Unification of Flavor SU(3) Analyses of Charmed Hadron Weak Decays

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Outline

- Motivation

- Two approach of SU(3) analysis: IRA and TDA

  $D \to PP,\ PV,\ VV$ decays

  Mismatch and equivalence between IRA and TDA

- Charm baryon decays

  $T_c \to T_8 P,\ T_8 V$ decays

- Summary

Motivation

Weak decays of heavy mesons and baryons carrying a bottom and/or a charm quark are of great interests and have been studied extensively on both experimental and theoretical sides.

Experiment Measurements

- **Rare decays → New Physics**
- **Branching fractions**
- **CP asymmetries**
- **Polarizations**

Theory

- **Quark model**
- **Sum rules**
- **pQCD**
- **HQET**
- **SU(3) analysis**

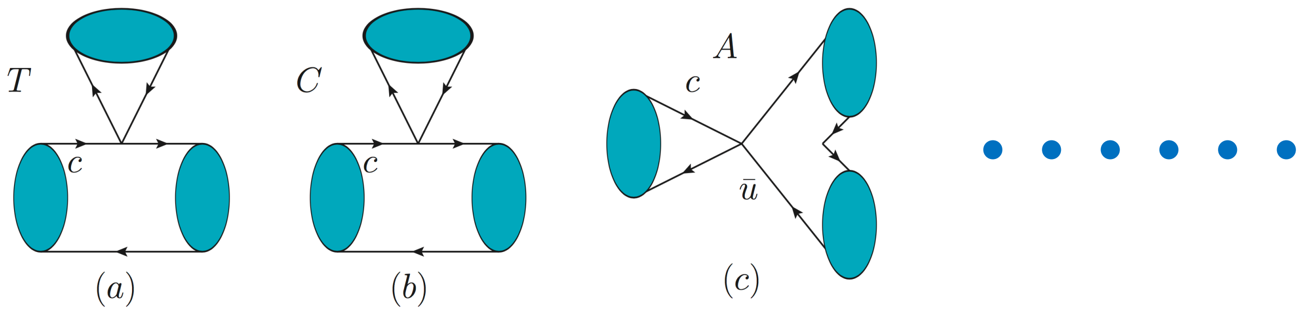
We know little about QCD at low energy

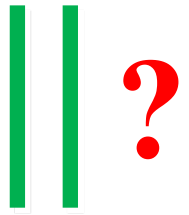
Not suit for charm quark

SU(3) analysis → Dynamic independent !

Two SU(3) approach

TDA: Draw diagrams





IRA: Write amplitudes

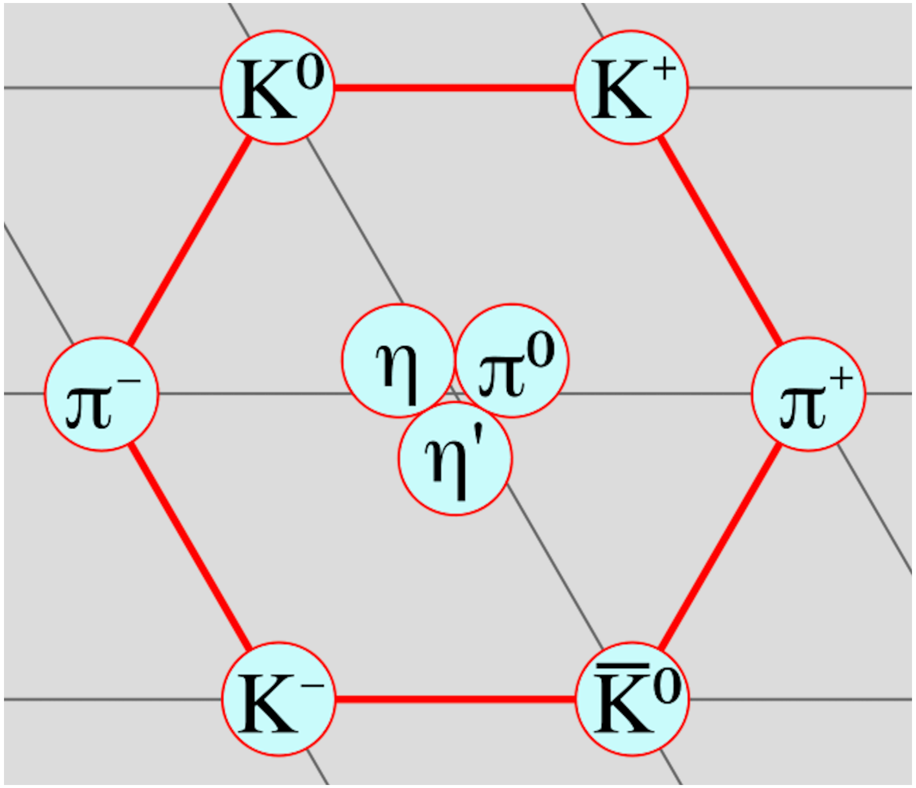
$$A_6^T D_i (H_6)_k^{[ij]} P_j^l P_l^k + C_6^T D_i (H_6)_k^{[jl]} P_j^i P_l^k + B_6^T D_i (H_6)_k^{[ij]} P_j^k P_l^l \;\cdots\cdots$$

