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Viscous Corrections to the Heavy-Quark Potential in QCD at High Temperatures

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- **References:**

JHEP 1709, 063 (2017)[arXiv:1706.08091] ; JHEP 1701, 123 (2017) [arXiv:1611.08379];

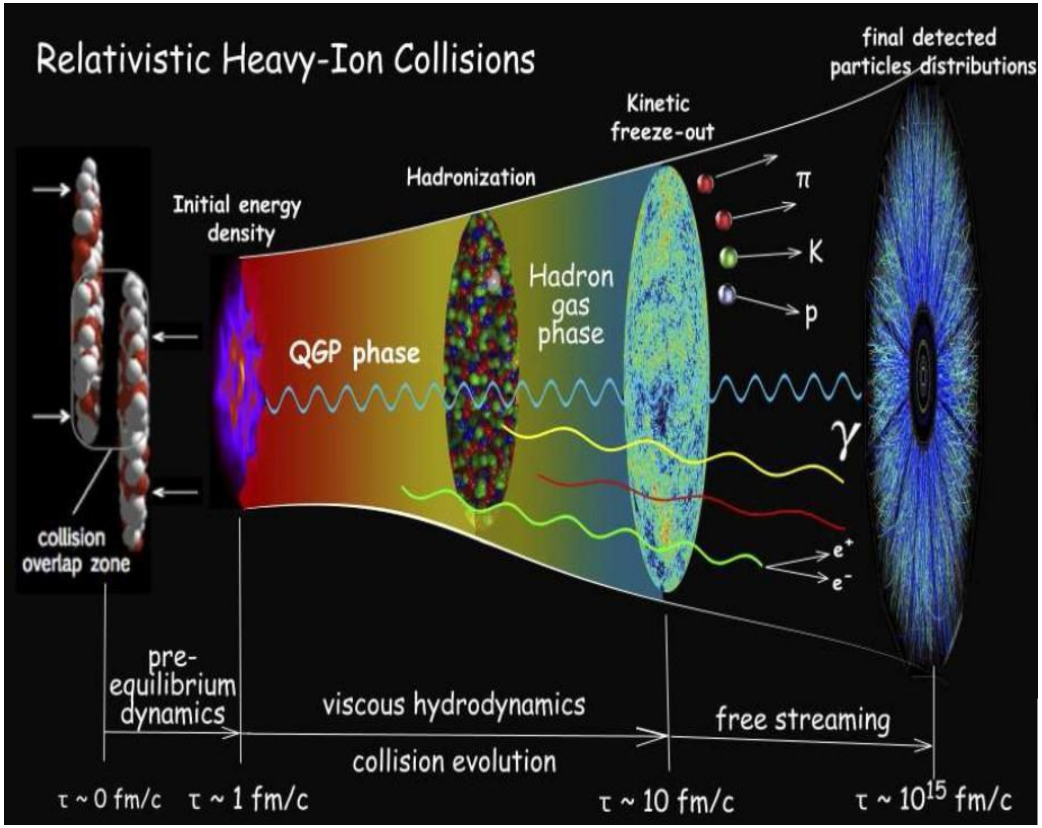
Phys. Rev. D 79, 114003 (2009) [arXiv:0903.4703]; Phys. Lett. B 662, 37 (2008) [arXiv:0711.4722]

QCD与夸克物质物理研讨会，广州 13 Nov. 2018

Outline：

- Introduction and Motivation
  - Why quarkonia?
  - The non-equilibrium quark gluon plasma (QGP)
- The Perturbative Heavy Quark Potential at Finite Temperature
  - Theoretical framework: real time formalism of thermal field theory
  - Some important results: from the equilibrium to the viscous corrections
- Summary

