Axial charge transports and dynamics

热烈祝贺华南师范大学量子物质研究院成立！

林树

SCNU, 2018/11/13

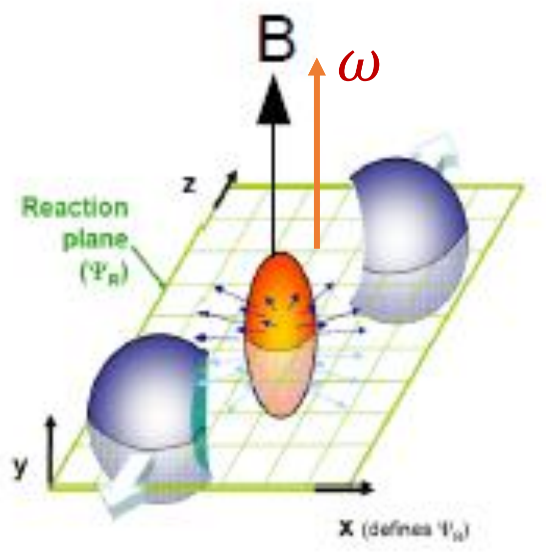
Outline

- Motivation
- Axial charge transports
- Axial charge dynamics
- Axial charge evolution from stochastic hydrodynamics
- Summary

Chiral Magnetic/Separation Effect(CME/CSE)

$$\boldsymbol{j} = C\mu_5 e\boldsymbol{B} \qquad \boldsymbol{j}_5 = C\mu e\boldsymbol{B}$$

Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008
Metlitski, Zhitnitsky, PRD 2005



Chiral Vortical Effect(VCVE/ACVE)

$$\boldsymbol{j} = C\mu_5\mu\boldsymbol{\omega} \qquad \boldsymbol{j}_5 = C(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3})\boldsymbol{\omega}$$

Vilenken, PRD 1980
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011

Talks by X.N. Wang, F. Wang, Liao

(Non)renormalization of anomalous transports

$$\boldsymbol{j} = C\mu_5 e\boldsymbol{B} + C\mu_5\mu\boldsymbol{\omega}$$

Nonrenormalizable!

Son, Surowka, PRL 2009

$$\boldsymbol{j}_5 = C\mu e\boldsymbol{B} + C(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3})\boldsymbol{\omega}$$

Renormalizable by interaction

Hou, Liu, Ren, PRD 2012

$$\partial_\mu j_5^\mu = CE \cdot B + bR \wedge R$$

renormalizable transport suggested to relate to gravitational anomaly

Landsteiner et al, PRL 2011

Renormalization of anomalous transport by mass

$$\partial_\mu j_5^\mu = C E\cdot B + b R\wedge R + 2 i m\bar{\psi}\gamma^5\psi$$

Renormalizable by mass(via interaction)

$$\boldsymbol{j}_5 = C\mu e\boldsymbol{B}$$

$$\langle j_5^3\rangle_\alpha = -\frac{\alpha e B\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

Golbar et al, PRD (2013)
Guo, SL, JHEP (2017)

$$\boldsymbol{j}_5 = C\left(\mu^2 + \frac{\pi^2 T^2}{3} - \frac{m^2}{2} + \cdots\right)\boldsymbol{\omega}$$

Flachi, Fukushima, 1702.04753
SL, Yang, 1810.02979

CSE/ACVE for general mass

$$\sigma_V = \sum_f \frac{N_c}{2\pi^2} \int_0^\infty dq \tilde{f}_+(\sqrt{q^2 + m_f^2}) \frac{2q^2 + m_f^2}{E_q},$$

$$\sigma_B = \sum_f \frac{eN_c}{2\pi^2} \int_0^\infty dq \tilde{f}_-(\sqrt{q^2 + m_f^2}).$$