SU(3) analysis for D to PP

SU(3) representation for hadron states:

$$(D_i) = (D^0(c\bar{u}), D^+(c\bar{d}), D_s^+(c\bar{s})) \quad \bar{3} \text{ representation}$$

P:



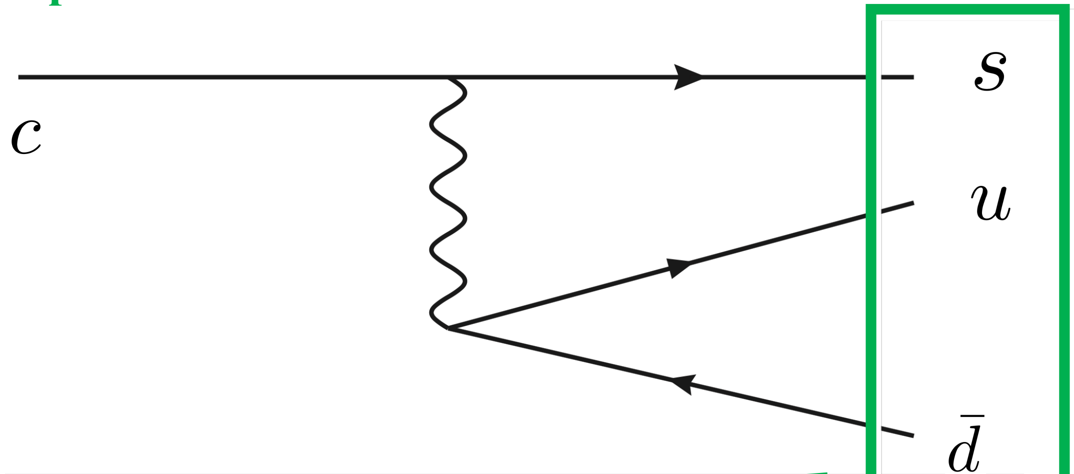
Including octets and singlet

Mixing angle:

$$\begin{aligned} \eta &= \cos\theta\eta_8 + \sin\theta\eta_1, \\ \eta' &= -\sin\theta\eta_8 + \cos\theta\eta_1. \end{aligned}$$

SU(3) analysis for D to PP (IRA)

Cabibbo-Allowed operator:



$$\bar{3}\otimes 3\otimes \bar{3} = \bar{3}\oplus\bar{3}\oplus 6\oplus\overline{15}$$

$$(H_6)^{31}_2 = -(H_6)^{13}_2 = 1, \quad (H_{\overline{15}})^{31}_2 = (H_{\overline{15}})^{13}_2 = 1$$

Doubly Cabibbo-Suppressed operator:

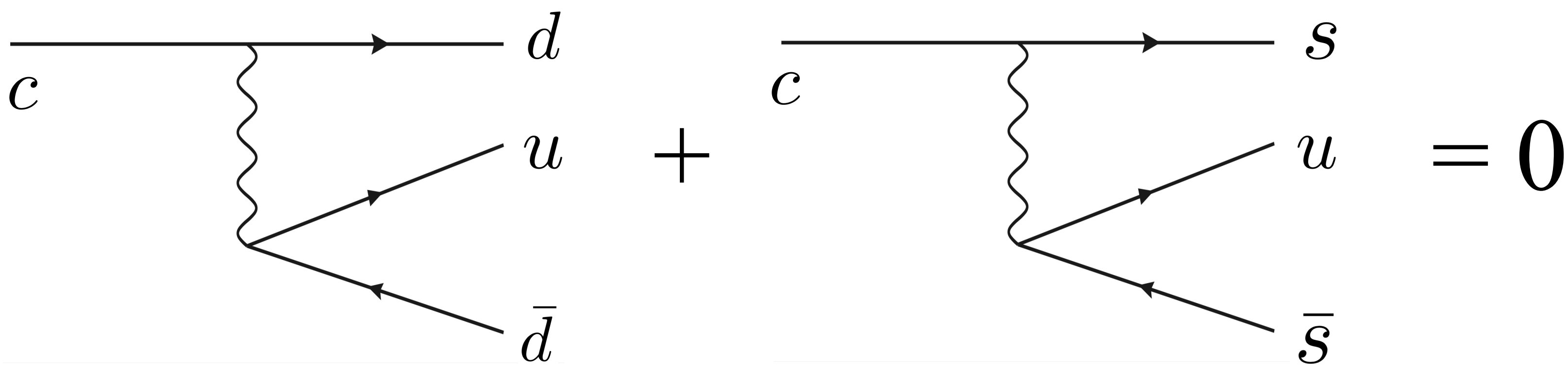
It has the same form except the inclusion of Cabibbo angel

$$(H_6)^{21}_3 = -(H_6)^{12}_3 = -\sin^2\theta_C, \quad (H_{\overline{15}})^{21}_3 = (H_{\overline{15}})^{12}_3 = -\sin^2\theta_C$$

