STRONG cosmic censorship
—— As subtle as ever

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arXiv: 1808.03635

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Outline

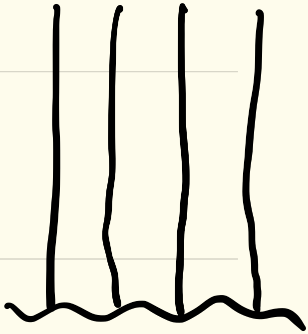
0. Singularity theorems
1. Cosmic Censorship

   □ Weak □ Strong

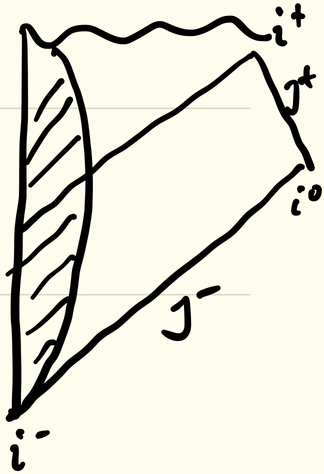
2. Strong Cosmic Censorship
3. our work
4. Speculations

Singularity theorems

Spacetime singularity is inevitable

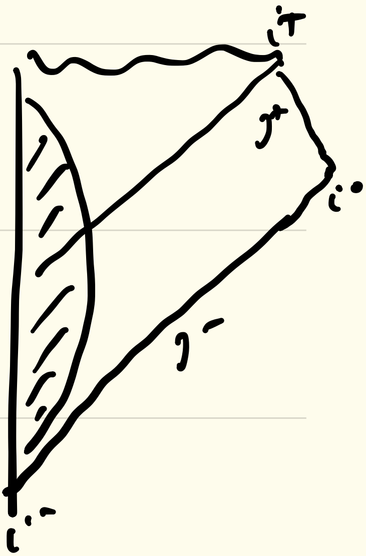


GR predicts its own death near the singularity and should be replaced by QG



Weak cosmic Censorship

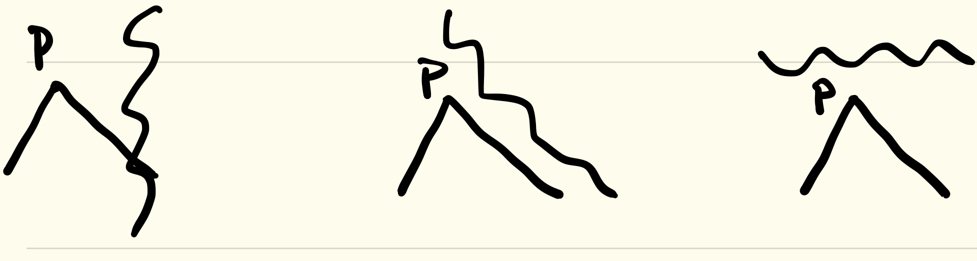
Gravitational collapse leads generically to no formation of naked singularity



God abhors naked singularity

Strong Cosmic Censorship

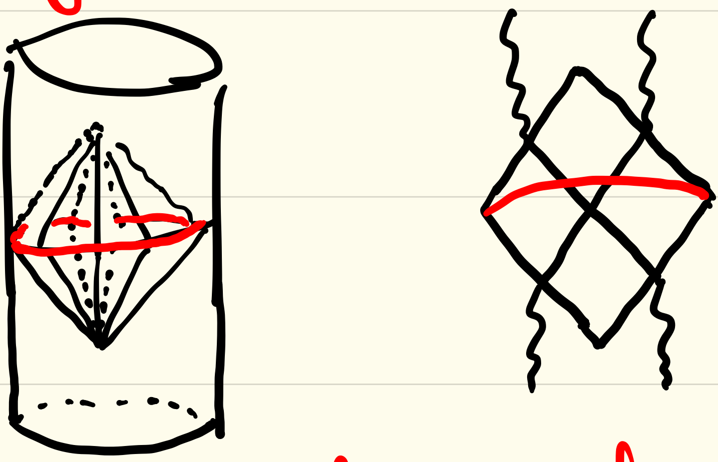
Gravitational collapse leads generically to no formation of timelike singularity



