Chirality production, Schwinger Mechanism and Anomalous Magnetohydrodynamics

Shi Pu

University of Science and Technology of China

2019.04 @ Chirality 2019

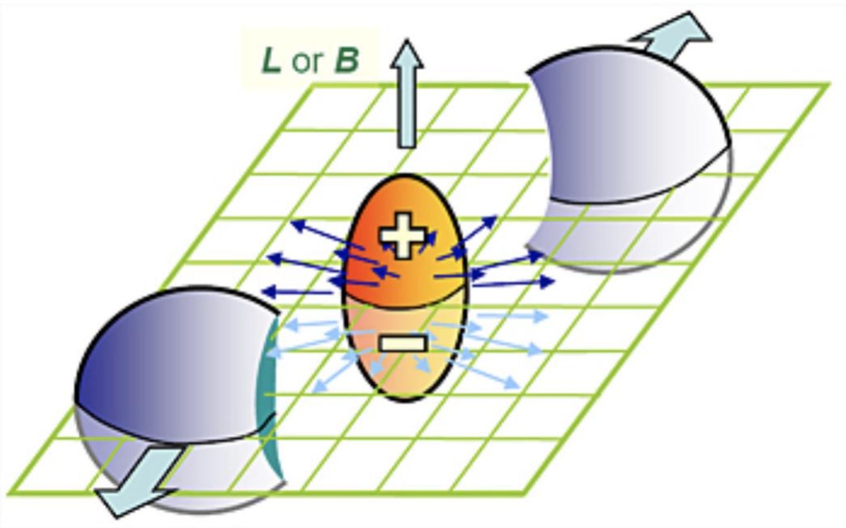
Outlines

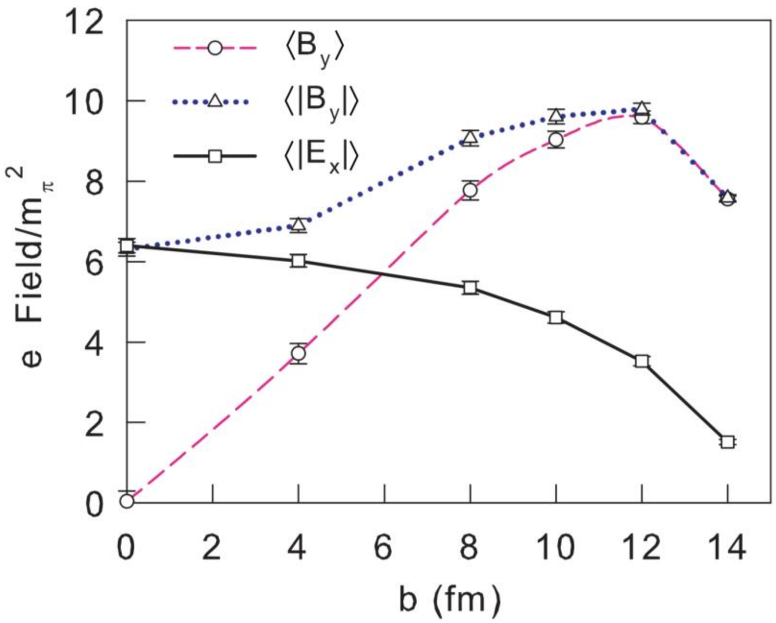
- Chirality production and Schwinger Mechanism
- Analytic solutions of Anomalous Magnetohydrodynamics
- Summary and outlook

Strong Electromagnetic fields in HIC

- Theoretical estimation:

