

Strong decays of doubly charmed baryons and higher charmonium states in 3P_0 model

PRD97,074005(2018)

EPJC78,605(2018)

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- hadron spectroscopy
- 3p_0 model
 - doubly charmed baryons
 - charmonium states
- summary and outlook

Hadron spectroscopy: we know 6 quarks & 6 leptons

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c ²	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times 10^{-9}$	0
e electron	0.000511	-1
ν_M middle neutrino*	$(0.009-0.13)\times 10^{-9}$	0
μ muon	0.106	-1
ν_H heaviest neutrino*	$(0.04-0.14)\times 10^{-9}$	0
τ tau	1.777	-1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

We know four types of interactions

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at	10^{-18} m	0.8	1	25
	3×10^{-17} m	10^{-4}	1	60

- Gravity is responsible for the structure of the Universe
- Electromagnetic interaction → the molecules and atoms
- Weak interaction → the stars shine
- Strong interaction → the structure of the nuclei, nucleons, hadronic matters from the building blocks ---quarks! **but HOW?**

We know quark pair makes mesons and three quarks makes baryons



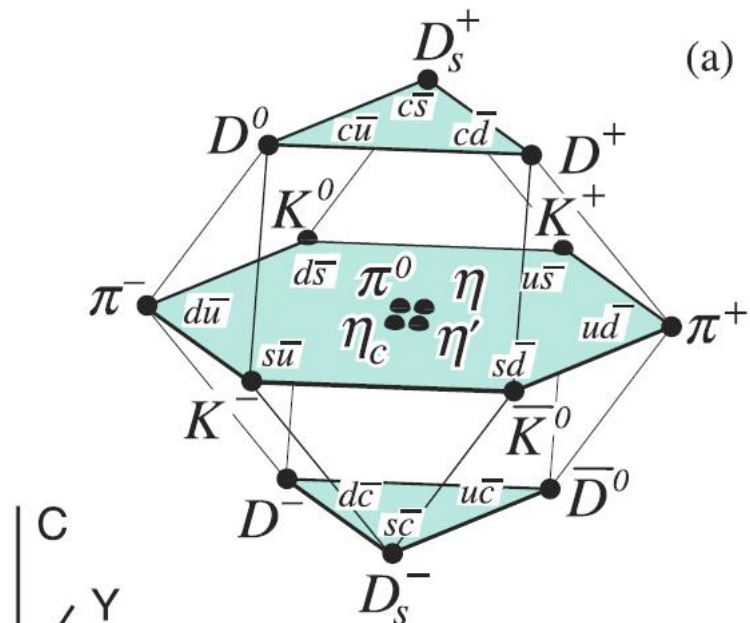
M. Gell-Mann

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" q and the members of the anti-triplet as anti-quarks \bar{q} . **Baryons** can now be constructed from quarks by using the combinations **(qqq) , $(qqqq\bar{q})$, etc.**, while **mesons** are made out of **$(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc.** It is assuming that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**.

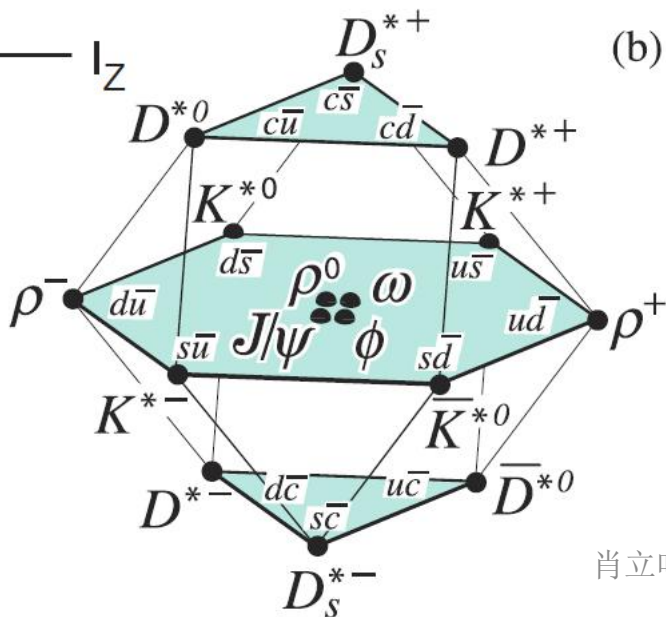
Published in Physics Letters 8, 214 (1964);
Similar idea by G. Zweig, CERN-TH-401 (1964).

We can put (all of) them in a simple picture!

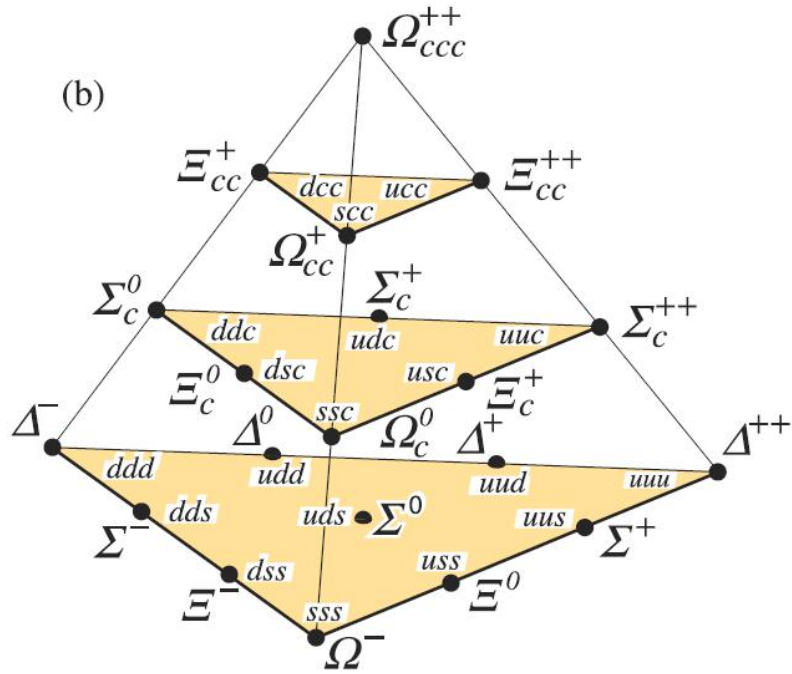
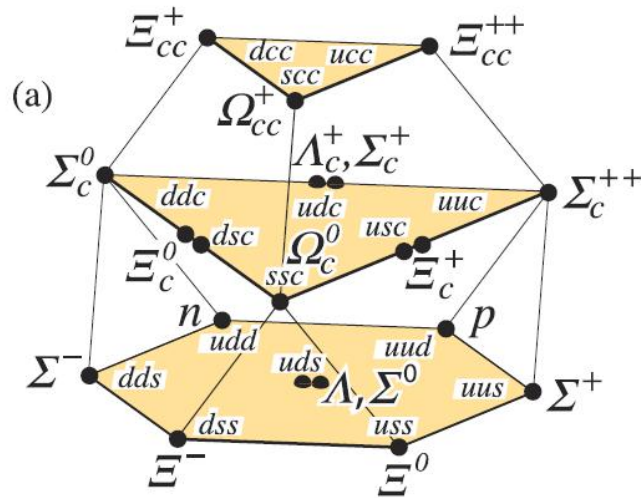
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J=1



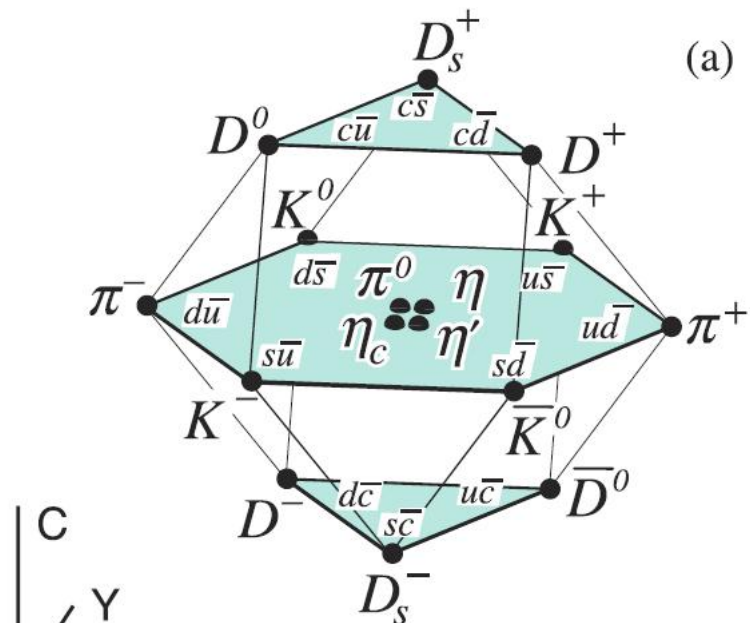
J=1/2



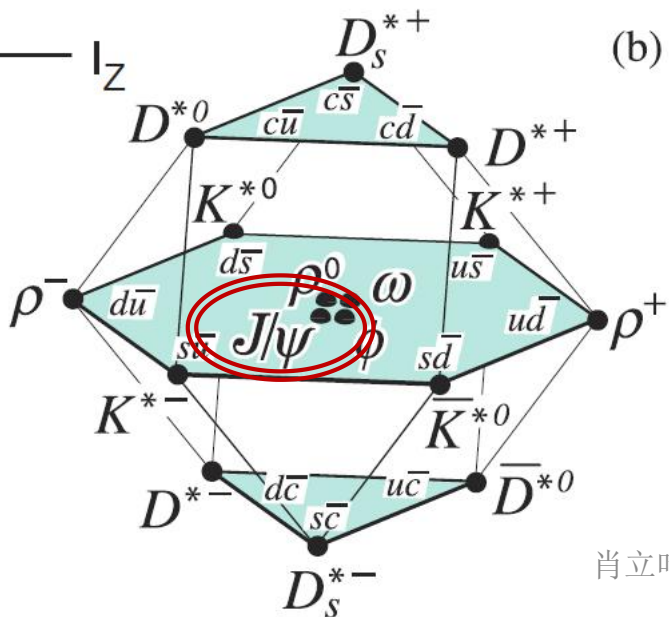
J=3/2

We can put (all of) them in a simple picture!

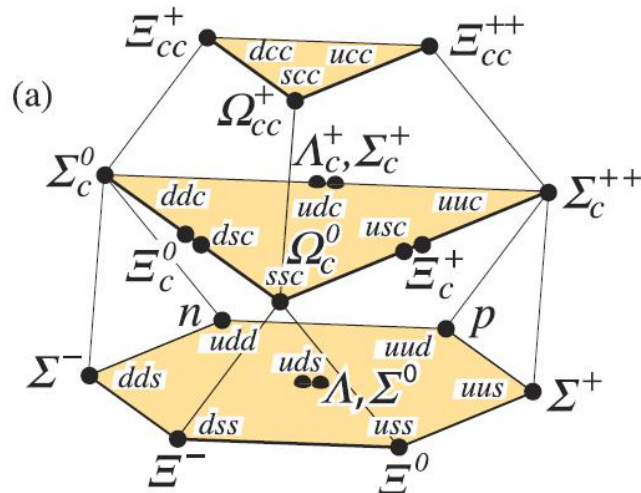
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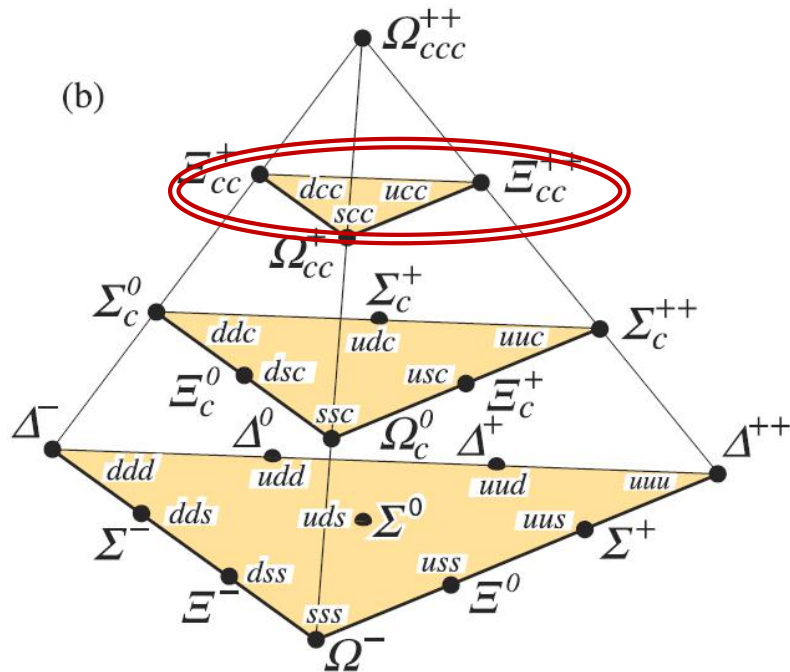
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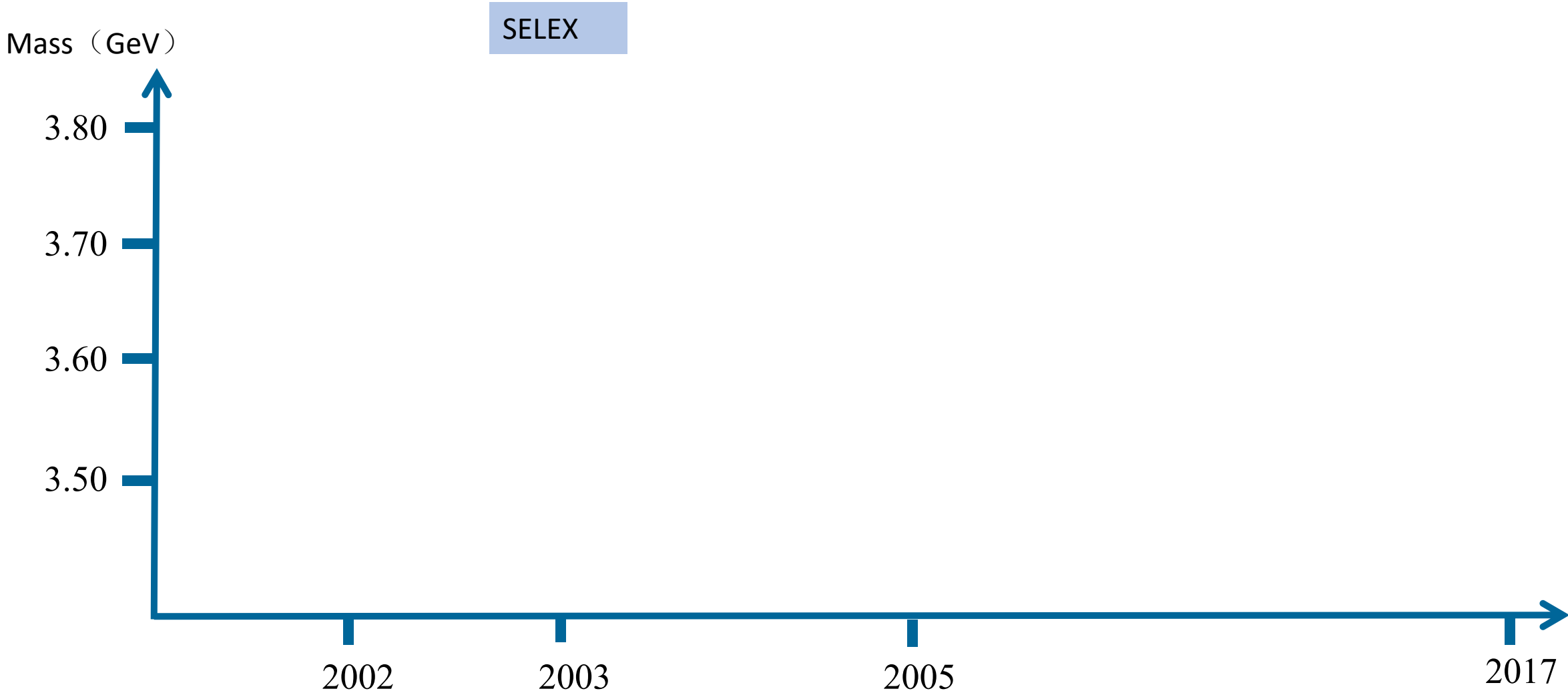
J=1/2



J=3/2

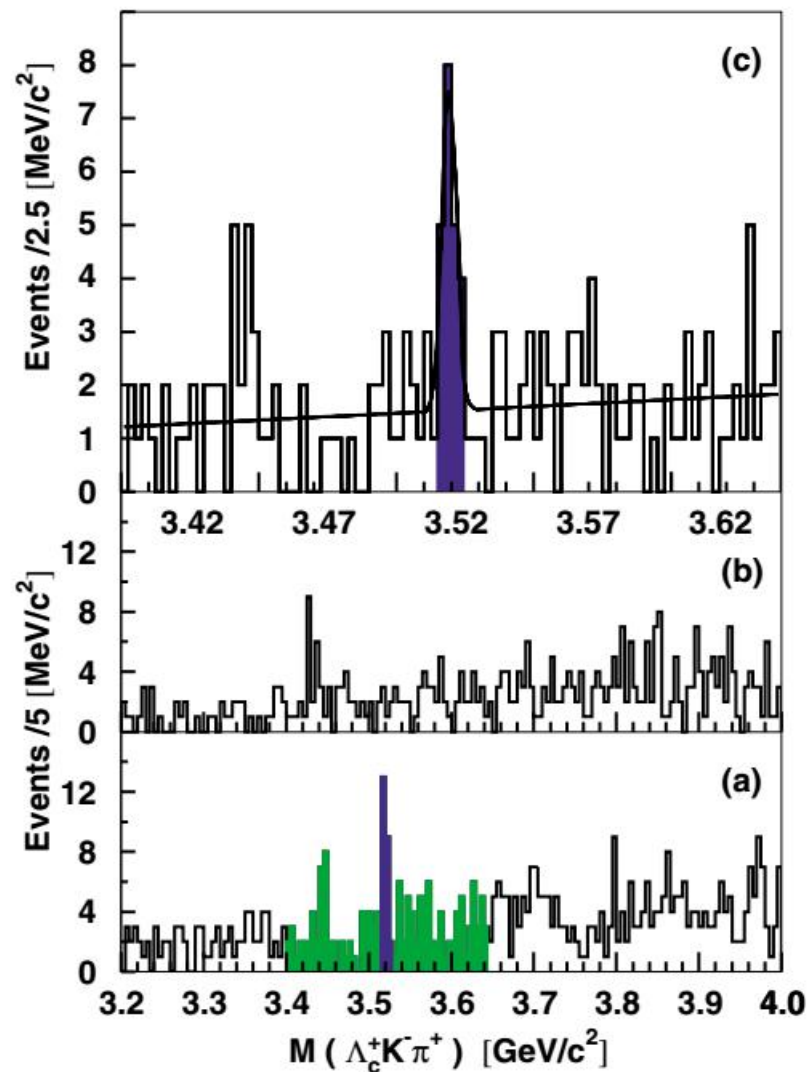
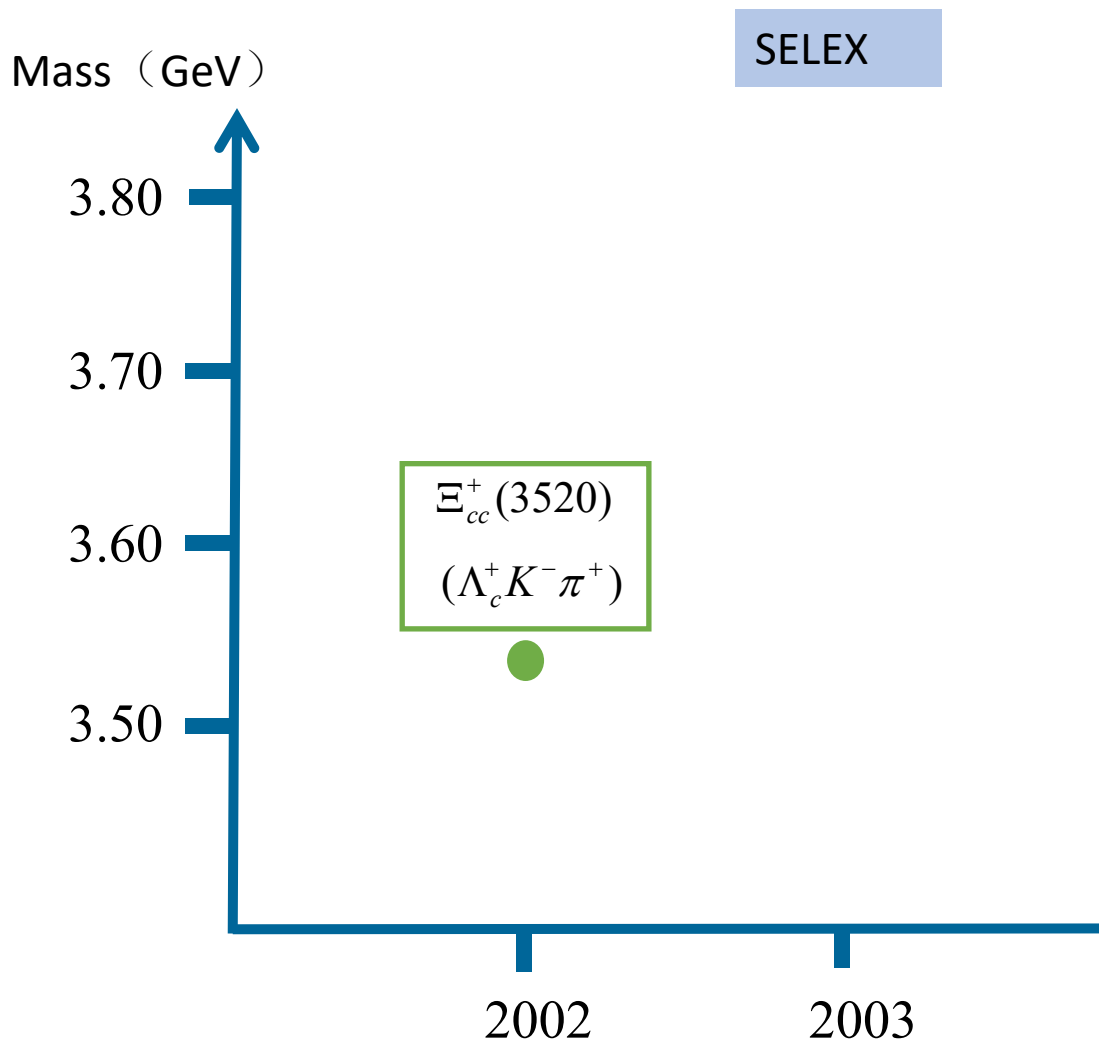


