Small-x Physics in Proton-Nucleus Collisions and at the Future Electron Ion Collider

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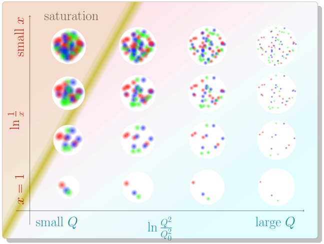
Small-x Physics

QCD matter (Color Glass Condensate) at extremely high gluon density



$$\sigma_0 \sum_{i=0}^{\infty} \alpha_s^i \left(L^i + C^{(i)}\right) \quad \text{ideal QCD expansion}$$

$$\begin{array}{c|c} \sigma_0 \sum_{i=0}^{n-1} \alpha_s^i L^i & \sigma_0 \sum_{i=0}^{n-1} \alpha_s^i C^{(i)} \\ \hline \sigma_0 \sum_{i=n}^{\infty} \alpha_s^i L^i & \sigma_0 \sum_{i=n}^{\infty} \alpha_s^i C^{(i)} \end{array} \quad \Leftarrow \text{pQCD}$$

$\Uparrow$ resummation

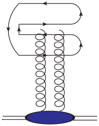
$\nwarrow$ negligible

- Gluon density rises rapidly in the low-x limit with fixed Q^2. When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine
  $\Rightarrow$ Non-linear QCD dynamics (BK equation) $\Rightarrow$ saturation in gluon distributions.
- From QCD expansion point of view, various types of resummations often is vital to get reliable results for a given physical processes.
- Core ingredients: Multiple interactions (tree) + Small-x (high energy) evolution (loop, Resummation of the $\alpha_s \ln \frac{1}{x}$).
- Introduce $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Gluons at small-x carry typical transverse momentum of order $Q_s(x)$. (Cf. Collinear pdf)

A Tale of Twin Gluon Distributions—-绝代双“胶”

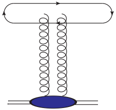
For many years, we have know that there are two different gluon distributions:
I. Weizsäcker Williams gluon distribution [McLerran, Venugopalan, 98; Kovchegov, Mueller, 98]

$xG^{(1)}$



II. Color Dipole gluon distribution [known for three decades]

$xG^{(2)}$



- Why there are two gluon distributions? How to distinguish them in experiment?
- [F. Dominguez, B. Xiao and F. Yuan, 2011]
- Quadrupole $\Rightarrow$ Weizsäcker Williams gluon distribution;
- Dipole $\Rightarrow$ Color Dipole gluon distribution.
- Small-x evolutions for these two gluon distributions are w.r.t. Quadrupole and Dipole objects in coordinate space.

A Tale of Two Gluon Distributions—-绝代双“胶”

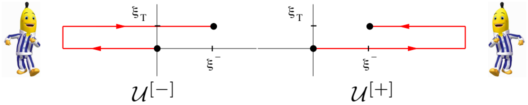
In terms of operators (TMD def. [Bomhof, Mulders and Pijlman, 06]), two gauge invariant gluon definitions: [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution: conventional gluon distributions

$$xG_{\mathrm{WW}}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG_{\mathrm{DP}}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



- Modified Universality for Gluon Distributions:

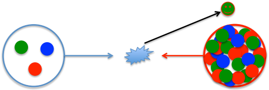
	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
xG_{WW}	$\times$	$\times$	$\surd$	$\times$	$\surd$
xG_{DP}	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

$\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

- Measurements in pA collisions and at the EIC are tightly connected with complementary physics missions.

