Axial charge dynamics in Heavy Ion Collisions

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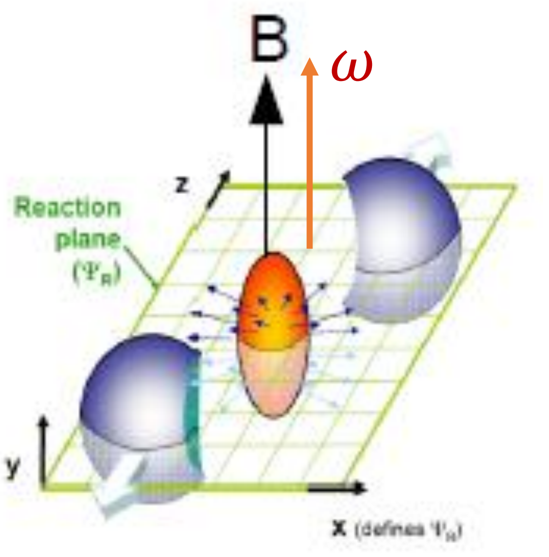
Thanks to: Hou, Iatrakis, Liang, Yan, Yin

Outline

- Motivation
- Axial charge is stochastic
- Fluctuation and dissipation of axial charge
- Stochastic hydrodynamics for axial charge
- Axial charge evolution in static flow and Bjorken flow
- CME from stochastic hydrodynamics
- Summary&Outlook

Experimental signature of anomalous effect



Chiral Magnetic Effect (CME)

$$\boldsymbol{j} = C\mu_5 e\boldsymbol{B}$$

Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008
Metlitski, Zhitnitsky, PRD 2005

Chiral Vortical Effect (CVE)

$$\boldsymbol{j} = C\mu_5\mu\boldsymbol{\omega}$$

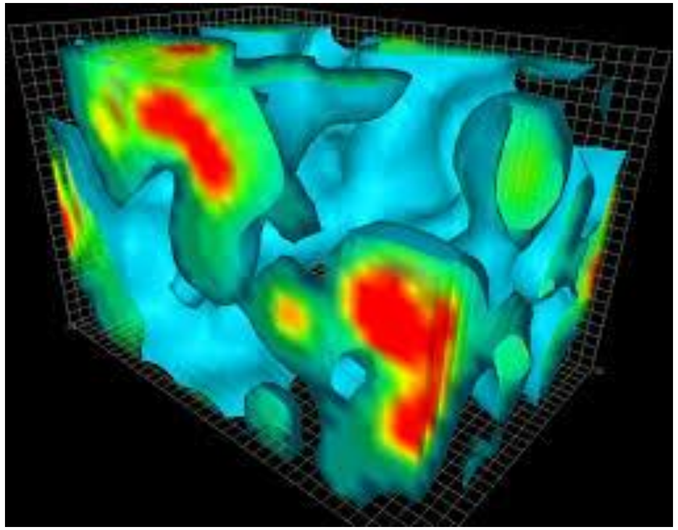
Talks by H. Huang, Ko, F. Wang et al

Vilenken, PRD 1980
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011

Why are we excited about anomalous effect?

$\boldsymbol{j} = C\mu_5 e\boldsymbol{B}$ $\qquad$ $\boldsymbol{j} = C\mu_5 \mu\boldsymbol{\omega}$

μ_5 characterizes chiral imbalance, originates from topological fluctuations in QCD



