2 \left[\Re\left(\left(\frac{q}{p}\right)^* \mathcal{A}_\alpha \overline{\mathcal{A}}^*_\alpha \right) \sinh y\,\Gamma t - \Im\left(\left(\frac{q}{p}\right)^* \mathcal{A}_\alpha \overline{\mathcal{A}}^*_\alpha \right) \sin x\,\Gamma t \right] \Big\} \, .$$

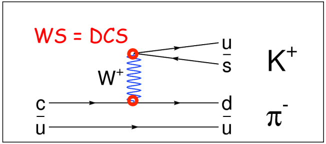


for $x, y \ll 1$ (valid for D^0, not for B_s):

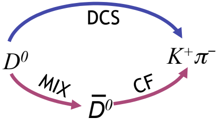
- **doubly Cabibbo-Suppressed (DCS) $\approx \propto e^{-\Gamma t}$;**
- **pure mixing $\propto e^{-\Gamma t} \times (\Gamma t)^2$**
- **interference $\approx \propto e^{-\Gamma t} \times \Gamma t$**

Time Evolution of $D^0 \to K\pi$





DCS and mixing amplitudes interfere to give a "quadratic" WS decay rate (x, y << 1):



$$\frac{\Gamma_{WS}(t)}{e^{-t/\tau}} \propto R_D + \sqrt{R_D}\, y' \left(\frac{t}{\tau}\right) + \left(\frac{x'^2 + y'^2}{4}\right)\left(\frac{t}{\tau}\right)^2$$

where $x' = x\cos\delta + y\sin\delta$ $\quad y' = y\cos\delta - x\sin\delta$

and δ is the phase difference between DCS and CF decays.

m_i, Γ_i ⇔ weak eigenstates ; $\quad x \equiv \dfrac{\Delta m}{\langle\Gamma\rangle}$; $\quad y \equiv \dfrac{\Delta m}{2\langle\Gamma\rangle}$; $\quad \tau \equiv \dfrac{1}{\langle\Gamma\rangle}$

CPV in Mixing

$$\langle D^0|H|\overline{D^0}\rangle = M_{12} - \frac{i}{2}\Gamma_{12}\ ; \quad \langle\overline{D^0}|H|D^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*\,,$$

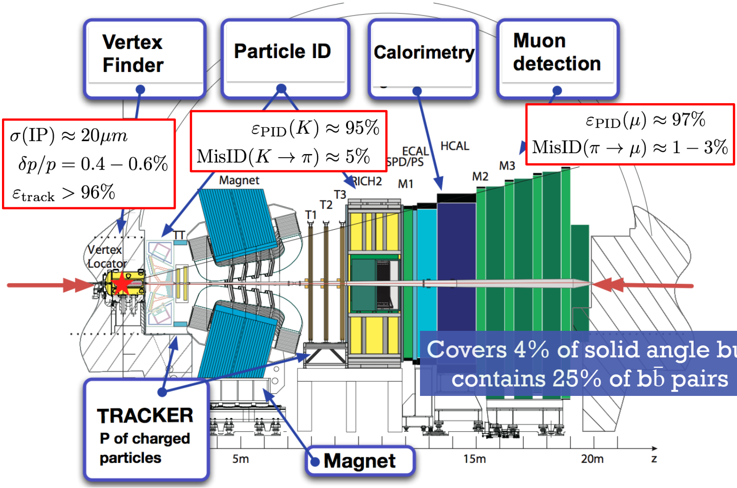
$$\frac{q}{p} = \frac{-2\,(M_{12}^* - \frac{1}{2}\Gamma_{12}^*)}{\Gamma\,(x - iy)}\ ; \quad \lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} = -\left|\frac{q}{p}\right| R_f\, e^{i(\phi+\Delta_f)} \quad \left(\rightarrow -\eta_f^{CP}\left|\frac{q}{p}\right| e^{i\phi}\right)$$

$$\left|\langle f|H\left|\overline{D}^{\,0}(t)\,\rangle\right|^2 \approx \frac{e^{-\Gamma t}}{2}\,|\mathcal{A}_f|^2\Big\{R_D + \left|\frac{p}{q}\right|\sqrt{R_D}\left[y\cos(\delta+\varphi) - x\sin(\delta+\varphi)\right](\Gamma t) + \left|\frac{p}{q}\right|^2\frac{x^2+y^2}{4}(\Gamma t)^2\Big\}$$

$$\left|\langle\bar{f}|H\left|D^0(t)\,\rangle\right|^2 \approx \frac{e^{-\Gamma t}}{2}\,|\overline{\mathcal{A}}_{\bar{f}}|^2\Big\{\overline{R}_D + \left|\frac{q}{p}\right|\sqrt{\overline{R}_D}\left[y\cos(\delta-\varphi) - x\sin(\delta-\varphi)\right](\Gamma t) + \left|\frac{q}{p}\right|^2\frac{x^2+y^2}{4}(\Gamma t)^2\Big\}\,.$$

no direct CPV $+\,x,\ y\ \ll 1\ \rightarrow$ $\tan\varphi \approx \left(1 - \left|\frac{q}{p}\right|\right)\frac{x}{y}$ $\left[|M_{12}|,\ |\Gamma_{12}|,\ \arg\left(\frac{\Gamma_{12}}{M_{12}}\right)\ \rightarrow\ x,\ y,\ \left|\frac{q}{p}\right|,\ \arg\left(\frac{q}{p}\right)\right]$

LHCb Detector [2008 JINST 3 S08005]



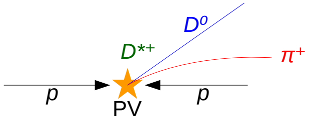
Covers 4% of solid angle but contains 25% of $b\bar{b}$ pairs