✗ ✓ ✓

Strong Cosmic Censorship

spacetime is globally hyperbolic



The maximal Cauchy development is inextendible

Strong Cosmic Censorship

C^0 extendible

arxiv: 1710.01722

Dafermos & Luk

C^2 inextendible

arXiv: 1702.05715

Luk & Oh

STRONG Cosmic Censorship

The maximal Cauchy development is inextendible as a **weak solution** to the equations of motion

- a spacetime with locally square integrable Christoffel symbols
- a scalar field in the Sobolev H^1_{loc} locally square integrable with a locally square integrable gradient

Strong cosmic censorship

Side Remark: Two other applications of weak solutions

- Shocks in a compressible fluid
- Quantum Fields as distributions

arXiv: 1501.04598 (RN)

Luk & Oh

arXiv: 1512.08260 (Kerr)

Dafermos & Shlapentokh-Rothman

strong cosmic censorship

$\Lambda < 0$ C^0 inextendible

arxiv: 1710.01722

Dafermos & Luk

$\Lambda > 0$ C^2 inextendible

PRD 1990 (RNdS), CQG 1994 (Kerrds)

Mellor & Moss | Chambers & Moss

gr-qc/9801032 | arxiv: 1201.1797

Brady, Moss & Myers | Dafermos

Our Work

arXiv: 1711.10502 (RNds)

Cardoso, Costa, Destounis, Hintz & Jansen

arXiv: 1801.09694 (Kerrds)

Dias, Eperson, Reall, Santos

arXiv: 1803.05443 (charged)

Hod

Our Work

RNdS black hole

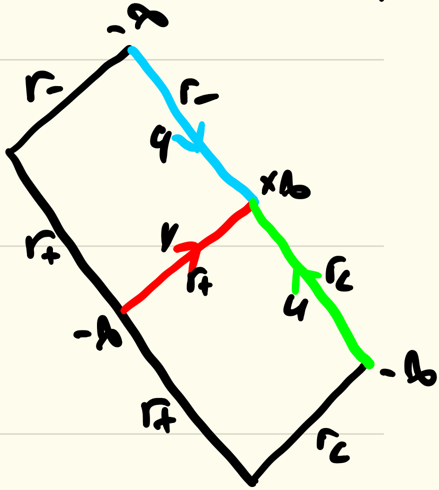
$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}$$

$$= \frac{\Lambda}{3r^2}(r_c - r)(r - r_+)(r - r_-)(r - r_o)$$

$$A_a = -\frac{Q}{r}(dt)_a$$

$$\kappa_h = \left|\frac{1}{2} f'(r_h)\right|$$



Our Work

Klein-Gordon equation

$$(\nabla_a - iqA_a)(\nabla^a - iqA^a)\Psi = 0$$

$$(A, \Psi) \rightarrow (A + d\lambda, e^{iq\lambda}\Psi)$$

$$\Psi = \frac{\psi(r)}{r} Y_{lm}(\theta, \phi) e^{-i\omega t}$$

Our Work

$$\frac{d^2\psi}{dr_*^2}+\left[\left(\omega-\Phi(r)\right)^2-V(r)\right]\psi=0$$

$$\Phi(r)=\frac{qQ}{r} \qquad r_*=\int\frac{dr}{f}$$

$$=-\frac{1}{2\kappa_c}\ln\left(1-\frac{r}{r_c}\right)+\frac{1}{2\kappa_+}\ln\left(\frac{r}{r_+}-1\right)$$
$$-\frac{1}{2\kappa_-}\ln\left(\frac{r}{r_-}-1\right)+\frac{1}{2\kappa_0}\ln\left(1-\frac{r}{r_0}\right)$$

$$V(r)=\frac{f\left(rf'+l(l+1)+m^2r^2\right)}{r^2}$$

$$r\to r_h,\quad \psi\sim e^{\pm i\left[\omega-\Phi(r_h)\right]r_*}$$

Our Work

Boundary Condition

$r \to r_+, \ \psi \sim e^{-i[\omega - \Phi(r)] r_*}$ ingoing

$r \to r_c, \ \psi \sim e^{i[\omega - \Phi(r)] r_*}$ outgoing

Quasi-normal Modes

$$(q, \omega_i) \longleftrightarrow (-q, -\overline{\omega_i})$$

Our Work

Analytic continuation to r_-

$$\Psi_{out} \sim e^{-i\omega u} \quad \left(d\lambda = \frac{Q}{r} d\Phi \right)$$