$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{8\pi^2} trG\tilde{G}$$

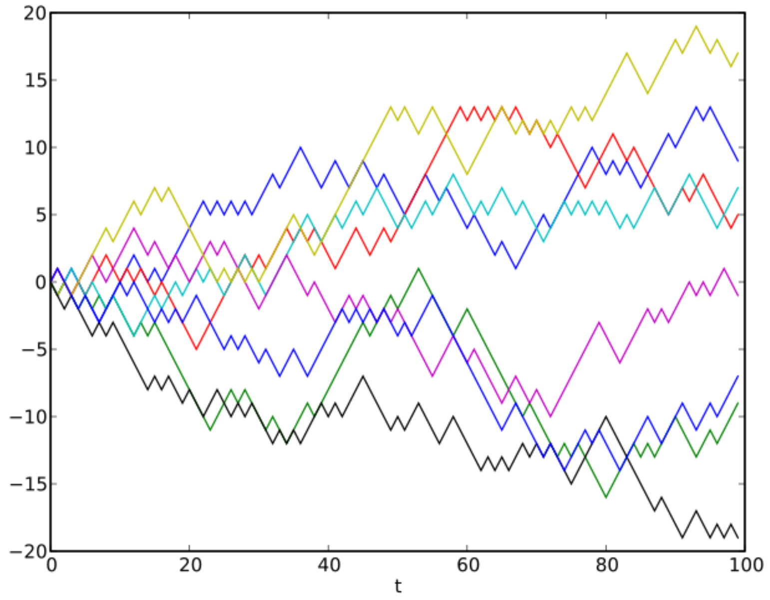
Axial charge is stochastic

Chiral imbalance generated by fluctuation: N_5 not a thermodynamic quantity

$\langle N_5 \rangle = 0, \langle {N_5}^2 \rangle \neq 0$ μ_5 effective chemical potential

N_5 stochastic!

schematic behavior of N_5

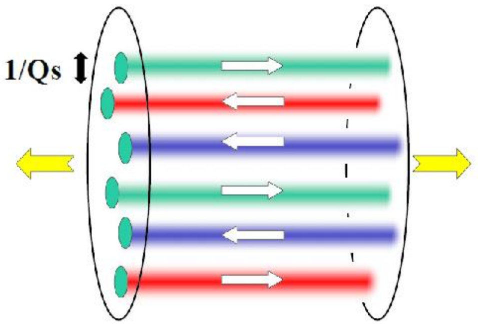


How should we quantify μ_5

initial generation

$\mu_5 \lesssim 100 MeV$

Fukushima, Kharzeev, Warringa (2010)
Hirono, Hirano, Kharzeev (2014)
Jiang, Shi, Yin, Liao (2016), (2017)
Mace, Schlichting, Venugopalan (2016)