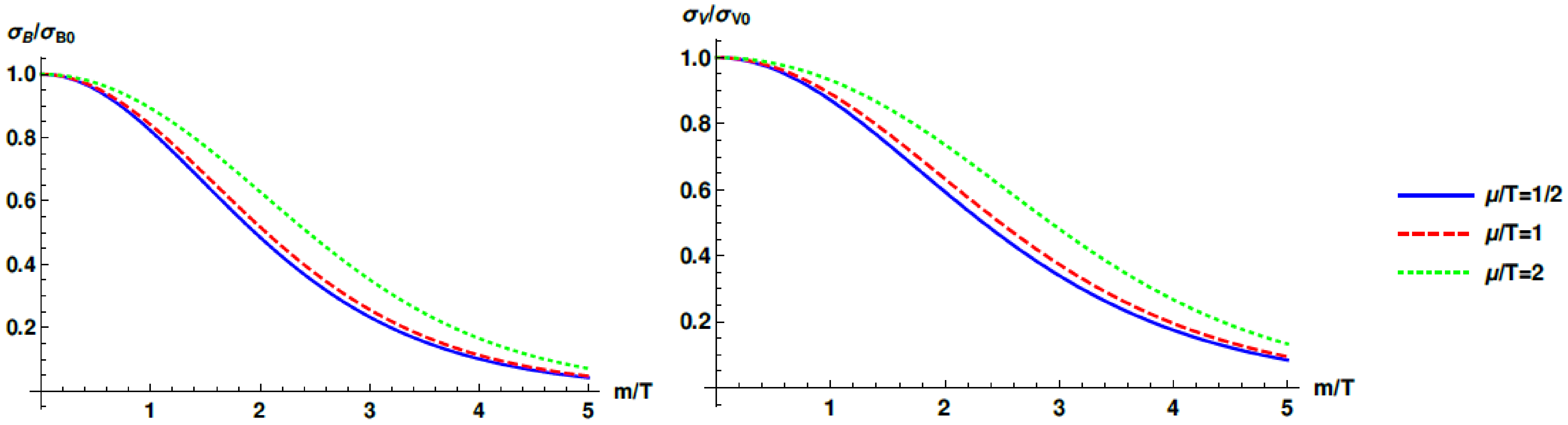
SL, Yang, 1810.02979

can be derived intuitively from the mass-corrected anomaly equation

$$\partial_\mu j_5^\mu = CE \cdot B + 2im\bar{\psi}\gamma^5\psi$$

$$\partial_\mu j_5^\mu = -\frac{1}{2\pi^2}(e^2 E \cdot B) C_1(m, \beta, \mu) - \frac{m^2}{2\pi^2}\beta(eE \cdot \omega) C_2(m, \beta, \mu).$$

Fang, Pang, Q. Wang, X.N. Wang,
PRD 2016



$$m_u = m_d \simeq 0,\ m_s = 100\text{MeV}$$

$$50\text{MeV} < \mu < 200\text{MeV} \qquad 200\text{MeV} < T < 400\text{MeV}$$

mass correction to σ_B and σ_V within 1%.
But could be significant near chiral transition

Existing anomalous hydro studies

$$\partial_\mu j^\mu = 0,$$
$$\partial_\mu j_5^\mu = -C E_\mu B^\mu,$$

$$j^\mu = n u^\mu + \kappa_B B^\mu,$$
$$j_5^\mu = n_5 u^\mu + \xi_B B^\mu,$$

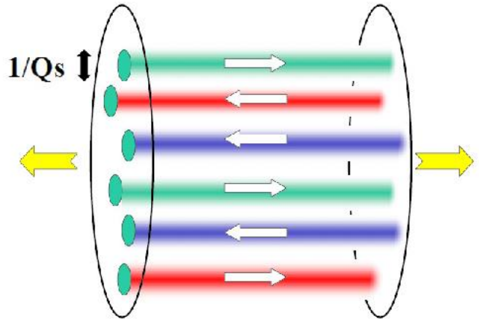
Hirono, Hirano, Kharzeev (2014)
Jiang, Shi, Yin, Liao (2016), (2017)

Talk by Liao

also chiral kinetic theory. Talk by Huang

initial condition for n_5

$\mu_5 \sim 100 MeV$

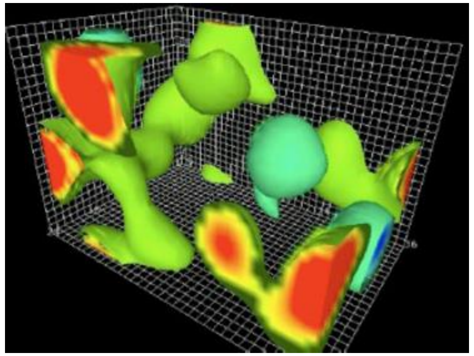


further evolution $E \cdot B = 0$

$N_5 = \int d^3 x n_5$ conserved

Fluctuation and dissipation of n_5

$$\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$



fluctuation and dissipation
exist throughout the evolution

Need to quantify their effects in hydrodynamic evolution!

