

# Quark matter under strong magnetic fields

Mei Huang

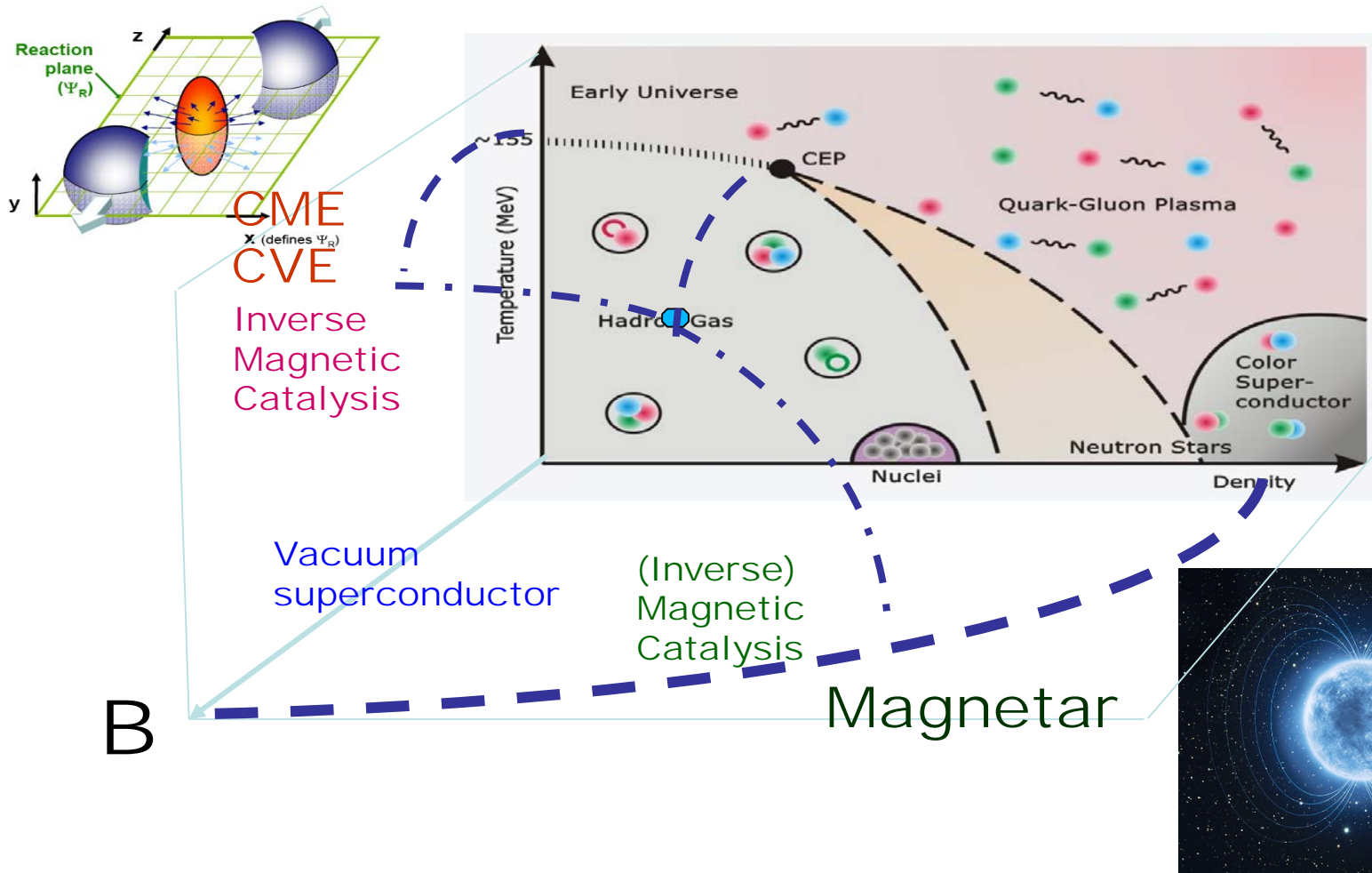


中国科学院大学

University of Chinese Academy of Sciences

Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions,  
Tsinghua Uni., Apr.8-12,2019

# Quark matter under magnetic fields (2014)



Recently, more interests on fast rotating system, finite size system, neutron star merge, gravitation wave.....