Doubly charmed baryons:



Doubly charmed baryons:

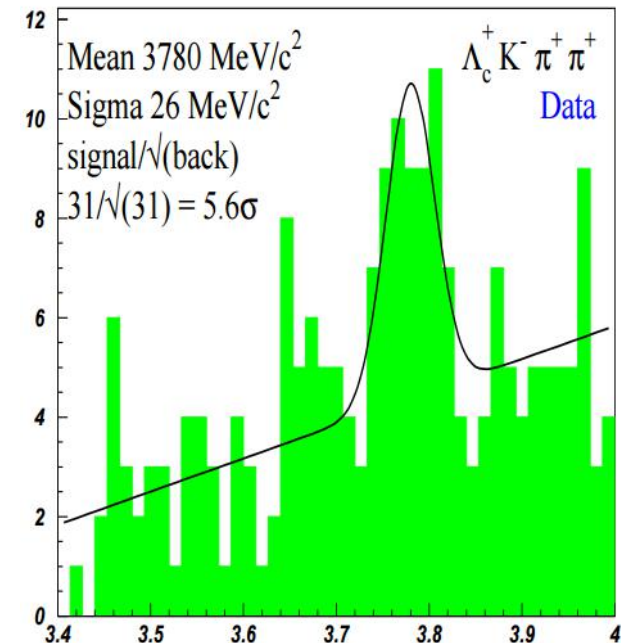
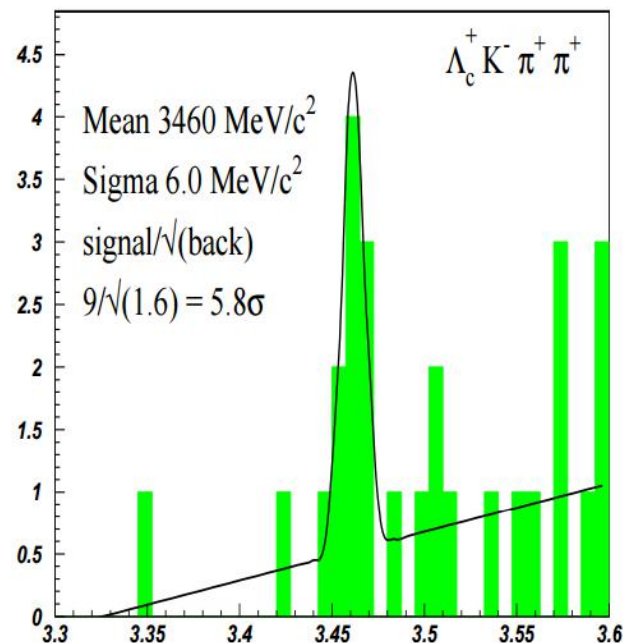
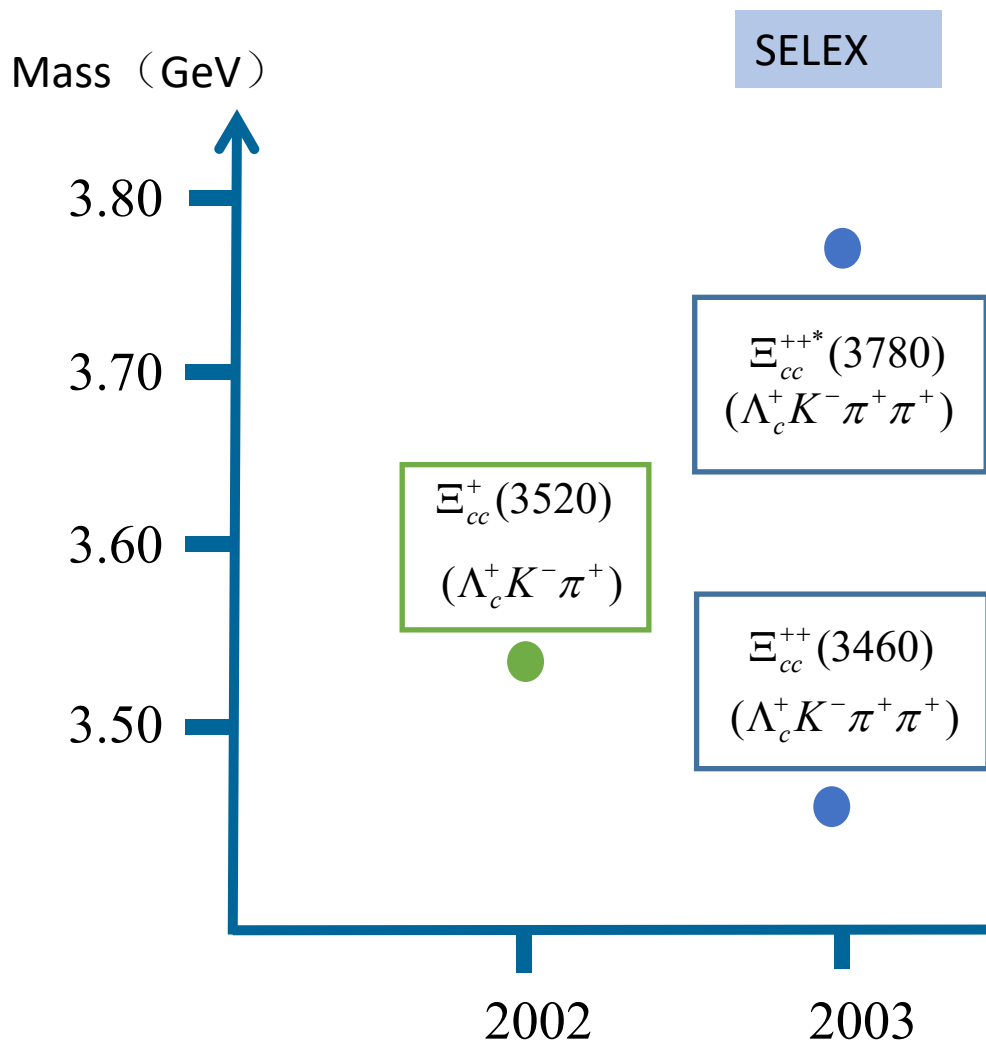
SELEX Collaboration PRL89,112001 (2002).



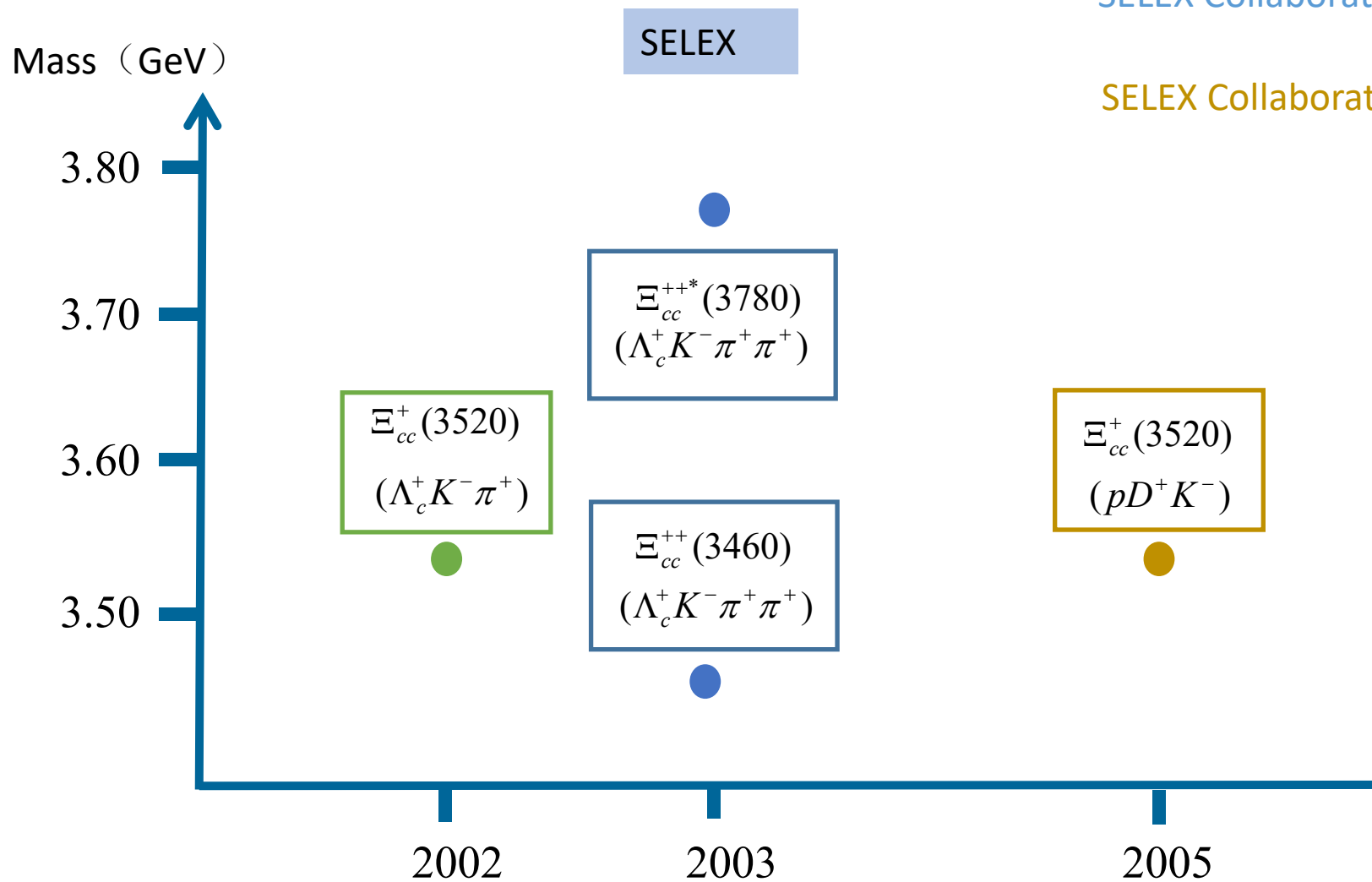
Doubly charmed baryons:

SELEX Collaboration
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PRL89,112001 (2002).
CJP53,B201 (2003).



Doubly charmed baryons:

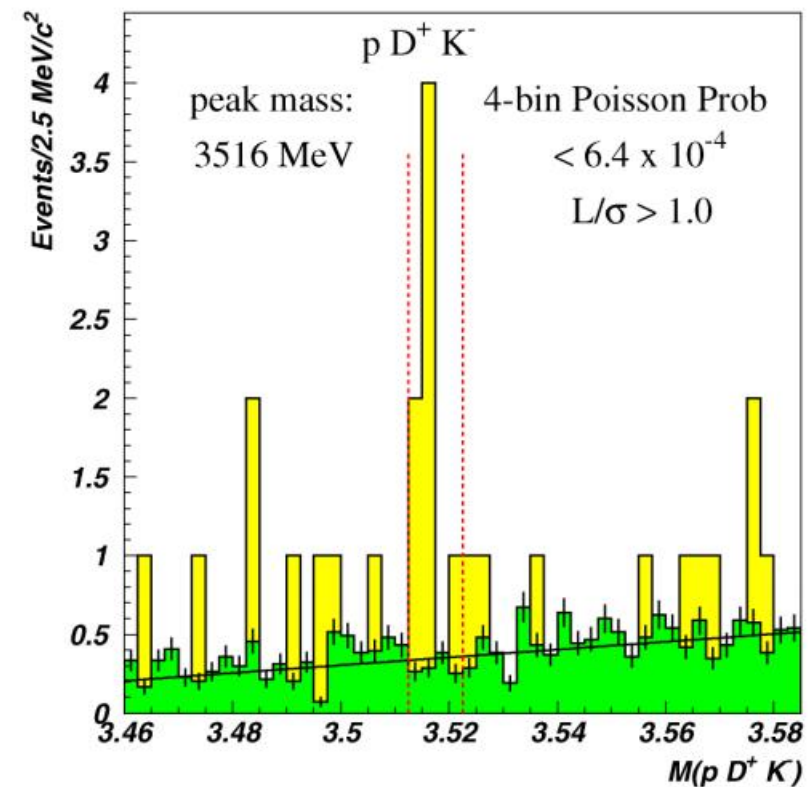


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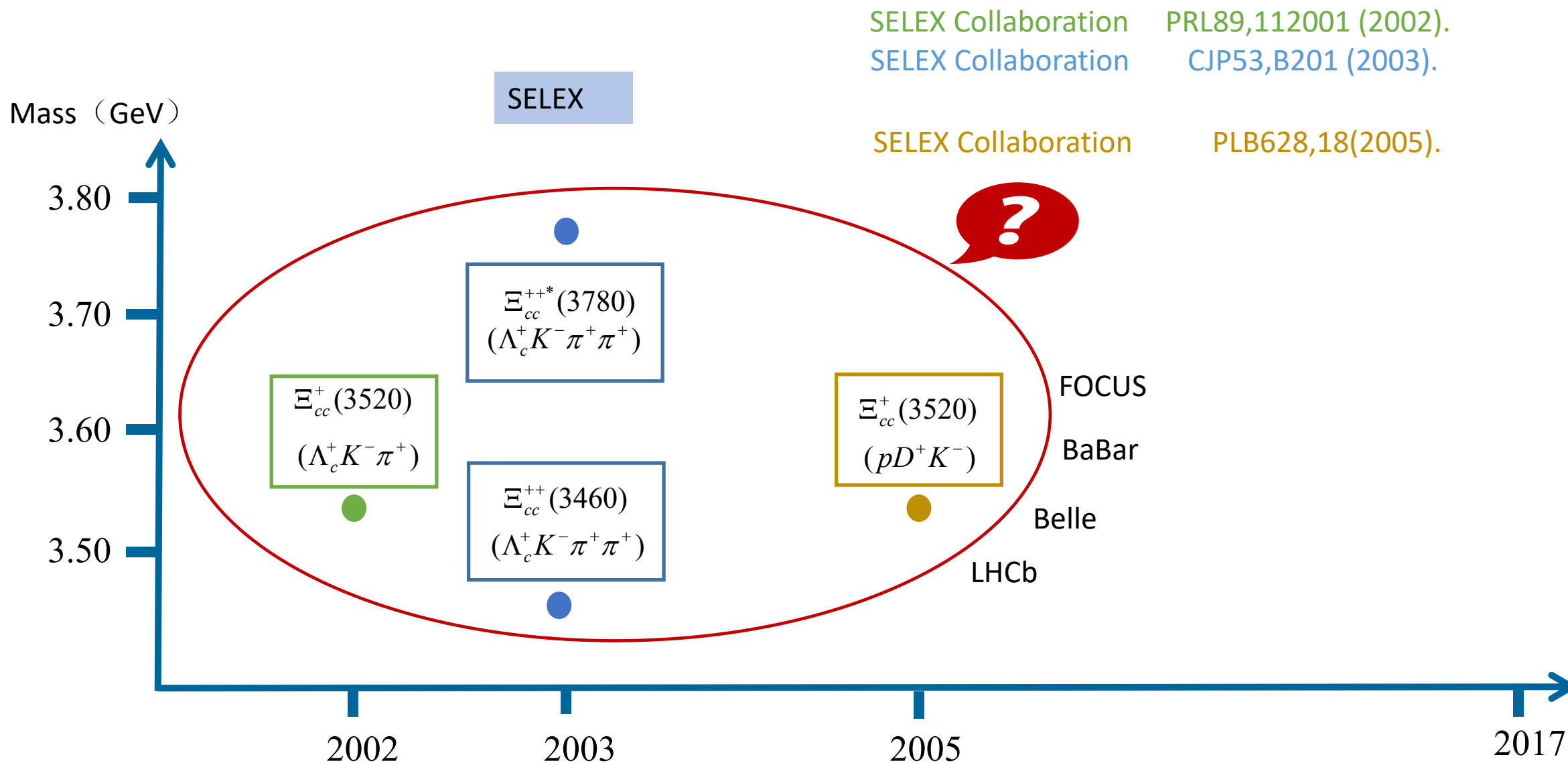
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SELEX Collaboration

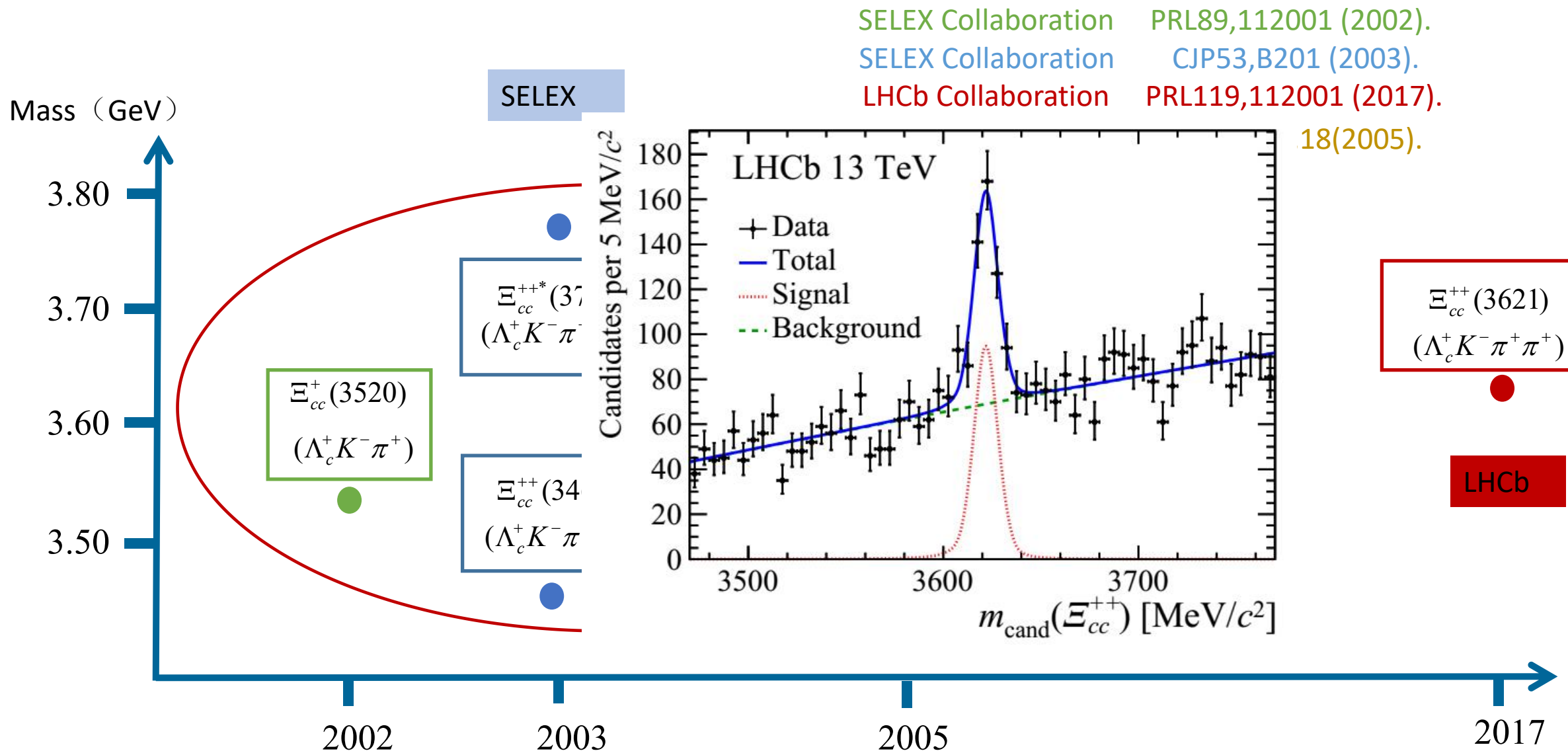
PLB628,18(2005).