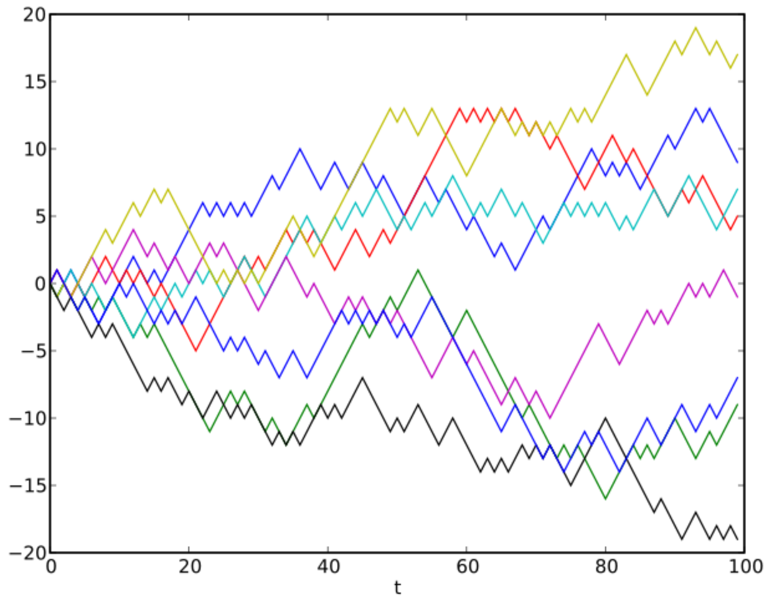
Schematic evolution of axial charge

$\langle N_5 \rangle = 0, \langle {N_5}^2 \rangle \neq 0$

N_5 stochastic variable

at late time

$\langle {N_5}^2 \rangle \to \chi T V$



Stochastic hydrodynamics for axial charge

$$\begin{cases} \partial_t n_5 + \nabla \cdot \mathbf{j}_5 = -2q, \\ \mathbf{j}_5 = -D\nabla n_5 + \xi, \\ q = \frac{n_5}{2\tau_{\mathrm{CS}}} + \xi_q, \end{cases}$$

thermal fluctuation

topological fluctuation

dissipation

Iatrakis, SL, Yin, JHEP 2015

coupling to vector current to be incldued

$$\partial_t N_5 = -\frac{N_5}{\tau_{CS}}$$

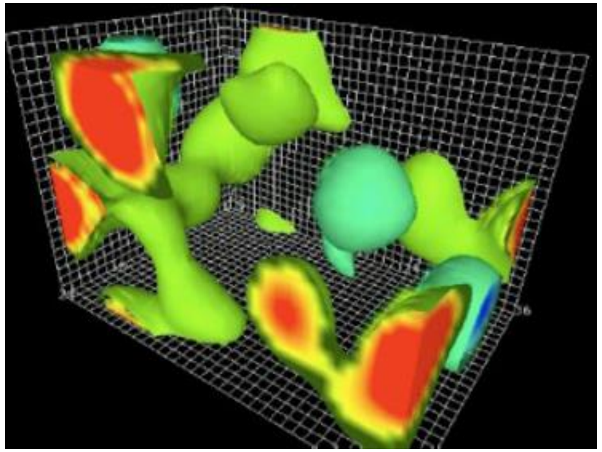
$$\langle \xi_q(t,\mathbf{x})\xi_q(t',\mathbf{x}')\rangle = \Gamma_{\mathrm{CS}}\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'),$$

$$\langle \xi_i(t,\mathbf{x})\xi_j(t',\mathbf{x}')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'),$$