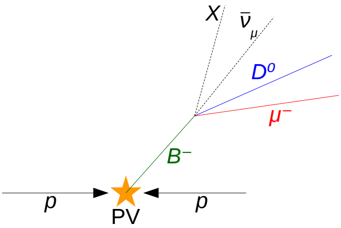
Production and tagging asymmetries

At LHCb, we use 2 independent tagging methods :

Prompt
$D^{*+} \to D^0 \pi^+$

Semileptonic
$B^- \to D^0 \mu^- \overline{\nu}_\mu X$





Some LHCb Measurements of Direct CPV

- Most precise measurements to date
  - Based on Run 1 data
  - Updated analyses with Run 2 data under way

$$A_{CP}(D^0 \to K^+K^-) = (0.4 \pm 1.2 \pm 1.0) \times 10^{-3}$$ [Phys. Lett. B 767 (2017), 177-187]

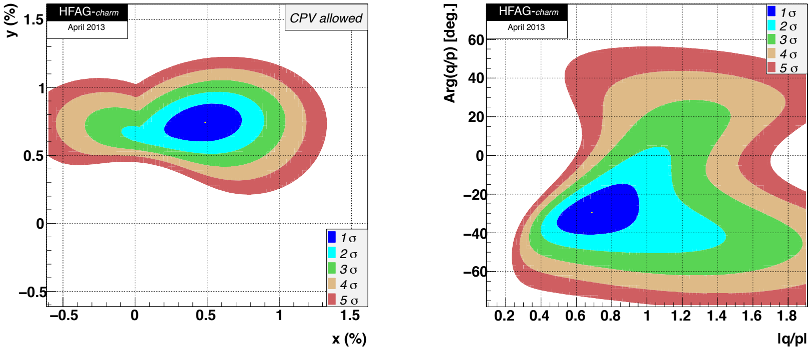
$$A_{CP}(D^0 \to \pi^+\pi^-) = (0.7 \pm 1.4 \pm 1.1) \times 10^{-3}$$ [Phys. Lett. B 767 (2017), 177-187]

$$\Delta A_{CP}(D^0 \to h^+h^-) = (1.0 \pm 0.8 \pm 0.3) \times 10^{-3}$$ [Phys. Rev. Lett. 116, 191601 (2016)]

- ΔA_{CP} measured first; then $A_{CP}(KK)$; then $A_{CP}(\pi\pi)$ extracted;
- systematic errors for ΔA_{CP} are smaller than for either channel alone;
- statistical errors are also smaller – tighter cuts were used to extract the absolute $A_{CP}(KK)$.

→ **next up – a historical context, and then on to the latest time-dependent results.**

Mixing + CPV: Context and History



The interpretation of experimental results often depends on prior knowledge and impact on underlying physics parameters.

These plots illustrate the status of charm mixing/CPV results compiled by the Heavy Flavor Averaging Group, **circa April 2013** (**before** LHCb's first $K\pi$ mixing + CPV results were announced [PRL 111 (2013) 251801].

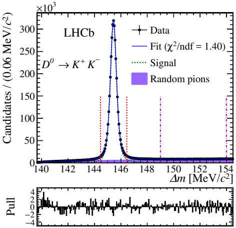
A_Γ with $D^0 \to hh$ decays

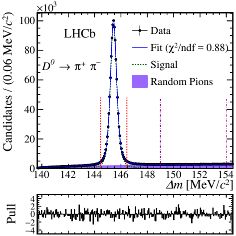
Phys. Rev. Lett 118, 261803 (2017)

$$A_{CP}(h^+h^-;t) \approx A_{CP}^{\rm dir}(h^+h^-) + A_\Gamma(h^+h^-)\left(\frac{t}{\tau}\right) + \left[< \mathcal{O}(10^{-6})\left(\frac{t}{\tau}\right)^2\right]$$

$$A_{CP}^{\rm dir}(h^+h^-) \equiv A_{CP}(t=0) = \frac{\left|\mathcal{A}(D^0\to h^+h^-)\right|^2 - \left|\mathcal{A}(\bar{D}^0\to h^+h^-)\right|^2}{\left|\mathcal{A}(D^0\to h^+h^-)\right|^2 + \left|\mathcal{A}(\bar{D}^0\to h^+h^-)\right|^2},$$

$$A_\Gamma(h^+h^-) = \frac{\eta_{CP}}{2}\left[y\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\varphi - x\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\varphi\right],$$





Dataset

- 9.0 M $D \to K^-K^+$ & 3.0 M $D \to \pi^-\pi^+$ from 3 fb^{-1} of Run 1 data (collected 2011-2012)
- prompt $D^{*+} \to D^-\pi^+$ + cc
- cut on $m(K\pi)$; study Δm
- combinatorial background is sideband-subtracted
- asymetry is measured in decay time intervals spanning $[0.6,\ 20]\ \tau(D^0)$.