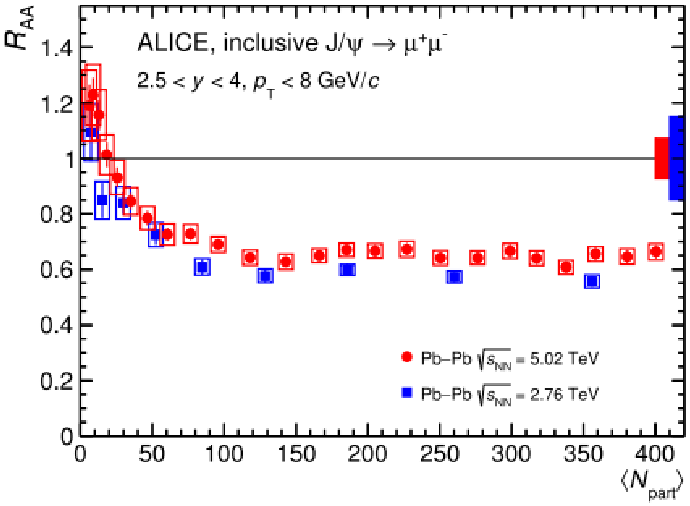
Introduction & Motivation



Probes that used to study the QGP:

- ***Quarkonia*;**
- **Jets;**
- **Strangeness… …**

The suppression of quarkonium production may indicate the formation of the QGP.

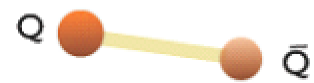


(ALICE Collaboration, PLB 2017)

Introduction & Motivation

- **Large quark mass**

$$M_c \approx 1.3\,\mathrm{GeV} \quad \text{and} \quad M_b \approx 4.7\,\mathrm{GeV} \gg \Lambda_{QCD} \approx 0.2\,\mathrm{GeV}$$



- **Tightly bound**

$$r_{J/\Psi} \approx 0.4\,\mathrm{fm} \quad \text{and} \quad r_\Upsilon \approx 0.2\,\mathrm{fm} \ll 1\,\mathrm{fm}$$

- **Quark velocity** $v \ll 1 \Longrightarrow$ **non-relativistic treatment**

- **QQbar properties obtained solving the Schrödinger equation**

$$\hat{H}\phi_v(\mathbf{x}) = E_v\,\phi_v(\mathbf{x}) \,,$$

$$\hat{H} = -\frac{\nabla^2}{2m_R} + V(\mathbf{x}) + m_1 + m_2 \,,$$

Potential describes the QQbar interaction

- **Heavy quark potential contains non-perturbative physics, constructed based on the Lattice simulations**

- **Short-distance behavior of the potential can be studied perturbatively**

this talk!

- **A deviation of the system from equilibrium:** $\delta f = f - f_{\rm id}$

The energy-momentum tensor

$$T_{ij} = P_{\rm eq}(\epsilon)\,\delta_{ij} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \zeta\delta_{ij}\nabla\cdot u$$

shear viscous correction | **bulk viscous correction**

- **The parton distribution function of the Quark-Gluon-Plasma**

$$f(\mathbf{p}) = f_{\rm id}(p) + \delta_{\rm bulk} f(p) + \delta_{\rm shear} f(\mathbf{p})$$

unlike the distributions at the thermal fixed point, non-equilibrium corrections are not universal

- **Anisotropy due to expansion and non-zero shear viscosity**

✓ The QGP created in HIC exhibits an anisotropy in momentum space.

$$f(\mathbf{p}) = f_{\rm id}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

(Romatschke & Strickland, PRD 2003)

$$\delta_{\rm shear} f(\mathbf{p}) = -\xi \frac{(\mathbf{p}\cdot\mathbf{n})^2}{2pT} f_{\rm id}(p)(1 \pm f_{\rm id}(p))$$

$$\xi = \frac{10}{T\tau}\frac{\eta}{s}.$$

(Asakawa, Bass & Müller, Prog. Theor. Phys. 2007)