SU(3) analysis for D to PP (IRA)

Cabibbo-Suppressed:

For charm decay the $\bar{3}$ operator vanishes since: $V_{cd}V_{ud}^* = -V_{cs}V_{us}^* - V_{cb}V_{ub}^* \approx -V_{cs}V_{us}^*$



$$(H_6)_3^{31} = -(H_6)_3^{13} = (H_6)_2^{12} = -(H_6)_2^{21} = \sin(\theta_C),$$
$$(H_{\overline{15}})_3^{31} = (H_{\overline{15}})_3^{13} = -(H_{\overline{15}})_2^{12} = -(H_{\overline{15}})_2^{21} = \sin(\theta_C).$$

SU(3) analysis for D to PP (IRA)

Irreducible representation tree amplitude:

$$\mathcal{A}_u^{IRA} = A_6^T D_i (H_6)_k^{[ij]} P_j^l P_l^k + C_6^T D_i (H_6)_k^{[jl]} P_j^i P_l^k + B_6^T D_i (H_6)_k^{[ij]} P_j^k P_l^l$$
$$+ A_{15}^T D_i (H_{\overline{15}})_k^{\{ij\}} P_j^l P_l^k + C_{15}^T D_i (H_{\overline{15}})_l^{\{jk\}} P_j^i P_k^l + B_{15}^T D_i (H_{\overline{15}})_k^{\{ij\}} P_j^k P_l^l$$

$$\mathcal{A} = V_{cs/d} V_{ud/s}^* \mathcal{A}_u^{IRA}$$

Cabibbo-Allowed channels:

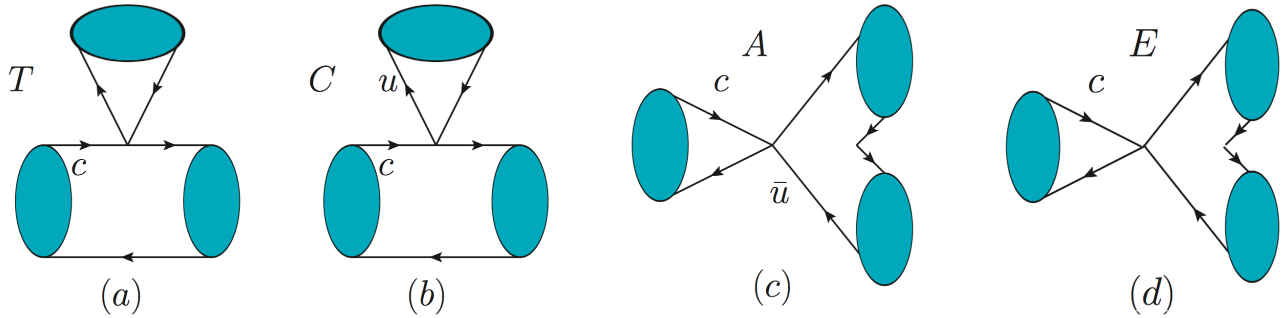
$V_{cs}V_{ud}^*$	IRA
$D^0 \to \pi^+ K^-$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$
$D^0 \to \pi^0 \overline{K}^0$	$(A_6^T - A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$
$D^0 \to \overline{K}^0 \eta_q$	$(-A_6^T + A_{15}^T - 2B_6^T + 2B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$
$D^0 \to \overline{K}^0 \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$
$D^+ \to \pi^+ \overline{K}^0$	$2C_{15}^T$
$D_s^+ \to \pi^+ \eta_q$	$\sqrt{2}\,(A_6^T + A_{15}^T + B_6^T + B_{15}^T)$
$D_s^+ \to \pi^+ \eta_s$	$B_6^T + B_{15}^T + C_6^T + C_{15}^T$
$D_s^+ \to K^+ \overline{K}^0$	$A_6^T + A_{15}^T - C_6^T + C_{15}^T$

There is one redundant degree of freedom !

$$C_6^{T\prime} = C_6^T - A_6^T$$
$$B_6^{T\prime} = B_6^T + A_6^T$$

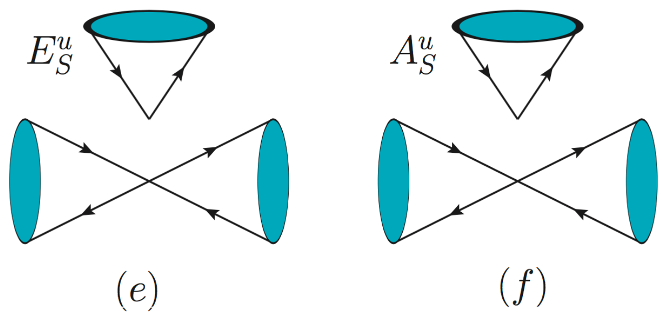
SU(3) analysis for D to PP (TDA)

In some literatures there's 4 topological diagrams:



However, we have 6 IRA amplitudes. One may wonder whether these 4 diagrams are equivalent IRA?