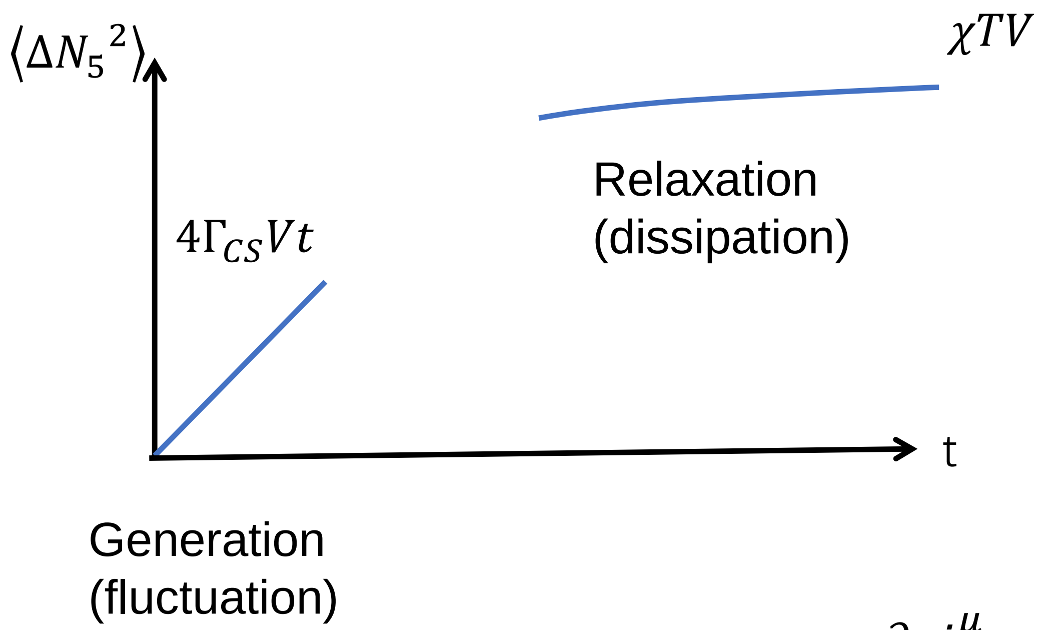


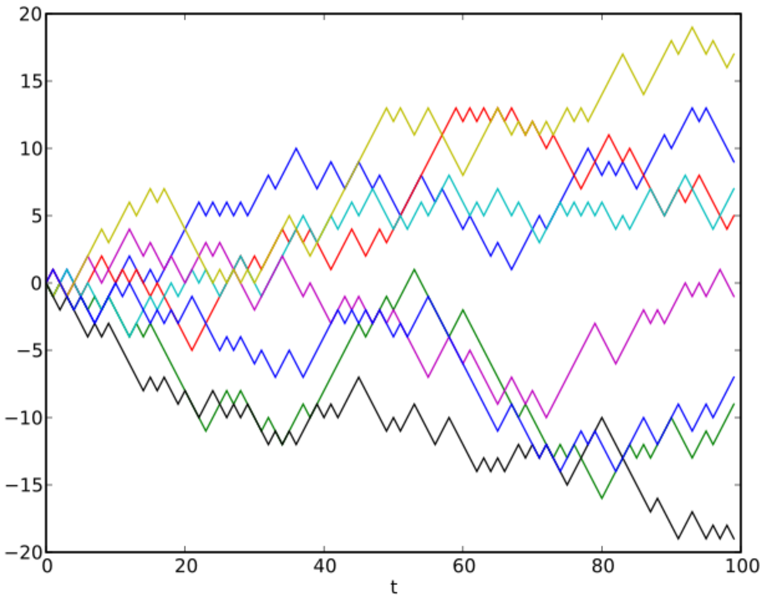
$$\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

further evolution: current studies assume $\partial_\mu j_5^\mu = 0$

$F\tilde{F} = 0$ in HIC
$trG\tilde{G}$ and $2im\bar{\psi}\gamma^5\psi$ can both change N_5: fluctuation and dissipation exist at all stages!

Gluonic fluctuation and dissipation of N_5 in equilibrium





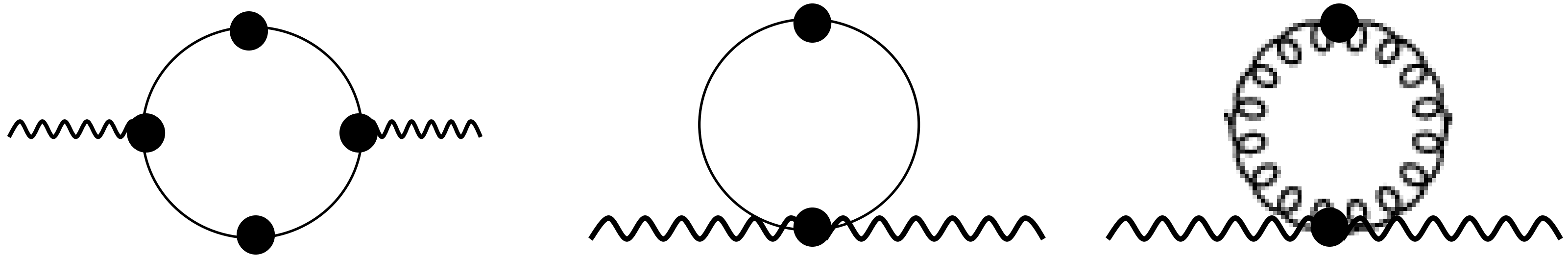
$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Γ_{CS}: rate of topological fluctuation (Chern-Simon diffusion)

$\tau_{CS} = \frac{\chi T}{2\Gamma_{CS}}$: relaxation time scale for N_5

Arnold, Moore, Yaffe et al 90s
Rubakov, Shaposhinikov (1996)

What about fluctuation and dissipation from mass term?



$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Hou, SL, PRD 2018
Guo, SL, PRD 2016