- **Bulk viscous correction to the parton distribution function**

$$\delta_{\rm bulk} f(k) = \left(\frac{k}{T}\right)^a \Phi\, f_{\rm id}(k)(1 \pm f_{\rm id}(k))$$

$$\Phi \sim \frac{P_{\rm bulk}}{P_{\rm id}} \qquad |\Phi| \sim \zeta$$

The Perturbative Heavy Quark Potential at Finite Temperature

- **The heavy quark(HQ) potential due to one-gluon exchange**

$$V(\mathbf{r}) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \left(D^{*L}(p_0 = 0, \mathbf{p})\right)_{11}$$

In the real time formalism of the thermal field theory, the physical 11 component of the propagator can be rewrite as $D_{11} = (D_R + D_A + D_F)/2$

$$\mathrm{Re}V(\mathbf{r}) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{1}{2}\left(D^{*L}_R + D^{*L}_A\right)$$

→ ***Binding energy***

$$\mathrm{Im}V(\mathbf{r}) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{1}{2} D^{*L}_F$$

→ ***Decay width***

The Perturbative Heavy Quark Potential at Finite Temperature

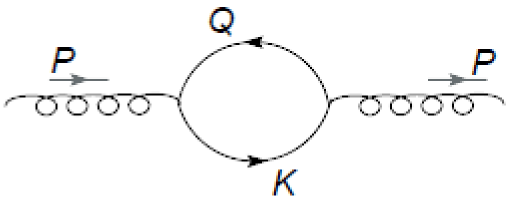
- **The Dyson-Schwinger equation**

$$\begin{bmatrix} D^*_{11} & D^*_{12} \\ D^*_{21} & D^*_{22} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} D^*_{11} & D^*_{12} \\ D^*_{21} & D^*_{22} \end{bmatrix}$$

$$D^*{}_R = D_R + D_R \Pi_R D^*{}_R \qquad D^*{}_F = D_F + D_R \Pi_R D^*{}_F + D_F \Pi_A D^*{}_A + D_R \Pi_F D^*{}_A$$

- **Gluon self-energy(SE) and Hard Thermal Loops(HTL)**

$$\Pi^{\mu\nu}(P) = -\frac{i}{2} N_f g^2 \int \frac{\mathrm{d}^4 K}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu S(Q) \gamma^\nu S(K)\right]$$



with HTL:

$$\Pi^L_R(P) = \frac{N_f g^2}{4\pi^3} \int k dk d\Omega_k\, f_F(\mathbf{k}) \frac{1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2}{(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}} + \frac{p_0 + i\,\epsilon}{p})^2}$$

$$K \sim T\,, \quad P \sim gT$$

$$\Pi^L_F(P) = 8 i N_f g^2 \pi^2 \int \frac{k^2 dk d\Omega}{(2\pi)^4} \frac{2}{p} f_F(\mathbf{k})(f_F(\mathbf{k}) - 1)\delta(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}} - \frac{p_0}{p})$$

The Perturbative Heavy Quark Potential at Finite Temperature

- **The ideal case:** $f_{\rm id}(p) = (e^{p/T} \pm 1)^{-1}$

$$\mathrm{Re}V(\mathbf{r}) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\frac{1}{p^2 + m_D^2} = -\frac{g^2 C_F}{4\pi}\left[m_D + \frac{e^{-\hat{r}}}{r}\right]$$

$$\mathrm{Im}V(\mathbf{r}) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\frac{-\pi T m_D^2}{p\,(p^2 + m_D^2)^2} = -\frac{g^2 C_F T}{4\pi}\phi(\hat{r})$$

with

$$\hat{r} \equiv r\, m_D$$

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^2 \frac{\mathrm{d}f_{\rm iso}(k)}{\mathrm{d}k} = \frac{g^2T^2}{6}(N_f + 2N_c)\,.$$

$$\phi(\hat{r}) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(z\hat{r})}{z\hat{r}}\right] \approx \frac{1}{3}\hat{r}^2 \ln\frac{1}{\hat{r}}$$

(Laine et al , JHEP 2007)