We discovered two missed diagrams:



Amplitudes for D to PP

$V_{cs}V_{ud}^*$	IRA	TDA
$D^0 \to \pi^+ K^-$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	E + T
$D^0 \to \pi^0 \overline{K}^0$	$(A_6^T - A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} - \mathrm{E})/\sqrt{2}$
$D^0 \to \overline{K}^0 \eta_q$	$(-A_6^T + A_{15}^T - 2B_6^T + 2B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} + 2\mathrm{E}_S^u + \mathrm{E})/\sqrt{2}$
$D^0 \to \overline{K}^0 \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$\mathrm{E}_S^u + \mathrm{E}$
$D^+ \to \pi^+ \overline{K}^0$	$2C_{15}^T$	C + T
$D_s^+ \to \pi^+ \eta_q$	$\sqrt{2}\,(A_6^T + A_{15}^T + B_6^T + B_{15}^T)$	$\sqrt{2}\,(\mathrm{A}_S^u + \mathrm{A})$
$D_s^+ \to \pi^+ \eta_s$	$B_6^T + B_{15}^T + C_6^T + C_{15}^T$	$\mathrm{A}_S^u + \mathrm{T}$
$D_s^+ \to K^+ \overline{K}^0$	$A_6^T + A_{15}^T - C_6^T + C_{15}^T$	A + C

Cabibbo-Allowed

Cabibbo-Suppressed

$V_{cs}V_{us}^*$	IRA	TDA
$D^0 \to \pi^+ \pi^-$	$A_6^T - A_{15}^T - C_6^T - C_{15}^T$	−E − T
$D^0 \to \pi^0 \pi^0$	$A_6^T - A_{15}^T - C_6^T + C_{15}^T$	C − E
$D^0 \to \pi^0 \eta_q$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$\mathrm{E}_S^u + \mathrm{E}$
$D^0 \to \pi^0 \eta_s$	$(-B_6^T + B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} + \mathrm{E}_S^u)/\sqrt{2}$
$D^0 \to K^+ K^-$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	E + T
$D^0 \to \eta_q \eta_q$	$A_6^T - A_{15}^T + 2B_6^T - 2B_{15}^T + C_6^T - C_{15}^T$	$-\mathrm{C} - 2\mathrm{E}_S^u - \mathrm{E}$
$D^0 \to \eta_q \eta_s$	$(-B_6^T + B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} + \mathrm{E}_S^u)/\sqrt{2}$
$D^0 \to \eta_s \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$\mathrm{E}_S^u + \mathrm{E}$
$D^+ \to \pi^+ \pi^0$	$\sqrt{2}C_{15}^T$	$(\mathrm{C} + \mathrm{T})/\sqrt{2}$
$D^+ \to \pi^+ \eta_q$	$-\sqrt{2}\,(A_6^T + A_{15}^T + B_6^T + B_{15}^T + C_{15}^T)$	$-(2\mathrm{A}_S^u + 2\mathrm{A} + \mathrm{C} + \mathrm{T})/\sqrt{2}$
$D^+ \to \pi^+ \eta_s$	$-B_6^T - B_{15}^T - C_6^T + C_{15}^T$	$\mathrm{C} - \mathrm{A}_S^u$
$D^+ \to K^+ \overline{K}^0$	$-A_6^T - A_{15}^T + C_6^T + C_{15}^T$	T − A
$D_s^+ \to \pi^+ K^0$	$A_6^T + A_{15}^T - C_6^T - C_{15}^T$	A − T
$D_s^+ \to \pi^0 K^+$	$(A_6^T + A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{A} + \mathrm{C})/\sqrt{2}$
$D_s^+ \to K^+ \eta_q$	$(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T + C_6^T - C_{15}^T)/\sqrt{2}$	$(2\mathrm{A}_S^u + \mathrm{A} - \mathrm{C})/\sqrt{2}$
$D_s^+ \to K^+ \eta_s$	$A_6^T + A_{15}^T + B_6^T + B_{15}^T + 2C_{15}^T$	$\mathrm{A}_S^u + \mathrm{A} + \mathrm{C} + \mathrm{T}$