  Lienard-Wiechert potential + Event-by-event



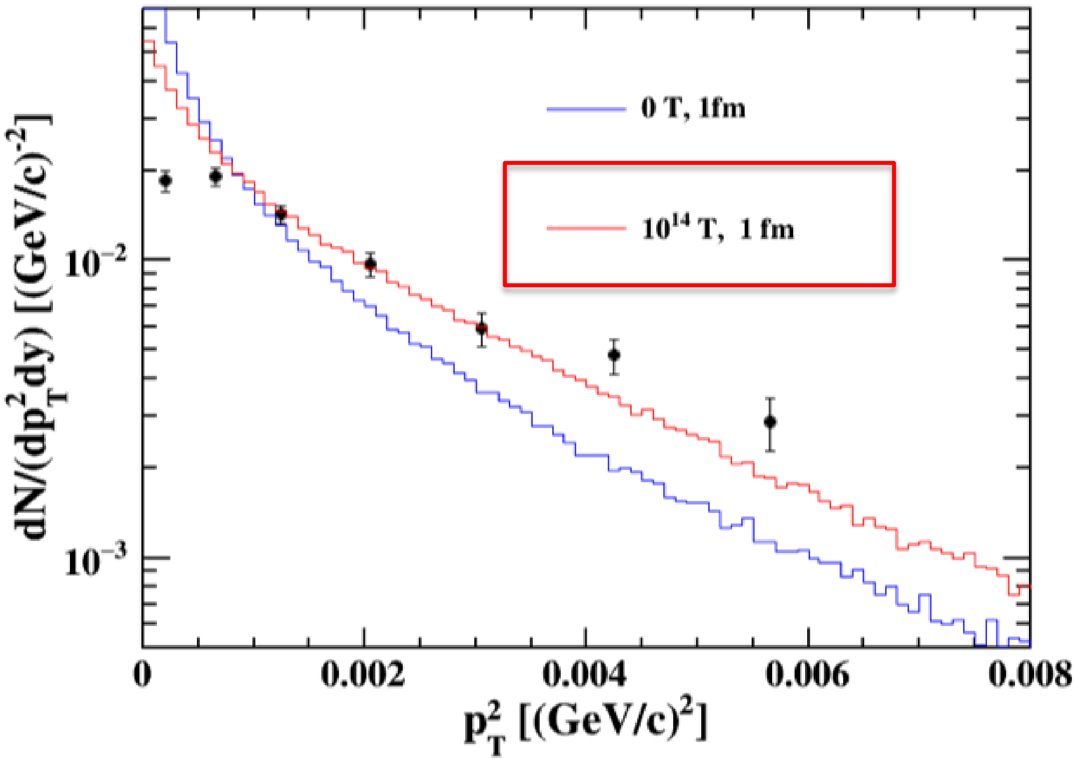


A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.l. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Strong Electromagnetic fields in HIC

- Experiential measurement

By assuming magnetic field ~ 10^14 T, it can Explain the data.



Cf. Prof. Zhangbu Xu's talk

1. Chirality production and Schwinger Mechanism

Ref: Patrick Copinger, Kenji Fukshima, SP, **Phys.Rev.Lett.** 121 (2018), 261602

Chirality production VS Schwinger Mechanism

- Axial Ward identity

$$\partial_\mu j_5^\mu = 2im\overline{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Chiral current **Pseudo-scalar** **~ E.B, Chiral anomaly**

- Volume integral

$$\Rightarrow \quad \frac{d}{dt}N_5 = \int d^3x \left(2im\overline{\psi}\gamma^5\psi - \frac{e^2}{2\pi^2}E\cdot B\right)$$

Pseudo-scalar

$$\partial_\mu j_5^\mu = 2im\overline{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

- Massless limit: Pseudo-scalar term->0
- Method:
  - Perturbative:

Cf. Jianhua Gao, Ziyue Wang, Xinli Sheng, Lixin Yang's talks

  - Non-perturbative: World-line formulism

World-line Formulism

- Spinor Feynman propagator at background fields:

$$S_A(x,y) =$$

$$= \left(i\not{D}_x + m\right)\underline{\Delta(x,y)}$$

- Path integral: (Homogenous, Constant E,B)

$$\Delta(x,y) = \int_0^\infty ds e^{-im^2 s}\frac{e^2 EB}{(4\pi)^2}\frac{\exp\left[-\frac{i}{2}eF\sigma s + \frac{i}{2}xeFy - \frac{i}{4}z\coth(eFs)eFz\right]}{\sinh(eEs)\sin(eBs)}$$

s: Schwinger proper time

Cf. N. Muller's Talk

M. D. Schwartz, Quantum Field theory and the standard model;
Christian Schubert: lecture note on the Worldline Formalism

“Well-known” Result done by Schwinger

- Homogenous Constant E,B at z direction

- Using world-line formulism (or original Schwinger’s methods):

$$< \partial_\mu j_5^\mu > = -2im < \overline{\psi}\gamma^5\psi > - \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} = 0$$ **?**

$$< \overline{\psi}\gamma^5\psi > = i\frac{1}{4\pi^2}\frac{EB}{m}$$

J. Schwinger, Phys. Rev. 82,5 (1951);
M. D. Schwartz, Quantum Field theory and the standard model;

Puzzle: All vanishing?