The Perturbative Heavy Quark Potential at Finite Temperature

- **Bulk viscous corrections:** $\delta_{\text{bulk}} f(k) = \left(\frac{k}{T}\right)^a \Phi \, f_{\text{id}}(k)(1 \pm f_{\text{id}}(k))$

assumptions:
- $|\Phi| \gg g^2$ *(neglect 2-loop corrections)*
- $a > 0$ *(HTL applicable)*

Corrections *isotropic*, only a change of the Debye mass in gluon SE

$$c_R^{(q,g)}(a) = \frac{1}{\Phi} \frac{\int k dk \, \delta_{\text{bulk}} f(k)}{\int k dk \, f_{\text{id}}(k)} = \begin{cases} \frac{12}{\pi^2}(1-2^{-a})\Gamma(2+a)\zeta(1+a) & \text{(fermion)} \\ \frac{6}{\pi^2}\Gamma(2+a)\zeta(1+a) & \text{(boson)} \, , \end{cases}$$

$$c_F^{(q,g)}(a) = \frac{1}{\Phi} \frac{\int dk k^2 \, \delta_{\text{bulk}} f(k)[1 \pm 2 f_{\text{id}}(k)]}{\int dk k^2 \, f_{\text{id}}(k)[1 \pm f_{\text{id}}(k)]} = \begin{cases} \frac{6}{\pi^2}(1-2^{-a})\Gamma(3+a)\zeta(1+a) & \text{(fermion)} \\ \frac{3}{\pi^2}\Gamma(3+a)\zeta(1+a) & \text{(boson)} \, . \end{cases}$$

Corrections NOT the same for retarded solution and symmetric solution

$$c_F^{(q,g)}(a) = \frac{1}{2}(2+a) c_R^{(q,g)}(a)$$

The Perturbative Heavy Quark Potential at Finite Temperature

- **Bulk viscous corrections to the HQ potential:**

$$\mathrm{Re}\,V(r) = -\frac{g^2 C_F}{4\pi r}\, e^{-r\sqrt{m_{R,D}^2+\delta m_{R,D}^2}} - \frac{g^2 C_F}{4\pi}\sqrt{m_{R,D}^2 + \delta m_{R,D}^2}$$

$$\mathrm{Im}\,V(r) = -\frac{g^2 C_F T}{4\pi}\,\frac{m_{F,D}^2 + \delta m_{F,D}^2}{m_{R,D}^2 + \delta m_{R,D}^2}\,\phi(\hat{r})$$

with

$$m_{R,D}^2 = m_{F,D}^2 = (2N_c + N_f)\frac{g^2 T^2}{6}$$

$$\delta m_{R,D}^2 = \Phi\left(2N_c c_R^{(g)}(a) + N_f c_R^{(q)}(a)\right)\frac{g^2 T^2}{6}$$

$$\delta m_{F,D}^2 = \Phi\left(2N_c c_F^{(g)}(a) + N_f c_F^{(q)}(a)\right)\frac{g^2 T^2}{6}$$

$\Phi < 0$

reduced screening and damping scales

✓ The real part has the same structure as in the ideal case with a modified retarded Debye mass

✓ The imaginary part is multiplied by a factor which equals 1 in the thermal equilibrium

The Perturbative Heavy Quark Potential at Finite Temperature

- **Shear viscous corrections:** $\delta_{\rm shear} f(\mathbf{p}) = -\xi \frac{(\mathbf{p}\cdot\mathbf{n})^2}{2pT} f_{\rm id}(p)(1 \pm f_{\rm id}(p))$

corrections *anisotropic*, can not be a redefinition of the Debye mass in SE

$$\delta\Pi^L_R(P) = \xi m_D^2 \left(\frac{1}{6} + \frac{\cos(2\theta_n)}{2}\right) + \xi \Pi^L_{R(0)}(P)\left[\cos(2\theta_n) - \frac{p_0^2}{2p^2}(1 + 3\cos(2\theta_n))\right]$$

$$\delta\Pi^L_F(P) = \xi \frac{3}{2}\pi i\, m_D^2 \frac{T}{p}\left(\sin^2\theta_n + (3\cos^2\theta_n - 1)\frac{p_0^2}{p^2}\right)\Theta(p^2 - p_0^2)$$