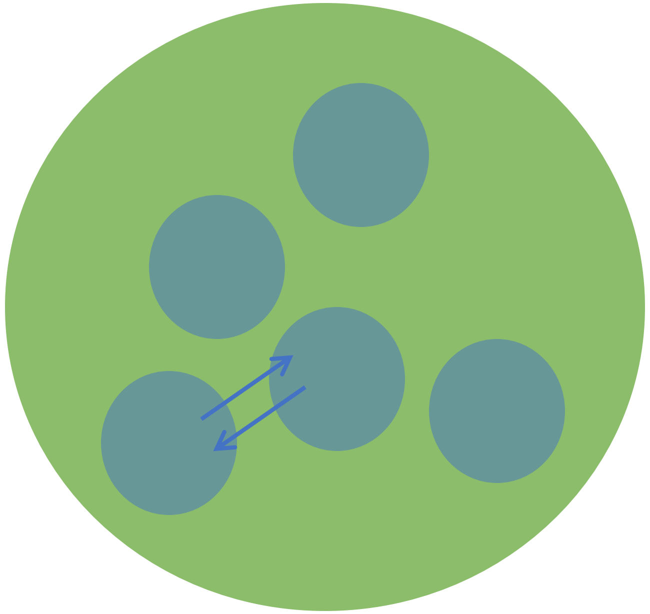
$$\langle \xi_i(t,\mathbf{x})\xi_q(t',\mathbf{x}')\rangle = 0.$$

topological noise vs thermal noise

topological noise
within fluid cell



between fluid cells



stochastic hydrodynamics include noises consistently

Covariant stochastic hydrodynamics

$$\begin{cases} \nabla_\mu J_5^\mu = -2q, \\ J_5^\mu = n_5 u^\mu - \sigma T P^{\mu\nu} \nabla_\nu \left(\frac{\mu_5}{T}\right) + P^{\mu\nu}\xi_\nu, \\ q = \frac{n_5}{2\tau_{\mathrm{CS}}} + \xi_q, \end{cases}$$

$$\langle P^{\mu\alpha}\xi_\alpha(x) P^{\nu\beta}\xi_\beta(x')\rangle = P^{\mu\alpha}P^{\nu\beta}g_{\alpha\beta} 2\sigma T \frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle \xi_q(x)\xi_q(x')\rangle = \Gamma_{\mathrm{CS}} \frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle P^{\mu\alpha}\xi_\alpha(x)\xi_q(x')\rangle = 0.$$

Einstein relations

$$\sigma = \chi D, \quad \tau_{\mathrm{CS}} = \frac{\chi T}{2\Gamma_{\mathrm{CS}}}$$

Apply to Bjorken flow

Choice of parameters in Bjorken flow

$$\tau_{\rm CS} \sim \frac{1}{T} \qquad \Gamma_{\rm CS} \sim T^4$$

$$T = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}, \quad \tau_{\rm CS} = \left(\frac{\tau}{\tau_0}\right)^{1/3} \tau_{\rm CS0}, \quad \Gamma_{\rm CS} = \Gamma_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}$$

$$\tau_0 = 0.6\text{fm}, \ T_0 = 350\text{MeV}, \ \Gamma_0 = 30\alpha_s^4 T_0^4 \qquad \alpha_s = 0.3.$$

Moore and Tassler, JHEP (2011)

$$\chi_0 = 3T_0^2$$

$$\tau_{\rm CS0} = \frac{\chi_0 T_0}{2\Gamma_0} \simeq 2.3\text{fm}.$$

Damping time scale at initial temperature, comparable to QGP evolution time. Damping important!

N_5 evolution from vanishing initial value

$\tau_1 = \tau_0$ and $N_5(\tau_1) = 0$.