$$< \partial_\mu j_5^\mu > = -2im < \overline{\psi}\gamma^5\psi > -\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} = 0$$

- Taking m->0 at the very beginning: Weyl fermions

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

- After all the calculations, taking m->0.

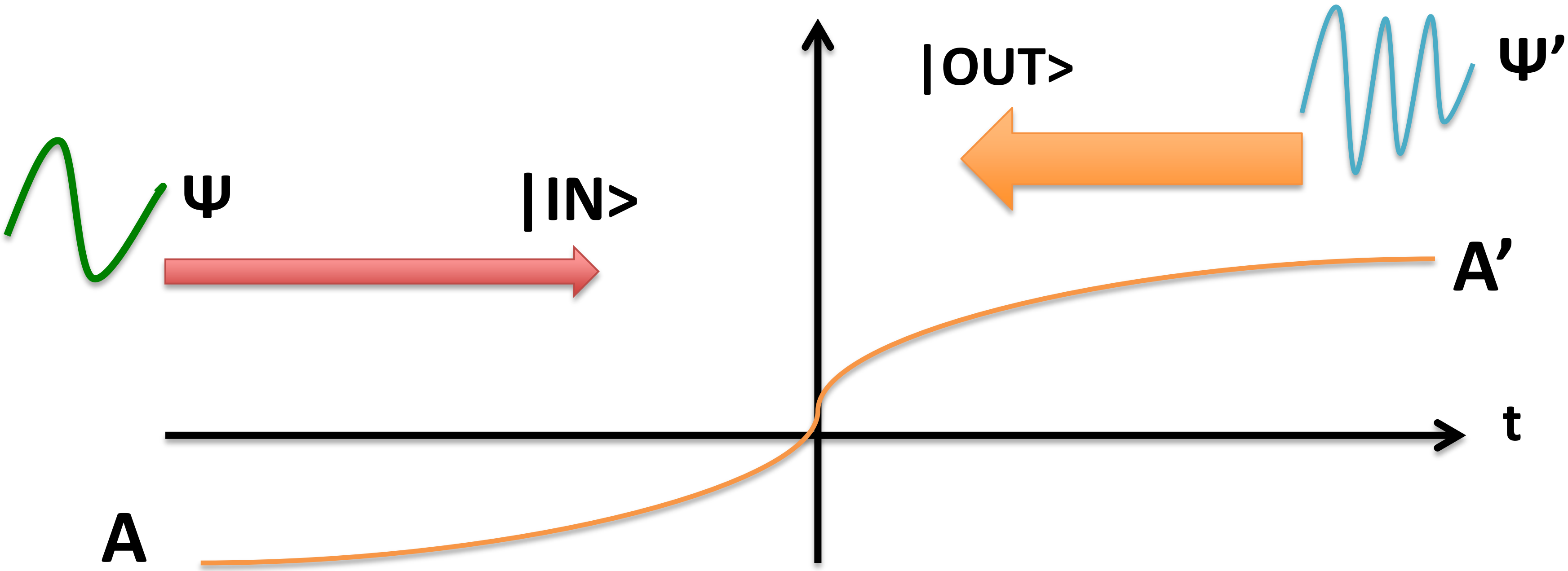
$$< \partial_\mu j_5^\mu > = -2im < \overline{\psi}\gamma^5\psi > -\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} = 0$$

Massless limit of massive Dirac equation	VS	Massless Dirac equation

IN and OUT States

- Homogenous Constant Ez,Bz field:

$$A^z(t) = eE_z t, \quad H = H(A(t)),$$



Unstable vacuum

- |0, IN> is NOT equal to |0, OUT>

$$| < 0, out \mid 0, in > |^2 \neq 1$$

- Schwinger Pair Production Rate:

$$P_0 = 1 - | < 0, out \mid 0, in > |^2 = \frac{e^2 E_z B_z}{4\pi^2} \coth\left(\frac{B_z}{E_z}\pi\right) \exp\left(-\frac{m^2\pi}{|eE_z|}\right)$$