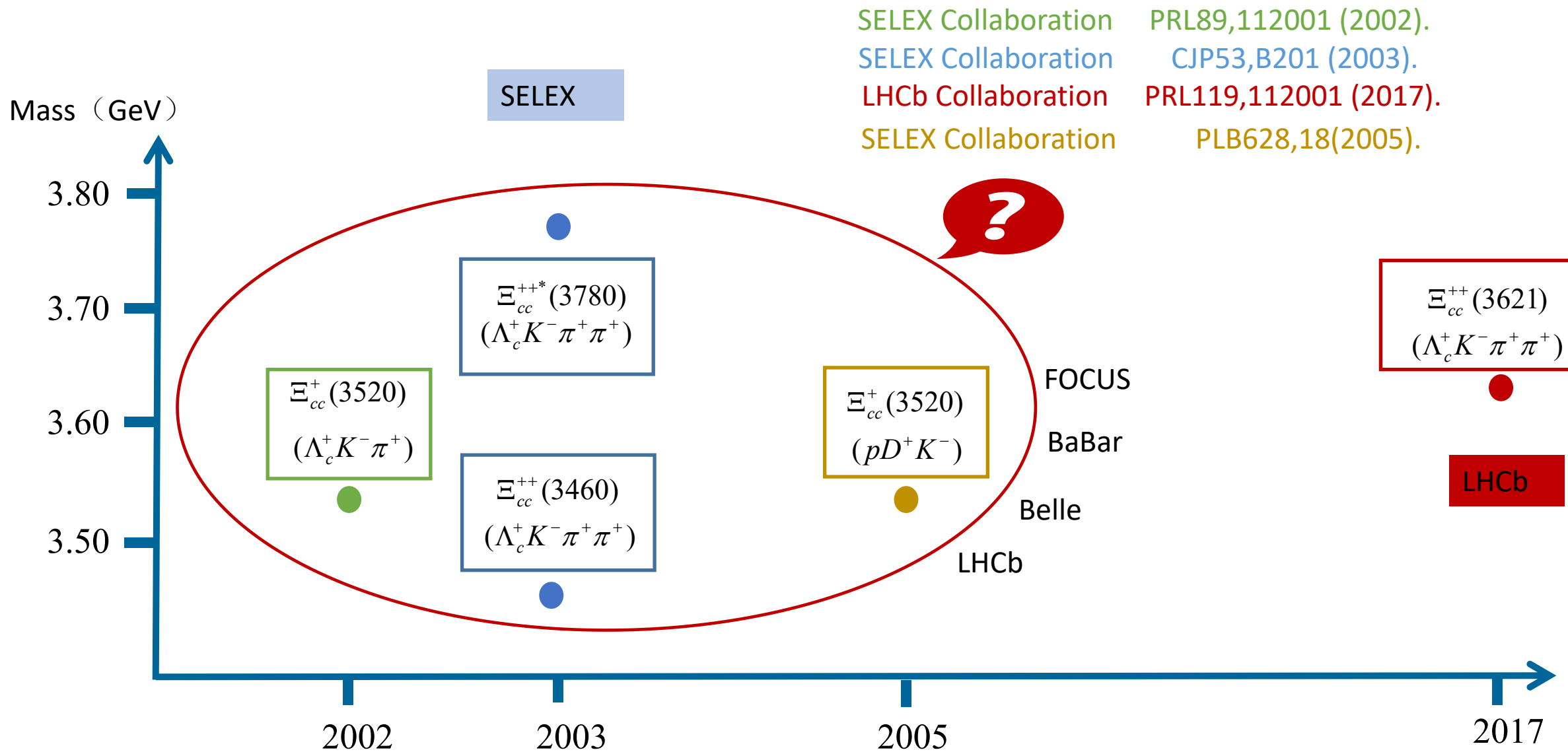
Doubly charmed baryons:



Doubly charmed baryons:



Doubly charmed baryons:



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1. First observation of the doubly charmed baryon Ξ_{cc}^{++}

SELEX Collaboration (M. Mattson (Carnegie Mellon U.) *et al.*). Aug 2002. 5 pp.

Published in **Phys.Rev.Lett.** **89 (2002) 112001**

FERMILAB-PUB-02-183-E

DOI: [10.1103/PhysRevLett.89.112001](https://doi.org/10.1103/PhysRevLett.89.112001)

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1. Observation of the doubly charmed baryon Ξ_{cc}^{++}

LHCb Collaboration (Roel Aaij (CERN) *et al.*). Jul 5, 2017. 10 pp.

Published in **Phys.Rev.Lett.** **119 (2017) no.11, 112001**

LHCB-PAPER-2017-018, CERN-EP-2017-156

DOI: [10.1103/PhysRevLett.119.112001](https://doi.org/10.1103/PhysRevLett.119.112001)

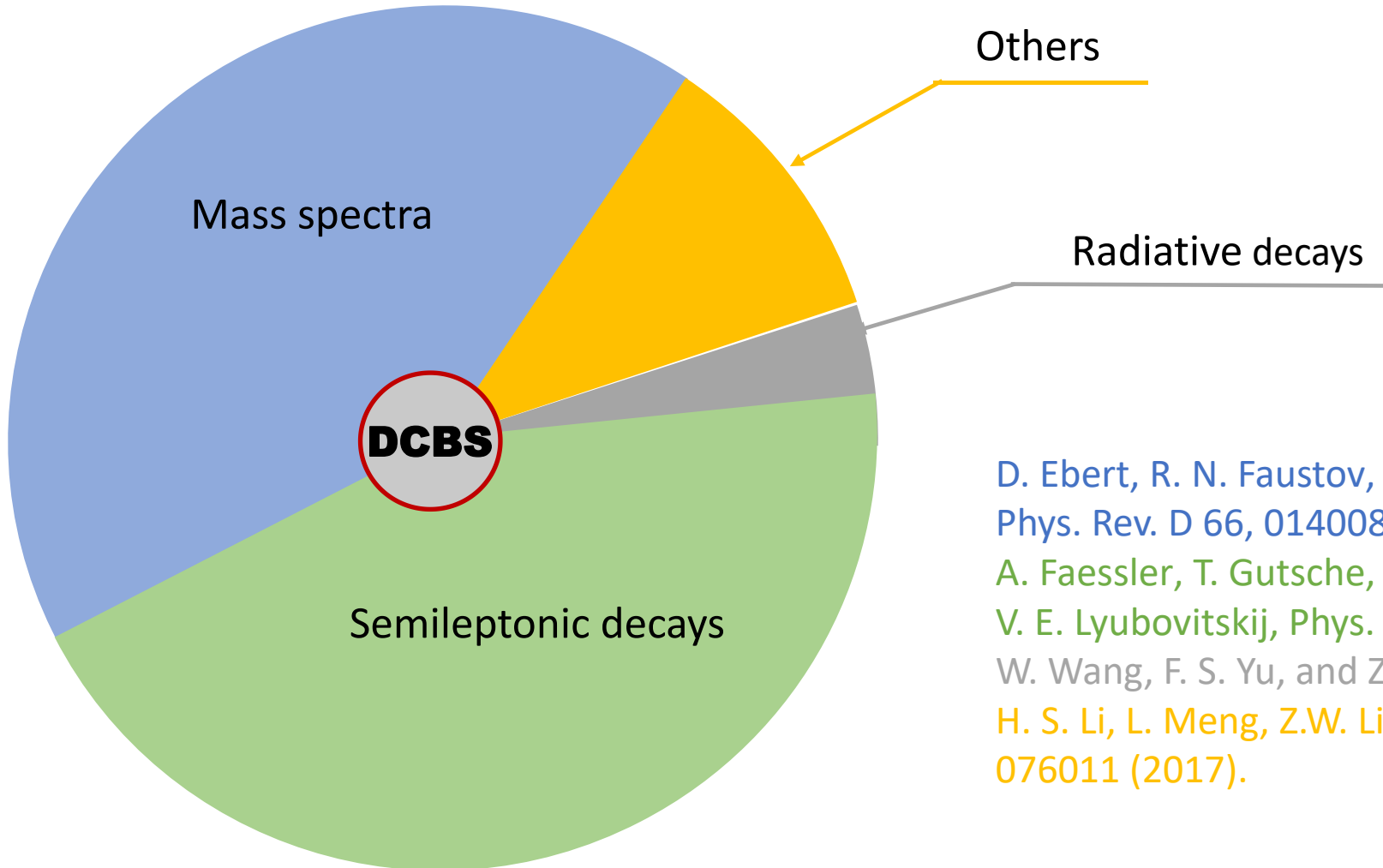
e-Print: [arXiv:1707.01621 \[hep-ex\]](#) | [PDF](#)

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W. Wang, F. S. Yu, and Z. X. Zhao, arXiv:1707.02834.

H. S. Li, L. Meng, Z.W. Liu, and S. L. Zhu, Phys. Rev. D 96, 076011 (2017).

TABLE I: Masses and possible two body strong decay channels of the $1P$ and $2D$ doubly charmed baryons (denoted by $|N^{2S+1}L_\sigma J^P\rangle$), where $|N^{2S+1}L_\sigma J^P\rangle = \sum_{L_z+S_z=J_z} \langle LL_z, SS_z | JJ_z \rangle^N \Psi_{LL_z}^\sigma \chi_{S_z} \phi$ [46]. The masses (MeV) are taken from the relativistic quark model [20].

State	Wave function	Mass [20]	Ξ_{cc} Strong decay channel	Ω_{cc} Strong decay channel
$ 0^2S \frac{1}{2}^+\rangle$	${}^0\Psi_{00}^S \chi_{S_z}^\lambda \phi$	3620
$ 0^4S \frac{3}{2}^+\rangle$	${}^0\Psi_{00}^S \chi_{S_z}^\lambda \phi$	3727
$ 1^2P_\rho \frac{1}{2}^-\rangle$	${}^1\Psi_{1L_z}^\rho \chi_{S_z}^\rho \phi$	3838
$ 1^2P_\rho \frac{3}{2}^-\rangle$		3959
$ 1^2P_\lambda \frac{1}{2}^-\rangle$	${}^1\Psi_{1L_z}^\lambda \chi_{S_z}^\lambda \phi$	4136	$\Xi_{cc}^{(*)} \pi$	$\Xi_{cc}^{(*)} K$
$ 1^2P_\lambda \frac{3}{2}^-\rangle$		4196	$\Xi_{cc}^{(*)} \pi$	$\Xi_{cc}^{(*)} K$
$ 1^4P_\lambda \frac{1}{2}^-\rangle$	${}^1\Psi_{1L_z}^\lambda \chi_{S_z}^s \phi$	4053	$\Xi_{cc}^{(*)} \pi$	$\Xi_{cc} K$
$ 1^4P_\lambda \frac{3}{2}^-\rangle$		4101	$\Xi_{cc}^{(*)} \pi$	$\Xi_{cc}^{(*)} K$
$ 1^4P_\lambda \frac{5}{2}^-\rangle$		4155	$\Xi_{cc}^{(*)} \pi$	$\Xi_{cc}^{(*)} K$
$ 2^2D_{\rho\rho} \frac{3}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\rho\rho} \chi_{S_z}^\lambda \phi$		$\Lambda_c D$	$\Xi_c D$
$ 2^2D_{\rho\rho} \frac{5}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D$	$\Xi_c D, \Xi_c^{(*)} D$
$ 2^4D_{\rho\rho} \frac{1}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\rho\rho} \chi_{S_z}^s \phi$		$\Lambda_c D$	$\Xi_c D$
$ 2^4D_{\rho\rho} \frac{3}{2}^+\rangle$			$\Lambda_c D, \Sigma_c D$	$\Xi_c D, \Xi_c' D$
$ 2^4D_{\rho\rho} \frac{5}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D$	$\Xi_c D, \Xi_c^{*} D$
$ 2^4D_{\rho\rho} \frac{7}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c' D_s$	$\Xi_c D, \Xi_c^{(*)} D, \Omega_c^{(*)} D_s$
$ 2^2D_{\lambda\lambda} \frac{3}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\lambda\lambda} \chi_{S_z}^\lambda \phi$		$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c^{(*)} D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{(*)} D, \Omega_c^{(*)} D_s, \Omega_{cc}^{(*)} \eta'$
$ 2^2D_{\lambda\lambda} \frac{5}{2}^+\rangle$			$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c^{(*)} D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{(*)} D, \Omega_c^{(*)} D_s, \Omega_{cc}^{(*)} \eta'$
$ 2^4D_{\lambda\lambda} \frac{1}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\lambda\lambda} \chi_{S_z}^s \phi$		$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c' D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{(*)} D, \Omega_c D_s$
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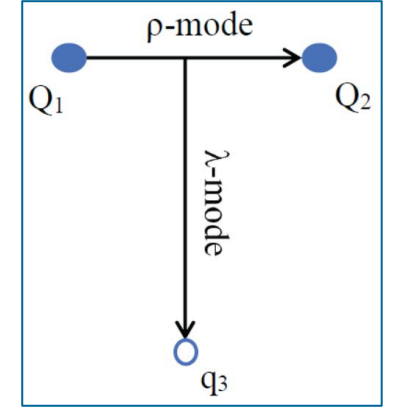
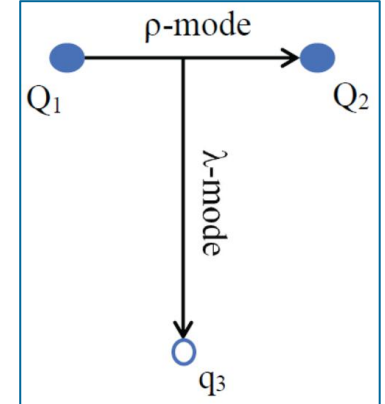


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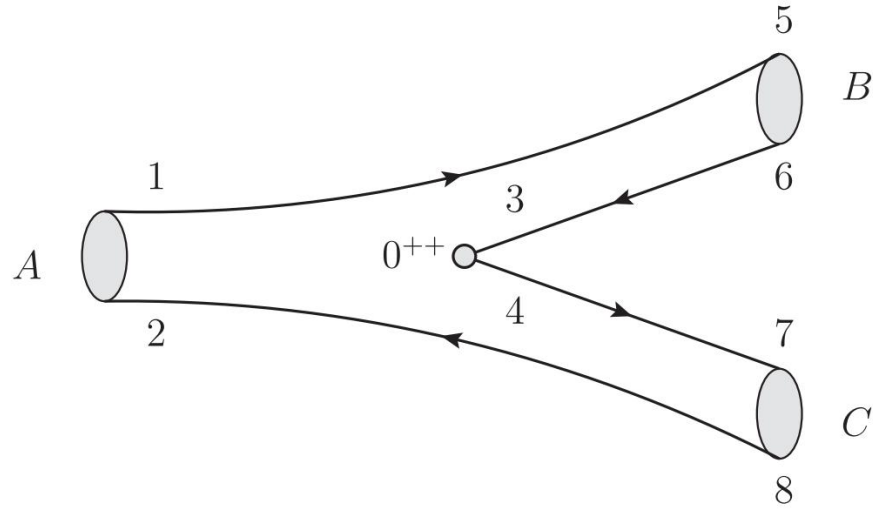
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$ 2^2D_{\rho\rho} \frac{5}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D$	$\Xi_c D, \Xi_c^{\prime(*)} D$
$ 2^4D_{\rho\rho} \frac{1}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\rho\rho} \chi_{S_z}^s \phi$		$\Lambda_c D$	$\Xi_c D$
$ 2^4D_{\rho\rho} \frac{3}{2}^+\rangle$			$\Lambda_c D, \Sigma_c D$	$\Xi_c D, \Xi_c' D$
$ 2^4D_{\rho\rho} \frac{5}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D$	$\Xi_c D, \Xi_c^{*} D$
$ 2^4D_{\rho\rho} \frac{7}{2}^+\rangle$			$\Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c' D_s$	$\Xi_c D, \Xi_c^{(*)} D, \Omega_c^{(*)} D_s$
$ 2^2D_{\lambda\lambda} \frac{3}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\lambda\lambda} \chi_{S_z}^\lambda \phi$		$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c^{\prime(*)} D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{\prime(*)} D, \Omega_c^{(*)} D_s, \Omega_{cc}^{(*)} \eta'$
$ 2^2D_{\lambda\lambda} \frac{5}{2}^+\rangle$			$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c^{\prime(*)} D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{\prime(*)} D, \Omega_c^{(*)} D_s, \Omega_{cc}^{(*)} \eta'$
$ 2^4D_{\lambda\lambda} \frac{1}{2}^+\rangle$	${}^2\Psi_{2L_z}^{\lambda\lambda} \chi_{S_z}^s \phi$		$\Xi_{cc}^{(*)} \pi, \Omega_{cc}^{(*)} K, \Lambda_c D, \Sigma_c^{(*)} D, \Xi_c D_s, \Xi_c' D_s$	$\Xi_{cc}^{(*)} K, \Omega_{cc}^{(*)} \eta, \Xi_c D, \Xi_c^{\prime(*)} D, \Omega_c D_s$
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3P₀ model



3P_0 model:

$A(\text{initial meson}) \rightarrow B(\text{final meson}) + C(\text{final meson})$



$0^{++} \begin{cases} \rightarrow l=1 \\ \rightarrow S=1 \end{cases}$

$\longrightarrow {}^{2S+1}L_J = {}^3P_0$

$$H_{q\bar{q}} = \gamma \sum_f 2m_f \int d^3x \bar{\psi}_f \psi_f$$

nonrelativistic limit \longrightarrow

$$T = -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d^3p_3 d^3p_4 \delta^3(p_3 + p_4) \\ \times \omega_0^{34} \varphi_0^{34} \chi_{1,-m}^{34} Y_1^m \left(\frac{p_3 - p_4}{2} \right) a_{3i}^+ b_{4j}^+$$

$$\omega_0^{34} = \frac{R_4 \bar{R}_3 + G_4 \bar{G}_3 + B_4 \bar{B}_3}{\sqrt{3}}$$