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2 x_\perp 2\Gamma_0 \tau_0 \tau_{\rm CS0} \left(1 - e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{\rm CS0}}\right)} \right)$$

At early time $\tau_0 \ll \tau_2 \ll \tau_{CS0}$

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2 x_\perp 6\Gamma_0 \tau_0^2 \left(\left(\frac{\tau_2}{\tau_0}\right)^{2/3} - 1 \right) \quad \sim \; {\tau_2}^{2/3}$$

as compared to $\sim t$ in static flow

At late time $\tau_0 \ll \tau_{CS0} \ll \tau_2$

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2 x_\perp 2\Gamma_0 \tau_0 \tau_{\rm CS0} = \int \tau_0 d\eta d^2 x_\perp \chi_0 T_0 \quad \sim \quad \chi T V$$

equilibrium limit

Estimate of μ_5 from equilibrium limit

$$N_5 \sim \left(\int d\eta d^2 x_\perp 2\Gamma_0 \tau_0 \tau_{\rm CS0} \right)^{1/2} = \left(\pi R^2 \Delta\eta 2\Gamma_0 \tau_0 \tau_{\rm CS0}\right)^{1/2},$$

$$\mu_5 = \frac{N_5}{\pi R^2 \tau \Delta\eta \chi}.$$

$\mu_5 \sim \tau^{-1/3}$ $\quad |\eta| < 2$

Centrality	70-80%	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
μ_5(MeV)	13.1	10.6	8.84	7.63	6.70	5.94	5.26	4.78	4.44

Fluctuation significant in small volume

N_5 evolution from large initial value

$$\langle N_5(\tau_2)^2\rangle = \langle N_5(\tau_1)^2\rangle e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{\mathrm{CS}0}}\right)} + \int d\eta d^2x_\perp 2\Gamma_0\tau_0\tau_{\mathrm{CS}0}\left(1 - e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{\mathrm{CS}0}}\right)}\right)$$

exponential decay of initial charge

growth of fluctuation

estimate of initial charge

$$\sqrt{\langle n_5(\tau_0)^2\rangle} \simeq \frac{Q_s^4(\pi\rho_{\mathrm{tube}}^2\tau_0)\sqrt{N_{\mathrm{coll}}}}{16\pi^2 A_{\mathrm{overlap}}}$$

$$\mu_5(\tau_0) \simeq 35\mathrm{MeV}$$

two terms become comparable

$\tau_2 \simeq 6.4\mathrm{fm}$ for $70-80\%$

$\tau_2 \simeq 12.5\mathrm{fm}$ for $0-5\%$

$\tau_2 \to \infty$, reach equilibrium, independent of initial condition

CME from axial charge in equilibrium limit

$$j = C_e \mu_5 eB$$

Axial charge induced electric charge dipole.
No backreaction

Centrality	70-80%	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
$e\mu_e$(MeV)	7.40	4.85	3.40	2.53	1.95	1.53	1.20	1.00	0.86

$$B = B_0 e^{-\tau/\tau_B}$$

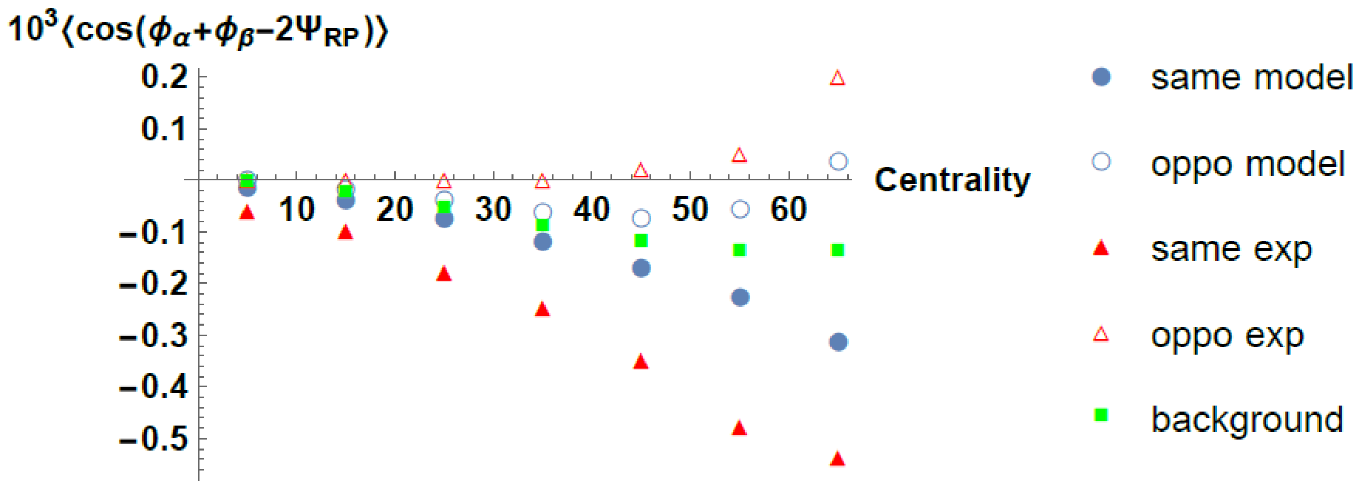
$$eB_0 = 10m_\pi^2 \text{ and } \tau_B = 3\text{fm}$$