Doubly Cabibbo-Suppressed

$V_{cd}V_{us}^*$	IRA	TDA
$D^0 \to \pi^0 K^0$	$(A_6^T - A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} - \mathrm{E})/\sqrt{2}$
$D^0 \to \pi^- K^+$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	E + T
$D^0 \to K^0 \eta_q$	$(-A_6^T + A_{15}^T - 2B_6^T + 2B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(\mathrm{C} + 2\mathrm{E}_S^u + \mathrm{E})/\sqrt{2}$
$D^0 \to K^0 \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$\mathrm{E}_S^u + \mathrm{E}$
$D^+ \to \pi^+ K^0$	$A_6^T + A_{15}^T - C_6^T + C_{15}^T$	A + C
$D^+ \to \pi^0 K^+$	$(A_6^T + A_{15}^T - C_6^T - C_{15}^T)/\sqrt{2}$	$(\mathrm{A} - \mathrm{T})/\sqrt{2}$
$D^+ \to K^+ \eta_q$	$(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T + C_6^T + C_{15}^T)/\sqrt{2}$	$(2\mathrm{A}_S^u + \mathrm{A} + \mathrm{T})/\sqrt{2}$
$D^+ \to K^+ \eta_s$	$A_6^T + A_{15}^T + B_6^T + B_{15}^T$	$\mathrm{A}_S^u + \mathrm{A}$
$D_s^+ \to K^+ K^0$	$2C_{15}^T$	C + T

We always find terms like:

T + E, C - E,

T – A = (T + E) - (A + E),

A + C = (A + E) +(C - E)

……

Equivalence between IRA and TDA

TDA also contain a redundant degree of freedom.

It's coefficients correspond to IRA coefficients one by one:

$$A_{15}^{T} = \frac{A+E}{2}, \quad B_{15}^{T} = \frac{A_{S}^{u}+E_{S}^{u}}{2}, \quad C_{15}^{T} = \frac{T+C}{2}, B_{6}^{\prime T} = \frac{A_{S}^{u}-E_{S}^{u}+A-E}{2}, \quad C_{6}^{\prime T} = \frac{T-C-A+E}{2}$$

The inverse relations are what we expected:

$$\boxed{T+E} = A_{15}^{T}+C_{6}^{\prime T}+C_{15}^{T}, \quad \boxed{C-E} = -A_{15}^{T}-C_{6}^{\prime T}+C_{15}^{T}, \quad \boxed{A+E} = 2A_{15}^{T},$$
$$\boxed{A_{S}^{u}-E} = -A_{15}^{T}+B_{6}^{\prime T}+B_{15}^{T}, \quad \boxed{E_{S}^{u}+E} = A_{15}^{T}-B_{6}^{\prime T}+B_{15}^{T}.$$

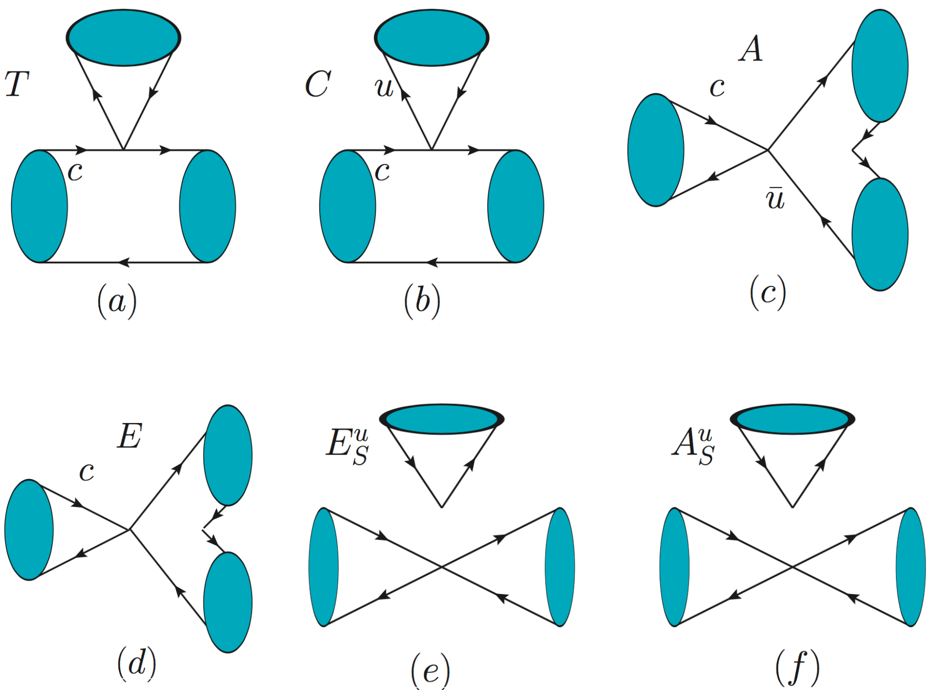
The TDA analysis must be equivalent with the IRA analysis.