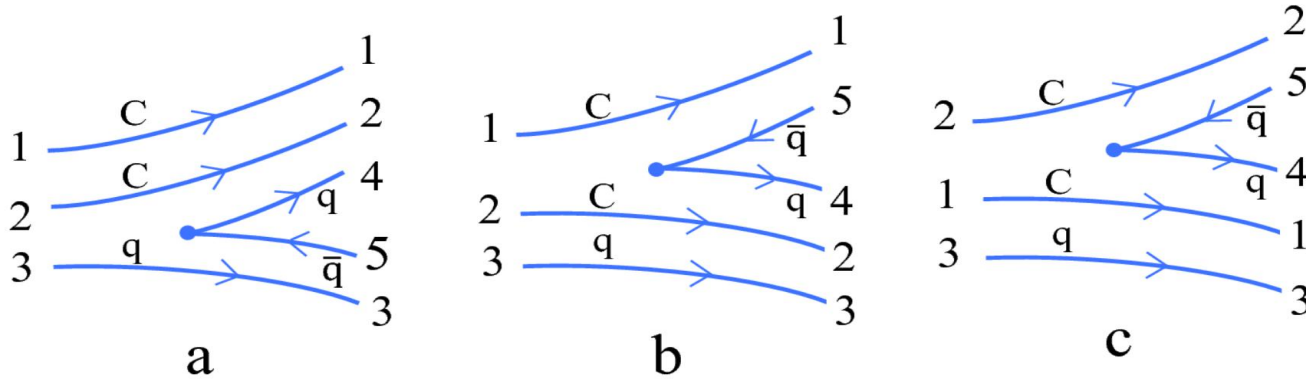
$$\varphi_0^{34} = \frac{u_4 \bar{u}_3 + d_4 \bar{d}_3 + s_4 \bar{s}_3}{\sqrt{3}}$$

$$Y_1^m(\vec{p}) = |\vec{p}| y_1^m(\theta, \varphi)$$

L. Micu, Nucl. Phys. B10,521(1969).

R. D. Carlitz and M. Kislinger, Phys. Rev. D2,336(1970).

3P_0 model:



$$T = -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d^3 p_4 d^3 p_5 \delta^3(p_4 + p_5) \\ \times \omega_0^{45} \varphi_0^{45} \chi_{1,-m}^{45} Y_1^m \left(\frac{p_4 - p_5}{2} \right) a_{4i}^+ b_{5j}^+$$

$$M^{M_{J_A} M_{J_B} M_{J_C}} (A \rightarrow B + C) = \sqrt{8E_A E_B E_C} \gamma \\ \sum \langle L_A M_{L_A}; S_A M_{S_A} | J_A M_{J_A} \rangle \langle L_B M_{L_B}; S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C}; S_C M_{S_C} | J_C M_{J_C} \rangle \\ \times \langle 1m; 1-m | 00 \rangle \langle \chi_{S_B M_{S_B}}^{124} \chi_{S_C M_{S_C}}^{35} | \chi_{S_A M_{S_A}}^{123} \chi_{1-m}^{45} \rangle \langle \varphi_B^{124} \varphi_C^{35} | \varphi_A^{123} \varphi_0^{45} \rangle I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}$$

$$I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m} (p) = \int d^3 \vec{p}_1 d^3 \vec{p}_2 d^3 \vec{p}_3 d^3 \vec{p}_4 d^3 \vec{p}_5 \delta^3(\vec{p}_4 + \vec{p}_5) \delta^3(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 - \vec{p}_A) \delta^3(\vec{p}_3 + \vec{p}_5 - \vec{p}_C) \delta^3(\vec{p}_1 + \vec{p}_2 + \vec{p}_4 - \vec{p}_B) \\ \times \psi_{n_B L_B M_{L_B}}^*(\vec{p}_1, \vec{p}_2, \vec{p}_4) \psi_{n_C L_C M_{L_C}}^*(\vec{p}_3, \vec{p}_5) \psi_{n_A L_A M_{L_A}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) Y_1^m \left(\frac{\vec{p}_4 - \vec{p}_5}{2} \right)$$

C. Hayne and N. Isgur, Phys.Rev. D25,1944(1982).

● baryon wave function

$$\begin{aligned}
 & \left| A(N_A^{2S_A+1} L_A J_A M_{J_A})(p_A) \right\rangle \\
 &= \sqrt{2E_A} \varphi_A^{123} \omega_A^{123} \sum_{M_{L_A}, M_{S_A}} \left\langle L_A M_{L_A}; S_A M_{S_A} \middle| J_A M_{J_A} \right\rangle \\
 & \times \int d^3 p_1 d^3 p_2 d^3 p_3 \delta^3(p_1 + p_2 + p_3 - p_A) \\
 & \times \psi_{N_A L_A M_{L_A}}(p_1, p_2, p_3) \chi_{S_A M_{S_A}}^{123} |q_1(p_1) q_2(p_2) q_3(p_3)\rangle
 \end{aligned}$$

● meson wave function

$$\begin{aligned}
 & \left| C(N_C^{2S_C+1} L_C J_C M_{J_C})(p_C) \right\rangle \\
 &= \sqrt{2E_C} \varphi_C^{ab} \omega_C^{ab} \sum_{M_{L_C}, M_{S_C}} \left\langle L_C M_{L_C}; S_C M_{S_C} \middle| J_C M_{J_C} \right\rangle \\
 & \times \int d^3 p_a d^3 p_b \delta^3(p_a + p_b - p_C) \\
 & \times \psi_{N_C L_C M_{L_C}}(p_a, p_b) \chi_{S_C M_{S_C}}^{ab} |q_a(p_a) q_b(p_b)\rangle
 \end{aligned}$$

spatial wave function

SHO

$$\psi_{lm}^0(\vec{p}) = (-i)^l \left[\frac{2^{l+2}}{\sqrt{\pi} (2l+1)!!} \right]^{\frac{1}{2}} \left(\frac{1}{\alpha} \right)^{l+\frac{3}{2}} \exp\left(-\frac{\vec{p}^2}{2\alpha^2}\right) Y_l^m(\vec{p})$$

$$\Gamma[A \rightarrow B + C] = \pi^2 \frac{|P|}{M_A^2} \frac{1}{2J_A + 1} \sum_{M_{J_A}, M_{J_B}, M_{J_C}} \left| M_{M_{J_A} M_{J_B} M_{J_C}} \right|^2$$

Result: $1P_\rho$

D. Ebert PRD66,014008 (2002).

$^{2S+1}L_{\rho/\lambda} J^P$	$\Xi_{cc}^{++2} P_\rho \frac{1}{2}^-$	$\Xi_{cc}^{++2} P_\rho \frac{3}{2}^-$	$\Omega_{cc}^{2} P_\rho \frac{1}{2}^-$	$\Omega_{cc}^{2} P_\rho \frac{3}{2}^-$
mass (MeV)	3838	3959	4002	4102

orthogonality of SHO function



doubly charmed baryon+light flavor meson

below the threshold



singly charmed baryon+heavy-light meson

The decay widths of the $1P_\rho$ states should be fairly narrow !

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4196	13.7	117	131	0.18
$ \Xi_{cc} \ ^4P_{\lambda\frac{1}{2}}^-\rangle$	4053	200	0.60	201	333
$ \Xi_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4101	4.43	127	131	0.03
$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4155	45.9	12.6	58.5	3.64

State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^-\rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4196	13.7	117	131	0.18
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State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^- \rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^- \rangle$	4252	2.02	154	156	0.01
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Result: $1P_\lambda$

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		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4196	13.7	117	131	0.18
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$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^- \rangle$	4155	45.9	12.6	58.5	3.64

State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^- \rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^- \rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^- \rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^{\frac{1}{2}-}\rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^{\frac{3}{2}-}\rangle$	4196	13.7	117	131	0.18
$ \Xi_{cc} \ ^4P_{\lambda\frac{1}{2}}^{\frac{1}{2}-}\rangle$	4053	200	0.60	201	333
$ \Xi_{cc} \ ^4P_{\lambda\frac{3}{2}}^{\frac{3}{2}-}\rangle$	4101	4.43	127	131	0.03
$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^{\frac{5}{2}-}\rangle$	4155	45.9	12.6	58.5	3.64

State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^{\frac{1}{2}-}\rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^{\frac{3}{2}-}\rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^{\frac{1}{2}-}\rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^{\frac{3}{2}-}\rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^{\frac{5}{2}-}\rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4196	13.7	117	131	0.18
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$ \Xi_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4101	4.43	127	131	0.03
$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4155	45.9	12.6	58.5	3.64

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		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^-\rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4196	13.7	117	131	0.18
$ \Xi_{cc} \ ^4P_{\lambda\frac{1}{2}}^- \rangle$	4053	200	0.60	201	333
$ \Xi_{cc} \ ^4P_{\lambda\frac{3}{2}}^- \rangle$	4101	4.43	127	131	0.03
$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^- \rangle$	4155	45.9	12.6	58.5	3.64

State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^- \rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^- \rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^- \rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^- \rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^- \rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

State	Mass	$\Gamma[\Xi_{cc}\pi]$	$\Gamma[\Xi_{cc}^*\pi]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Xi_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4136	21.9	18.6	40.5	1.18
$ \Xi_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4196	13.7	117	131	0.18
$ \Xi_{cc} \ ^4P_{\lambda\frac{1}{2}}^-\rangle$	4053	200	0.60	201	333
$ \Xi_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4101	4.43	127	131	0.03
$ \Xi_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4155	45.9	12.6	58.5	3.64

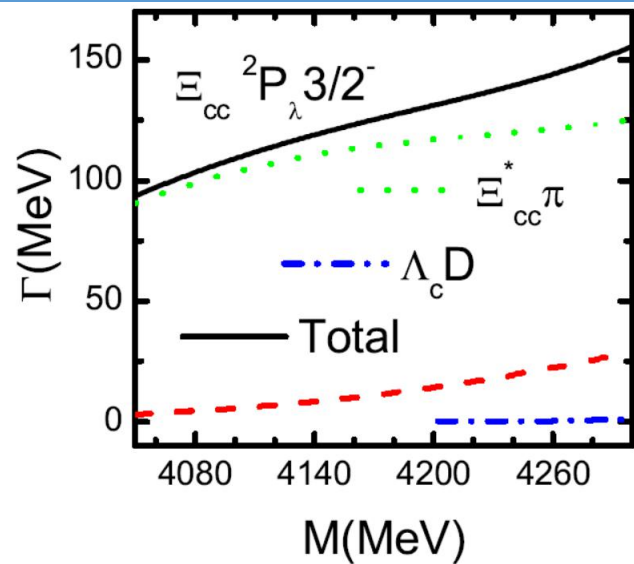
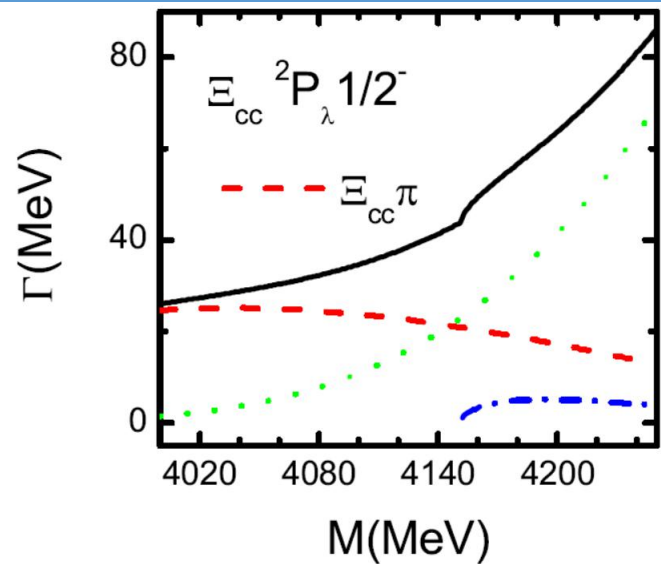
State	Mass	$\Gamma[\Xi_{cc}K]$	$\Gamma[\Xi_{cc}^*K]$	Total	\mathcal{B}
		3P_0	3P_0	3P_0	3P_0
$ \Omega_{cc} \ ^2P_{\lambda\frac{1}{2}}^-\rangle$	4271	49.3	1.53	50.8	32.2
$ \Omega_{cc} \ ^2P_{\lambda\frac{3}{2}}^-\rangle$	4325	8.50	199	208	0.04
$ \Omega_{cc} \ ^4P_{\lambda\frac{1}{2}}^-\rangle$	4208	378	...	378	...
$ \Omega_{cc} \ ^4P_{\lambda\frac{3}{2}}^-\rangle$	4252	2.02	154	156	0.01
$ \Omega_{cc} \ ^4P_{\lambda\frac{5}{2}}^-\rangle$	4303	29.1	2.62	31.7	11.1

Result: $1P_\lambda$

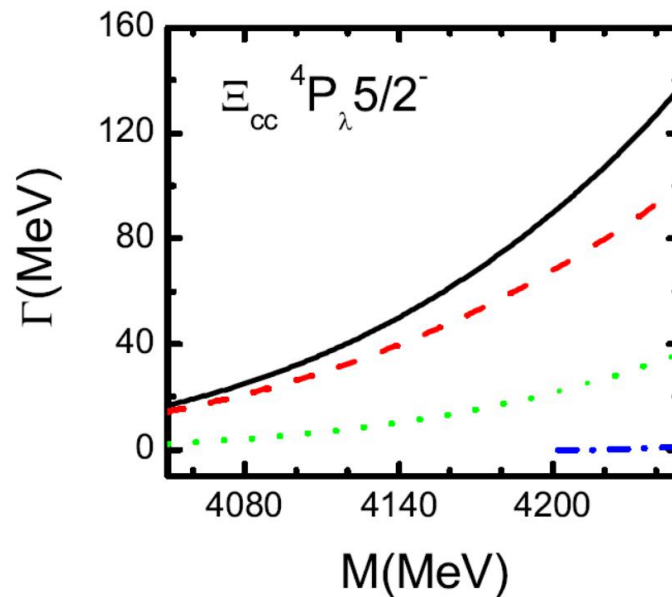
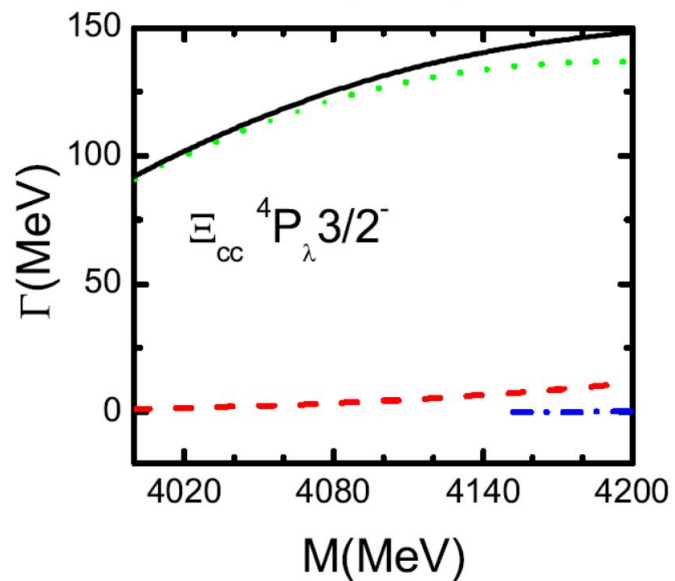
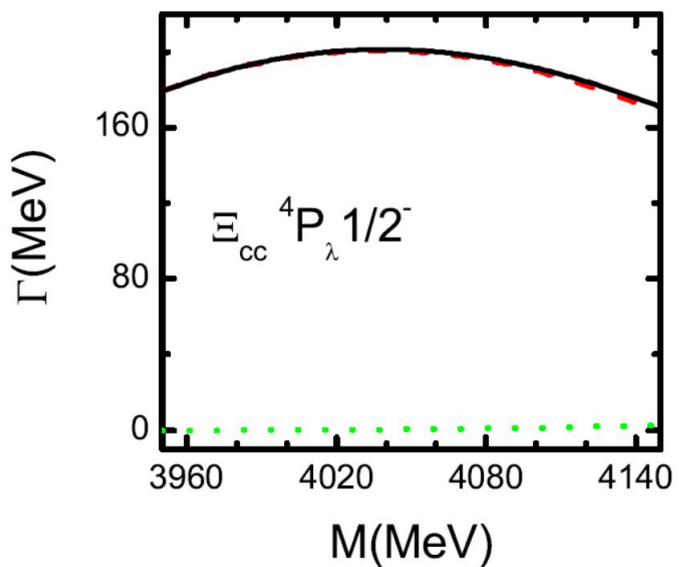
State	Mass	$\Gamma[\Xi_{cc}\pi]$		$\Gamma[\Xi_{cc}^*\pi]$		Total		\mathcal{B}	
		3P_0	CQM	3P_0	CQM	3P_0	CQM	3P_0	CQM
$ \Xi_{cc}^2P_{\lambda\frac{1}{2}}^-\rangle$	4136	21.9	15.6	18.6	33.9	40.5	49.5	1.18	0.46
$ \Xi_{cc}^2P_{\lambda\frac{3}{2}}^-\rangle$	4196	13.7	21.6	117	101	131	123	0.18	0.21
$ \Xi_{cc}^4P_{\lambda\frac{1}{2}}^-\rangle$	4053	200	133	0.60	1.22	201	134	333	110
$ \Xi_{cc}^4P_{\lambda\frac{3}{2}}^-\rangle$	4101	4.43	7.63	127	84.6	131	92.2	0.03	0.09
$ \Xi_{cc}^4P_{\lambda\frac{5}{2}}^-\rangle$	4155	45.9	75.3	12.6	22.8	58.5	98.1	3.64	3.30

State	Mass	$\Gamma[\Xi_{cc}K]$		$\Gamma[\Xi_{cc}^*K]$		Total		\mathcal{B}	
		3P_0	CQM	3P_0	CQM	3P_0	CQM	3P_0	CQM
$ \Omega_{cc}^2P_{\lambda\frac{1}{2}}^-\rangle$	4271	49.3	33.1	1.53	2.36	50.8	35.5	32.2	14.0
$ \Omega_{cc}^2P_{\lambda\frac{3}{2}}^-\rangle$	4325	8.50	11.4	199	174	208	185	0.04	0.06
$ \Omega_{cc}^4P_{\lambda\frac{1}{2}}^-\rangle$	4208	378	323	378	323
$ \Omega_{cc}^4P_{\lambda\frac{3}{2}}^-\rangle$	4252	2.02	3.08	154	137	156	140	0.01	0.02
$ \Omega_{cc}^4P_{\lambda\frac{5}{2}}^-\rangle$	4303	29.1	41.5	2.62	4.38	31.7	45.9	11.1	9.47

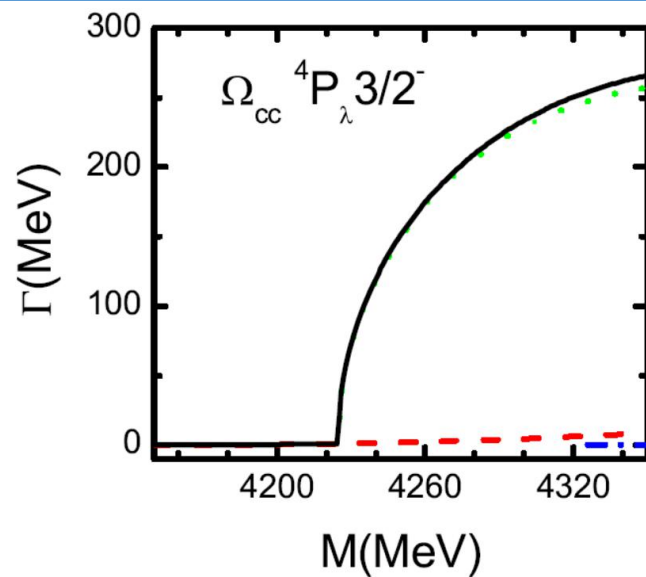
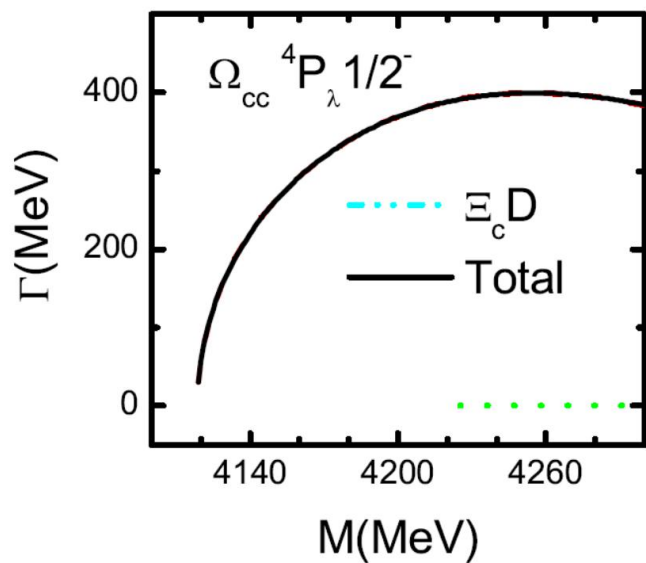
Result: $\Xi_{cc}^{++} 1P_{\lambda}$