Forward hadron production in pA collisions

Single inclusive forward hadrons in pA collisions, [Dumitru, Jalilian-Marian, 02]

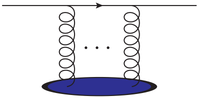


projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

Dilute-dense factorization at forward rapidity

$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2} \left[\sum_f x_p q_f(x_p) \mathcal{F}(k_\perp) D_{h/q}(z) + x_p g(x_p) \tilde{\mathcal{F}}(k_\perp) D_{h/g}(z) \right].$$



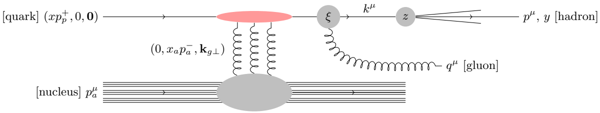
$$\Rightarrow \quad U(x_\perp) = \mathcal{P} \exp \left\{ i g_S \int_{-\infty}^{+\infty} \mathrm{d}x^+ \, T^c A_c^-(x^+, x_\perp) \right\},$$

$$\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp).$$

- Proton PDFs are under control at large-x, use collinear PDFs and FFs.
- Dense gluons at low-x in the nucleus target is described by saturation or CGC.
- $S_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c} \left\langle {\rm Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle_Y$ with $Y \sim \ln 1/x_g$.

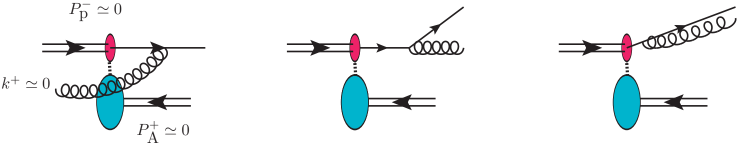
Forward rapidity single hadron productions in pA collisions

Dilute-Dense factorizations: large x proton or γ^* $\rightarrow$ as dilute probe:



$$x_p = \frac{k_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{dilute}$$
$$x_A = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{dense}$$

- LO [Dumitru, Jalilian-Marian, 02]: probing $xG_{\rm DP}(x, k_\perp)$ at small-x.
- NLO Cutoff[Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11]
- NLO Complete NLO in DR: [Chirilli, BX and Yuan, 12].
  1. 1. soft, collinear to the target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$. Subtraction scheme is not Unique but highly constrained.
  2. 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs. $\bar{MS}$ scheme.
  3. 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs, $\bar{MS}$ scheme.
  4. The importance of subtraction: systematic resummation of large logarithms. $(\alpha_s \ln 1/x_g)$, which allows us to have $\mathcal{H} \sim \mathcal{O}(\alpha_s)$. Interesting recent development: RG approach and threshold resummation.



Hard Factors

One-loop corrections to the single hadron productions in the $q \to q$ channel:

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right.$$
$$\left.+\int\frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\} \quad \text{where}$$

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right)-3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2k_\perp^2}$$
$$-(2C_F-N_c)e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{\left(1+\xi^2\right)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

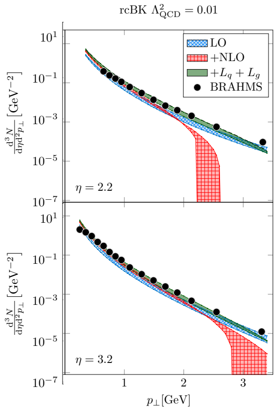
$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right.$$
$$\left.-\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\},$$

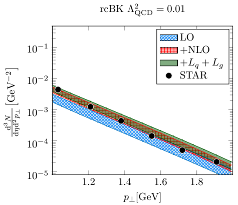
where $$\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\right]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}.$$

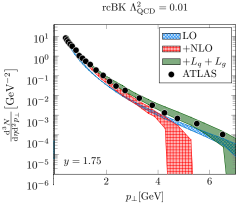
with $\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)$ and additional L_q terms.

Numerical implementation of the NLO result

SOLO (Saturation physics at One Loop Order) results [Stasto, Xiao, Zaslavsky, 13; Watanabe, Xiao, Yuan, Zaslavsky, 15]



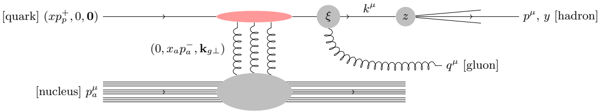




- Agree with RHIC and LHC data in low $p_\perp \leq Q_s$ region where pQCD does not apply.
- SOLO (1.0 and 2.0) break down in the large $p_\perp \geq Q_s$ region($k_\perp \gg Q_s$).
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14; Kang, Vitev and Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller and Triantafyllopoulos, 16]
- Another idea: threshold resummation! The resummation of plus-functions or $\bar{\alpha}_s \ln(1 - x_p) < 0$.