# **Update information about:**

**I. Inverse magnetic catalysis**

**II. Vacuum superconductor**

**III. Phase structure under B**

**IV. Summary**

# **I. Inverse magnetic catalysis**

# Magnetic catalysis at zero temperature

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S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);  
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

$$\mathcal{L} = \bar{\Psi} i\gamma^\mu D_\mu \Psi + \frac{G}{2} \left[ (\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\Psi)^2 \right]$$

$$D_\mu = \partial_\mu - ieA_\mu^{\text{ext}}, \quad \mathbf{A}^{\text{ext}} = (0, Bx^1, 0)$$

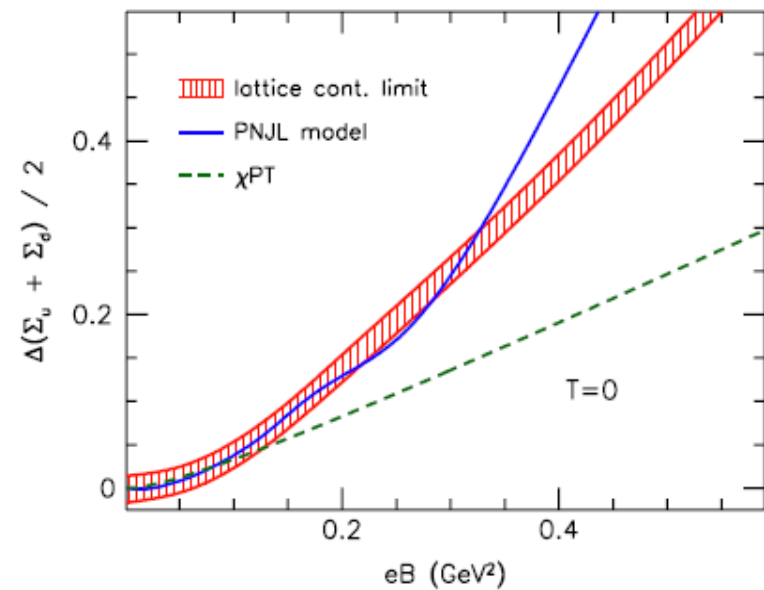
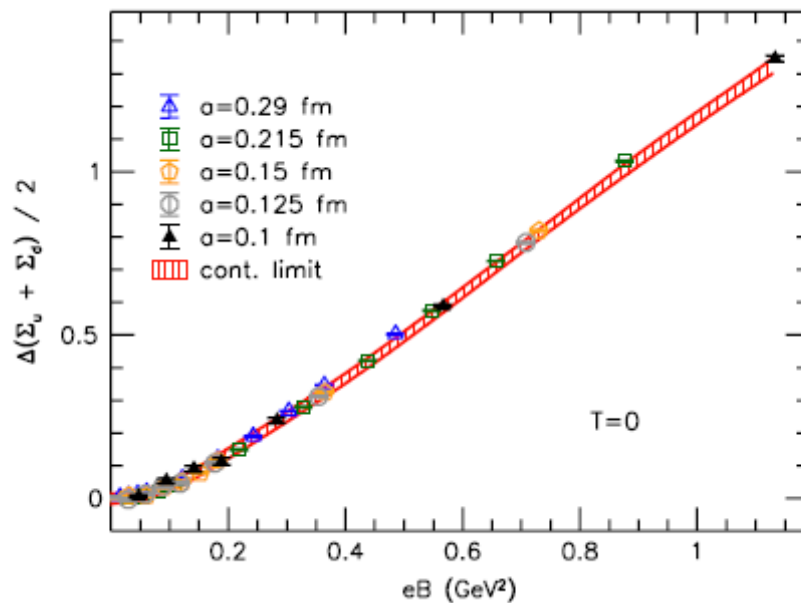
$$m = G \text{tr}[S(x,x)] \approx \frac{Gm}{(2\pi)^2} \left( \Lambda^2 + |eB| \ln \frac{|eB|}{\pi m^2} + O(m^2) \right)$$