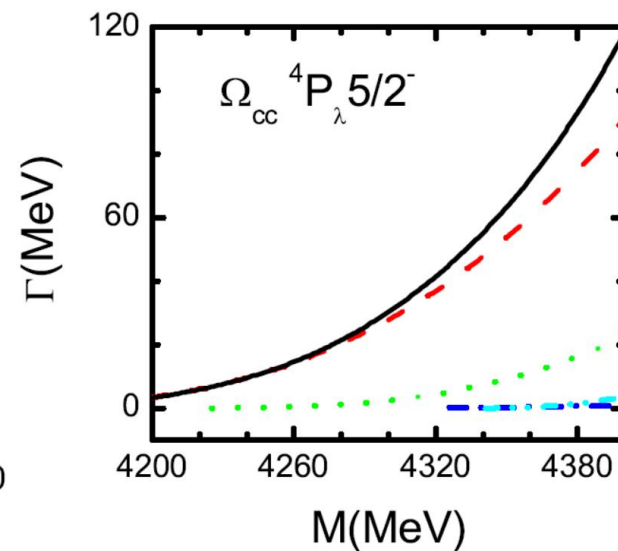
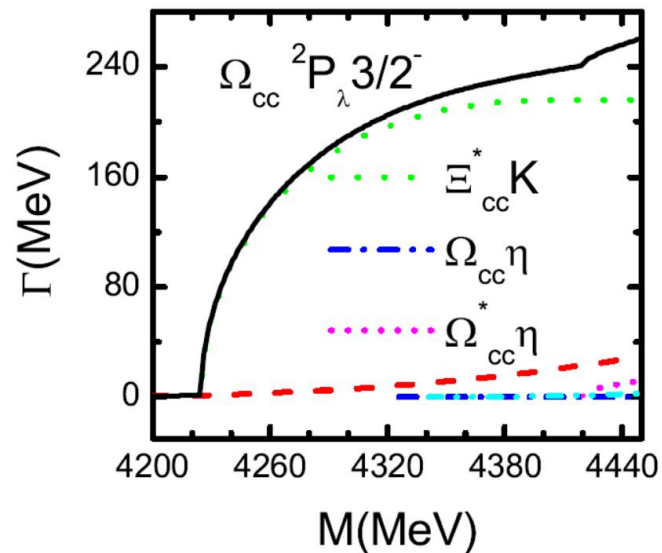
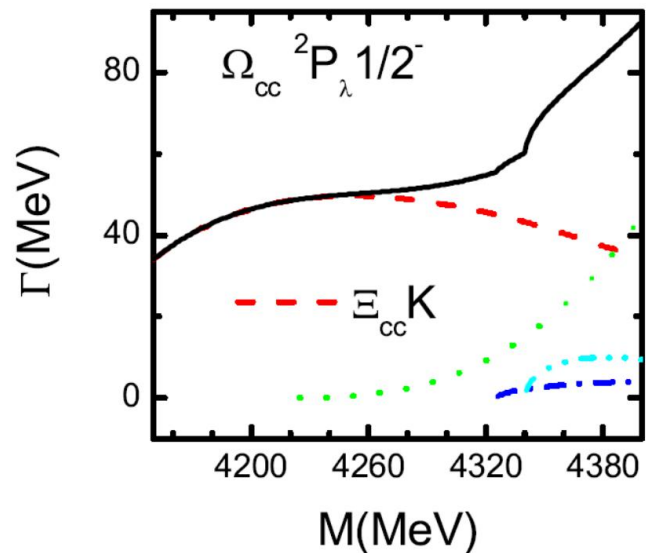
states	${}^2P_{\lambda} \frac{1}{2}^{-}$	${}^2P_{\lambda} \frac{3}{2}^{-}$	${}^4P_{\lambda} \frac{1}{2}^{-}$	${}^4P_{\lambda} \frac{3}{2}^{-}$	${}^4P_{\lambda} \frac{5}{2}^{-}$
mass(MeV)	4136	4196	4053	4101	4155



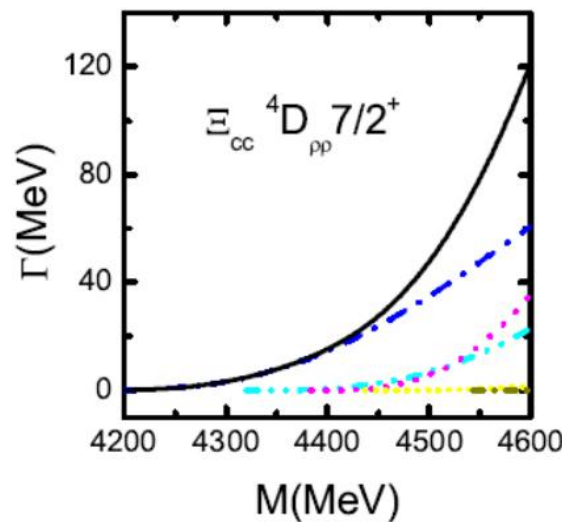
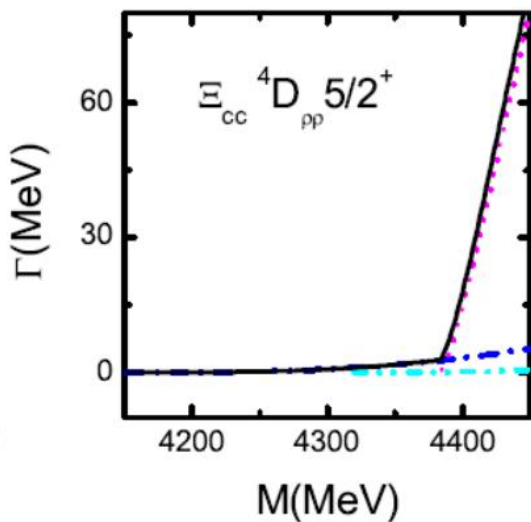
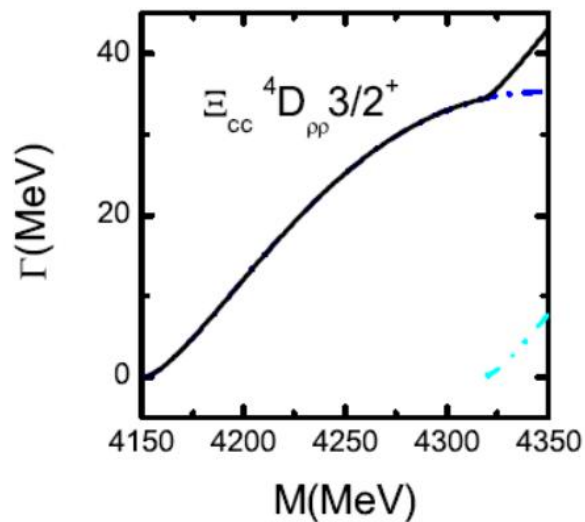
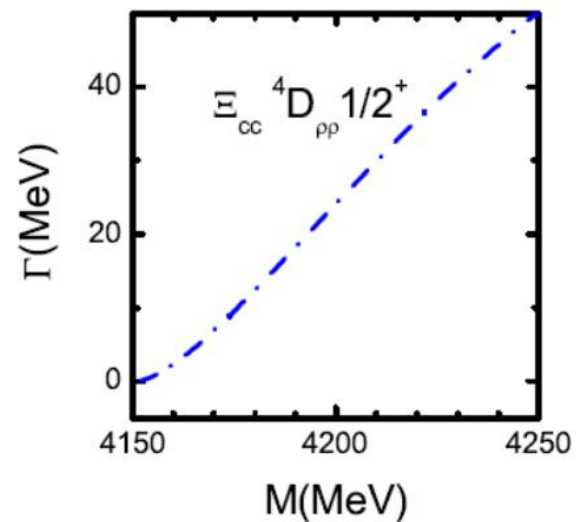
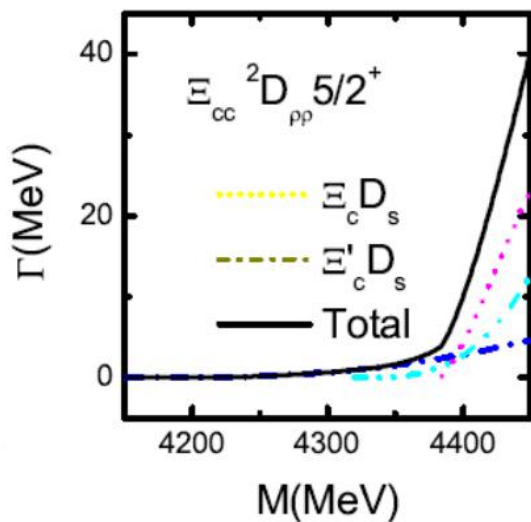
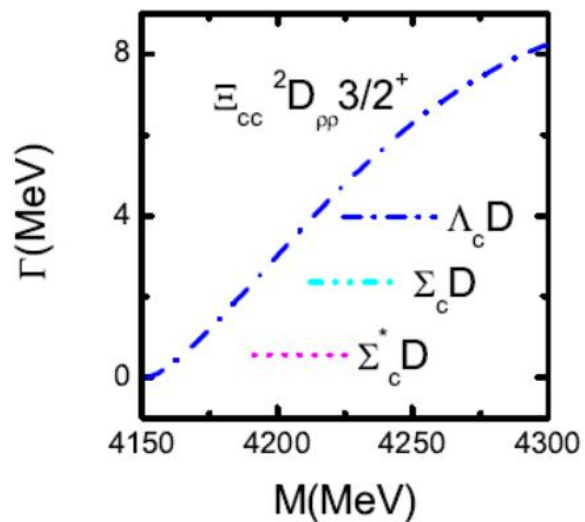
Result: $\Omega_{cc}1P_\lambda$



states	${}^2P_\lambda \frac{1}{2}^-$	${}^2P_\lambda \frac{3}{2}^-$	${}^4P_\lambda \frac{1}{2}^-$	${}^4P_\lambda \frac{3}{2}^-$	${}^4P_\lambda \frac{5}{2}^-$
mass(MeV)	4271	4325	4208	4252	4303

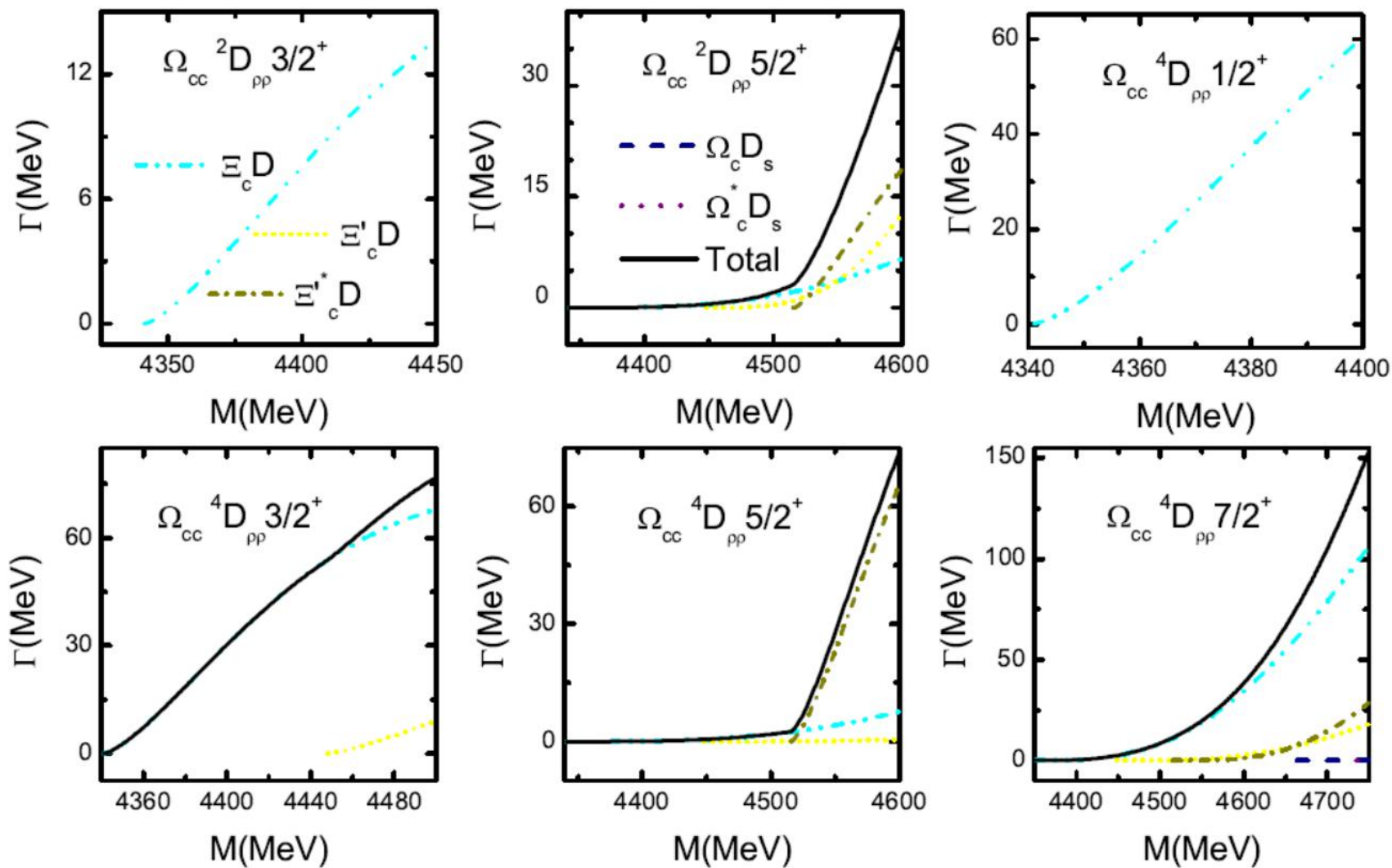


Result: $\Xi_{cc}^{++} 2D_{pp}$



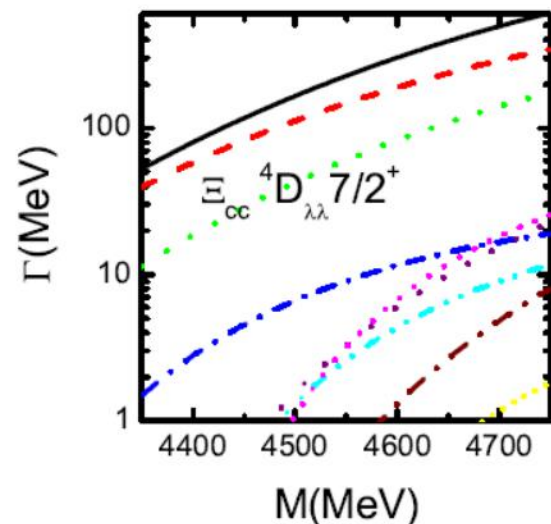
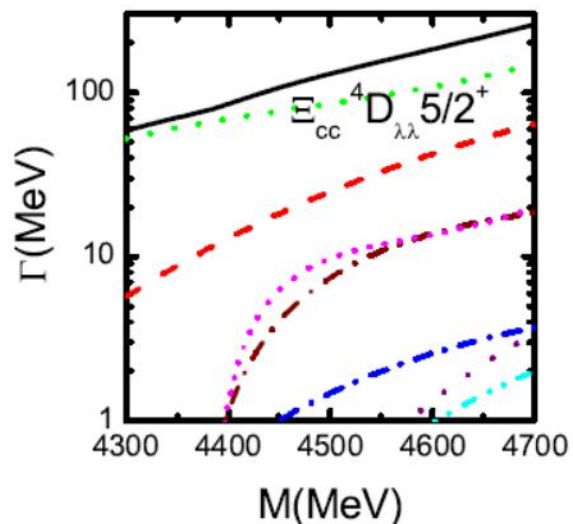
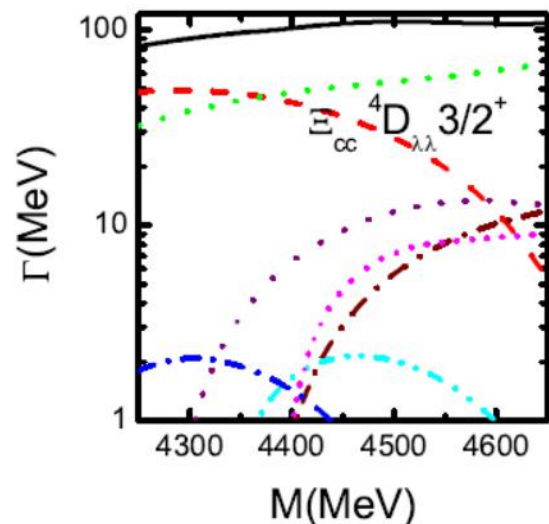
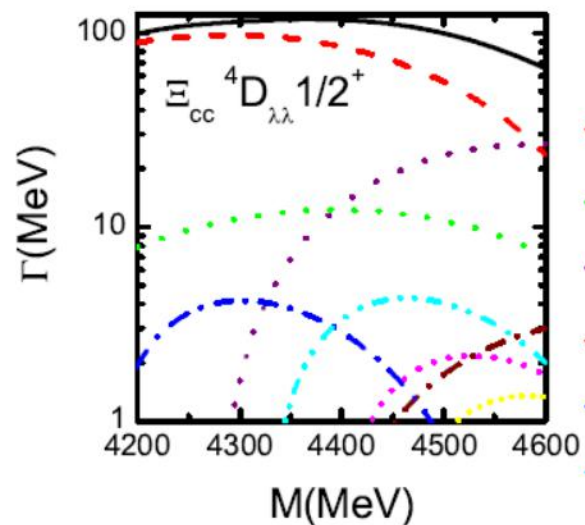
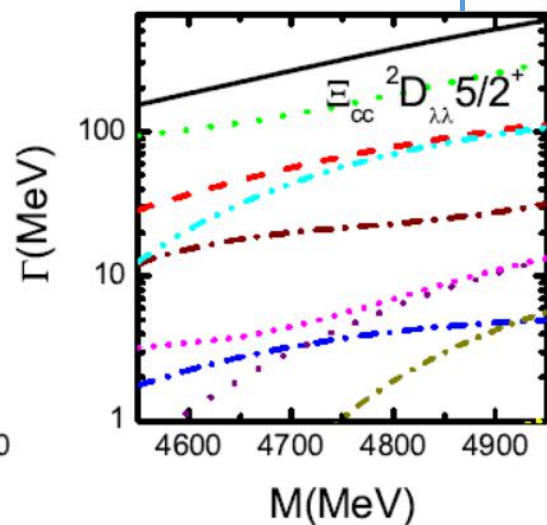
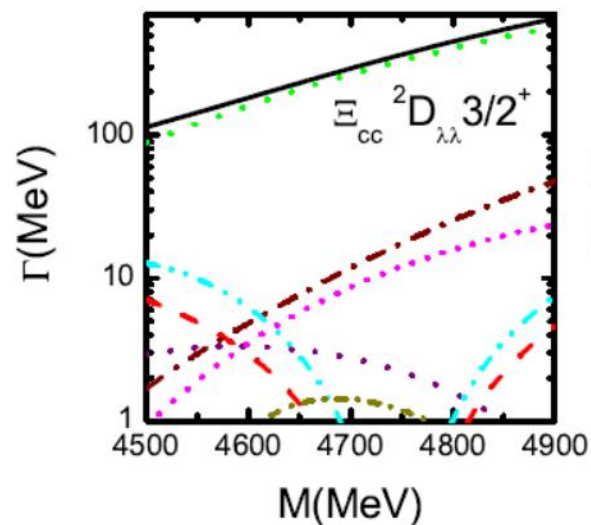
The main decay channels are sensitive to the masses.

Result: $\Omega_{cc} 2D_{pp}$



The main decay channels are sensitive to the masses.