Power counting: $\omega \sim gT$

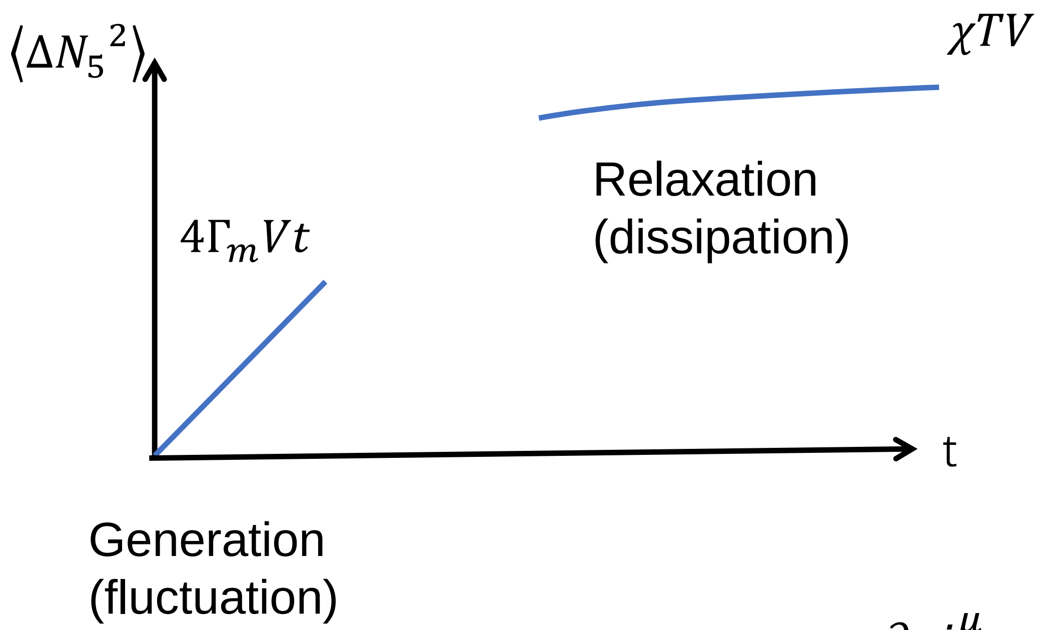
assume current mass: $m \sim gT$

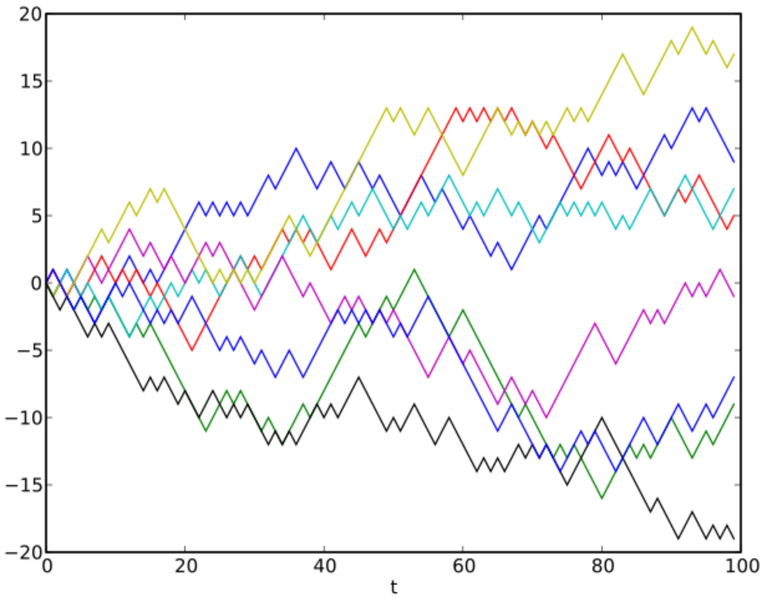
$\mathrm{Im}G^R \sim \frac{i\Gamma_m T}{\omega}$, as $\omega \to 0$

Random walk behavior with $\Gamma_m \simeq 0.013 m^2 m_f^2.$

m_f: thermal mass

Quark fluctuation and dissipation of N_5 in equilibrium





$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Γ_m: rate of mass diffusion

$\tau_m = \frac{\chi T}{2\Gamma_m}$: relaxation time scale for N_5

Hou, SL, PRD 2018

Guo, SL, PRD 2016

Gluonic fluctuation vs Quark fluctuation

weak coupling results

$\Gamma_{CS} \sim 30\alpha_s^4 T^4$ Moore, Tassler JHEP 2011

$\Gamma_m \simeq 0.013 m^2 m_f^2.$ Hou, SL, PRD 2018

For $T = 350$MeV and $\alpha_s = 0.3$, $\Gamma_{CS} \gg \Gamma_m$ Gluonic fluctuation dominates

$$\tau_{CS} = \frac{\chi T}{2N_f^2 \Gamma_{CS}} \lesssim 0.8fm$$

N_5 relaxation significant

$$\tau_{CS} \sim \frac{1}{N_f T}$$

Add Fluc/Diss to anomalous hydrodynamics

$$\partial_\mu j^\mu = 0,$$
$$\partial_\mu j_5^\mu = -CE_\mu B^\mu,$$
$$j^\mu = nu^\mu + \kappa_B B^\mu,$$
$$j_5^\mu = n_5 u^\mu + \xi_B B^\mu,$$