(n=1 world-line instanton)

Expectation Value: IN-IN states

- Transition amplitude

$$< 0, out|\partial_\mu j_5^\mu|0, in >$$

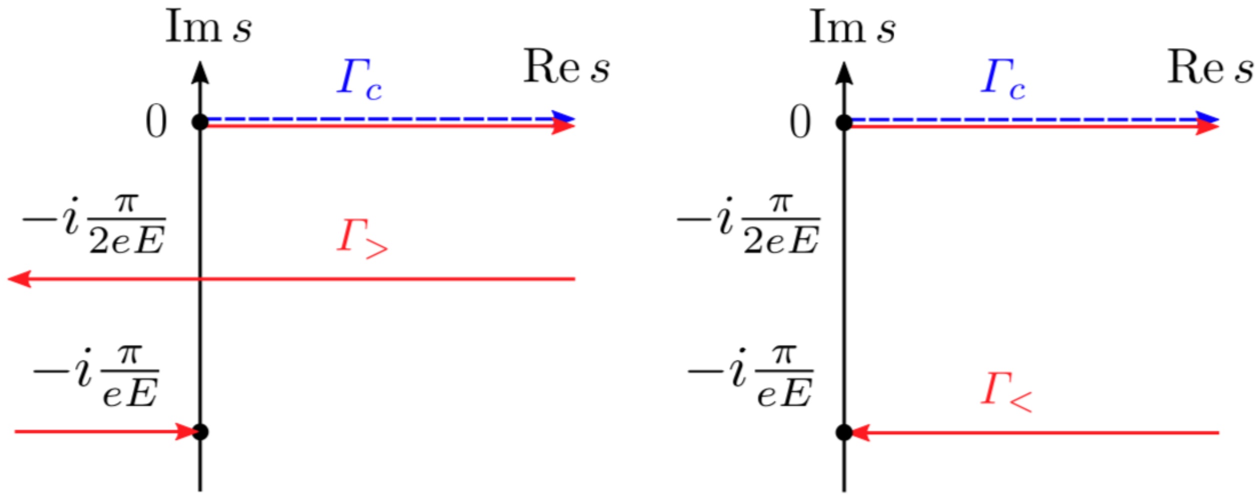
- Expectation value

$$< 0, in|\partial_\mu j_5^\mu|0, in >$$

Review: F. Gelis, N. Tanji 2015; N. Tanji 2009

Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman: Quantum Electrodynamic with Unstable Vacuum, 1991

Feynman Propagator for IN-IN



IN-OUT Propagator Path in Blue

$$S_A(x,y) = \left(i\not{D}_x + m\right)\Delta(x,y)$$

$$\Delta(x,y) = \left[\theta(x_3 - y_3)\int_{\Gamma^C} + \theta(y_3 - x_2)\int_{\Gamma^C}\right] ds \times e^{-im^2 s} g(x,y,s),$$

IN-IN Propagator Path in Red

$$S_{in}(x,y) = \left(i\not{D}_x + m\right)\Delta_{in}(x,y)$$

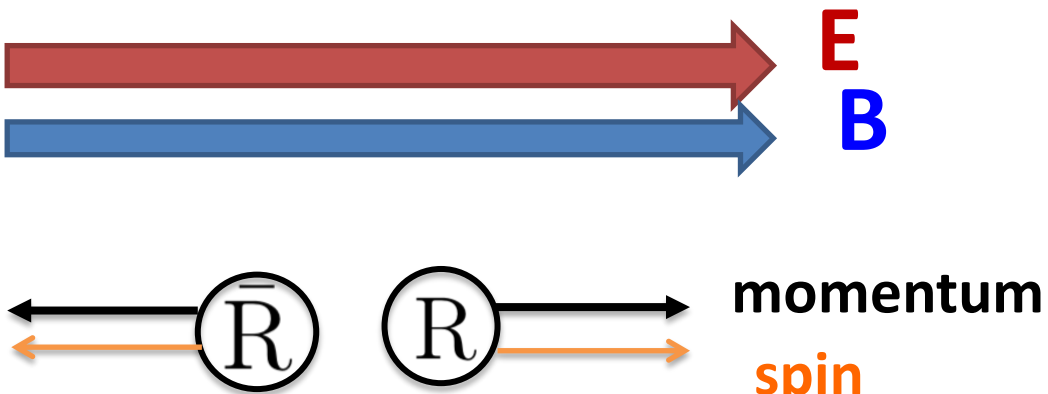
$$\Delta_{in}(x,y) = \left[\theta(x_3 - y_3)\int_{\Gamma^>} + \theta(y_3 - x_2)\int_{\Gamma^<}\right] ds \times e^{-im^2 s} g(x,y,s),$$

Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman: Quantum Electrodynamic with Unstable Vacuum, 1991

Chirality Production

$$\partial_\mu j_5^\mu = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right)$$

- m -> 0, Chiral anomaly



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

K. Fukushima, D.Kharzeev, H. Warringa PRL 2010

Mass correction to CME

- E,B at z direction:

$$j^3 = \frac{e^2 EB}{2\pi^2} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right) t$$

- Non-perturbative: ~1/(eE)
- Sum over all Landau levels: Coth(B/E \pi)

Summary for world-line formulism

- Introduce a new method to compute non-perturbative dynamical quantities in strong EB fields.

e.g.

- Axial Ward identity, correct mass correction!
- Mass correction to CME
- Dynamical chiral condense

Cf. Prof. Kenji Fukushima's Talk

2. Anomalous Magnetohydrodynamics

Ref: Irfan Siddique, Ren-jie Wang, Shi Pu, and Qun Wang, arXiv: 1904.01807

Anomalous MHD

- Conservation equations:
  - Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = T_F^{\mu\nu} + T_{EM}^{\mu\nu}.$$

Fluid part — Electromagnetic part

  - (anomalous) currents conservation

$$\partial_\mu j_e^\mu = 0,$$

Electric Charge current

$$\partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

Chiral current

- Maxwell's equation:

$$\partial_\mu F^{\mu\nu} = j_e^\nu,$$

$$\partial_\mu (\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0.$$

Previous Studies: ideal MHD without CME

- 1+1 D ideal MHD Bjorken flow

V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Lett. B750, 45

  - With magnetization effects

SP, V. Roy, L. Rezzolla, D. Rischke,Phys.Rev. D93, 074022

- 2+1 D ideal MHD Bjorken flow (perturbative)

SP, Di-Lun Yang, Phys.Rev. D93, 054042

- Background Magnetic field: contribution to v2

V.Roy, SP, L. Rezzolla, D. Rischke, Phy.Rev. C96, 054909

Cf. Prof. D.H. Rischke's Talk
Shi's Talk on AVFD for simulations

ideal limit of MHD

- Ideal:
  – Electric conductivity is infinite $\sigma \to \infty$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \Rightarrow \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Maxwell's equation

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B})$$

- No space for the CME

$$\nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E}$$

- Anomalous MHD needs finite conductivity

Constitution eqs. for Anomalous MHD

$$\partial_\mu T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = (\varepsilon + p + E^2 + B^2)u^\mu u^\nu - (p + \frac{1}{2}E^2 + \frac{1}{2}B^2)g^{\mu\nu}$$
$$-E^\mu E^\nu - B^\mu B^\nu - u^\mu \epsilon^{\nu\lambda\alpha\beta}E_\lambda B_\alpha u_\beta - u^\nu \epsilon^{\mu\lambda\alpha\beta}E_\lambda B_\alpha u_\beta,$$

$$\partial_\mu j_e^\mu = 0,$$

$$\partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

Electric Conducting flow

CME

$$j_e^\mu = n_e u^\mu + \sigma E^\mu + \xi B^\mu,$$
$$j_5^\mu = n_5 u^\mu + \sigma_5 E^\mu + \xi_5 B^\mu,$$

CSE

$$\partial_\mu F^{\mu\nu} = j_e^\nu,$$

$$\partial_\mu (\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0.$$

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta,$$

Equation of States (EoS)

- High (chiral) chemical potential:

Energy density → ε; pressure → p

$$\varepsilon = c_s^{-2} p,$$

$$n_e = a\mu_e(\mu_e^2 + 3\mu_5^2),$$

$$n_5 = a\mu_5(\mu_5^2 + 3\mu_e^2),$$

Chemical potential → μ_e; Chiral chemical potential → μ_5

- High temperature:

$$\varepsilon = c_s^{-2} p,$$

$$n_e = a\mu_e T^2,$$

$$n_5 = a\mu_5 T^2,$$

temperature → T

Bjorken boost invariance

- Profound Bjorken velocity

$$u^\mu = \gamma(1, 0, 0, z/t),$$

- Bjorken invariance: all observed quantity are independent on rapidity.