A_Γ with $D^0 \to hh$ decays: Experimental Challenges

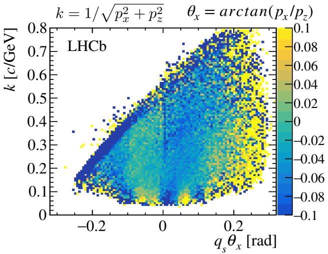
Phys. Rev. Lett 118, 261803 (2017)

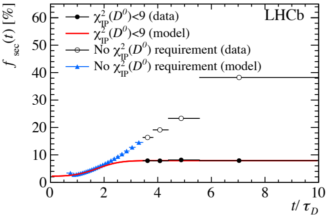
Instrumental Asymmetries

- **Soft pion charge reconstruction asymmetry**
  Time dependent correction due to correlation between soft pion kinematics and D^0 decay time
- **Reweighed the soft pion kinematic to recover left-right asymmetry of the detector**
  Validated on $D^0 \to K^- \pi^+$ decays

D^0 from B decays (Secondaries)

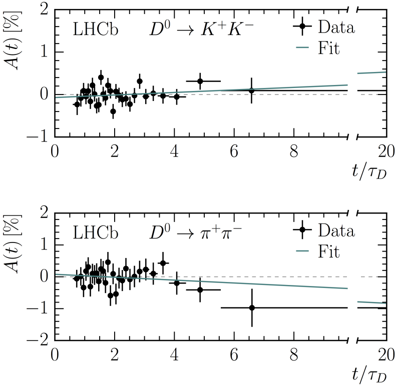
- **Undetected B decays mimic a larger D^0 decay time**
  Dilutes the asymmetry
- **Applied requirement of the D^0 pointing to PV**
  Residual background from B decays estimated with a model calibrated by the yield of secondaries at higher decay time





A_Γ with $D^0 \to hh$ decays: Results

Phys. Rev. Lett 118, 261803 (2017) + JHEP 04 (2015) 043



The data are consistent with hypothesis that *CP* symmetry is exact (in this measurement) at the level of 3×10^{-4}.

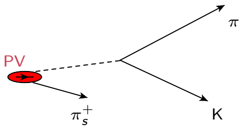
- $A_\Gamma(KK) = (-3.0 \pm 3.2 \pm 1.0) \times 10^{-4}$
- $A_\Gamma(\pi\pi) = (-4.6 \pm 5.8 \pm 1.2) \times 10^{-4}$

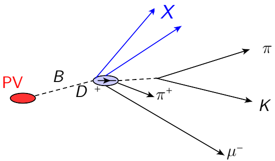
A complementary analysis of the same data using per-event acceptance calculations produces compatible results.

Combining these results with those from a statisically independent sample ($B \to D^0\mu^- X$)

- $A_\Gamma = (-2.9 \pm 2.8) \times 10^{-4}$

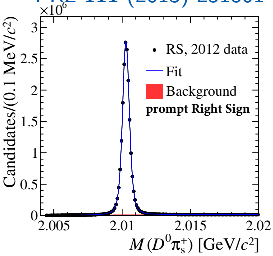
$D^0 \to K\pi$ Samples: Prompt and Doubly-Tagged (DT)



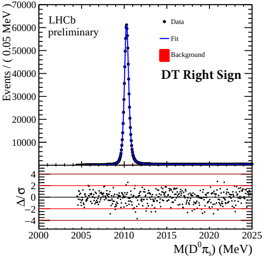


- prompt signal trigger becomes "fully" efficient well above one lifetime;
- doubly-tagged trigger is $\sim$ independent of D^0 decay time;

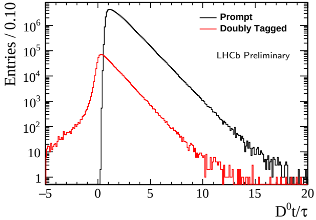
PRL **111** (2013) 251801



PRD 95, 052004 (2017)



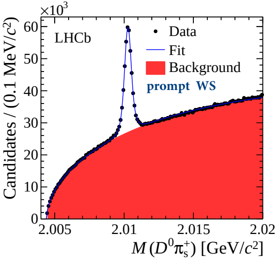
PRD 95, 052004 (2017)

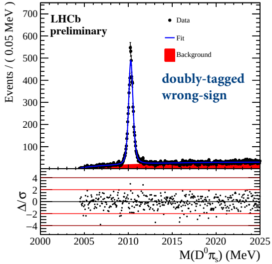


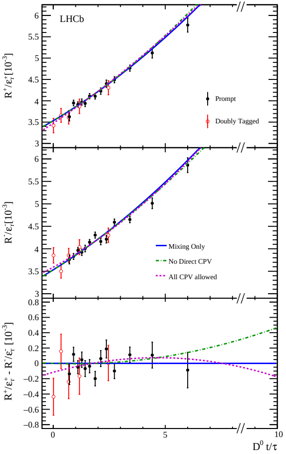
$D^0 \to K\pi$ Mixing and CPV Measurements

$$R^{\pm}(t) = \frac{\mathrm{WS}(t)}{\mathrm{RS}(t)} = R_D^{\pm} + \sqrt{R_D^{\pm}}\, y' \left(\frac{t}{\tau}\right) + \left(\frac{x'^{\pm 2} + y'^{\pm 2}}{4}\right)\left(\frac{t}{\tau}\right)^2$$

PRL **111** (2013) 251801; PRD 95, 052004 (2017)







- $\sim$ 54 M prompt RS, $\sim$ 1.7 M DT RS;
- $\sim$ 230 K WS, $\sim$ 6 K WS DT;
- **D^0, $\overline{D}^0$ mixing rates are equal, $\pm$5%.**
- adding DT sample [$\mathcal{O}(3\%)$] improves precision by $(10 - 20)\%$.