Hirono, Hirano, Kharzeev (2014)
Jiang, Shi, Yin, Liao (2016), (2017)

Talk by Shi

Fluctuation generates stochastic n_5, dissipation drives n_5 to equilibrium

Hydro: near equilibrium dynamics, use equilibrium fluc/diss in hydro

Stochastic hydrodynamics for n_5

$$\begin{cases} \partial_t n_5 + \nabla \cdot \mathbf{j}_5 = -2q, \\ \mathbf{j}_5 = -D\nabla n_5 + \xi, \\ q = \frac{n_5}{2\tau_{\rm CS}} + \xi_q, \end{cases}$$

thermal fluctuation

topological fluctuation

dissipation

Iatrakis, SL, Yin, JHEP 2015

coupling to vector current:
Huang, Liao, Lin in preparation

$$\partial_t N_5 = -\frac{N_5}{\tau_{CS}}$$

$$\langle \xi_q(t,\mathbf{x})\xi_q(t',\mathbf{x}')\rangle = \Gamma_{\rm CS}\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'),$$

$$\langle \xi_i(t,\mathbf{x})\xi_j(t',\mathbf{x}')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'),$$

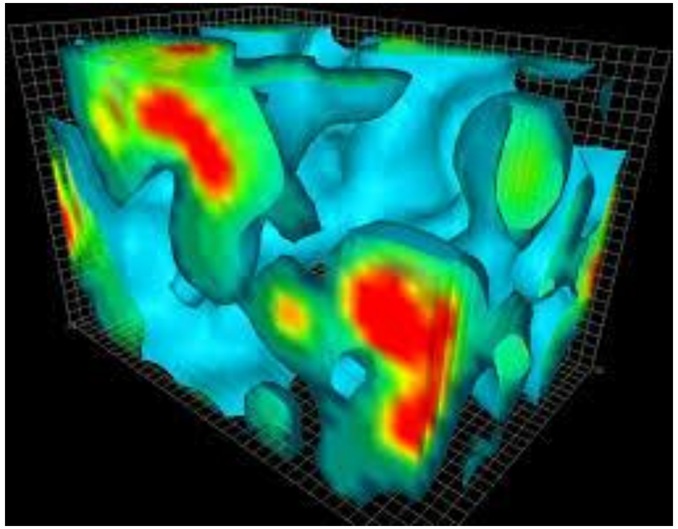
$$\langle \xi_i(t,\mathbf{x})\xi_q(t',\mathbf{x}')\rangle = 0.$$

Einstein relations

$$\sigma = \chi D, \quad \tau_{\rm CS} = \frac{\chi T}{2\Gamma_{\rm CS}}$$

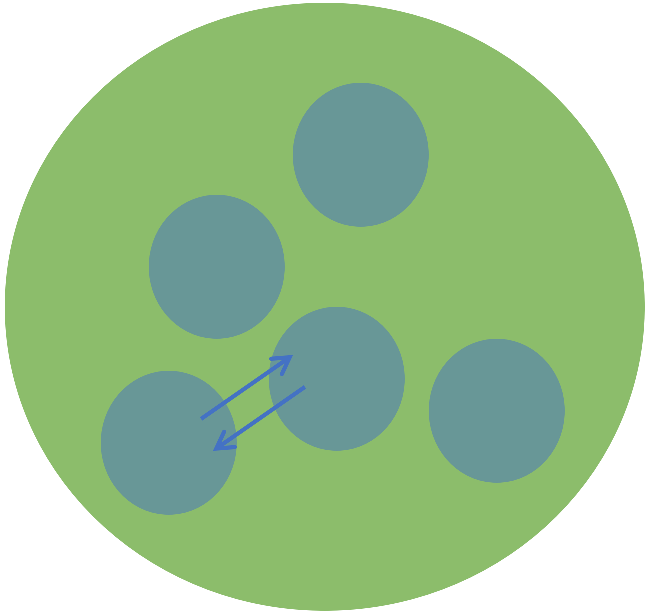
Topological noise vs Thermal noise

Topological noise
within fluid cell



unique to n_5

Thermal noise
between fluid cells



exists for n_5 and n

Example: axial charge dynamics in static fluid

$$C_{nn}(t,x) \equiv \langle [n_5(t,x) - n_5(0,x)][n_5(t,0) - n_5(0,0)] \rangle$$