- Q: could the Bjorken velocity hold in electromagnetic fields?

Simplification

- Neutral fluid:
  - Electric field will not accelerate the fluid
- Force-free-like magnetic field:
- Configuration of Electromagnetic fields:

$$E^{\mu} = (0, 0, \chi E(\tau), 0), \;\; B^{\mu} = (0, 0, B(\tau), 0),$$

$$\chi = \pm 1$$

τ: proper time

$$E^{\mu} = F^{\mu\nu} u_{\nu}, \;\; B^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_{\nu} F_{\alpha\beta},$$

Not EM fields
In lab frame!

Results: High temperature EoS

- Analytic solutions: (at the order of hbar):

chiral density

$$n_5(\tau) = n_{5,0}\left(\frac{\tau_0}{\tau}\right)\{1 + a_2 e^{\sigma\tau_0}[\mathrm{E}_1(\sigma\tau_0) - \mathrm{E}_1(\sigma\tau)]\},$$

Energy density

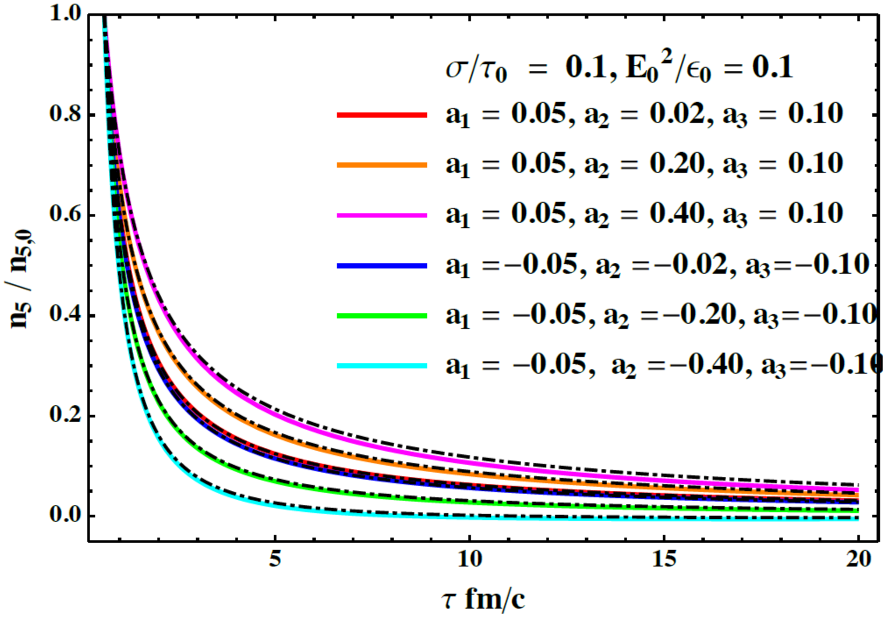
τ: proper time
σ: electric conductivity

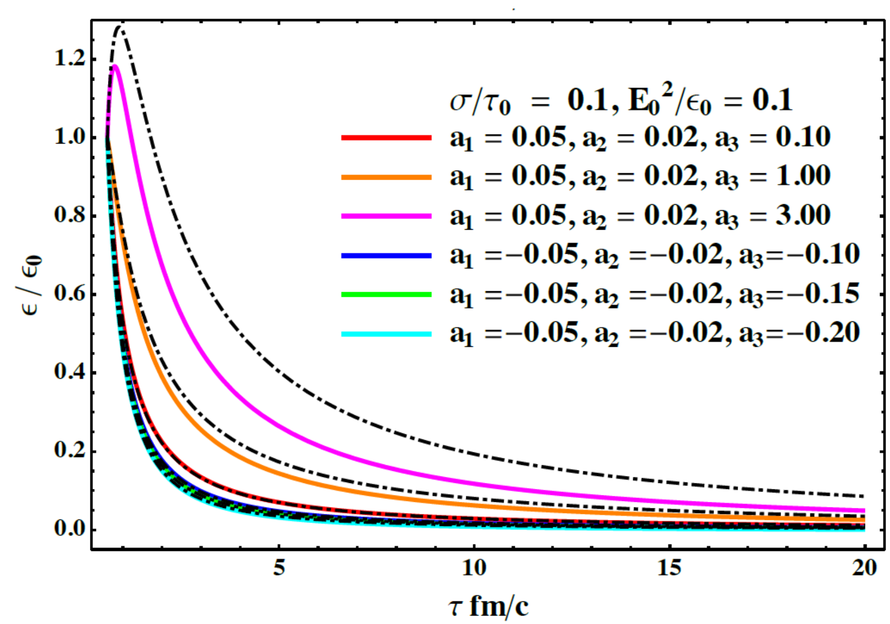