$$\Psi_{in} \sim e^{-i\omega u} (r - r_-)^{\frac{i[\omega - \Phi(r_-)]}{\kappa_-}}$$

extendible iff

Ψ_{in} has a locally square integrable derivative, belonging to H^1_{loc}

$$\beta \equiv -\frac{\mathrm{Im}(\omega)}{\kappa_-} > \frac{1}{2}$$

Our Work

Double null Coordinates

$$ds^2 = -f\,du\,dv + r^2 d\Omega^2$$

$$u = t - r_*, \quad v = t + r_*$$

$$d\lambda = \frac{Q(2r - r_c - r_+)}{r(r_c - r_+)} dr_*$$

$$A_a = \frac{Q(r - r_+)}{r(r_c - r_+)} (du)_a + \frac{Q(r - r_c)}{r(r_c - r_+)} (dv)_a$$

Our Work

$$\Psi = \frac{\psi(u,v)}{r} Y_{lm}(\theta,\phi)$$

$$-4\partial_u\partial_v\psi - 4i\Phi(r)\left[\frac{r_c - r}{r_c - r_+}\partial_u\psi + \frac{r - r_+}{r_c - r_+}\partial_v\psi\right]$$

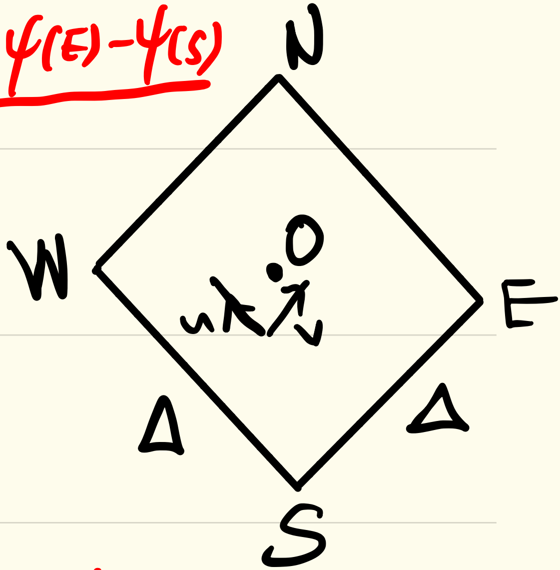
$$- U(r)\psi = 0$$

$$U(r) = \frac{4\Phi^2(r)(r-r_c)(r-r_+)}{(r_c-r_+)^2} + \frac{f}{r^2}\left[l(l+1) + rf' + m^2 r^2 + i\frac{\Phi(r)\, r(r_c+r_+)}{r_c - r_+}\right]$$

Our Work

$$\partial_u \partial_v \psi\Big|_0 = \frac{\psi(N)-\psi(E)-\psi(W)+\psi(S)}{\Delta^2}$$

$$\partial_u \psi\Big|_0 = \frac{\psi(N)+\psi(W)-\psi(E)-\psi(S)}{2\Delta}$$



$$\partial_v \psi\Big|_0 = \frac{\psi(N)+\psi(E)-\psi(W)-\psi(S)}{2\Delta}$$ (u_0, v_0)