DS equation

θ_n is the angle between $\mathbf{p}$ and $\mathbf{n}$

$$\delta D^{*L}_R(p_0 = 0) = \frac{\xi m_D^2}{6} \frac{1 - 3\cos(2\theta_n)}{(p^2 + m_D^2)^2}$$

$$\delta D^{*L}_F(p_0 = 0) = \xi \frac{3\pi i T m_D^2}{2p\,(p^2 + m_D^2)^2}\sin^2\theta_n - \xi \frac{4\pi i T m_D^4}{p\,(p^2 + m_D^2)^3}(\sin^2\theta_n - \frac{1}{3})$$

Beyond linear approximation, analytical results for the static gluon propagator can be also obtained.

(Nopoush, Guo & Strickland, ArXiv:1706.08091)

(Dumitru, Guo & Strickland, PLB, 2008)

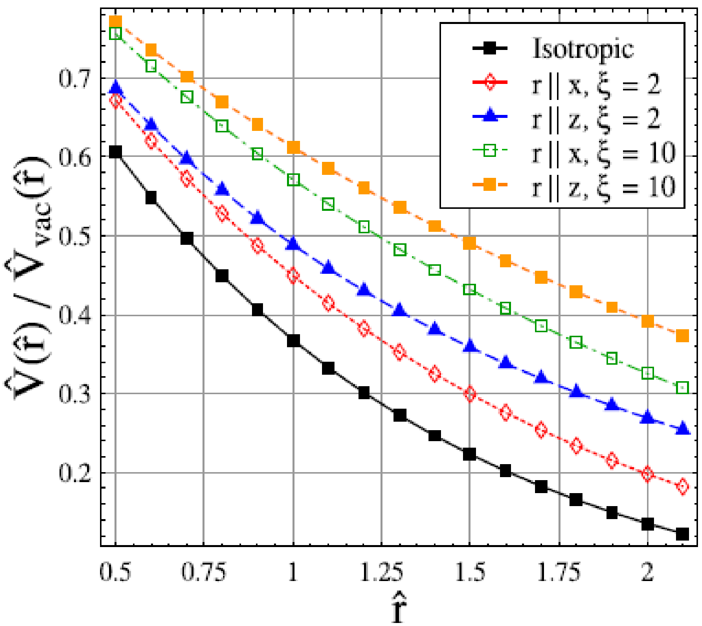
- **Shear viscous corrections to the HQ potential:**

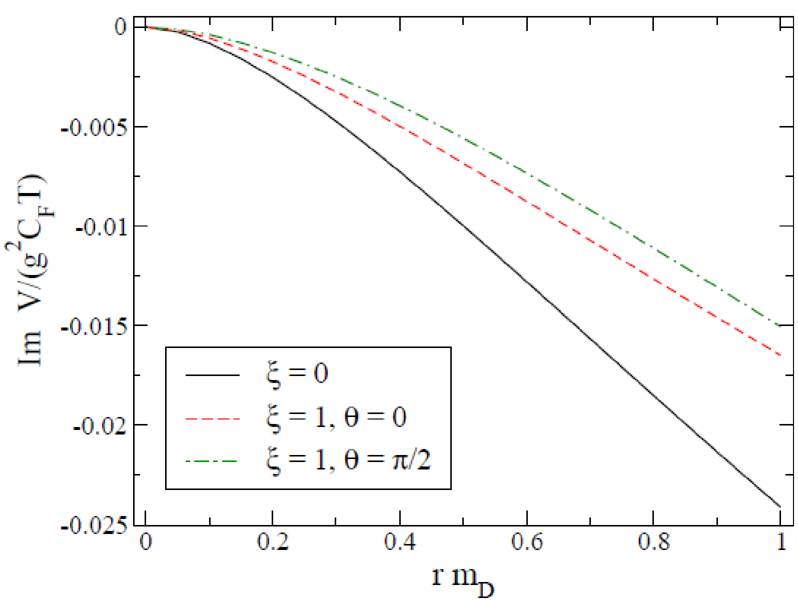
$$\delta\mathrm{Re}\,V(\mathbf{r}) = \xi\frac{g^2C_F}{4\pi}\left[\frac{m_D}{6}+\frac{e^{-\hat{r}}}{r}\mathcal{F}(\hat{r},\theta)\right] \approx \xi\frac{g^2C_F}{4\pi}\frac{e^{-\hat{r}}}{r}\left[\frac{\hat{r}}{6}(e^{\hat{r}}-1)-\frac{\hat{r}^2}{48}(1+3\cos(2\theta))\right]$$