Threshold resummation in the saturation formalism

Dilute-Dense factorizations: large x proton or γ^* $\rightarrow$ as dilute probe:



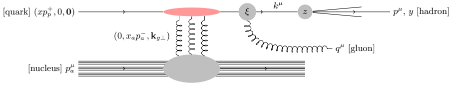
$$x_p = \frac{k_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{dilute}$$
$$x_A = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{dense}$$

- Plus-functions. [Stasto, Zaslavsky, 16]: $\int_{x_p}^1 \frac{d\xi}{(1-\xi)_+} f(\xi) \sim f(1) \ln(1-x_p)$
- It is also the resummation of logarithm $\bar{\alpha}_s \ln(1-x_p) < 0$. For example: let $X = \bar{\alpha}_s \ln(1-x_p)$, $e^X = 1 + X + \frac{1}{2}X^2 + \cdots$
- Tightly related to DGLAP physics due to subtractions of collinear singularities.
- Mellin transform is the technique used to perform resummation. $\frac{1}{(1-\xi)_+}\xi^N \sim\simeq -\ln N$
- Resummation of plus functions can be approximately carried out analytically (RGE or M.T.)

$$f_t(\tau, \mu_b) = \frac{e^{\gamma_\beta - \gamma_E \gamma_{\mu,b_\perp}}}{\Gamma[\gamma_{\mu,b_\perp}]} \int_\tau^1 \frac{dx}{x} f(x,\mu) \left[\ln \frac{x}{\tau}\right]_*^{\gamma_{\mu,b_\perp}-1}$$

Threshold resummation in the saturation formalism

[Xiao, Yuan, 18; Wei, Xiao, Yuan, *et al*, numerical work in process]



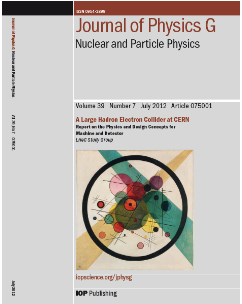
- Near the threshold at fixed forward rapidity, the gluon emission is forced to be soft. The intricate interplay between the soft and collinear emission of gluons also leads to interesting Sudakov like double logs as well.
- Note: this is not traditional Sudakov, which needs two kinematical scales.
- The objective is to identify large logarithms $\ln(1-x_p)$ and $\ln k_\perp^2/Q_s^2$ in the large $k_\perp$ region ($k_\perp \gg Q_s$) near threshold at fixed rapidity. In fact, these two logs seem to always appear together in our calculation.
- Many different threshold resummation formalism. We find remarkable similarities between the threshold resummation in pA collisions in the small-x formalism and threshold resummation in SCET[Becher, Neubert, 06].
- The forward threshold jet function $\Delta(\mu^2, \mu_b^2, z)$ satisfies a RGE

$$\frac{d\Delta(\mu^2, \mu_b^2, z)}{d\ln\mu} = -\frac{2\alpha_s N_c}{\pi}\left[\ln z + \beta_0\right]\Delta(\mu^2, \mu_b^2, z) + \frac{2\alpha_s N_c}{\pi}\int_0^z dz' \frac{\Delta(\mu^2, \mu_b^2, z) - \Delta(\mu^2, \mu_b^2, z')}{z - z'}.$$

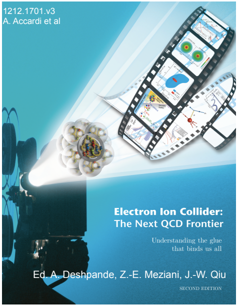
Into the future

Electron Ion Collider (LHeC)

arXiv:1108.1713



arXiv:1206.2913

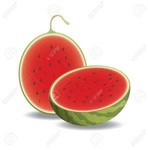
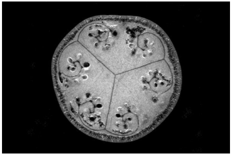


arXiv:1212.1701

- The proposed cutting-edge EIC can give us the opportunity to understand proton spin puzzle and discover the gluon saturation phenomenon.
- EIC will be a fantastic stereoscopic “camera” with extremely high resolution, which allows us to visualise protons and nuclei in a multi-dimensional fashion. The ultimate goal in the hadronic structure physics research.