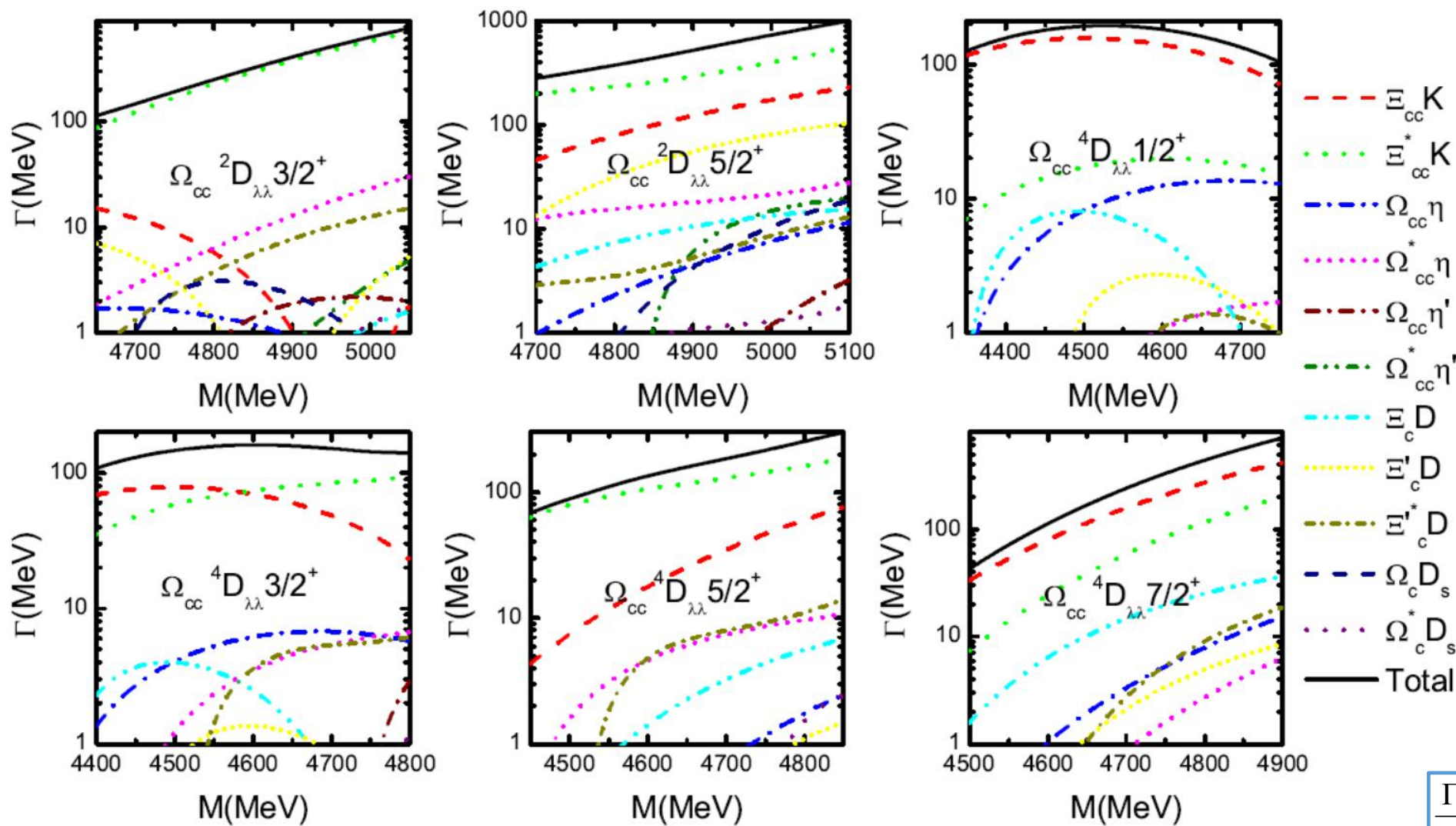
Result: $\Xi_{cc}^{++} 2D_{\lambda\lambda}$



$$\frac{\Gamma[\Sigma_c D]}{\Gamma[total]} \approx (8-18)\%$$

- $\Xi_{cc} \pi$
- ... $\Xi_{cc}^+ \pi$
- ... $\Omega_{cc} K$
- $\Omega_{cc}^+ K$
- $\Lambda_c D$
- $\Sigma_c D$
- ... $\Sigma_c^+ D$
- ... $\Xi_c D_s$
- ... $\Xi_c^+ D_s$
- $\Xi_c^+ D_s$
- Total

Result: $\Omega_{cc} 2D_{\lambda\lambda}$



$$\frac{\Gamma[E_c D]}{\Gamma[E_{cc} K]} \approx (4.6-8.7)\%$$

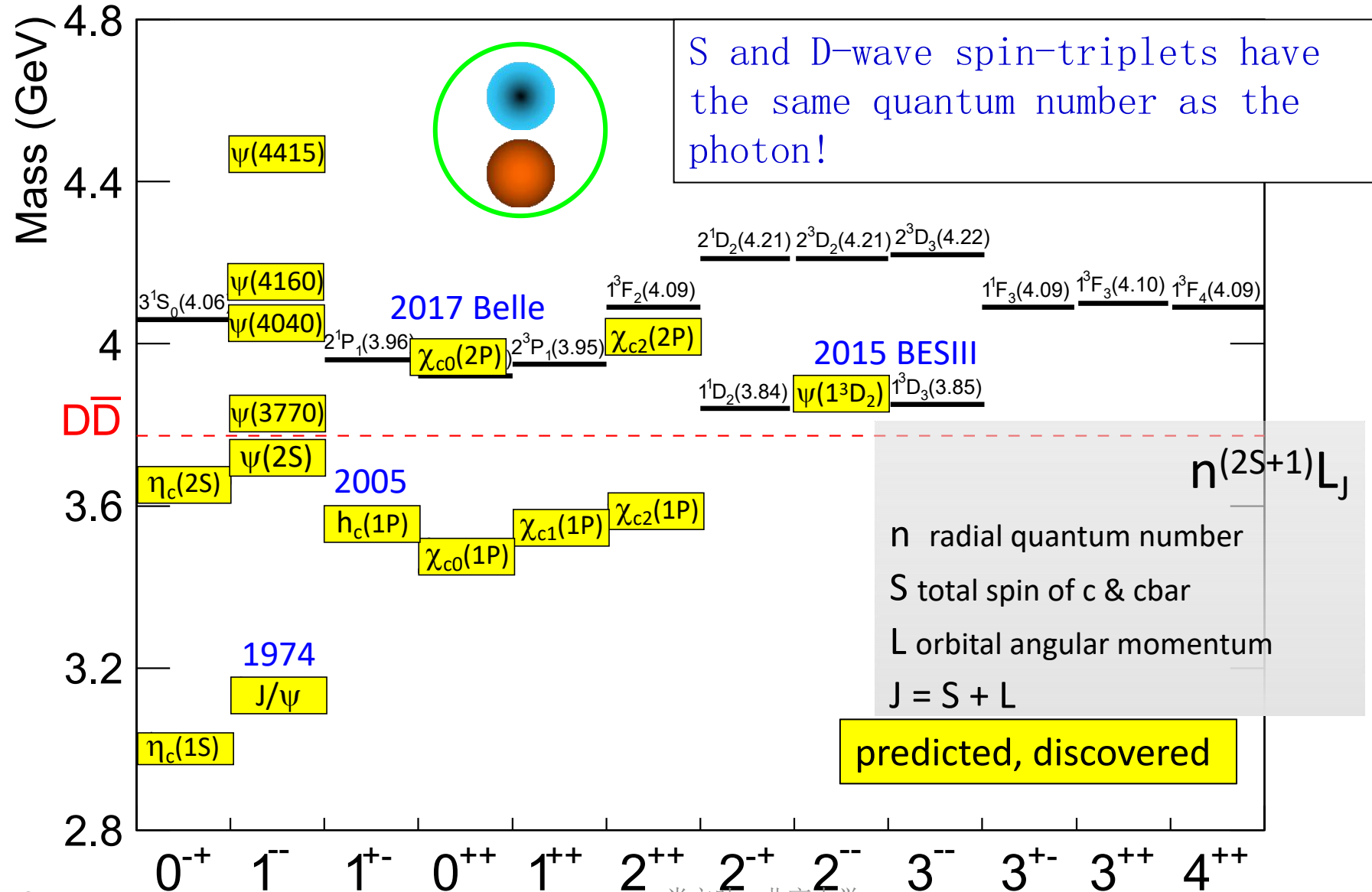
Summary: Doubly charmed baryons with 3P_0 model

The strong decay properties of the doubly charmed baryons: Ξ_{cc}^{++} and Ω_{cc}

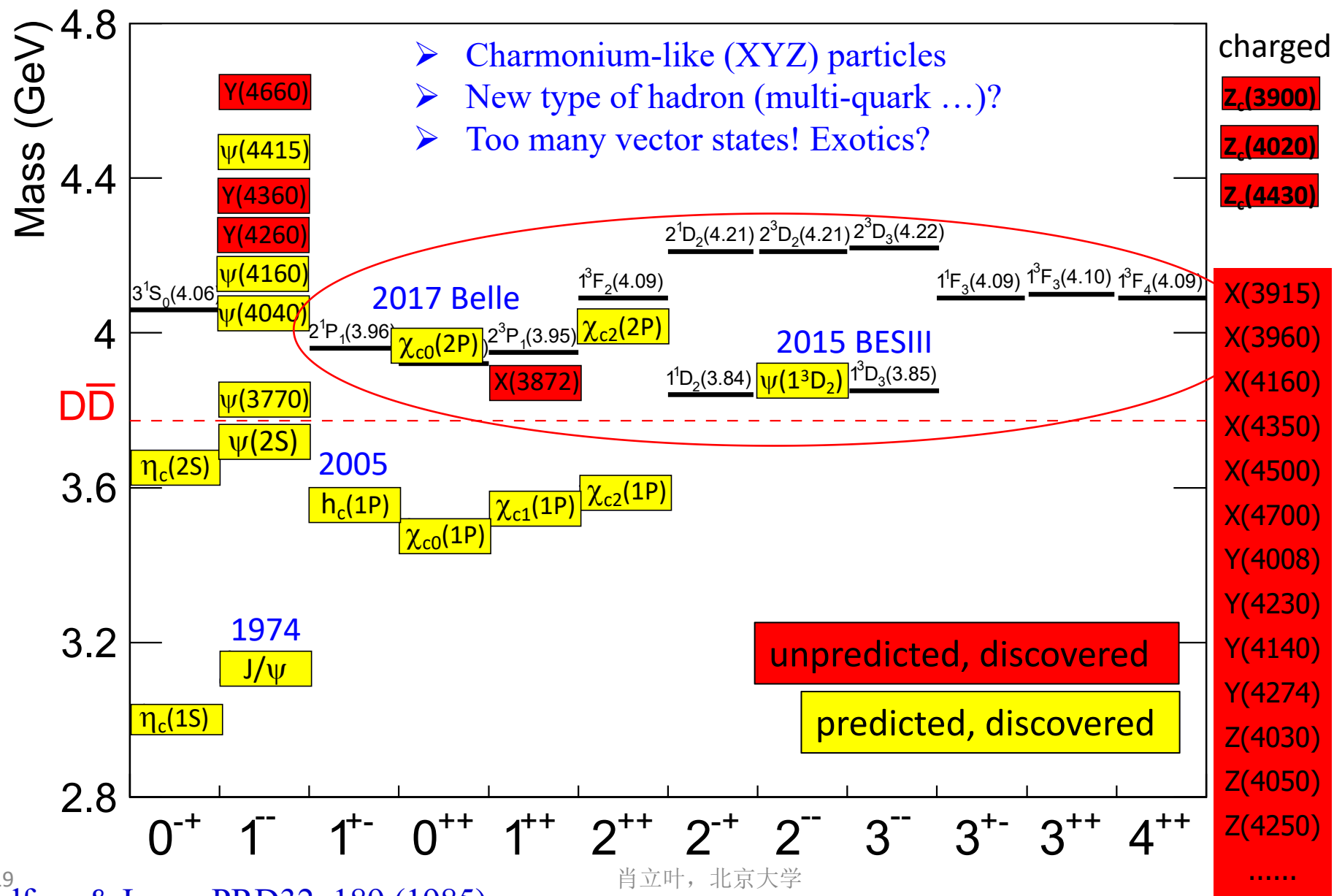
- For the $1P_\rho$ mode states: $\Gamma \sim$ fairly narrow
- For the $1P_\lambda$ mode states: $\Gamma \sim 100$ MeV
- For the $2D_{\rho\rho}$ states: They mainly decay via emitting a heavy-light meson
 $\Gamma \sim$ several tens MeV
- For the $2D_{\lambda\lambda}$ states: $\Gamma > 100$ MeV

$$Y(4660) \rightarrow \Lambda_c \bar{\Lambda}_c$$

Charmonium spectroscopy



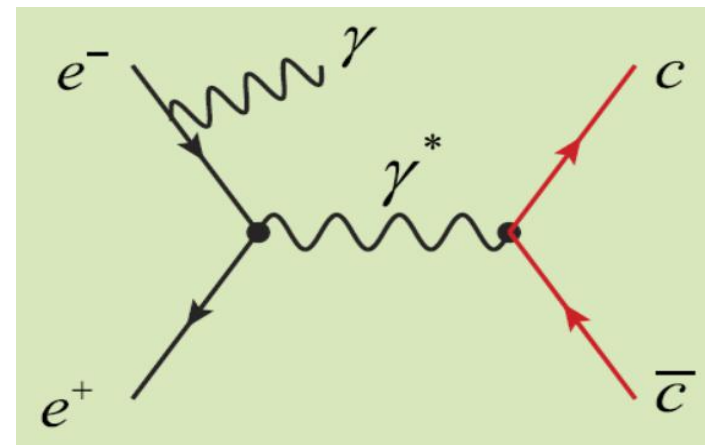
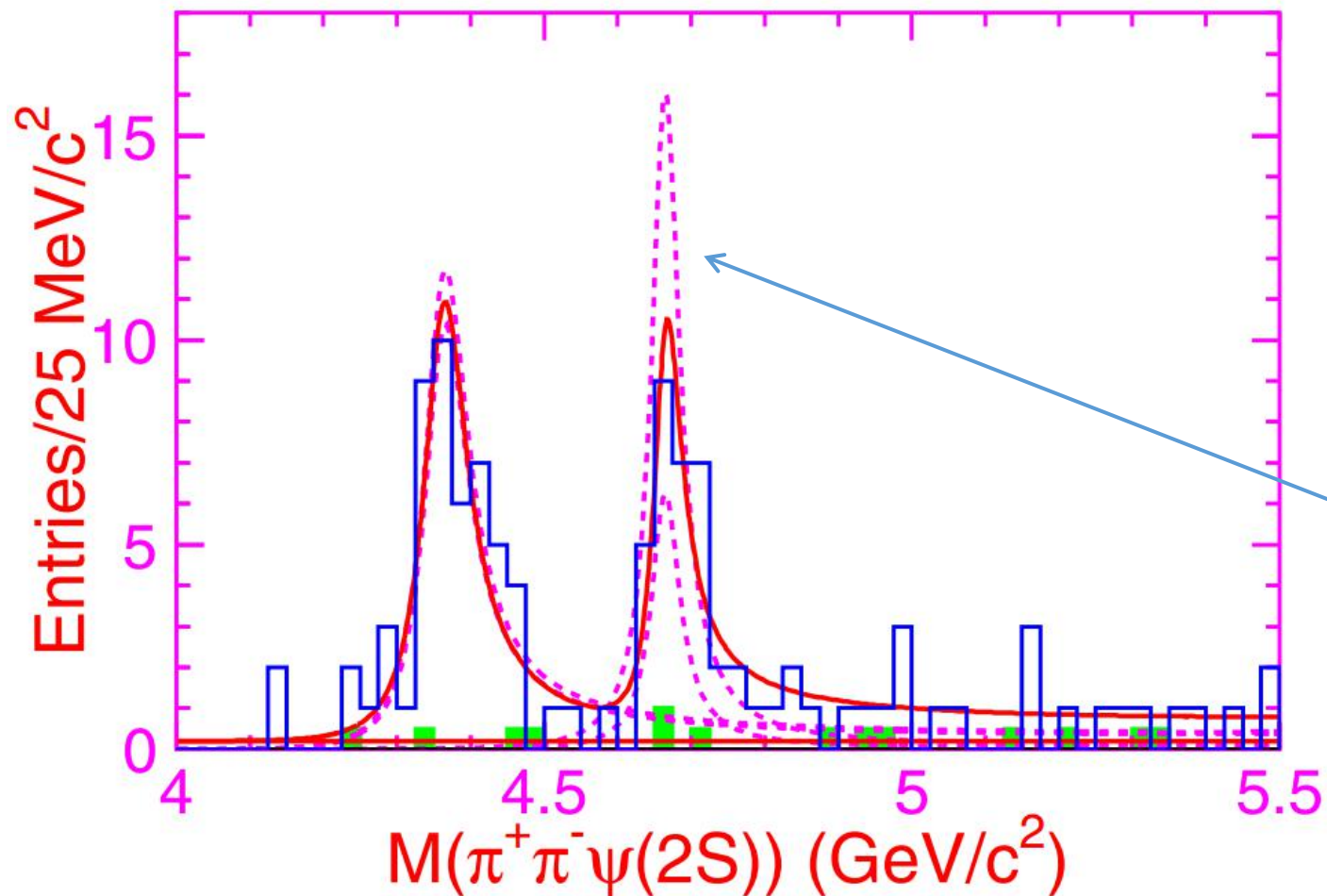
Charmonium(like) spectroscopy



Y(4660) : first observation at Belle

PRL99 142002(2007)

673/fb data



$$M(Y(4660)) = 4664 \pm 11 \pm 5 \text{ MeV}$$

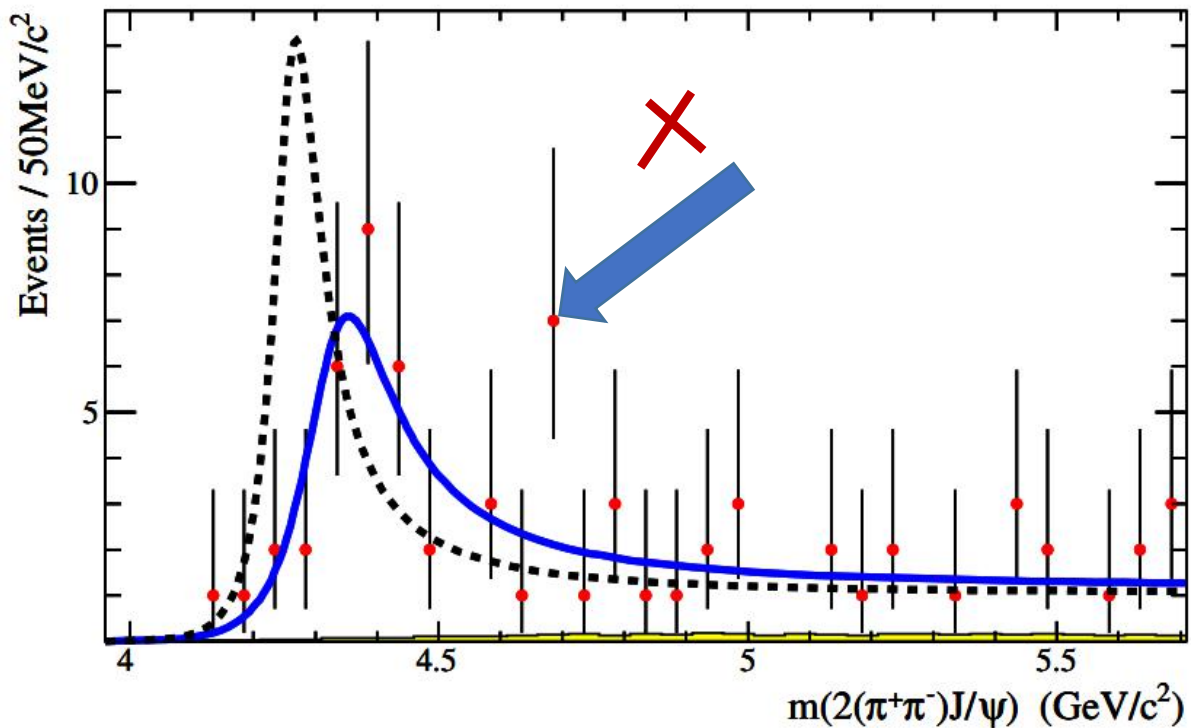
$$\Gamma_{tot}(Y(4660)) = 48 \pm 15 \pm 3 \text{ MeV}$$

5.8 σ

Y(4660) : not be confirmed by BaBar

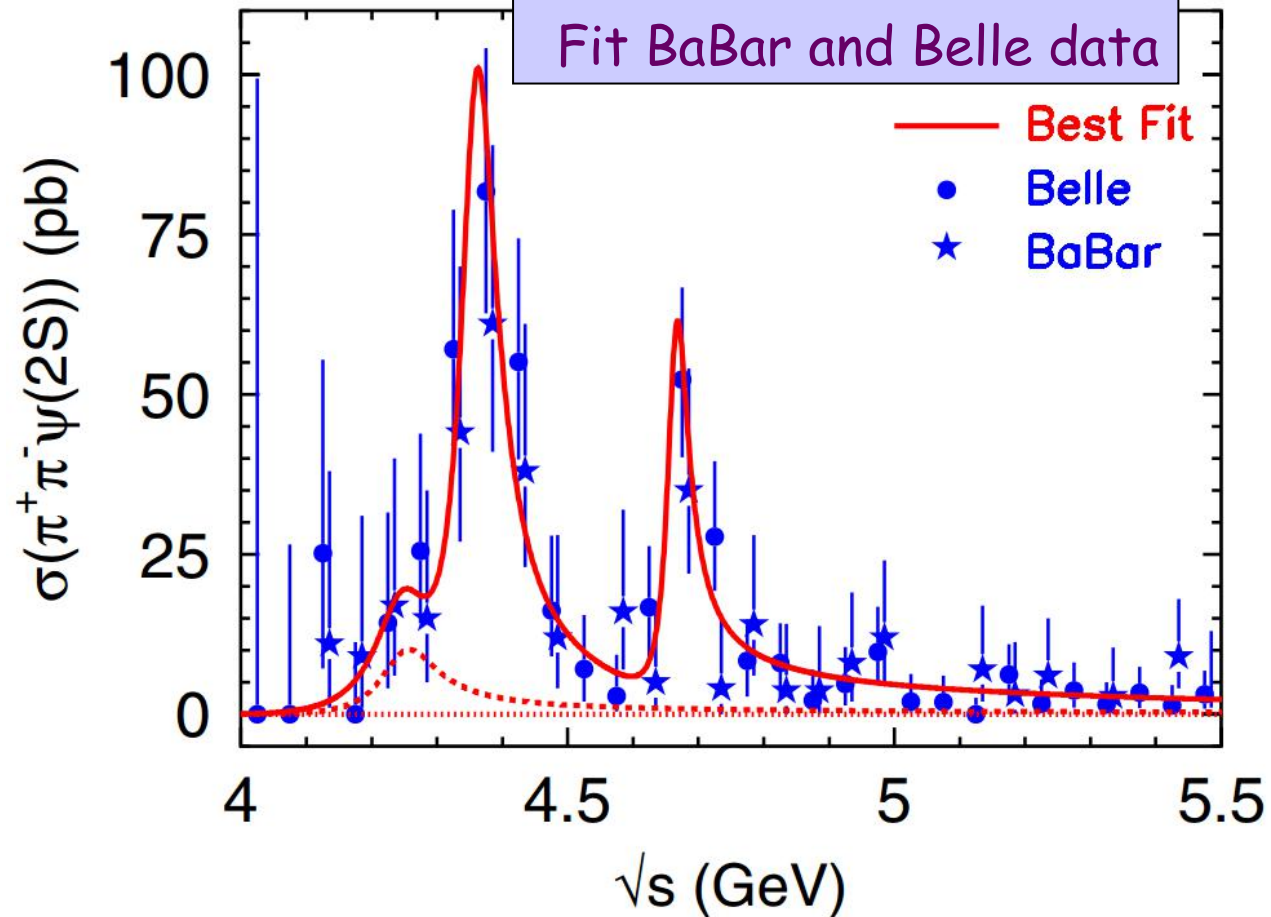
PRL98 212001(2007)