$D^0 \to K\pi$ Mixing and CPV Measurements

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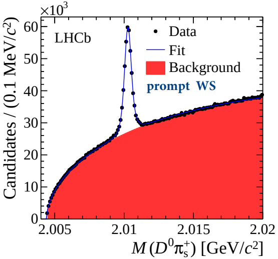
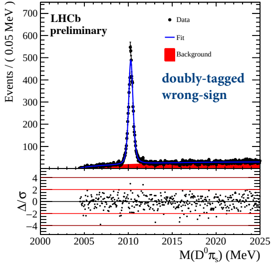
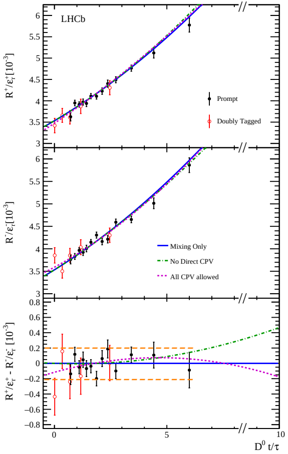
PRL **111** (2013) 251801; PRD 95, 052004 (2017)







- $\sim$ 54 M prompt RS, $\sim$ 1.7 M DT RS;
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- **D^0, $\overline{D}^0$ mixing rates are equal, $\pm 5\%$.**
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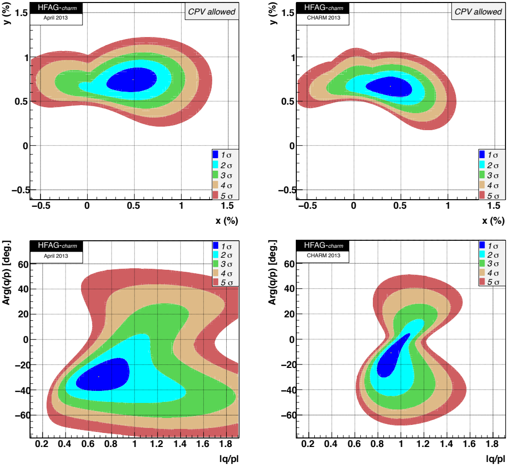
Impact: Run 1 $K\pi$ Mixing + CPV Measurement

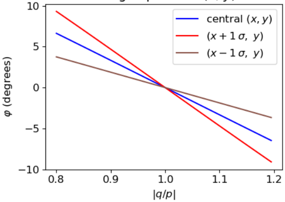
[PRL 111 (2013) 251801]



HFLAV World Averages

April 2013	w/o constraint	with constraint
$\lvert q/p \rvert$	$0.69^{+0.17}_{-0.14}$	$1.04^{+0.07}_{-0.06}$
$\varphi\,(^\circ)$	$-29.6^{+8.9}_{-7.5}$	$-1.6^{+2.4}_{-2.5}$
Sept 2013		
$\lvert q/p \rvert$	$0.91^{+0.11}_{-0.09}$	$1.008^{+0.014}_{-0.014}$
$\varphi\,(^\circ)$	$-10.8^{+10.5}_{-12.3}$	$-0.3^{+0.5}_{-0.6}$

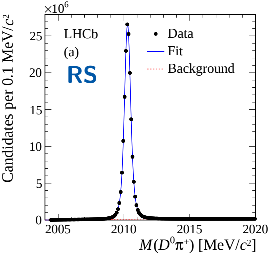
$\tan\varphi = (1 - \lvert q/p \rvert)(x/y)$
using Sept. 2013 (x, y)

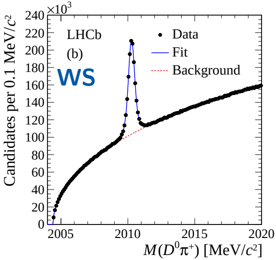


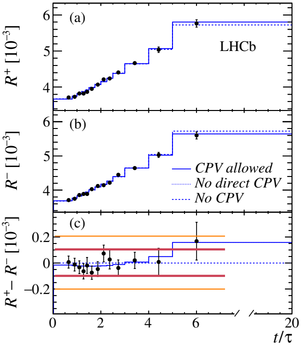
$D^0 \to K\pi$ Mixing and CPV Measurements – 2018 Update

$$R^{\pm}(t) = \frac{\mathrm{WS}(t)}{\mathrm{RS}(t)} = R_D^{\pm} + \sqrt{R_D^{\pm}}\, y' \left(\frac{t}{\tau}\right) + \left(\frac{x'^{\pm 2} + y'^{\pm 2}}{4}\right) \left(\frac{t}{\tau}\right)^2$$

PRD 97, 031101 (2018)



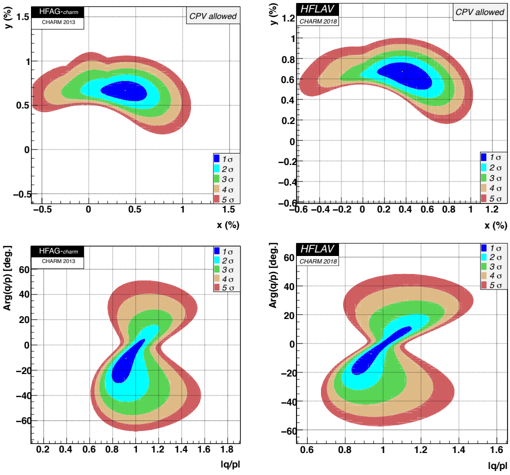




- $3\ \mathrm{fb}^{-1}$ @ 7/8 TeV $+$ $2\ \mathrm{fb}^{-1}$ @ 13 TeV;
- $\sim$ 177 M prompt RS, (was $\sim$ 54 M);
- $\sim$ 720 K prompt WS, (was $\sim$ 230 K);
- fit range extended to $t/\tau = 20$

Impact: 5 fb^{-1} $K\pi$ Mixing + CPV Measurement

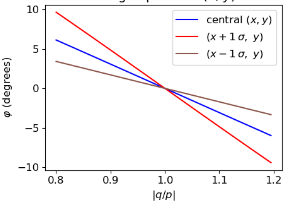
[PRD 97 (2018) 031101]