$$\varepsilon(\tau) = \epsilon_0\left(\frac{\tau_0}{\tau}\right)^{1+c_s^2}\left\{1 + \sigma\frac{E_0^2}{\varepsilon_0}e^{2\sigma\tau_0}[\tau_0\mathrm{E}_{1-c_s^2}(2\sigma\tau_0) - \tau\left(\frac{\tau}{\tau_0}\right)^{c_s^2-1}\mathrm{E}_{1-c_s^2}(2\sigma\tau')]\right.$$
$$\left. + \frac{a_3}{\tau_0}e^{\sigma\tau_0}[\tau_0\mathrm{E}_{2-3c_s^2}(\sigma\tau_0) - \tau\left(\frac{\tau_0}{\tau}\right)^{2-3c_s^2}\mathrm{E}_{2-3c_s^2}(\sigma\tau)]\right\}.$$

a1,a2,a3 are related to the initial EM fields and chirality density

En(x): the generalized exponential integral. $\mathrm{E}_n(z) \equiv \int_1^\infty dt t^{-n} e^{-zt}$

Analytic solution VS numerical results





Solid line: numerical results / Dashed line: analytic solutions

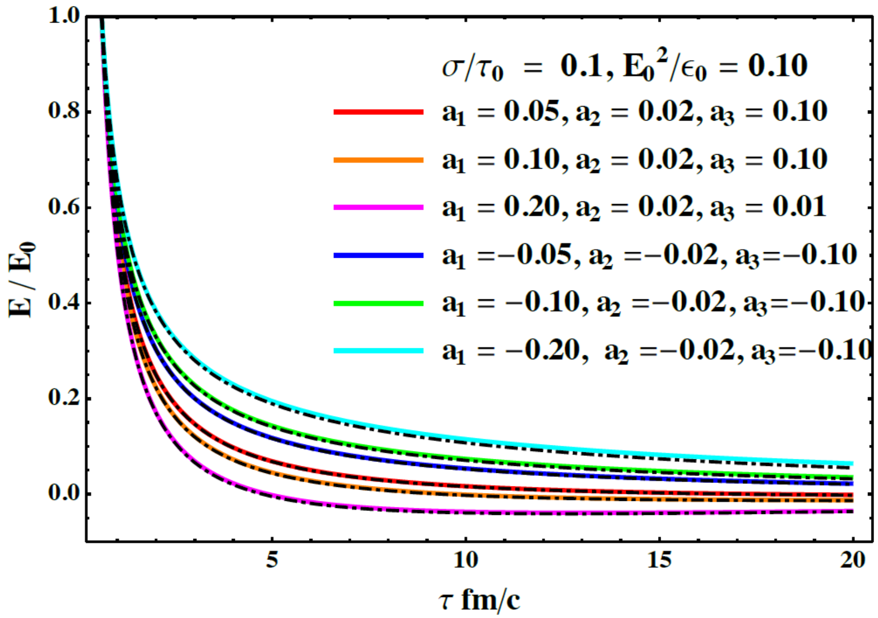
$$a_1 = eC\chi \frac{B_0 n_{5,0}}{aT_0^2 E_0}\tau_0, \qquad a_2 = \frac{e^2 C\chi E_0 B_0}{n_{5,0}}\tau_0,$$