298/fb data



PRD78 014032(2008)

Fit BaBar and Belle data



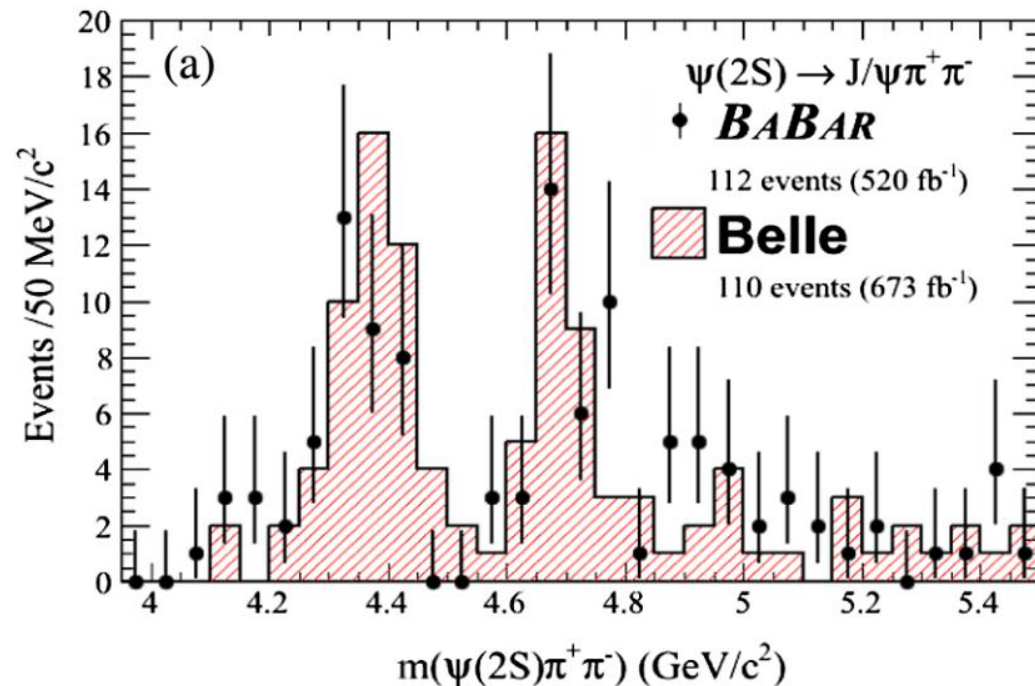
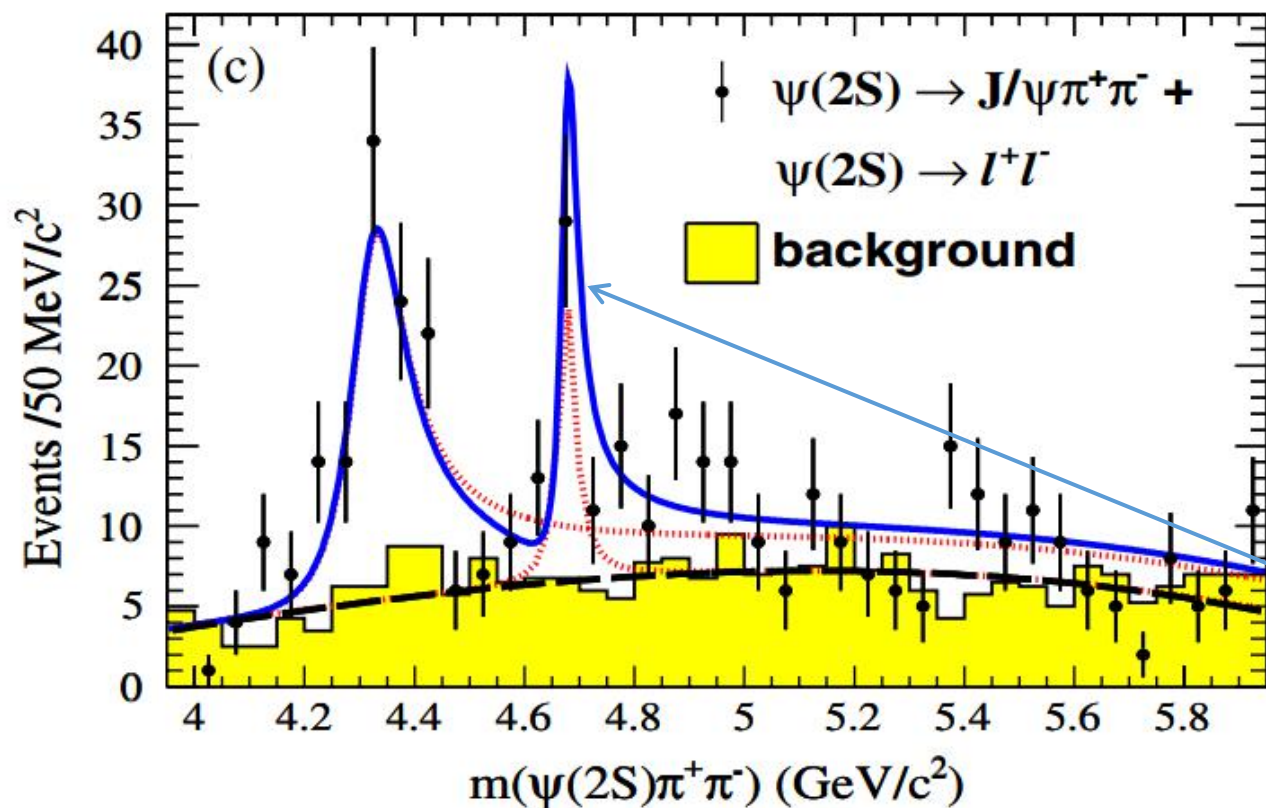
$$M(Y(4660)) = 4661_{-8}^{+9} \pm 6 \text{ MeV}$$

$$\Gamma(Y(4660)) = 42_{-12}^{+17} \pm 6 \text{ MeV}$$

Y(4660) : confirmation by BaBar in 2012

PRD89 111103(2014)

520/fb data



$$M(Y(4660)) = 4669 \pm 21 \pm 3 \text{ MeV}$$

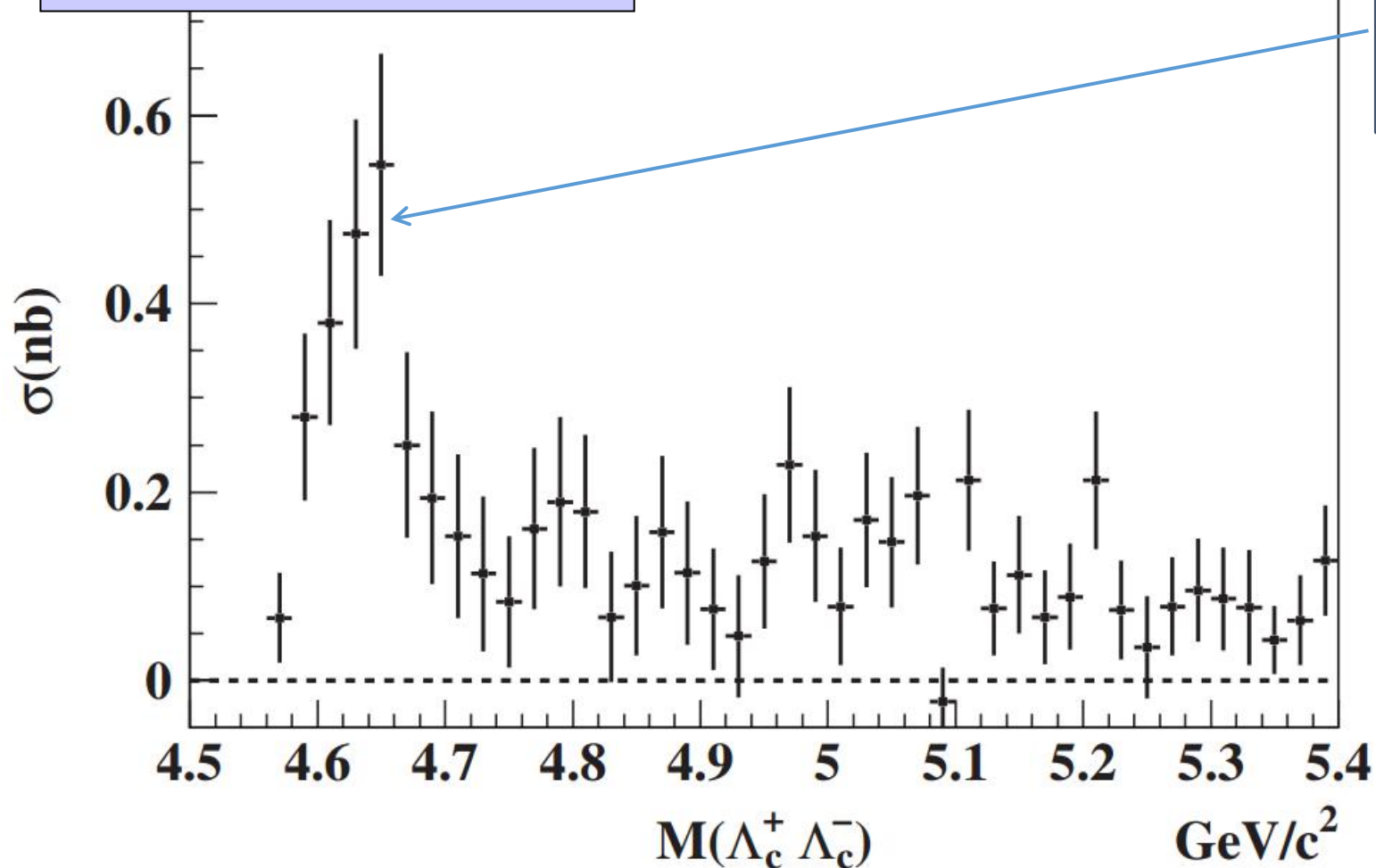
$$\Gamma_{tot}(Y(4660)) = 104 \pm 48 \pm 10 \text{ MeV}$$

5.7σ

Y(4630) : first observation at Belle

PRL101 172001(2008)

695/fb data



$$M(Y(4630)) = 4634_{-7}^{+8} (stat)_{-8}^{+5} (syst) \text{MeV}$$

$$\Gamma_{tot}(Y(4630)) = 92_{-24}^{+40} (stat)_{-21}^{+10} (syst) \text{MeV}$$

8.2 σ

consistent with Y(4660)

Y(4660) and Y(4630): interpretation in theory

Y(4660)

- As charmonium state:

- $\psi(6S)$ ➤ $\psi(5D)$
- $\psi(5S)$ ➤ $\psi(4D)$
- $\psi(4S)$ ➤ $\psi(3D)$

- As exotic state:

- $\psi'f_0(980)$ bound state
- tetraquark state
- hadro-charmonium state

Y(4630)

- As exotic state:

- $\Lambda_c \bar{\Lambda}_c$ bound state
- tetraquark state

Y(4660) and Y(4630)

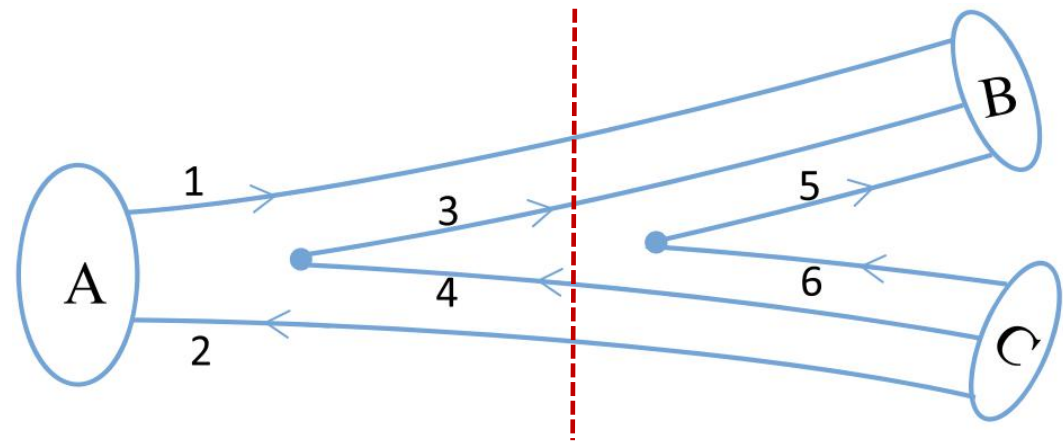
- As charmonium state:

- tetraquark state
- $\psi'f_0(980)$ bound state

Phys. Rept. 639, 1-121 (2016)

3P_0 model: $\psi(nS/nD) \rightarrow \Lambda_c \bar{\Lambda}_c$

$A(\text{initial meson}) \rightarrow B(\text{final baryon}) + C(\text{final baryon})$



$$H_{q\bar{q}} = \gamma \sum_f 2m_f \int d^3x \bar{\psi}_f \psi_f$$

$$\delta^3(\mathbf{p}_A - \mathbf{p}_B - \mathbf{p}_C) M^{M_{J_A} M_{J_B} M_{J_C}} = \frac{\sum_k \langle BC | H_{q\bar{q}} | k \rangle \langle k | H_{q\bar{q}} | A \rangle}{E_k - E_A}$$

hadron level: $J^P=1^{--}$
 PRD89 114010(20014)

$DD_1, D^* \bar{D}_1, D^* \bar{D}_0, D^* \bar{D}_2, J/\psi f_0(500) / f_0(600),$
 $h_c(1P)\eta, h_c(1P)\pi, \chi_{c0}(1P)\omega(782), \chi_{c2}(1P)\omega(782)$

$\sim 4.0\text{GeV}$

quark level: intermediate state - initial state = two quarks

$$E_k - E_A \approx 0.6\text{GeV} \approx 2m_q \quad \text{as a constant}$$

3P_0 model: $\psi(nS/nD) \rightarrow \Lambda_c \bar{\Lambda}_c$

$$H_{q\bar{q}} = \gamma \sum_f 2m_f \int d^3x \bar{\psi}_f \psi_f$$

$$T = \frac{9\gamma^2}{4m_\mu} \sum_{m,m'} \langle 1m; 1-m | 00 \rangle \langle 1m'; 1-m' | 00 \rangle$$

$$\times \int d^3\mathbf{p}_3 d^3\mathbf{p}_4 d^3\mathbf{p}_5 d^3\mathbf{p}_6 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \delta^3(\mathbf{p}_5 + \mathbf{p}_6)$$

$$\times \varphi_0^{34} \omega_0^{34} \chi_{1,-m}^{34} \mathcal{Y}_1^m \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) a_{3i}^\dagger b_{4j}^\dagger$$

$$\times \varphi_0^{56} \omega_0^{56} \chi_{1,-m'}^{56} \mathcal{Y}_1^{m'} \left(\frac{\mathbf{p}_5 - \mathbf{p}_6}{2} \right) a_{5i}^\dagger b_{6j}^\dagger,$$

$$\delta^3(\mathbf{p}_A - \mathbf{p}_B - \mathbf{p}_C) M^{M_{J_A} M_{J_B} M_{J_B}}$$

$$= \frac{\sum_k \langle BC | H_{q\bar{q}} | k \rangle \langle k | H_{q\bar{q}} | A \rangle}{E_k - E_A}.$$

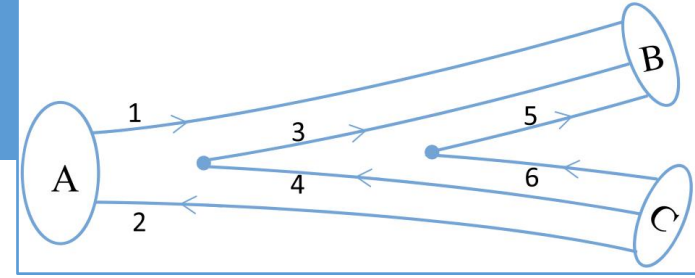


← nonrelativistic limit $\frac{\langle BC | H_{q\bar{q}} H_{q\bar{q}} | A \rangle}{2m_q}$

$$E_k - E_A \approx 0.6 \text{ GeV} \approx 2m_q \quad \text{as a constant}$$

3P_0 model: $\psi(nS/nD) \rightarrow \Lambda_c \bar{\Lambda}_c$

C. Hayne and N. Isgur, Phys.Rev. D25,1944(1982).



meson wave function

$$|A(N_A \ 2S_A+1 \ L_A \ J_A \ M_{J_A})(\mathbf{p}_A)\rangle = \sqrt{2E_A} \varphi_A^{12} \omega_A^{12} \\ \times \sum_{M_{L_A}, M_{S_A}} \langle L_A M_{L_A}; S_A M_{S_A} | J_A M_{J_A} \rangle \\ \times \int d^3 \mathbf{p}_1 d^3 \mathbf{p}_2 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_A) \\ \times \Psi_{N_A L_A M_{L_A}}(\mathbf{p}_1, \mathbf{p}_2) \chi_{S_A M_{S_A}}^{12} |q_1(\mathbf{p}_1) q_2(\mathbf{p}_2)\rangle,$$

SHO wave function

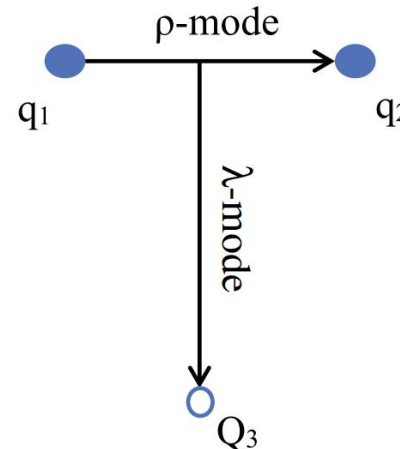
spatial wave function

$$\psi_{lm}^0(\mathbf{p}) = (-i)^l \left[\frac{2^{l+2}}{\sqrt{\pi} (2l+1)!!} \right]^{\frac{1}{2}} \left(\frac{1}{\beta} \right)^{l+\frac{3}{2}} \\ \times \exp\left(-\frac{\mathbf{p}_R^2}{2\beta^2}\right) \mathcal{Y}_l^m(\mathbf{p}),$$

baryon wave function

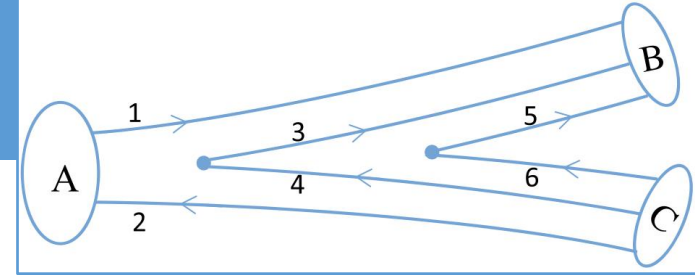
$$|B(N_B \ 2S_B+1 \ L_B \ J_B \ M_{J_B})(\mathbf{p}_B)\rangle = \sqrt{2E_B} \varphi_B^{135} \omega_B^{135} \\ \times \sum_{M_{L_B}, M_{S_B}} \langle L_B M_{L_B}; S_B M_{S_B} | J_B M_{J_B} \rangle \\ \times \int d^3 \mathbf{p}_1 d^3 \mathbf{p}_3 d^3 \mathbf{p}_5 \delta^3(\mathbf{p}_1 + \mathbf{p}_3 + \mathbf{p}_5 - \mathbf{p}_B) \\ \times \Psi_{N_B L_B M_{L_B}}(\mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_5) \chi_{S_B M_{S_B}}^{135} |q_1(\mathbf{p}_1) q_3(\mathbf{p}_3) q_5(\mathbf{p}_5)\rangle.$$

$$\psi_{0,0} = 3^{\frac{3}{4}} \left(\frac{1}{\pi \alpha_\rho^2} \right)^{\frac{3}{4}} \left(\frac{1}{\pi \alpha_\lambda^2} \right)^{\frac{3}{4}} \exp\left(-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right)$$



$$\rho = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \\ \lambda = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}} \\ \alpha_\lambda = \left(\frac{3m_Q}{2m_q + m_Q} \right)^{1/4} \alpha_\rho$$