HFLAV World Averages

Sept 2013	w/o constraint	with constraint
$\lvert q/p \rvert$	$0.91\,^{+0.11}_{-0.09}$	$1.008\,^{+0.014}_{-0.014}$
$\varphi\,(^\circ)$	$-10.8\,^{+10.5}_{-12.3}$	$-0.3\,^{+0.5}_{-0.6}$
May 2018		
$\lvert q/p \rvert$	$0.94\,^{+0.17}_{-0.07}$	$0.998\,^{+0.007}_{-0.008}$
$\varphi\,(^\circ)$	$-7.2\,^{+14.7}_{-9.6}$	$0.09\,^{+0.32}_{-0.32}$

$\tan\varphi = (1 - |q/p|)\,(x/y)$
using Sept. 2013 (x, y)



Time-Dependent Amplitude Analysis of $D \to K\pi\pi\pi$

[Prospects: Dominik Muller, CERN-THESIS-2017-257]

In the decays $D \to K^{\mp}\pi^{\mp}\pi^{\pm}\pi^{\pm}$, the strong phase depends on the position in phase space.

- The variation of the strong phase can be determined by doing an amplitude analysis of the CF (DCS) decays. [See, for example, LHCb, Eur.Phys.J. C78 (2018) no.6, 443.]
- The phase corresponding to δ in the equation below [derived for $D \to K\pi$] depends on both the strong phase variation across phase space of the CF and DCS amplitudes **and** an overall phase difference that cannot be determined from uncorrelated decays.
- Because $y'' = y\cos(\delta+\varphi) - x\sin(\delta+\varphi)$ varies across phase space, mixing measurements become **linearly sensitive to both x and y**.

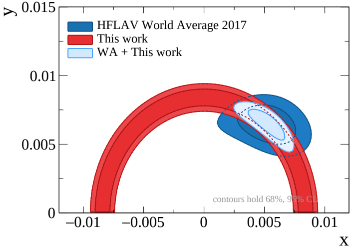
$$\left|\langle f|H\left|\overline{D}^{0}(t)\rangle\right.\right|^2 \approx \frac{e^{-\Gamma t}}{2}|\mathcal{A}_f|^2\left\{R_D + \left|\frac{p}{q}\right|\sqrt{R_D}\left[y\cos(\delta+\varphi) - x\sin(\delta+\varphi)\right](\Gamma t) + \left|\frac{p}{q}\right|^2\frac{x^2+y^2}{4}(\Gamma t)^2\right\}$$

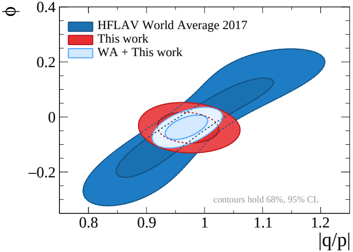
Time-Dependent Amplitude Analysis of $D \to K\pi\pi\pi$

[Prospects: Dominik Muller, CERN-THESIS-2017-257]

Physics reach studied by simulating the LHCb 2015/2016 data set:

- The numbers of right- and wrong-sign decays, and the S:B ratios, are taken from the data, for which the integrated luminosity is $\sim 2/\mathrm{fb}$.
- The acceptance for the WS sample is determined from the signal in the RS sample and the RS model extracted from Run1 data [LHCb, Eur.Phys.J. C78 (2018) no.6, 443].
- The WS backgrounds are modeled from RS signal and sidebands.
- Systematic uncertainties are studied from many sources, then doubled.





Correlated $\Psi(3770) \to D\overline{D}$ Decays

Measure Relative Strong Phases of CF and DCS Amplitudes

Measuring correlated $D\overline{D}$ decay rates at the $\Psi(3770)$ allows the determination of relative strong phases such as $\delta_{K\pi}$, $\delta_{K\pi\pi^0}$, and $\delta_{K\pi\pi\pi}$.

We want to calculate the correlated amplitude for the D and the $\overline{D}$ to decay to the states α and β at times t_1 and t_2 respectively, where the times are measured in the center-of-mass (CM) system and $t = 0$ is the time of the $e^+e^- \to c\bar{c}$ production. Because the $\Psi(3770)$ is $J^{PC} = 1^{--}$ state, we **anti-symmetrize the amplitude with respect to charge conjugation**.

$$\mathcal{M} = \frac{1}{\sqrt{2}} \left[\langle\alpha|\mathcal{H}|D^0(t_1)\rangle\langle\beta|\mathcal{H}|\overline{D}^0(t_2)\rangle - \langle\beta|\mathcal{H}|D^0(t_2)\rangle\langle\alpha|\mathcal{H}|\overline{D}^0(t_1)\rangle \right] \tag{4}$$

The time evolution of the D^0–$\overline{D}^0$ system is described by

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{5}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$.

Flavor Eigenstates Evolve with Time (again)

The two eigenstates D_1 and D_2 of the effective Hamiltonian are

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle\ , \quad |p|^2 + |q|^2 = 1\,. \tag{6}$$

The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2}\Gamma_{1,2} = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right), \tag{7}$$

where $m_{1,2}$, $\Gamma_{1,2}$ are the masses and decay widths. The proper time evolution of the eigenstates of Eq. 5 is

$$|D_{1,2}(t)\rangle = e_{1,2}(t)|D_{1,2}\rangle,\ e_{1,2}(t) = e^{[-i(m_{1,2} - \frac{i\Gamma_{1,2}}{2})t]}. \tag{8}$$