$$\psi\Big|_0 = \frac{\psi(E)+\psi(W)}{2}$$

Our Work

$$\psi(N) = \left(1 + i\frac{\Phi(r)\Delta}{2}\right)^{-1}$$

$$\left[-\left(1 - i\frac{\Phi(r)\Delta}{2}\right)\psi(S)\right.$$

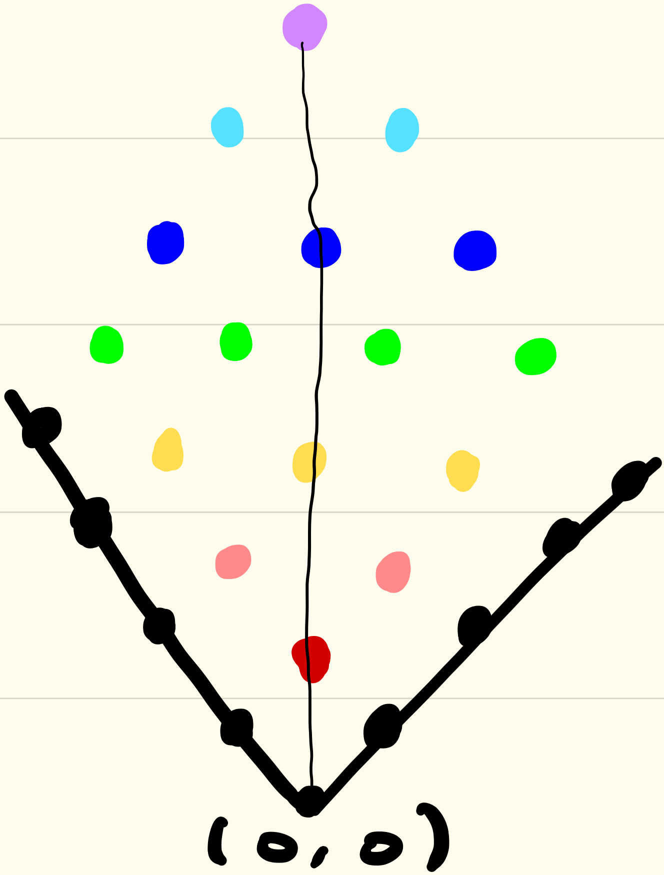
$$-i\frac{(2r - r_c - r_+)\Delta}{2(r_c - r_+)}\left(\psi(E) - \psi(W)\right)$$

$$\left.+\left(1 - \frac{U(r)\Delta^2}{8}\right)\left(\psi(E) + \psi(W)\right)\right]$$

$$\psi(0, v) = 0, \quad \psi(u, 0) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u - u_c)^2}{2\sigma^2}}$$

Our Work

Diamond parallel evolution



Our Work

Prony Method

$$\psi_N \equiv \Psi(t_N) = \sum_{i=1}^{M} c_i e^{-i\omega_i \Delta t N}$$

$$= \sum_{i=1}^{M} c_i Z_i^N$$

$$Z^M - P_1 Z^{M-1} - P_2 Z^{M-2} - \cdots - P_M$$

$$= \prod_{i=1}^{M} (Z - Z_i)$$

Our Work

Prony Method

$$\begin{pmatrix} \psi_M \\ \vdots \\ \psi_{N-1} \end{pmatrix} = \begin{pmatrix} \psi_{M-1} & \cdots & \psi_0 \\ \vdots & \ddots & \vdots \\ \psi_{N-2} & \cdots & \psi_{N-M-1} \end{pmatrix} \begin{pmatrix} p_1 \\ \vdots \\ p_M \end{pmatrix}$$

$N \geq 2M$

$$\begin{pmatrix} \psi_0 \\ \vdots \\ \psi_L \end{pmatrix} = \begin{pmatrix} z_1^0 & \cdots & z_m^0 \\ \vdots & \ddots & \vdots \\ z_1^L & \cdots & z_m^L \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_m \end{pmatrix}$$

$L \geq M$

Nyquist sampling criteria $\mathrm{Re}(\omega_i) < \frac{\pi}{\Delta t}$

Our Work

Numerical Results

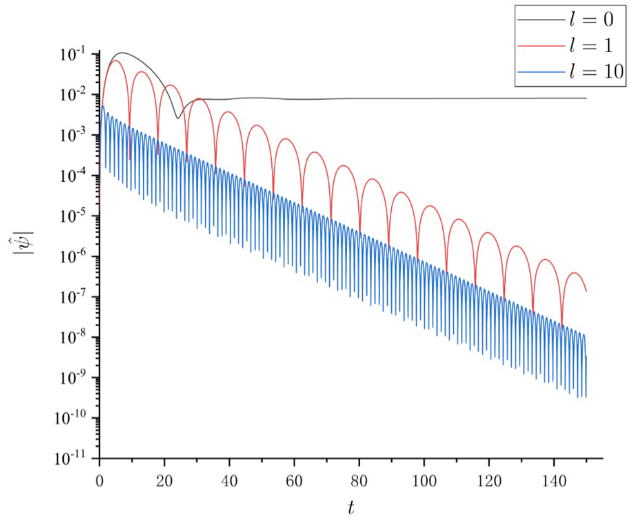


FIG. 4: The temporal evolution of $|\hat{\psi}(t)|$ of $q = 0.1$ for $\Lambda M^2 = 0.02$ and $Q/Q_m = 0.9910$.