SU(3) analysis for D to VP

It's almost the same as D to PP. The IRA amplitude is doubled:

$$\begin{aligned}\mathcal{A}_u^{IRA} &= A_6^{T1}D_i(H_6)^{[ij]}_k P^l_j V^k_l + A_6^{T2}D_i(H_6)^{[ij]}_k V^l_j P^k_l + C_6^{T1}D_i(H_6)^{[jl]}_k P^i_j V^k_l + C_6^{T2}D_i(H_6)^{[jl]}_k V^i_j P^k_l \\ &= +B_6^{T1}D_i(H_6)^{[ij]}_k P^k_j V^l_l + B_6^{T2}D_i(H_6)^{[ij]}_k V^k_j P^l_l + A_{15}^{T1}D_i(H_{\overline{15}})^{\{ij\}}_k P^l_j V^k_l + A_{15}^{T2}D_i(H_{\overline{15}})^{\{ij\}}_k V^l_j P^k_l \\ &\quad +C_{15}^{T1}D_i(H_{\overline{15}})^{\{jk\}}_l P^i_j V^l_k + C_{15}^{T2}D_i(H_{\overline{15}})^{\{jk\}}_l V^i_j P^l_k + B_{15}^{T1}D_i(H_{\overline{15}})^{\{ij\}}_k P^k_j V^l_l + B_{15}^{T2}D_i(H_{\overline{15}})^{\{ij\}}_k V^k_j P^l_l\end{aligned}$$

While the independent topological diagrams remain the same:



SU(3) analysis for D to VP

Some useful relations for decay widths:

Cabibblo allowed channels:

$$\Gamma(D_s^+ \to \rho^+\pi^0) = \Gamma(D_s^+ \to \rho^0\pi^+)$$

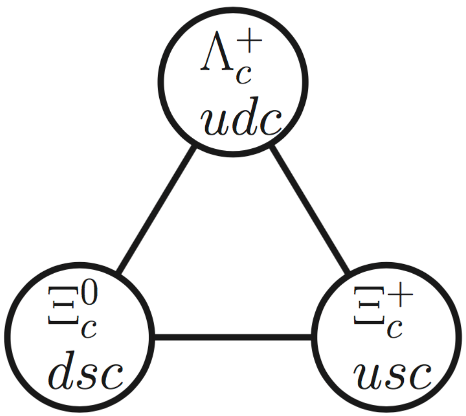
Singly Cabibblo suppressed channels:

$$\Gamma(D^0 \to \rho^+\pi^-) = \Gamma(D^0 \to K^{*+}K^-), \quad \Gamma(D^0 \to \rho^-\pi^+) = \Gamma(D^0 \to K^{*-}K^+),$$
$$\Gamma(D^+ \to K^{*+}\overline{K}^0) = \Gamma(D_s^+ \to \rho^+K^0), \quad \Gamma(D^+ \to \overline{K}^{*0}K^+) = \Gamma(D_s^+ \to K^{*0}\pi^+),$$
$$\Gamma(D^0 \to \overline{K}^{*0}K^0) = \Gamma(D^0 \to K^{*0}\overline{K}^0).$$

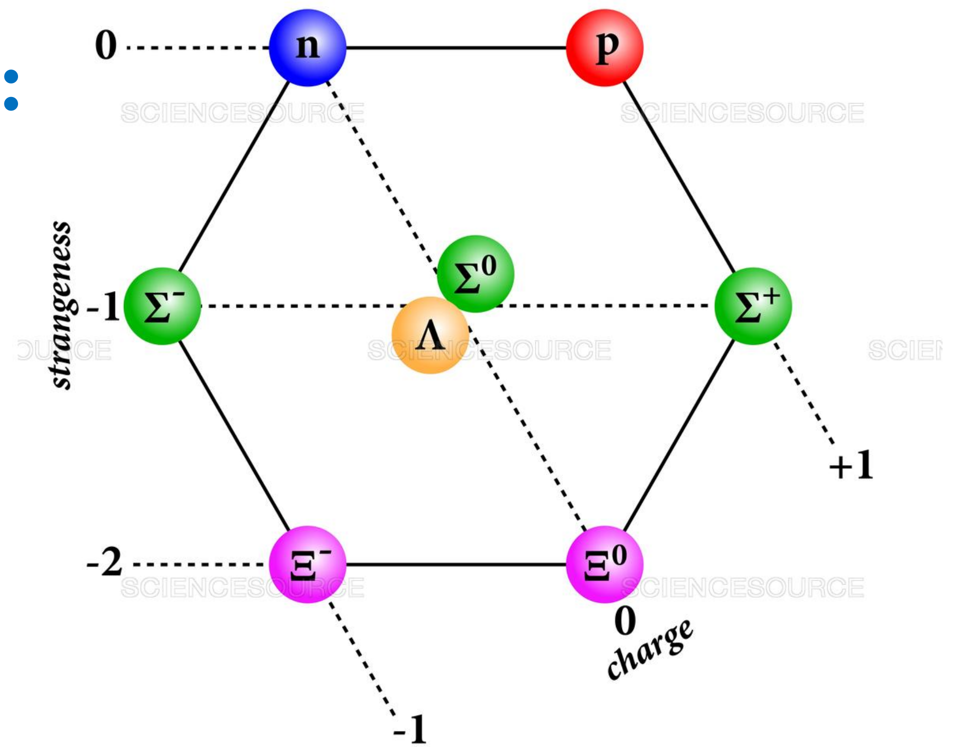
SU(3) analysis for Tc to T8 P

Charmed baryons with two light quarks can form an anti-triplet or a sextet. Most members of the sextet except Ω_c can decay via strong interactions or electromagnetic interactions.