$$m \propto \exp\left(-\frac{2\pi^2}{G|eB|}\right)$$

**nonzero mass for arbitrary small G**

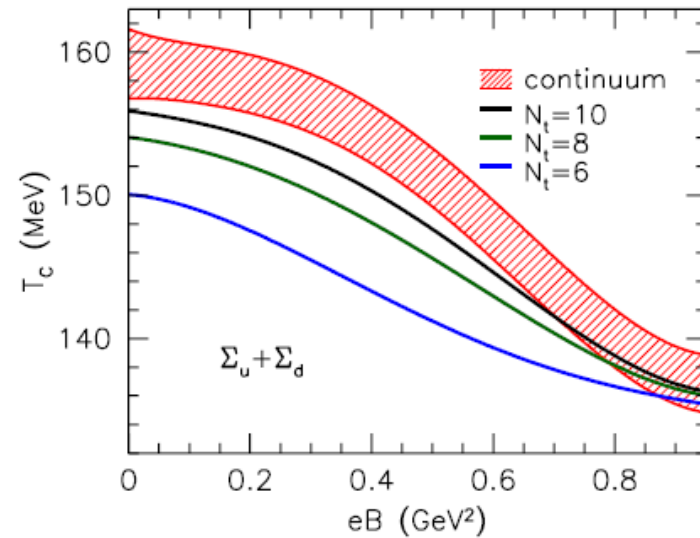
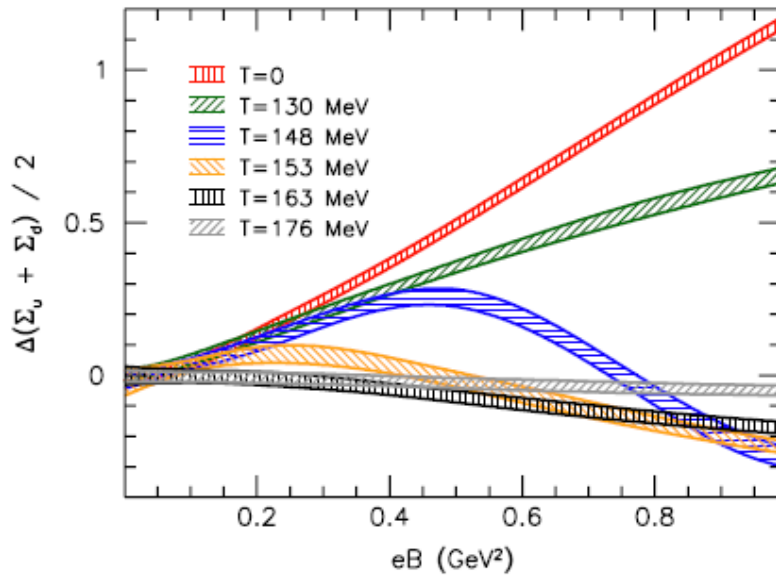
# Magnetic catalysis at zero temperature

Bali et al. arXiv:1206.4205 [hep-lat]



# Inverse magnetic catalysis at nonzero temperature

Bali et al. arXiv:1206.4205 [hep-lat]



# How to understand inverse magnetic catalysis ?

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**1) Magnetic inhibition** K. Fukushima, Y. Hidaka, PRL 110, 031601 (2013)

**Contribution from neutral pions**

**2) Contribution from sea quarks**

Bruckmann et.al. arXiv:1303.3972

**3) Polyakov holomoly**

Nowak et.al. arXiv:1304.6020

**4) Running coupling**

M. Ferreira, et.al. arXiv:1404.5577

**5) Chirality imbalance**

**Sphaleron transition**

Jingyi Chao, Pengcheng Chu, MH,  
arXiv:1305.1100, PRD88(2013)