Our Work

Numerical Results

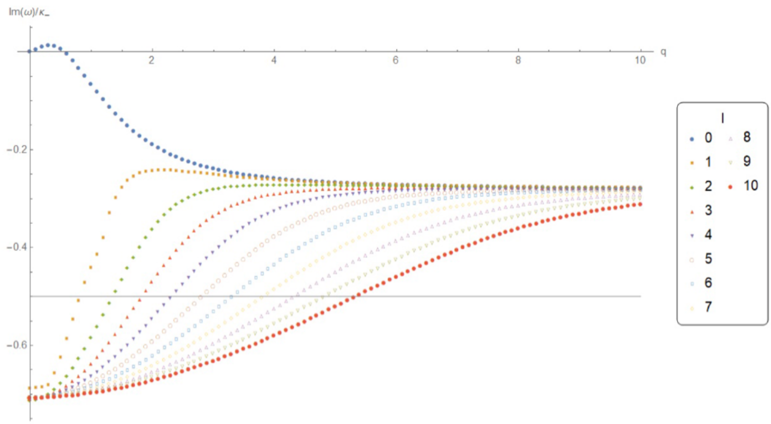


FIG. 5: The lowest-lying QNMs for $\Lambda M^2 = 0.02$ and $Q/Q_m = 0.9950$.

Our Work

Numerical Results

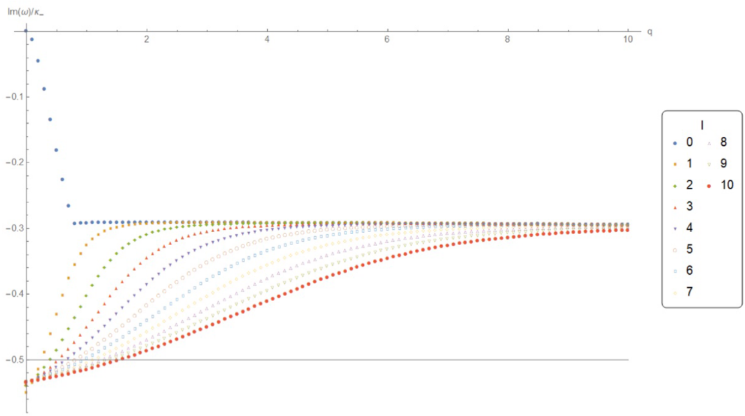


FIG. 6: The lowest-lying QNMs for $\Lambda M^2 = 0.14$ and $Q/Q_m = 0.9950$.

Our Work

Numerical Results

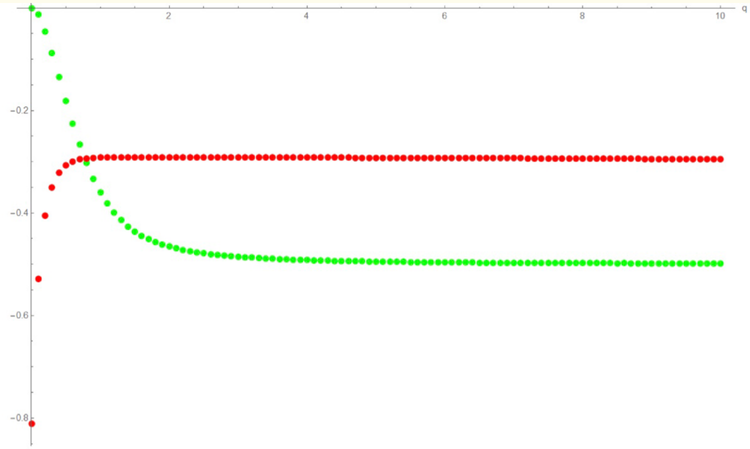


FIG. 7: The $l = 0$ dominant and sub-dominant QNMs for $\Lambda M^2 = 0.14$ and $Q/Q_m = 0.9950$, where the green points and red points denote the zero mode and near extremal mode, respectively.

Our Work

Numerical Results

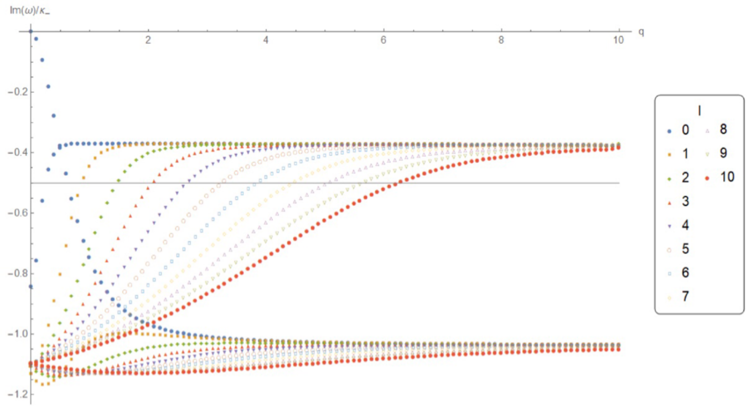


FIG. 8: The dominant and sub-dominant QNMs for $\Lambda M^2 = 0.14$ and $Q/Q_m = 0.9985$.