$$a_3 = \frac{eC\chi}{a}\frac{n_{5,0}E_0B_0}{\varepsilon_0 T_0^2}\tau_0.$$

C: chiral anomaly

Coefficients = $\hbar/(2\pi^2)$

Electromagnetic fields in the Lab frame



$$\mathbf{E}_L = (\gamma v^z B(\tau),\ \chi\gamma E(\tau),\ 0),$$

$$\mathbf{B}_L = (-\gamma v^z \chi E(\tau),\ \gamma B(\tau),\ 0),$$

τ: proper time

σ: electric conductivity

$$E(\tau) = E_0\left(\frac{\tau_0}{\tau}\right)\left\{e^{-\sigma(\tau-\tau_0)} - a_1 e^{-\sigma\tau}\left[\mathrm{E}_{1-2c_s^2}(-\sigma\tau_0) - \left(\frac{\tau}{\tau_0}\right)^{2c_s^2}\mathrm{E}_{1-2c_s^2}(-\sigma\tau)\right]\right\},$$

$$B(\tau) = B_0\frac{\tau_0}{\tau},$$

$$a_1 = eC\chi\frac{B_0 n_{5,0}}{aT_0^2 E_0}\tau_0,$$

In the Lab frame:

By decays ~ 1/τ

Bx decays ~ exp(- σ τ)/ τ

Much slower than decaying in vacuum

Why Bz and Ez vanish?

- We have checked the Maxwell's eq. in Lab frame.

Space and time derivatives of Bz, Ez are zero.

- Key point: the currents is different with static case!

$$\nabla \times \mathbf{B}_L = \mathbf{j}_e + \partial_t \mathbf{E}_L.$$

$$\mathbf{j}_{e,\parallel} = \sigma \mathbf{E}_{L,\parallel} + \xi \mathbf{B}_{L,\parallel},$$

v: three vector of fluid velocity

$$\mathbf{j}_{e,\perp} = \sigma\gamma(\mathbf{E}_L + \mathbf{v} \times \mathbf{B}_L)_\perp + \xi\gamma(\mathbf{B}_L - \mathbf{v} \times \mathbf{E}_L)_\perp,$$

- Similar to the force-free EM fields in classical electrodynamics (e.g. Woltjer states)

Qin, Liu, Li, Squire, PRL 109, 235001 (2012); Xia, Qin, Q. Wang, PRD(2016)

Summary of Anomalous MHD

- Anomalous MHD:

Hydrodynamic eq. + Maxwell's eq. + Chiral anomalous currents

- Analytic solutions of anomalous MHD in Bjorken flow with transverse EM fields
- In lab frame, B field decays much slower than in the vacuum

By decays like ~ $1/\tau$, Bx decays like~ $\exp(-\sigma\tau)/\tau$

Summary

- Introduction a new method to compute the dynamical quantities in non-perturbative way, in strong electromagnetic fields

- Analytic solutions of anomalous MHD

Thank you for your time!

Backup

World-line Formulism (I)

- Schwinger proper time:

$$\frac{i}{A+i\varepsilon} = \int_0^\infty dT e^{iT(A+i\varepsilon)}$$

- Spinor Feynman propagator at background fields:

$$G_A(x,y) =$$ 

$$= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{\not{D} - m + i\varepsilon}$$

$$= \int_0^\infty dT e^{-im^2 T} < y|(\not{D}+m)e^{-i\hat{H}_A T}|x>,$$

$$\hat{H}_A = -(\hat{p}^\mu - eA^\mu)^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}.$$

M. D. Schwartz, Quantum Field theory and the standard model;
Christian Schubert: lecture note on the Worldline Formalism

Lienard-Weichart formula

- Estimations from classic electromagnetic dynamics

$$\vec{E}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{R}_i - R_i\vec{v}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2), \quad (1)$$

$$\vec{B}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{v}_i \times \vec{R}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2), \quad (2)$$

- Position of charged particles:

e.g. MC-Glauber Model + Woods-Saxon nuclear distributions