Tc:

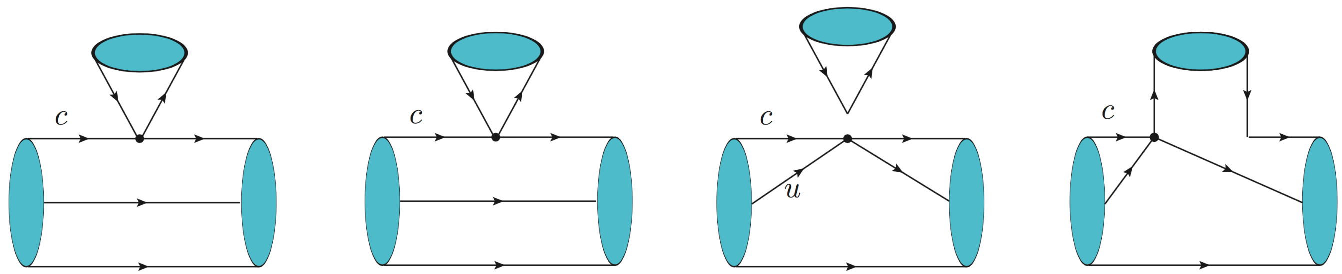


T8:

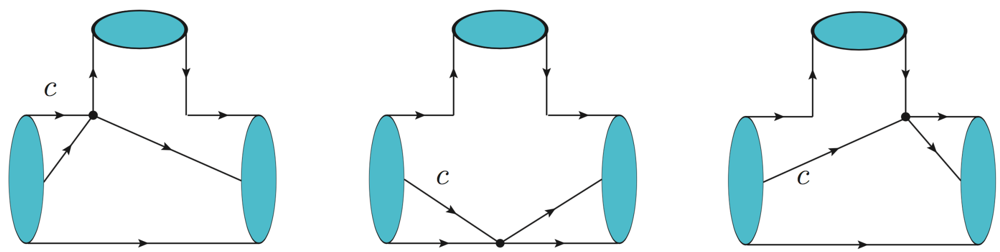


SU(3) analysis for Tc to T8 P

We can find 7 topological diagrams:



$(\bar{a}_1, \bar{a}_{15})$ $(\bar{a}_2, \bar{a}_{16})$ $(\bar{a}_3, \bar{a}_5, \bar{a}_9)$ $(\bar{a}_4, \bar{a}_6, \bar{a}_{10})$

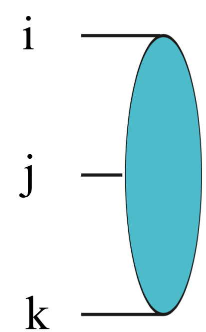


$(\bar{a}_7, \bar{a}_8, \bar{a}_{11})$ $(\bar{a}_{12}, \bar{a}_{13}, \bar{a}_{14})$ $(\bar{a}_{17}, \bar{a}_{18}, \bar{a}_{19})$

There are more than one amplitudes corresponding to one topological diagram. Totally 19 amplitudes.

$$(\overline{T}_8)_{ijk}$$

Neither symmetric nor antisymmetric



SU(3) analysis for Tc to T8 P

However, we have 10 IRA amplitudes:

$$\begin{aligned}\mathcal{A}_u^{IRA} &= A_6^T(T_{c\bar{3}})_i(H_6)_j^{[ik]}(\overline{T}_8)_k^j P_l^l + B_6^T(T_{c\bar{3}})_i(H_6)_j^{[ik]}(\overline{T}_8)_k^l P_l^j + C_6^T(T_{c\bar{3}})_i(H_6)_j^{[ik]}(\overline{T}_8)_l^j P_k^l \\ &+ E_6^T(T_{c\bar{3}})_i(H_6)_l^{[jk]}(\overline{T}_8)_j^i P_k^l + D_6^T(T_{c\bar{3}})_i(H_6)_l^{[jk]}(\overline{T}_8)_j^l P_k^i + A_{15}^T(T_{c\bar{3}})_i(H_{\overline{15}})_j^{\{ik\}}(\overline{T}_8)_k^j P_l^l \\ &+ B_{15}^T(T_{c\bar{3}})_i(H_{\overline{15}})_j^{\{ik\}}(\overline{T}_8)_k^l P_l^j + C_{15}^T(T_{c\bar{3}})_i(H_{\overline{15}})_j^{\{ik\}}(\overline{T}_8)_l^j P_k^l + E_{15}^T(T_{c\bar{3}})_i(H_{\overline{15}})_l^{\{jk\}}(\overline{T}_8)_j^i P_k^l \\ &+ D_{15}^T(T_{c\bar{3}})_i(H_{\overline{15}})_l^{\{ik\}}(\overline{T}_8)_j^l P_k^i.\end{aligned}$$