$$C_{nn}(t,x) = \chi T[\delta^3(x) - \frac{1}{(8\pi D t)^{\frac{3}{2}}} e^{-\frac{2t}{\tau_{CS}} - \frac{|x|^2}{8Dt}}]$$

within cell | across cells

Early time $t \ll \tau_{CS}$ | $C_{nn}(t,x) = 4\Gamma_{CS} t \delta^3(x)$ | random walk growth

Late time $t \gg \tau_{CS}$ | $C_{nn}(t,x) = \chi T \delta^3(x)$ | thermodynamic limit

Covariant stochastic hydrodynamics

$$\begin{cases} \nabla_\mu J_5^\mu = -2q, \\ J_5^\mu = n_5 u^\mu - \sigma T P^{\mu\nu} \nabla_\nu \left(\frac{\mu_5}{T}\right) + P^{\mu\nu}\xi_\nu, \\ q = \frac{n_5}{2\tau_{\rm CS}} + \xi_q, \end{cases}$$

SL, Yan, Liang, PRC 2018

$$\langle P^{\mu\alpha}\xi_\alpha(x) P^{\nu\beta}\xi_\beta(x')\rangle = P^{\mu\alpha}P^{\nu\beta}g_{\alpha\beta}2\sigma T\frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle \xi_q(x)\xi_q(x')\rangle = \Gamma_{\rm CS}\frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle P^{\mu\alpha}\xi_\alpha(x)\xi_q(x')\rangle = 0.$$

Apply to Bjorken flow

N_5 dynamics in Bjorken: vanishing initial value

$\tau_1 = \tau_0$ and $N_5(\tau_1) = 0.$

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2x_\perp 2\Gamma_0 \tau_0 \tau_{\rm CS0} \left(1 - e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{\rm CS0}}\right)}\right)$$

Early time $\tau_0 \ll \tau_2 \ll \tau_{CS0}$

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2x_\perp 6\Gamma_0 \tau_0^2 \left(\left(\frac{\tau_2}{\tau_0}\right)^{2/3} - 1\right) \quad \sim \ {\tau_2}^{2/3}$$

compared to $\sim t$ in static flow

Late time $\tau_0 \ll \tau_{CS0} \ll \tau_2$

$$\langle (N_5(\tau_2)^2) \rangle = \int d\eta d^2x_\perp 2\Gamma_0 \tau_0 \tau_{\rm CS0} = \int \tau_0 d\eta d^2x_\perp \chi_0 T_0 \ \sim \ \chi T V$$

thermodynamic limit

N_5 dynamics in Bjorken: large initial value

$$\langle N_5(\tau_2)^2\rangle = \langle N_5(\tau_1)^2\rangle e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{CS0}}\right)} + \int d\eta d^2x_\perp 2\Gamma_0\tau_0\tau_{CS0}\left(1 - e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{CS0}}\right)}\right)$$

exponential decay of initial charge

growth of fluctuation

estimate of initial charge

$$\sqrt{\langle n_5(\tau_0)^2\rangle} \simeq \frac{Q_s^4(\pi\rho_{\rm tube}^2\tau_0)\sqrt{N_{\rm coll}}}{16\pi^2 A_{\rm overlap}}$$

$$\mu_5(\tau_0) \simeq 35{\rm MeV}$$

Jiang, Shi, Yin, Liao (2016)

two terms become comparable

$$\tau_2 \simeq 3.2{\rm fm \ for\ } 70-80\%$$

$$\tau_2 \simeq 6.3{\rm fm \ for\ } 0-5\%$$

Late time: thermodynamic limit, independent of initial condition

Estimate of μ_5 from thermodynamic limit

$$N_5 \sim \left(\int d\eta d^2x_\perp 2\Gamma_0\tau_0\tau_{\rm CS0}\right)^{1/2} = \left(\pi R^2\Delta\eta 2\Gamma_0\tau_0\tau_{\rm CS0}\right)^{1/2},$$

$$\mu_5 = \frac{N_5}{\pi R^2\tau\Delta\eta\chi}.$$

$\mu_5 \sim \tau^{-1/3}$ $\quad |\eta| < 2$

Centrality	70-80%	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
μ_5(MeV)	13.1	10.6	8.84	7.63	6.70	5.94	5.26	4.78	4.44