3P_0 model: $\psi(nS/nD) \rightarrow \Lambda_c \bar{\Lambda}_c$



$$\Gamma[A \rightarrow BC] = \pi^2 \frac{|\mathbf{p}|}{M_A^2} \frac{1}{2J_A + 1} \times \sum_{M_{J_A}, M_{J_B}, M_{J_C}} |\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}|^2$$

$$\mathcal{M}_{4S} = \frac{1}{3\sqrt{35}} \left(15\beta \frac{\partial}{\partial\beta} + 6\beta^2 \frac{\partial^2}{\partial\beta^2} + 2\beta^3 \frac{\partial^3}{\partial\beta^3} \right) \mathcal{M}_{1S},$$

$$\mathcal{M}_{5S} = \frac{1}{18\sqrt{70}} \left(63 + 72\beta \frac{\partial}{\partial\beta} + 96\beta^2 \frac{\partial^2}{\partial\beta^2} + 24\beta^3 \frac{\partial^3}{\partial\beta^3} + 4\beta^4 \frac{\partial^4}{\partial\beta^4} \right) \mathcal{M}_{1S},$$

$$\mathcal{M}_{6S} = \frac{1}{45\sqrt{77}} \left(\frac{675}{2}\beta \frac{\partial}{\partial\beta} + 240\beta^2 \frac{\partial^2}{\partial\beta^2} + 120\beta^3 \frac{\partial^3}{\partial\beta^3} + 20\beta^4 \frac{\partial^4}{\partial\beta^4} + 2\beta^5 \frac{\partial^5}{\partial\beta^5} \right) \mathcal{M}_{1S},$$

$$\mathcal{M}_{3D} = \frac{1}{3\sqrt{14}} \left(7 + 2\beta \frac{\partial}{\partial\beta} + 2\beta^2 \frac{\partial^2}{\partial\beta^2} \right) \mathcal{M}_{1D},$$

$$\mathcal{M}_{4D} = \frac{1}{3\sqrt{231}} \left(27\beta \frac{\partial}{\partial\beta} + 6\beta^2 \frac{\partial^2}{\partial\beta^2} + 2\beta^3 \frac{\partial^3}{\partial\beta^3} \right) \mathcal{M}_{1D},$$

$$\mathcal{M}_{5D} = \frac{1}{6\sqrt{6006}} \left(231 + 120\beta \frac{\partial}{\partial\beta} + 144\beta^2 \frac{\partial^2}{\partial\beta^2} + 24\beta^3 \frac{\partial^3}{\partial\beta^3} + 4\beta^4 \frac{\partial^4}{\partial\beta^4} \right) \mathcal{M}_{1D}.$$

Result:

$$M(Y(4660)) = 4643 \pm 9 \text{ MeV}$$

$$\Gamma_{tot}(Y(4660)) = 72 \pm 11 \text{ MeV}$$

TABLE I: The $\Lambda_c \bar{\Lambda}_c$ partial decay widths (MeV) of the vector charmonium with a mass of $M = 4643$ MeV. \mathcal{B} represents the branching ratio of the $\Lambda_c \bar{\Lambda}_c$ pair.

State	$\psi(4^3S_1)$	\mathcal{B}	$\psi(5^3S_1)$	\mathcal{B}	$\psi(6^3S_1)$	\mathcal{B}
$\Gamma_{\Lambda_c \bar{\Lambda}_c}$	6.57	9%	2.44	3%	0.84	1%
State	$\psi(3^3D_1)$	\mathcal{B}	$\psi(4^3D_1)$	\mathcal{B}	$\psi(5^3D_1)$	\mathcal{B}
$\Gamma_{\Lambda_c \bar{\Lambda}_c}$	0.33	0.4%	0.19	0.3%	0.09	0.1%

$$\Gamma(\psi(4S/5S/6S) \rightarrow \Lambda_c \bar{\Lambda}_c) \sim \text{a few MeV}$$

$$\Gamma(\psi(3D/4D/5D) \rightarrow \Lambda_c \bar{\Lambda}_c) \sim \text{less than one MeV}$$

If the enhancement $Y(4630)$ in the $\Lambda_c \bar{\Lambda}_c$ invariant-mass distribution is the same structure as $Y(4660)$, the $Y(4660)$ resonance is most likely to be a S-wave charmonium state.

TABLE II: The possible assignments of the $Y(4660)$ with the predicted masses (MeV) from various models.

State	QM [47]	QM [48]	QM [49]	SSE/EA[50]	NR/GI [51]	SP [10]	LP/SP [52]
$\psi(4^3S_1)$	4625	4450	4389	4398/4426	4406/4450	4273	4412/4281
$\psi(5^3S_1)$	4641	4642/4672	...	4463	4711/4472
$\psi(6^3S_1)$	4804/4828	...	4608	...
$\psi(3^3D_1)$...	4520	4426	4464/4477	...	4317	4478/4336
$\psi(4^3D_1)$	4641	4690/4707
$\psi(5^3D_1)$	4840/4855

10、 PRD79,094004 (2009).

50、 arXiv:0810.2875.

47、 PRD72,094004 (2005).

51、 Phys. Atom. Nucl.72,638 (2009).

48、 PRD21,203 (1980).

52、 PRD72,054026 (2005).

49、 PRD32,189 (1985).

Result:

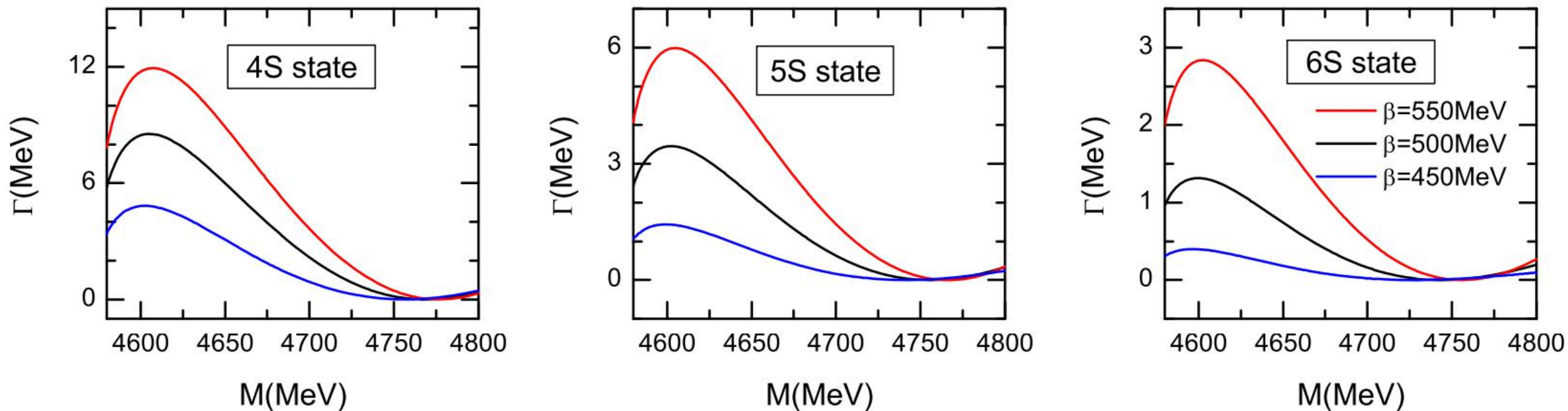
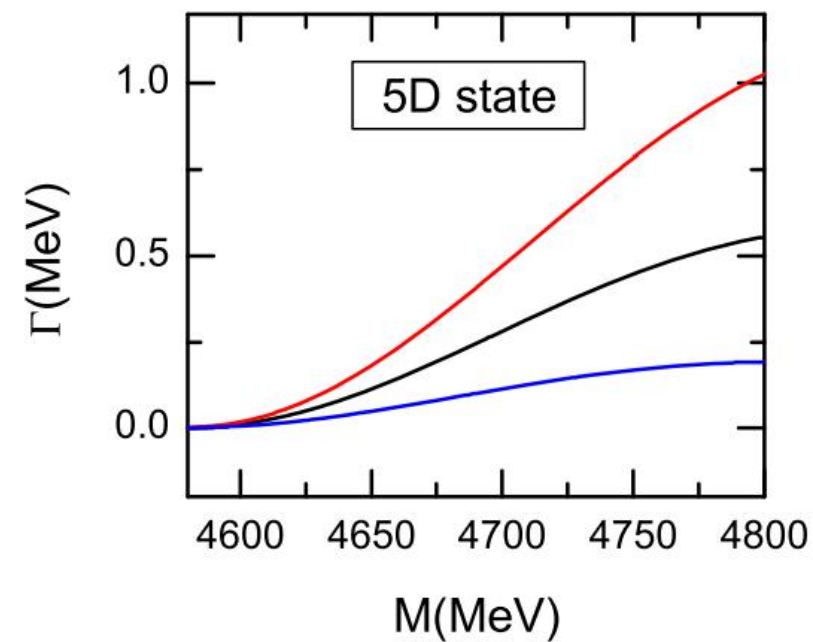
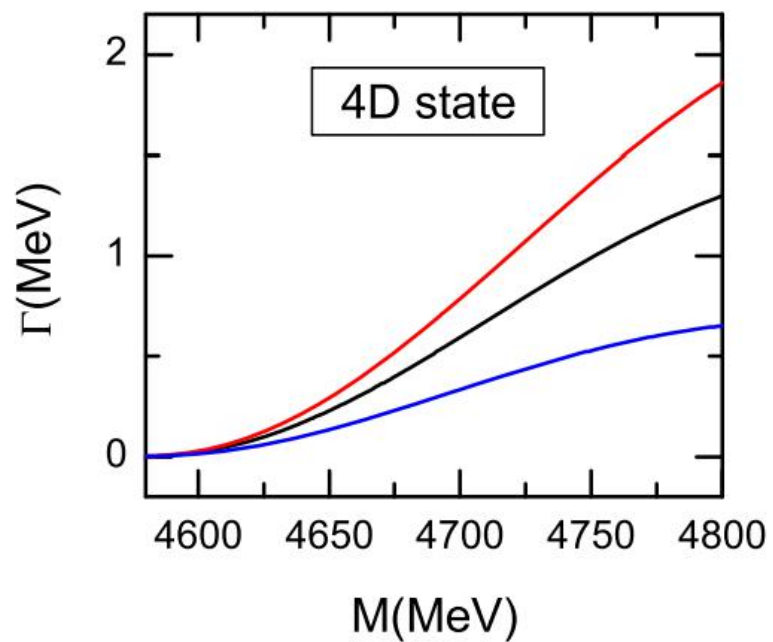
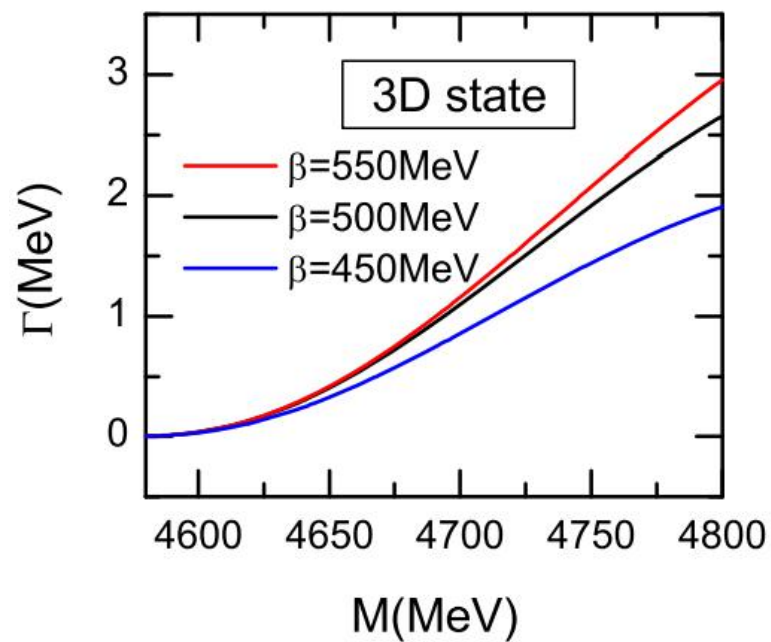


Fig. 3 The variation of the $\Lambda_c \bar{\Lambda}_c$ decay width with the mass of the S-wave vector charmonium. The blue, black, and red lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500, \text{ and } 550$ MeV, respectively

Result:

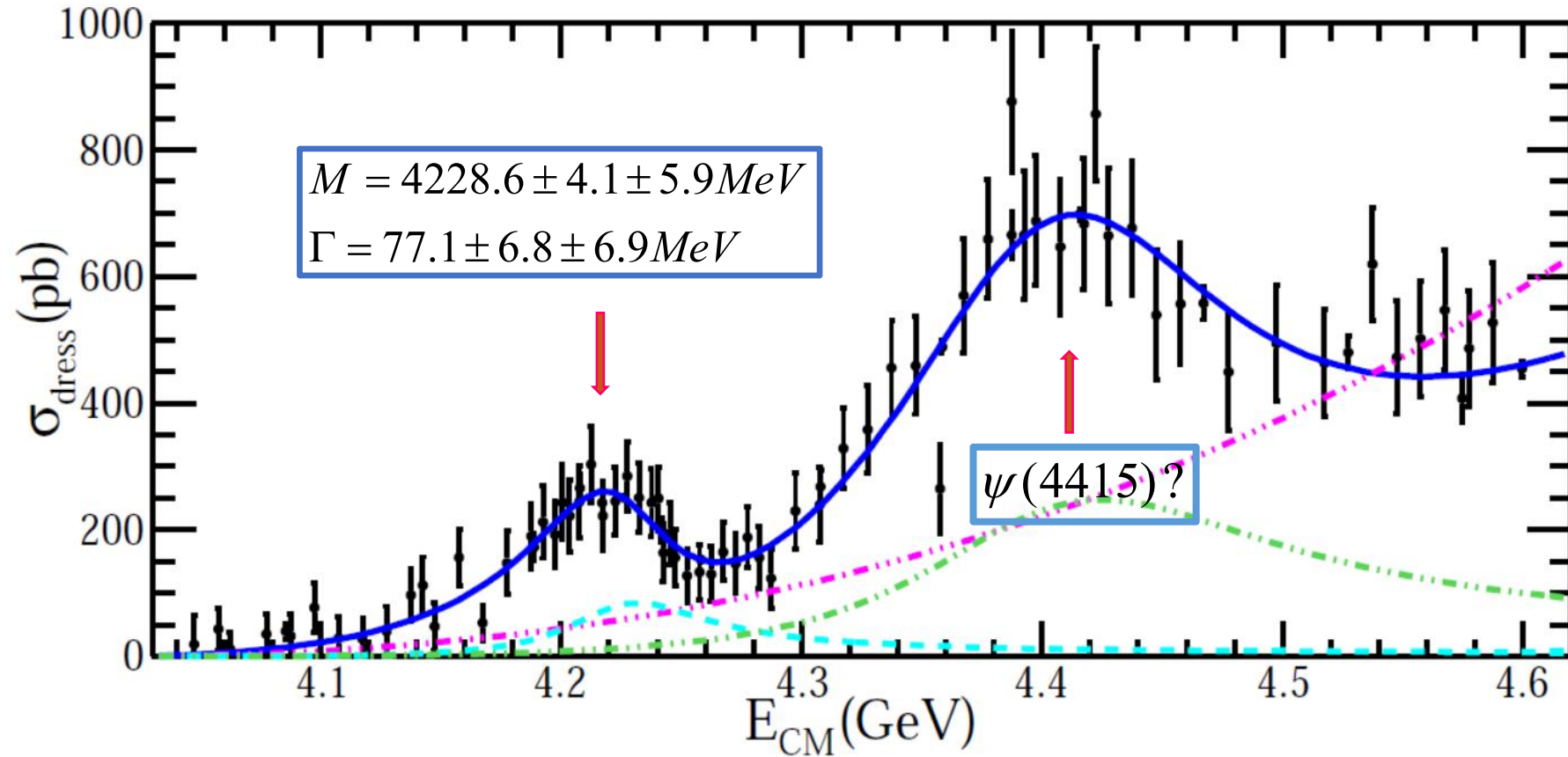


- The $\Lambda_c \bar{\Lambda}_c$ partial decay width of the excited vector charmonium states around 4.6 GeV:
 - ★ $\Gamma(\psi(4S / 5S / 6S) \rightarrow \Lambda_c \bar{\Lambda}_c) \sim$ a few MeV
 - ★ $\Gamma(\psi(3D / 4D / 5D) \rightarrow \Lambda_c \bar{\Lambda}_c) \sim$ less than one MeV
- If the enhancement $Y(4630)$ in the $\Lambda_c \bar{\Lambda}_c$ invariant-mass distribution is the same structure as $Y(4660)$, the $Y(4660)$ resonance is most likely to be a S-wave charmonium state.
- This OZI allowed mode provides a new tool to explore the higher charmonium, which can be produced abundantly at Belle-II.

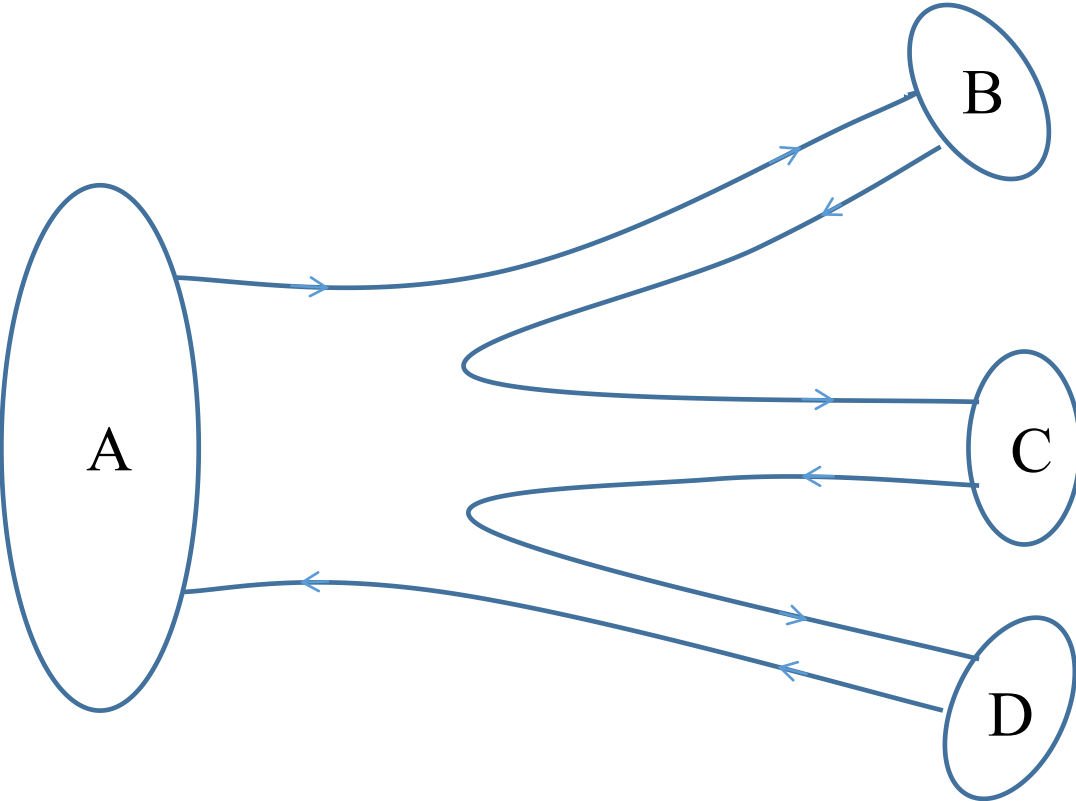
Evidence of a resonant structure in the $e^+e^- \rightarrow \pi^+D^0D^{*-}$ cross section between 4.05 and 4.60 GeV

(BESIII Collaboration)

[arXiv:1808.02847](https://arxiv.org/abs/1808.02847)



Outlook



D^0	D^+	D_s^+
$\pi^0/\pi^+/k^+$	$\pi^-/\pi^0/\bar{k}^0$	$k^-/k^0/\eta$
$\bar{D}^0/D^-/D_s^-$		

Thanks