Cooper-Frye freezeout

$$\delta N_Q = \frac{Q}{(2\pi)^2} \int dy \int m_\perp^2 dm_\perp \tau_f d\eta d^2x_\perp \cosh(\eta - y) e^{-m_\perp \cosh(\eta-y)/T_f + \mu_\pi/T_f} \delta\mu_e$$

$$|y| < 1 \quad |\eta| < 2$$

CME from stochastic hydrodynamics



Au-Au 200GeV

STAR, PRC (2010)

momentum/charge conservation background not included, signal less than measured correlation

Liang, SL, Yan, in preparation

n_5 evolution from vanishing initial value

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\tau_2 n_5(\tau_2,0)\rangle = \chi_0 T_0 \tau_0 \int \frac{dk_\eta}{2\pi} e^{-ik_\eta \eta}\left(1 - e^{-2(c+bk_\eta^2)}|_{\tau=\tau_1}\right)$$

$$= \chi_0 T_0 \tau_0 \left(\delta(\eta) - e^{-2c}\frac{e^{-\eta^2/(8b)}}{2\sqrt{2\pi b}}|_{\tau=\tau_1}\right).$$

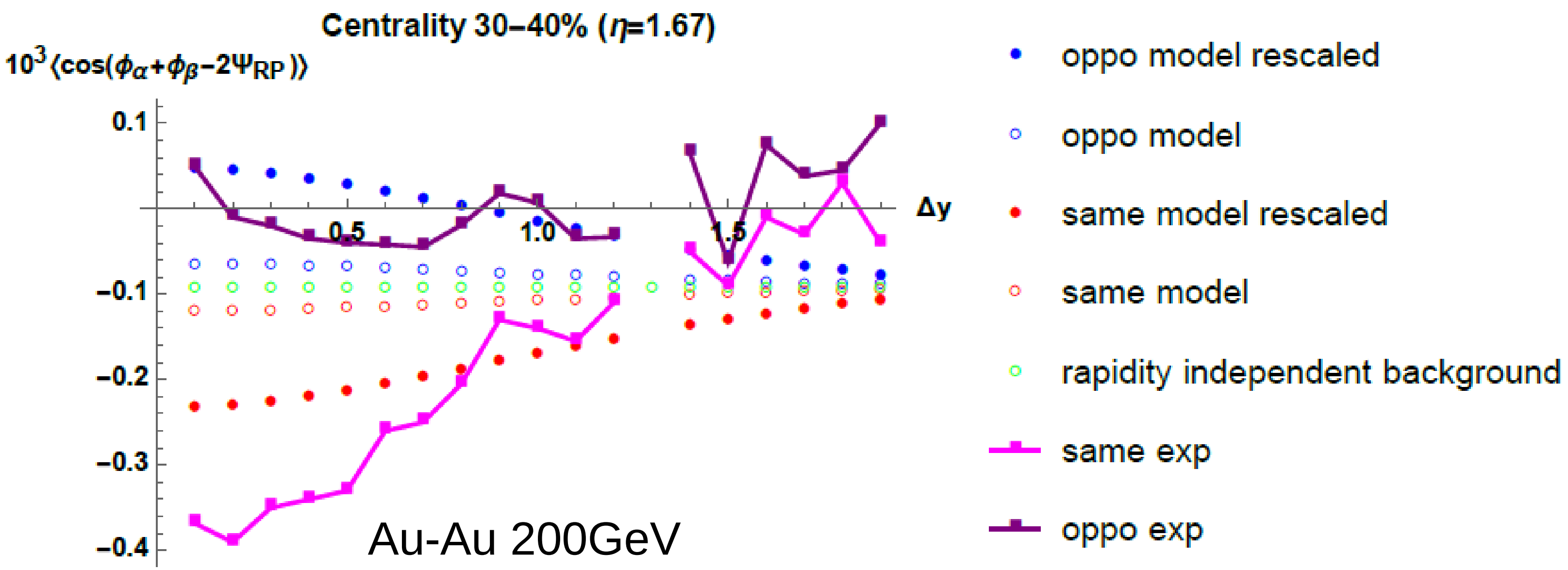
localized in rapidity

from diffusion

from topological fluctuation

Can be used to calculate pseudo-rapidity dependence of CME

Rapidity-dependence of CME signal



STAR, PRC (2010)

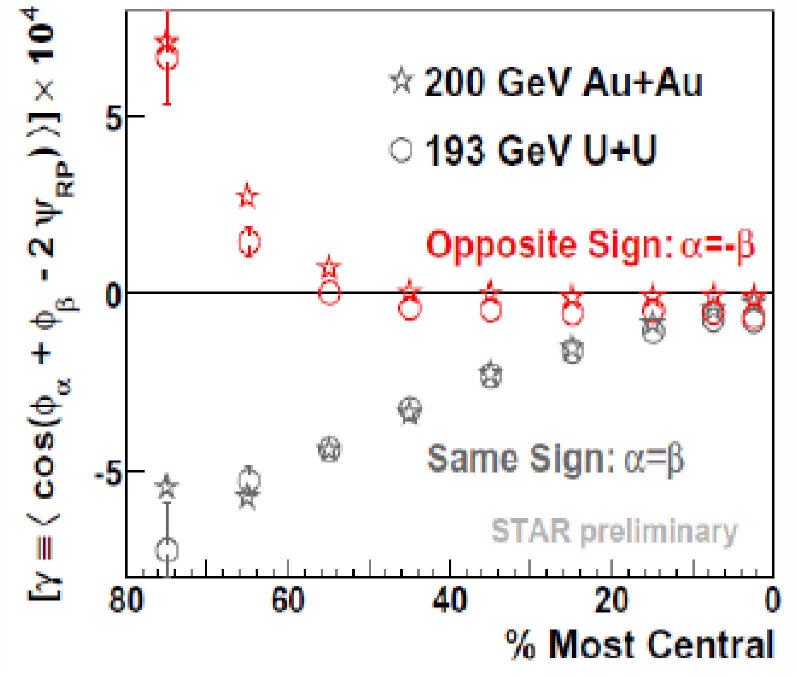
momentum/charge conservation background not included, signal less than measured correlation

Liang, SL, Yan, in preparation

Summary

- Axial charge transports renormalizable by mass and interaction.
- Mass correction negligible for HIC phenomenology.
- Axial charge is stochastic.
- Use stochastic hydrodynamics to include topological noise and thermal noise. Damping important for axial charge.
- Axial charge fluctuation at thermal equilibrium may provide a reasonable estimate for CME.