Also with one redundant degree of freedom:

$$A_6^{T\prime} = A_6^T + B_6^T, \quad B_6^{T\prime} = B_6^T - C_6^T, \quad C_6^{T\prime} = C_6^T - E_6^T, \quad D_6^{T\prime} = C_6^T + D_6^T$$

Equivalent ?

TDA with 19 amplitudes ⟷ IRA with 10-1=9 amplitudes

SU(3) analysis for Tc to T8 P

TDA must be equivalent with IRA and the relations between them are:

$$
\begin{aligned}
A_6^T &= \frac{1}{2}(\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - \bar{a}_{13} + \bar{a}_{14}), \quad B_6^T = \frac{1}{2}(\bar{a}_{13} - \bar{a}_{14} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
C_6^T &= \frac{1}{2}(\bar{a}_4 - \bar{a}_7 - \bar{a}_{10} + \bar{a}_{11} - 2\bar{a}_{12}), \quad D_6^T = \frac{1}{2}(\bar{a}_6 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} + \bar{a}_{14}), \\
E_6^T &= \frac{1}{2}(2\bar{a}_1 - 2\bar{a}_2 - \bar{a}_6 + \bar{a}_8 - \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
A_{15}^T &= \frac{1}{2}(\bar{a}_3 + \bar{a}_5 - \bar{a}_{13} - \bar{a}_{14}), \quad B_{15}^T = \frac{1}{2}(\bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{18}), \\
C_{15}^T &= \frac{1}{2}(\bar{a}_4 + \bar{a}_7 - \bar{a}_{10} - \bar{a}_{11}), \quad D_{15}^T = \frac{1}{2}(\bar{a}_6 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} + \bar{a}_{14}), \\
E_{15}^T &= \frac{1}{2}(2\bar{a}_1 + 2\bar{a}_2 - \bar{a}_6 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18}).
\end{aligned}
$$

Compared with experiment

From SU(3) approach, there is one relation between the different channels:

$$\Gamma(\Lambda_c^+ \to \Sigma^+\pi^0) = \Gamma(\Lambda_c^+ \to \Sigma^0\pi^+)$$

This result fits well with the data:

$$\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0) = 1.24 \pm 0.10\%, \quad \mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+) = 1.28 \pm 0.07\%$$

Relation with fitting results

In the paper: C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D 97, no. 7, 073006 (2018) a global fit was conducted, We relate our SU(3) parameters with these fitting values:

$$-A_6^T + D_6^T = h = (0.105 \pm 0.073)\ \mathrm{GeV}^3, \quad -B_6^T + E_6^T = a_1 = (0.244 \pm 0.006)\ \mathrm{GeV}^3,$$
$$-C_6^T - D_6^T = a_2 = (0.115 \pm 0.014)\ \mathrm{GeV}^3, \quad E_6^T + D_6^T = a_3 = (0.088 \pm 0.019)\ \mathrm{GeV}^3.$$

$a_1,\ a_2,\ a_3,\ h$ are SU(3) parameters used in this paper:

$$T(\mathbf{B}_c \to \mathbf{B}_n M) = T(\mathcal{O}_6) + \boxed{T(\mathcal{O}_{\overline{15}})}$$

Neglected in fitting

$$T(\mathcal{O}_6) = a_1 H_{ij}(6) T^{ik} (\mathbf{B}_n)^l_k (M)^j_l + a_2 H_{ij}(6) T^{ik} (M)^l_k (\mathbf{B}_n)^j_l$$
$$+ a_3 H_{ij}(6) (\mathbf{B}_n)^i_k (M)^j_l T^{kl} + h H_{ij}(6) T^{ik} (\mathbf{B}_n)^j_k (M)^l_l\,,$$

Summary

- In some literatures where TDA method is used, some topological diagrams are missed. Furthermore, among these diagrams, a redundant degree of freedom exists.
- We have shown that TDA analysis is actually equal to IRA analysis.
- For charmed baryon decays, although TDA approach seems very intuitive, it suffers the difficulty in providing the independent amplitudes. So IRA approach become more reliable.
- We have obtained a complete list of decay amplitudes in terms of SU(3) parameters, as well as relations between some of them. Most of them are waiting to be tested by experiments.
- All the analysis mentioned above can also be applied to bottom decays, which are also included in our work.

Thank you for your attention !