$$\delta\mathrm{Im}\,V(\mathbf{r}) = \xi\frac{g^2C_FT}{4\pi}\left[\psi_1(\hat{r},\theta)+\psi_2(\hat{r},\theta)\right] \approx \xi\frac{g^2C_FT}{4\pi}\hat{r}^2\ln\frac{1}{\hat{r}}\frac{3-\cos(2\theta)}{20}$$

✓ **Numerical results:**

θ is the angle between $\mathbf{r}$ and $\mathbf{n}$





Results and Discussion

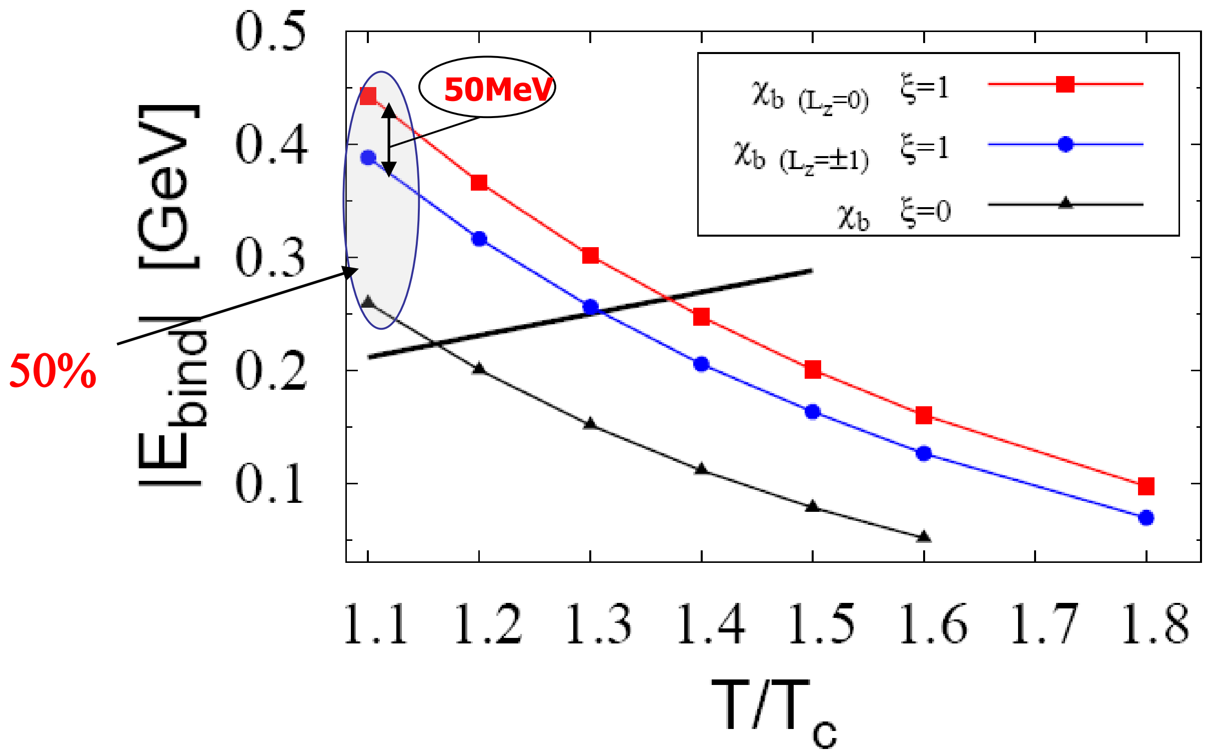


FIG. 6: Temperature-dependence of the binding energy for the $1P$ state of bottomonium for two values of the plasma anisotropy parameter ξ. The straight line corresponds to a binding energy equal to the temperature.

$$\rho \sim \exp\left(-\frac{E_{\text{bind}}}{T}\right) ,$$

At $T \sim T_c$, the population of the state with $L_z = 0$ is about 30% higher than that of either one of the $L_z = \pm 1$ states.