A state that is prepared as a flavor eigenstate $|D^0\rangle$ or $|\overline{D}^0\rangle$ at $t = 0$ will evolve

$$|D^0(t)\rangle = \frac{1}{2p}\left[p(e_1(t)+e_2(t))|D^0\rangle + q(e_1(t)-e_2(t))|\overline{D}^0\rangle\right] \tag{9}$$

$$|\overline{D}^0(t)\rangle = \frac{1}{2q}\left[p(e_1(t)-e_2(t))|D^0\rangle + q(e_1(t)+e_2(t))|\overline{D}^0\rangle\right]\ . \tag{10}$$

Forms of $\mathcal{M}$ and $|\mathcal{M}|^2$

After a bit of algebra we can write the matrix element as

$$
\begin{aligned}
2\sqrt{2}\,\mathcal{M} &= \left(\frac{q}{p}\,\overline{\mathcal{A}}_\alpha\overline{\mathcal{A}}_\beta - \frac{p}{q}\,\mathcal{A}_\alpha\mathcal{A}_\beta\right)[e_1(t_1)e_2(t_2) - e_1(t_2)e_2(t_1)] \\
&+ (\mathcal{A}_\alpha\overline{\mathcal{A}}_\beta - \overline{\mathcal{A}}_\alpha\mathcal{A}_\beta)\,[e_1(t_1)e_2(t_2) + e_1(t_2)e_2(t_1)]
\end{aligned}
\tag{11}
$$

which has the form

$$
2\sqrt{2}\,\mathcal{M} = X(e_{11}e_{22} - e_{12}e_{21}) \; + \; Y(e_{11}e_{22} + e_{12}e_{21})\,. \tag{12}
$$

From this one calculates

$$
\begin{aligned}
8|\mathcal{M}|^2 &= e^{-\Gamma(t_1+t_2)} \times \{\; XX^*\,(\cosh\, y\Gamma\Delta t - \cos\, x\Gamma\Delta t) \\
&- 2\,\Re(XY^*)\,\sinh\, y\boldsymbol{\Gamma\Delta t} + \; 2\,\Im(XY^*)\,\sin x\boldsymbol{\Gamma\Delta t} \\
&+ YY^*\,(\cosh y\Gamma\Delta t + \cos x\Gamma\Delta t\;\}
\end{aligned}
\tag{13}
$$

where $\boldsymbol{\Delta t = t_2 - t_1}$ **is the (signed) difference of proper decay times**.

$|\mathcal{M}|^2$ in the Small (x, y) Limit

For $x\Gamma\Delta t$, $y\Gamma\Delta t \ll 1$ this can be approximated by

$$
\begin{aligned}
4|\mathcal{M}|^2 &= e^{-\Gamma(t_1+t_2)} \times \left\{ \; XX^* \left[\frac{(x^2+y^2)}{4} (\Gamma\Delta t)^2 \right] \right. \\
&- \Re(XY^*)\, y\Gamma\Delta t + \Im(XY^*)\, x\Gamma\Delta t \\
&+ \left. YY^* \left[1 + \frac{(y^2-x^2)}{4} (\Gamma\Delta t)^2 \right] \right\}
\end{aligned}
\quad (14)
$$

- Y is the unmixed amplitude
- X is the mixing amplitude
- XY^* controls the interference terms in the mixing rate

This can be compared to the WS rate for a D whose flavor is tagged at $t = 0$:

$$
\left| \langle \bar{f} | H \left| D^0(t) \right\rangle \right|^2 \approx \frac{e^{-\Gamma t}}{2} |\overline{\mathcal{A}}_{\bar{f}}|^2 \left\{ \overline{R}_D + \left| \frac{q}{p} \right| \sqrt{\overline{R}_D} \left[y \cos(\delta - \varphi) - x \sin(\delta - \varphi) \right] (\Gamma t) + \left| \frac{q}{p} \right|^2 \frac{x^2+y^2}{4} (\Gamma t)^2 \right\} .
$$

Some General Observations

- Each of X and Y is the difference of two products of amplitudes; the difference reflects the charge conjugation symmetry of the initial $D^0\overline{D}^0$ state.
- The components of the decay rate proportional to the real and imaginary parts of XY^* corresponds to the interference of the direct and mixing amplitudes to a common final state.
- The relative time-dependence dependence of the interference term is proportional to $y' \Gamma \Delta t$ where $y' = y \cos\delta + x \sin\delta$ with $XY^* = Ce^{i\delta}$ (C and δ real).
- The phase δ depends upon the phase of p/q and also on both the final state α and the final state β.
- The interference term is odd in $\Gamma\Delta t$ while the pure mixing and unmixed terms are even in $\Gamma\Delta t$. Thus, the interference term disappears when considering only time-integrated decay rates.

CP Eigenstates versus $K\pi$

Correlated decays to a CP eigenstate and a hadronic non-CP eigenstate are somewhat more complicated. Consider the final state $(K^-\pi^+, K^-K^+)$.

$$\begin{aligned}
\mathcal{A}_\alpha &= \mathcal{A}(D^0 \to K^-\pi^+) = ae^{i(\delta+\phi)} \\
\overline{\mathcal{A}}_\alpha &= ke^{i\delta_{K\pi}}\mathcal{A}_\alpha \\
\mathcal{A}_\beta &= \mathcal{A}(D^0 \to K^-K^+) \\
\overline{\mathcal{A}}_\beta &= \mathcal{A}_\beta
\end{aligned}$$

The mixing and direct amplitudes for $(K^-\pi^+, K^-K^+)$ are

$$\begin{aligned}
X &= \left(\frac{q}{p}ke^{i\delta_{K\pi}} - \frac{p}{q}\right)\mathcal{A}_\alpha\mathcal{A}_\beta \\
Y &= (1 - ke^{i\delta_{K\pi}})\mathcal{A}_\alpha\mathcal{A}_\beta
\end{aligned}$$

As is well-known, the time-integrated rate is dominated by the term

$$YY^* = (1 - 2k\cos\delta_{K\pi} + k^2)\,\mathcal{A}_\alpha\overline{\mathcal{A}}^*_\alpha\mathcal{A}_\beta\overline{\mathcal{A}}^*_\beta$$

which depends linearly on $\cos\delta_{K\pi}$. Using some external inputs and several CP eigenstates from 2.9 fb^{-1} of data, **BES-III reports a preliminary value** $\cos\delta_{K\pi} = 1.03 \pm 0.12 \pm 0.04 \pm 0.01$ [Hai-Bo Li, Moriond QCD, 2016].