Fluctuation significant in small volume

CME from n_5 in thermodynamic limit

$j = C_e \mu_5 eB$ Axial charge induced electric charge dipole.

Centrality	70-80%	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
$e\mu_e$(MeV)	7.40	4.85	3.40	2.53	1.95	1.53	1.20	1.00	0.86

$$B = B_0 e^{-\tau/\tau_B}$$

$$eB_0 = 10 m_\pi^2 \text{ and } \tau_B = 3\text{fm}$$

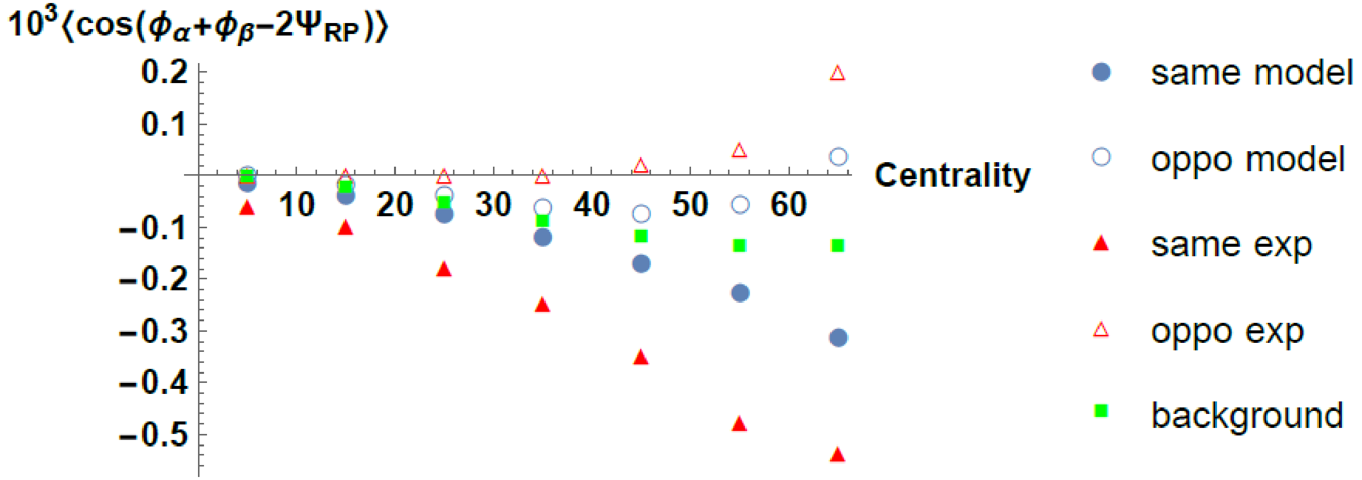
B field uncertainty: talks by Ma, K. Xu

Cooper-Frye freezeout

$$\delta N_Q = \frac{Q}{(2\pi)^2} \int dy \int m_\perp^2 dm_\perp \tau_f d\eta d^2 x_\perp \cosh(\eta - y) e^{-m_\perp \cosh(\eta - y)/T_f + \mu_\pi / T_f} \delta\mu_e$$

$|y| < 1 \quad |\eta| < 2$

CME from stochastic hydrodynamics



Au-Au 200GeV

STAR, PRC (2010)

- background from $v1^2$ from hydro
- background from momentum/charge conservation not included
- signal less than measured correlation