Measurement of CME from electric charge correlation



$$j = \frac{N_c \mu_5}{2\pi^2} eB$$

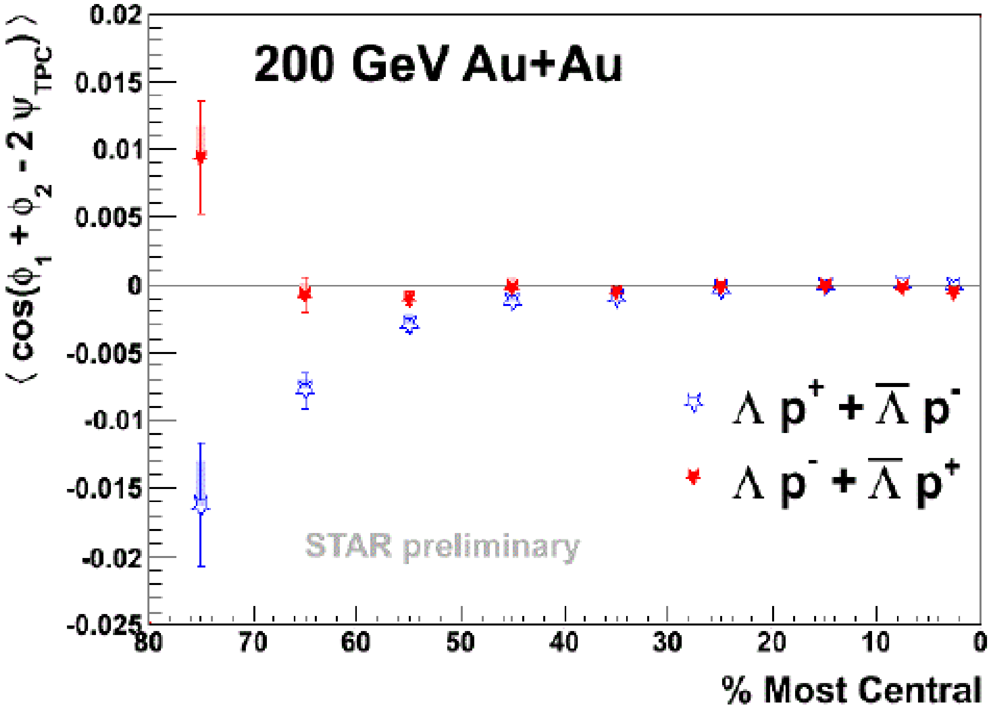
Chiral imbalance characterized by μ_5, originates from fluctuation of n_5

$$\langle n_5 \rangle = 0, \langle {n_5}^2 \rangle \neq 0$$

Same electric charge correlation enhanced than opposite electric charge correlation due to CME

STAR collaboration, PRL (2014), 1404.1433

Measurement of CVE from baryon charge correlation



$$j_B = \frac{N_c \mu_5 \mu}{2\pi^2} \omega$$

$$\langle N_5 \rangle = 0, \langle {N_5}^2 \rangle \neq 0$$

CVE also from fluctuation of n_5

More appropriate quantity n_5?

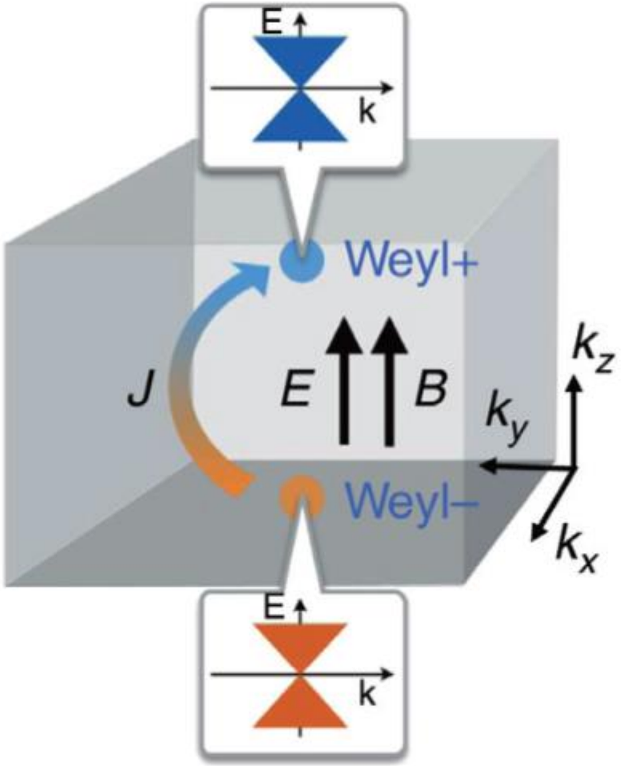
Same baryon charge correlation enhanced than opposite baryon charge correlation due to CVE

Liwen Wen (STAR), RHIC AGS meeting 2015

Sources of axial charge generation

$$\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Weyl Semi-metal



Li, Kharzeev et al Nature. Phys. (2016)

Successful frameworks

Hydrodynamics (axial charge)

Son, Surowka, PRL (2009)
Neiman, Oz, JHEP (2011)
Landsteiner et al, PRL (2011)

Chiral kinetic theory (Berry curvature)

Son, Yamamoto, PRL (2012)
Stephanov, Yin, PRL (2012)
Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRL (2012), (2013), PRD (2014)
Huang, Guo, Jiang, Liao, Zhuang (2017)