CP eigenstates versus $K\pi\pi^0$

With the notation

$$\begin{aligned}\mathcal{A}(D^0 \to K^-\pi^+\pi^0) &= A_r\zeta(s_{12}, s_{13}) \\ \overline{\mathcal{A}}(\overline{D}^0 \to K^-\pi^+\pi^0) &= \overline{A}_r\overline{\zeta}(s_{12}, s_{13}) = ke^{i\delta_{K\pi\pi^0}} A_r\overline{\zeta}(s_{12}, s_{13})\end{aligned}$$

the time-integrated rate for the final state $(K^-K^+, K^-\pi^+\pi^0)$ will be dominated by

$$YY^* = \left(\zeta\zeta^* + k^2\overline{\zeta}\overline{\zeta}^* - 2k\left[\Re(\zeta\overline{\zeta}^*)\cos\delta_{K\pi\pi^0} + \Im(\zeta\overline{\zeta}^*)\sin\delta_{K\pi\pi^0}\right]\right)|A_\beta|^2|A_r|^2 .$$

Notionally, A_r is an overall (integrated) magnitude of the amplitude for the CF decay and ζ describes the variation over phase space. Note that **this measurement is sensitive to both $\cos\delta_{K\pi\pi^0}$ and $\sin\delta_{K\pi\pi^0}$.** .

Presumably, the amplitudes $A_r\zeta(s_{12}, s_{13})$ and $\overline{A}_r\overline{\zeta}(s_{12}, s_{13})$ will be measured with good precision by Belle-II.

Conceptually, the corresponding equation applies for $(K^-K^+, K^-\pi^-\pi^+\pi^+)$ with $\delta_{K\pi\pi^0} \to \delta_{K\pi\pi\pi}$ and $\zeta(s_{12}, s_13) \to \zeta(\vec{p}_i)$ where $\vec{p}_i$ denotes the location of the decay in its 5-dimensional phase space.

$D\overline{D} \to (K^-\pi^+, K^-\pi^+\pi^0)$ [same-sign hadronic]

Here we will write

$$\begin{aligned}
\mathcal{A}_\alpha &= \mathcal{A}(D^0 \to K^-\pi^+) \\
\overline{\mathcal{A}}_\alpha &= k_1 e^{i\delta_1} \mathcal{A}_\alpha \\
\mathcal{A}_\beta &= \mathcal{A}(D^0 \to K^-\pi^+\pi^0) = A_r \zeta \\
\overline{\mathcal{A}}_\beta &= k_2 e^{i\delta_2} A_r \overline{\zeta}
\end{aligned}$$

so that

$$\begin{aligned}
X &= \left(\frac{q}{p} k_1 k_2 \, e^{i(\delta_1+\delta_2)} \, \overline{\zeta} - \frac{p}{q} \zeta \right) A_r \mathcal{A}_\beta \\
Y &= \left(k_2 \, e^{i\delta_2} \, \overline{\zeta} - k_1 \, e^{i\delta_1} \, \zeta \right) A_r \mathcal{A}_\beta \, .
\end{aligned}$$

It follows that

$$\begin{aligned}
\boldsymbol{YY^*} &= \Big(k_2^2 \, \overline{\zeta}\overline{\zeta}^* + k_1^2 \zeta\zeta^* \\
&\quad -2 \, k_1 k_2 \left[\Re(\zeta\overline{\zeta}^*) \cos(\delta_1 - \delta_2) - \Im(\zeta\overline{\zeta}^*) \sin(\delta_1 - \delta_2) \right] \Big) \, |\mathcal{A}_\alpha|^2 \, |A_r|^2 \, .
\end{aligned}$$

The corresponding equation applies for $(K^-\pi^+, K^-\pi^-\pi^+\pi^+)$ with $\delta_{K\pi\pi^0} \to \delta_{K\pi\pi\pi}$ and $\zeta(s_{12}, s_13) \to \zeta(\vec{p}_i)$ where $\vec{p}_i$ denotes the location of the decay in its 5-dimensional phase space.

Measuring Relative Strong Phases Simultaneously

The extension of the coherent decays formalism to other $D\overline{D} \to$ (multi-body, multi-body) final states such as $(K^-\pi^+\pi^0, K^-\pi^-\pi^+, \pi^+)$, $(K^-\pi^-\pi^+\pi^+, K^0_S\pi^-\pi^+)$, $(K^-\pi^-\pi^+\pi^+, K^0_L\pi^-\pi^+)$, etc., is generally tedious, but straight-forward.

Final states where one D is observed decaying into $K^0_{S\,(L)}\,\pi^-\pi^+$ are special. The relative strong phase in such decays is constrained by the CP eigenvalue of intermediate resonances such as $K^0_{S\,(L)}\,\rho^0$, so is not determined empirically.