Our Work

Numerical Results

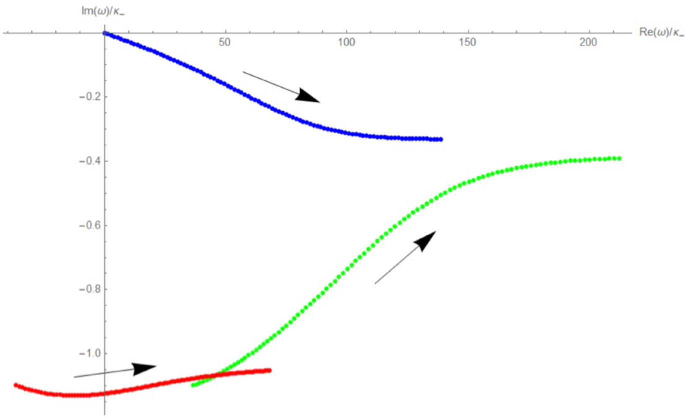


FIG. 9: The symmetry breaking of the $l = 10$ left and right photon sphere modes for $\Lambda M^2 = 0.14$ and $Q/Q_m = 0.9985$. The red points denote the left photon sphere modes ω_L, the green points denote the right photon sphere modes ω_R, and the blue points denote $(\omega_L + \overline{\omega_R})/2$, where the arrow indicates the increase of the charge q from 0 to 10.

Our Work

Numerical Results

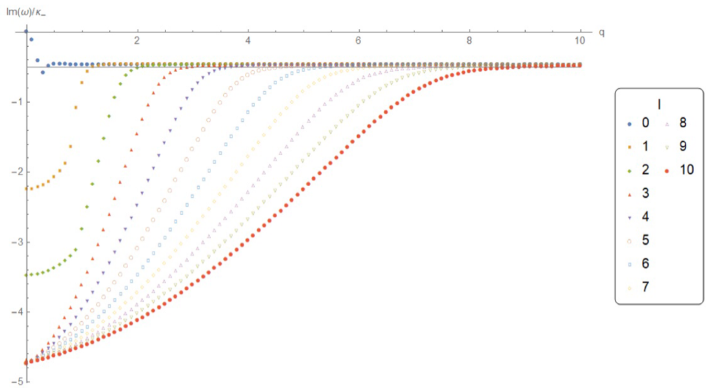


FIG. 10: The lowest-lying QNMs for $\Lambda M^2 = 0.14$ and $Q/Q_m = 0.9999$.

Our Work

Two related papers

arXiv:1808.03631

Cardoso, Costa, Destounis, Hintz & Jansen

arXiv:1808.04832

Dias, Reall & Santos

Speculations

- Rough version !

  arxiv: 1805.08764

  Dafermos & Shlapentokh-Rothman

  arxiv: 1808.02895

  Dias, Reall, & Santos

- Quantum Gravity ?

  private Communication with Vafa

  charged fermion ? (on going)

Thank you

for your listening

Appendix

Generalized eigenvalue method

$$(M_0 + \omega M_1 + \cdots + \omega^p M_p) X = 0$$

$$\tilde{M}_0 = \begin{pmatrix} M_0, & M_1 & \cdots & M_{p-1} \\ 0 & & & \\ 0 & & \mathbb{I} & \\ \vdots & & & \\ 0 & & & \end{pmatrix}$$

$$\tilde{M}_1 = \begin{pmatrix} 0, & 0, & \cdots & M_p \\ & & & 0 \\ & -\mathbb{I} & & \vdots \\ & & & 0 \end{pmatrix}$$

$$(\tilde{M}_0 + \omega \tilde{M}_1) \tilde{X} = 0,$$

$$\tilde{x} = (\omega^0 x, \omega^1 X, \cdots, \omega^{p-1} X)$$