Summary

- **HQ potential is an important quantity to study the physics of quakonia in hot and dense medium**

- **Bulk viscous corrections to HQ potential**
  - **for the real part: a redefinition of the Debye mass**
  - **for the imaginary part: an extra factor depends on the corrections to Debye masses**

- **Shear viscous corrections to HQ potential**
  - **for the real part:**

  (1)deeper and closer to the vacuum potential, reduced screening and stronger binding;

  (2)not uniform in the polar angle, different binding energies for quarkonia with different polarization

  - **for the imaginary part: decreases with increasing viscosity and smaller decay width**

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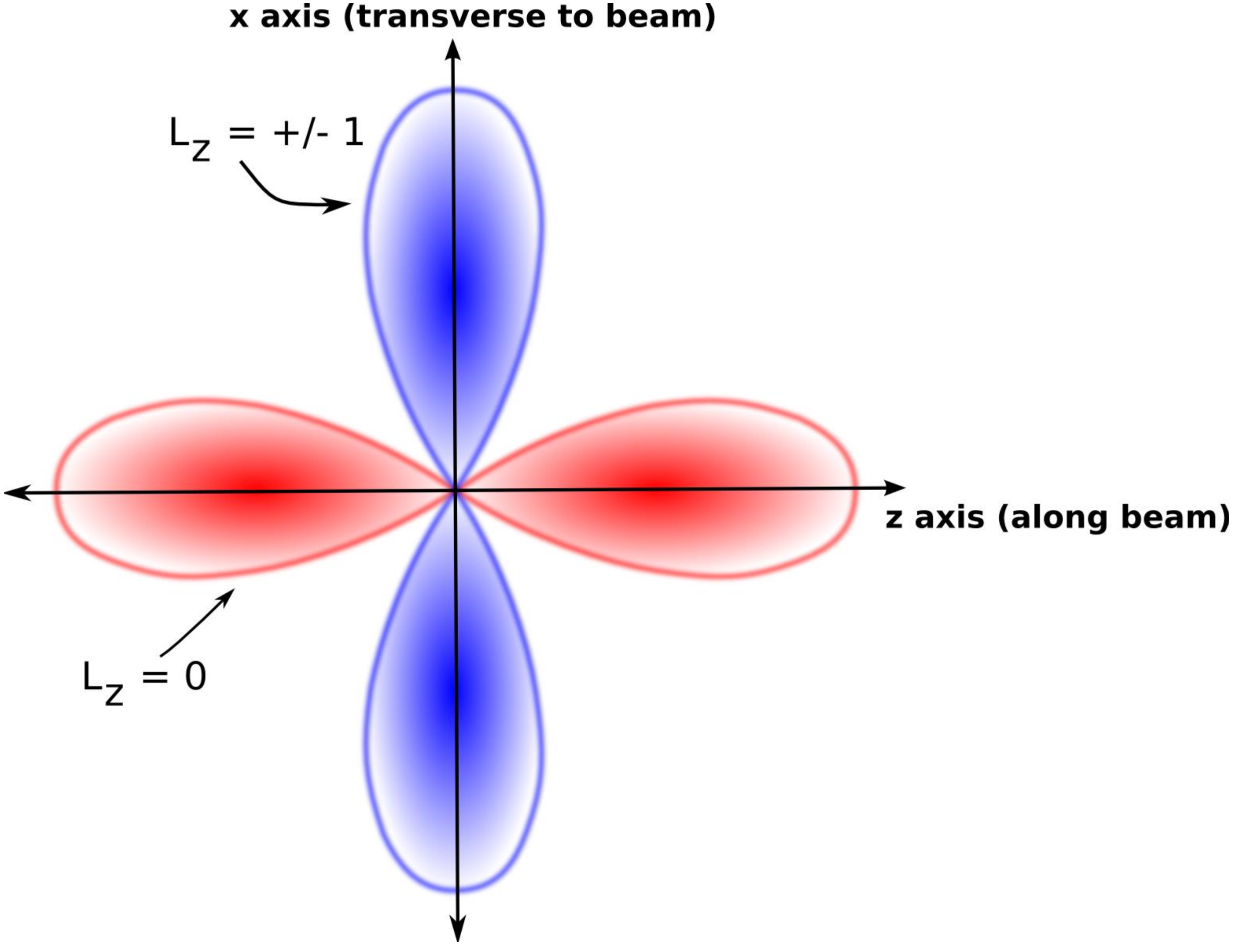
成立！