Three-dimensional Imaging

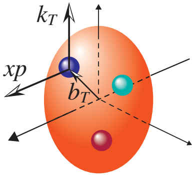
Let us view a proton as a watermelon: seeds $\rightarrow$ quarks and pulps $\rightarrow$ gluons

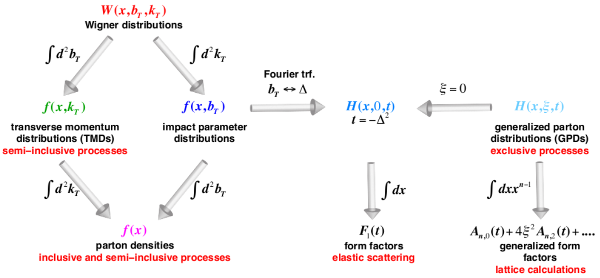




- DIS (smashing)$\rightarrow$discovery of quarks, measurement of PDFs.
- SIDIS (cutting) $\rightarrow$ multiple-dimensional information of parton distribution
- Diffractive scattering (CT scanning) $\rightarrow$ see the inside of a watermelon without cutting! Also provides 3D digital information!
- 3D imaging and spin [Hatta, Xiao, Yuan, 16; Zhou, 16; Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Xiao, Yuan, Zhao, 16]

3D Tomography of Proton

Wigner distributions ingeniously encode all quantum information of how partons are distributed inside hadrons. [Ji, 03; Belitsky, Ji, Yuan, 03]

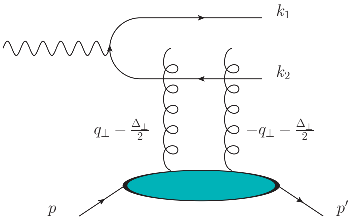




- Small-x gluon distributions $\Leftrightarrow$ gluon Wigner distributions? [Ji, 03]
- TMDs and GPDs can be studied and measured in various processes.
- Can we measure the gluon Wigner distribution at small-x? Yes, we can!
- Impact on the spin side of EIC: gluon OAM [Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Yuan, Zhao, 16, Bhatttacharya, Metz, Zhou, 17]

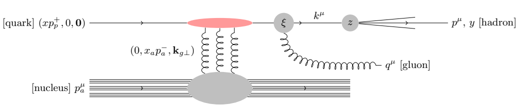
Probing 3D Tomography of Proton at small-x

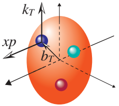
Diffractive back-to-back dijet productions in DIS [Hatta, Xiao, Yuan, 16]



- Find connections between dipole amplitude and Wigner distribution at low-x.
- Measure final state proton recoil $\Delta_\perp$ as well as dijet momentum $k_{1\perp}$ and $k_{2\perp}$.
- We can approximately access $|xG_{\rm DP}(x, q_\perp, \Delta_\perp)|^2$ in the back-to-back limit in which $q_\perp \simeq P_\perp \equiv \frac{1}{2}(k_{2\perp} - k_{1\perp}) \gg \Delta_\perp$.
- Cross-Sections are positive-definite, although Wigner distributions may not be.
- WW Wigner (WWW) distribution may be also defined and measured.

Summary





- Rich physics in dilute-dense factorization formalism. (Multiple scattering, small-x resummation $\alpha_s \ln 1/x_g$, collinear logarithms $\alpha_s \ln \frac{Q^2}{\mu^2}$, Sudakov resummation $\alpha_s \ln^2 \frac{P_\perp^2}{q_\perp^2}$ and threshold resummation $\alpha_s \ln(1 - x_p)$, etc.)
- Reliable higher order calculations and robust predictions, as well as new ideas are emerging. Threshold Resummation!
- EIC will be a superb "stereoscopic camera", which allows us to depict 3D the internal structure of protons and heavy nuclei with unprecedented precision and significantly advance our knowledge of hadron structure.
- Complementary studies in pA collisions and the future EIC can give us the opportunity to discover the gluon saturation phenomenon.
- 热烈祝贺“量子物质研究院” 的成立！.