The relative strong phase between the CF and DCS amplitudes for a final state f, δ_f, can be measured using the events where that final state is observed along with CP eigenstate. **The ensemble of $\cos\delta_f$ and $\sin\delta_f$ for a variety of final states f can be additionally constrained by using same-sign events $D\overline{D} \to (f, f')$.**

For example, the strong phases δ_f for $f = K\pi\pi\pi$, $K\pi$, $K\pi\pi^0$ can be measured using

- $(K^-\pi^-\pi^+\pi^+, K^-K^+)$
- $(K^-\pi^-\pi^+\pi^+, \pi^-\pi^+)$
- $(K^-\pi^-\pi^+\pi^+, K^-\pi^+)$
- $(K^-\pi^-\pi^+\pi^+, K^-\pi^+\pi^0)$
- $(K^-\pi^-\pi^+\pi^+, K^0_{S\,(L)}\pi^-\pi^+)$

and all the corresponding decays with $K^-\pi^-\pi^+\pi^+$ replaced by $K^-\pi^+$ and $K^-\pi^+\pi^0$.

Overview: Mixing and CPV in Neutral Charm Decays

- **Time-dependent measurements** are required to measure mixing and CPV.
  - Continued studies of $D \to K\pi$, KK, & $\pi\pi$ decays by **LHCb** will provide high precision constraints.
  - A time-dependent amplitude analysis of $D \to K\pi\pi\pi$ by **LHCb** promises to provide very competitive measurements.
  - Time-dependent amplitude analyses of $D \to K^0_S\pi^+\pi^-$ by **LHCb and Belle-II** will provide very sensitive measurements. CPV parameters.
  - A time-dependent amplitude analysis of $D \to K\pi\pi^0$ by **Belle-II** should provide sompetitie measurements.
- **Correlated $D\overline{D}$ measurements** by **BES-III** are critical for measuring (x, y) rather than $(x\cos\delta_f, y\sin\delta_f)$. These measurements will allow $\cos\delta_f$ and $\sin\delta_f$ to be extracted with good precision.
  - Targeted final states (f) include $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$.
  - The targeted final states must be measured pairwise, with each other and with CP eigenstates.

Simultaneously Fitting All the Data

Building an open-source framework for simultaneously fitting all the data, and publishing the underlying data, has several **potential benefits**:

- model-dependent systematic uncertainties become properly correlated;
- as improved amplitude models are developed, the ensemble of results can be refit transparently, across all measurements;
- using common decay descriptors for amplitudes minimizes errors when two measurements (nominally) use the same amplitude model;
- using common, open software encourages careful review of code and models, by members of the experiments *and* others who are interested;
- using common, open software and publishing the underlying data will allow more stringent consistency checks across data sets.

Using such a framework has many **social/political challenges**, including:

- to what extent do experiments collaborate when adding data sets?
- when will experiments using the software publish their underlying data?
- who decides when new versions of the software are merged/released?
- how is credit allocated when new results are published?

A common framework could be built using GooFit

already used for $\pi\pi\pi^0$, $K^0_S\pi^-\pi^+$, and $K\pi\pi\pi$ studies; has Python bindings

GooFit
CUDA/OpenMP
Fitting Framework
for C++ & Python

GooFit is a massively-parallel framework, written using Thrust for CUDA and OpenMP, for doing maximum-likelihood fits with a familiar syntax.

What's new • Tutorials • API documentation • 2.0 upgrade • 2.1 upgrade • 2.2 upgrade • Build recipes • Python

Requirements

- A recent version of CMake is required. The minimum is 3.4, but tested primarily with 3.6 and newer. CMake is incredibly easy to install, you can even use `pip` (see the system install page). GooFit developers have supplied patches to CMake 3.12, so that is highly recommended.
- A ROOT 6 build highly recommended -- GooFit will use the included Minuit2 submodule if ROOT is not found, and the Minuit1 based fitter will not be available. Supports 6.06-6.14 (6.10+ recommended).

▸ If using CUDA: (click to expand)
▸ If using OpenMP: (click to expand)
▸ If using CPP: (click to expand)

To Take Away

- We are measuring **direct CPV** in charm decays with sensitivities in the range $\mathbf{10^{-3} - 10^{-2}}$. Standard Model predictions are in the range $10^{-4} - 10^{-3}$.
- We are measuring the particle – antiparticle **differences in mixing rates (CPV in mixing) in $D^0 \to K\pi$ at the few percent level.**
- The **super-weak constraint** (that all CPV in mixing originates in $|M_{12}|$, $|\Gamma_{12}|$, and $\arg(\Gamma_{12}/M_{12})$ **dramatically reduces the uncertainties on both $|q/p|$ and $\arg(q/p)$** . This constraint should apply for mixing with CF and DCS final states.
- The limits from these measurements **constrain BSM physics at high mass scales** and complement the limits from direct searches.
- We anticipate $> 4\times$ as much reconstructed charm in Run 2 as in Run 1, and **another $10\times - 50\times$** as much in Run 3.
- **Measuring CPV in mixing at the per mille level** will require complementary contributions from **LHCb**, **Belle-II**, and **BES-III**

Special Thanks

As usual, all work in experimental particle physics requires contributions from too many people, over too much time, to properly acknowledge all their contributions. However, I would especially like to thank:

- **Prof. Sun Liang**
  - for inviting me to Wuhan,
  - for his pioneering contributions to measuring mixing using $D \to \pi^- \pi^+ \pi^0$ decays from BaBar, and
  - his seminal contributions to measuring CPV in $D \to K\pi$ decays with LHCb data.
- **Rolf Andreassen and Richard Gass**
  - for their contributions for developing and studying the time-dependent formalism for correlated D-decays.
- **Christoph Hasse and Dominik Muller**
  - for their pioneering studies of the physics reach of $D \to K\pi\pi\pi$ for measuring mixing and CPV parameters.