Backup



- **Transport coefficients in QCD**

✓ Parametric behavior: weak coupling and massless quarks

$$\eta \sim \frac{T^3}{\alpha_s^2 \log \frac{1}{\alpha_s}} \qquad \zeta \sim \frac{\alpha_s^2 T^3}{\log \frac{1}{\alpha_s}}$$

(Arnold, Moore & Yaffe, JHEP 2000, 2003)

(Arnold, Dogan & Moore , Phys. Rev. D 2006)

Bulk viscosity: suppressed by two powers of $\sim \alpha_s^2$

✓ the bulk viscosity to entropy density ratio increase, as T approaches Tc

(Kharzeev & Tuchin, JHEP 2008)

(Karsch, Kharzeev & Tuchin, Phys. Letts. B 2008)

The Perturbative Heavy Quark Potential at Finite Temperature

- **Shear viscous corrections to the binding energy and decay width of heavy quarkonia:**

for extremely heavy bound states: Bohr radii $\sim 1/(g^2 M_Q) \ll$ screening length $\sim 1/m_D$

Coulombic contribution dominates

✓ **treating the medium effect as a perturbation of the vacuum Coulomb potential provides an estimate for the binding energy**

$$V(\mathbf{r}) \;=\; V_{\mathrm{vac}}(r) + \alpha m_D - \frac{\alpha \xi m_D}{6} + \ldots \quad \Longrightarrow \quad E_{\mathrm{bin}} \approx E_{\mathrm{vac}} + \alpha m_D - \frac{\alpha \xi m_D}{6}$$

✓ **treating the (imaginary) potential as a perturbation of the vacuum Coulomb potential provides an estimate for the decay width**

$$\Gamma \;=\; \frac{g^2 C_F T}{4\pi} \int \mathrm{d}^3\mathrm{r}\, |\Psi(r)|^2\, \hat{r}^2 \ln\frac{1}{\hat{r}} \left(\frac{1}{3} - \xi\frac{3-\cos(2\theta)}{20}\right) \;=\; \frac{16\pi T}{g^2 C_F}\frac{m_D^2}{M_Q^2}\left(1-\frac{\xi}{2}\right)\ln\frac{g^2 C_F M_Q}{8\pi m_D}$$

Density evolution **Iso vs. Aniso**