Liang, SL, Yan, in preparation

n_5 dynamics in Bjorken: vanishing initial value

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\tau_2 n_5(\tau_2,0)\rangle = \chi_0 T_0 \tau_0 \int \frac{dk_\eta}{2\pi} e^{-ik_\eta \eta}\left(1 - e^{-2(c+bk_\eta^2)}|_{\tau=\tau_1}\right)$$
$$= \chi_0 T_0 \tau_0 \left(\delta(\eta) - e^{-2c}\frac{e^{-\eta^2/(8b)}}{2\sqrt{2\pi b}}|_{\tau=\tau_1}\right).$$

localized in rapidity

from diffusion

from topological fluctuation

can be used to calculate pseudo-rapidity dependence of CME signal

Summary&Outlook

- Axial charge dynamics includes fluctuation and dissipation.
- Gluonic fluctuation (topological) >> quark fluctuation (mass)
- Stochastic hydrodynamics: near equilibrium hydrodynamics with equilibrium noises.
- Stochastic hydrodynamics in Bjorken flow: damping effect significant. Axial charge in thermodynamic limit may be a reasonable approximation for CME.

- More realistic stochastic hydrodynamics for axial/vector charge
- Fluctuations/dissipation away from equilibrium: applicable to hydro and possibly chiral kinetic theory

Thank you!

Choice of parameters in Bjorken flow

$\tau_{\rm CS} \sim \frac{1}{T} \qquad \Gamma_{\rm CS} \sim T^4$

$$T = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}, \quad \tau_{\rm CS} = \left(\frac{\tau}{\tau_0}\right)^{1/3} \tau_{\rm CS0}, \quad \Gamma_{\rm CS} = \Gamma_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}$$

$$\tau_0 = 0.6\text{fm}, \; T_0 = 350\text{MeV}, \; \Gamma_0 = 30\alpha_s^4 T_0^4 \qquad \alpha_s = 0.3.$$

Moore and Tassler, JHEP (2011)

$$\chi_0 = 3T_0^2$$

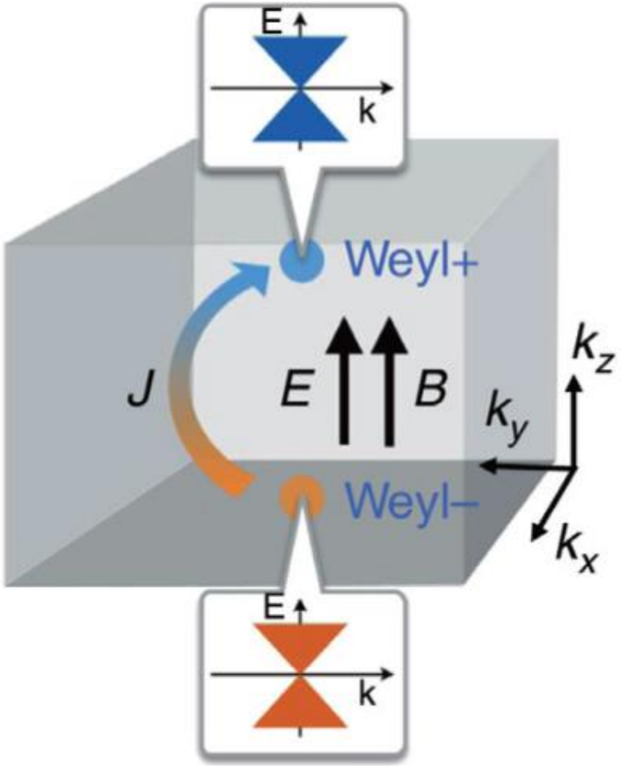
$$\tau_{CS} = \frac{\chi T}{2N_f^2 \Gamma_{CS}} \lesssim 0.8fm$$

Damping time scale at initial temperature, comparable to QGP evolution time. Damping important!

Sources of axial charge generation

$$\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Weyl Semi-metal



Li, Kharzeev et al
Nature. Phys. (2016)

γ correlator

$$\gamma_{\alpha\beta} = \langle v_1^2 \cos 2(\Psi_1 - \Psi_{RP}) \rangle - \frac{\pi^2}{16} \frac{\langle \Delta_\alpha \Delta_\beta \rangle}{\langle N_\alpha \rangle \langle N_\beta \rangle} = \langle v_1^2 \cos 2(\Psi_1 - \Psi_{RP}) \rangle - a_{\alpha\beta}.$$

$v1^2$ background | CME

$v1^2$ background calculated from viscous hydro with fluctuating initial condition

CME calculated from stochastic hydrodynamics