**Instanton-anti-instanton pairing condensate**

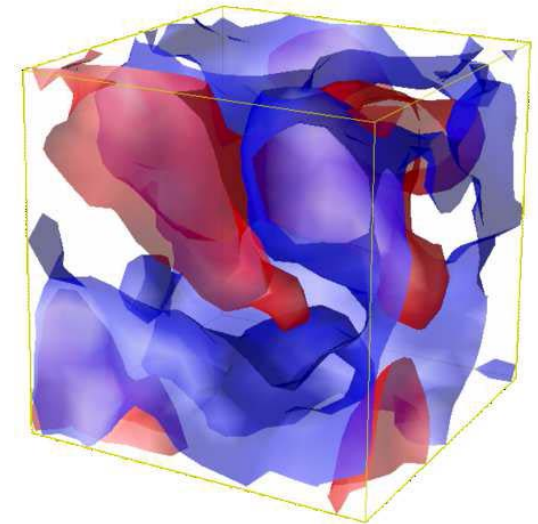
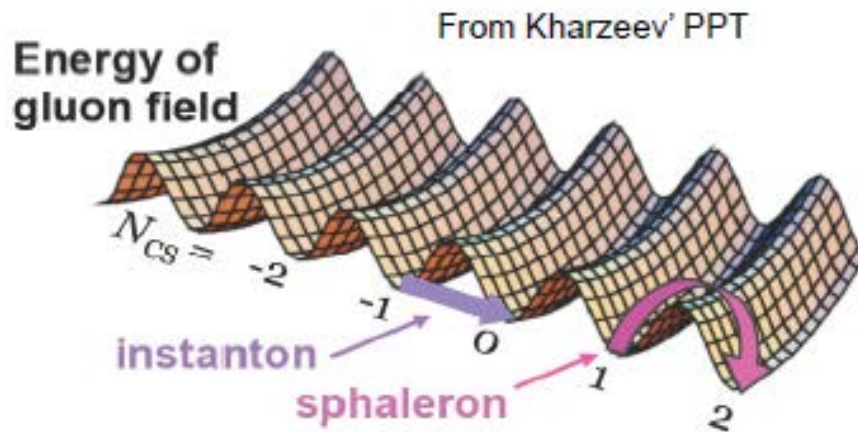
Lang Yu, Hao Liu, MH, arXiv:1404.6969,  
PRD90(2014)



# Theta vacuum, instantons and sphalerons

QCD vacuum has non-trivial topological structure characterized by an integer valued Chern-Simons number

Buividovich et al. arXiv:1111.6733

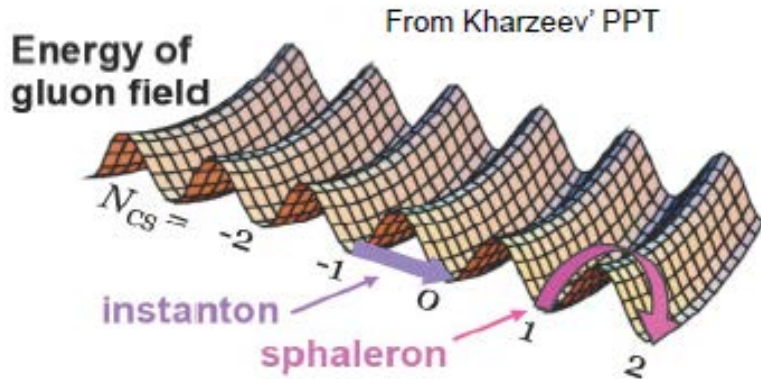


$$\Delta N_{CS} = \frac{g^2}{32\pi^2} \int d^4x \text{Tr}[F_{\alpha\mu\nu} \tilde{F}^{\alpha\mu\nu}]$$

Induce chirality imbalance:

$$(N_R - N_L)_{t=+\infty} - (N_R - N_L)_{t=-\infty} = -2N_f \Delta N_{CS}$$

# Inverse magnetic catalysis induced by chirality imbalance



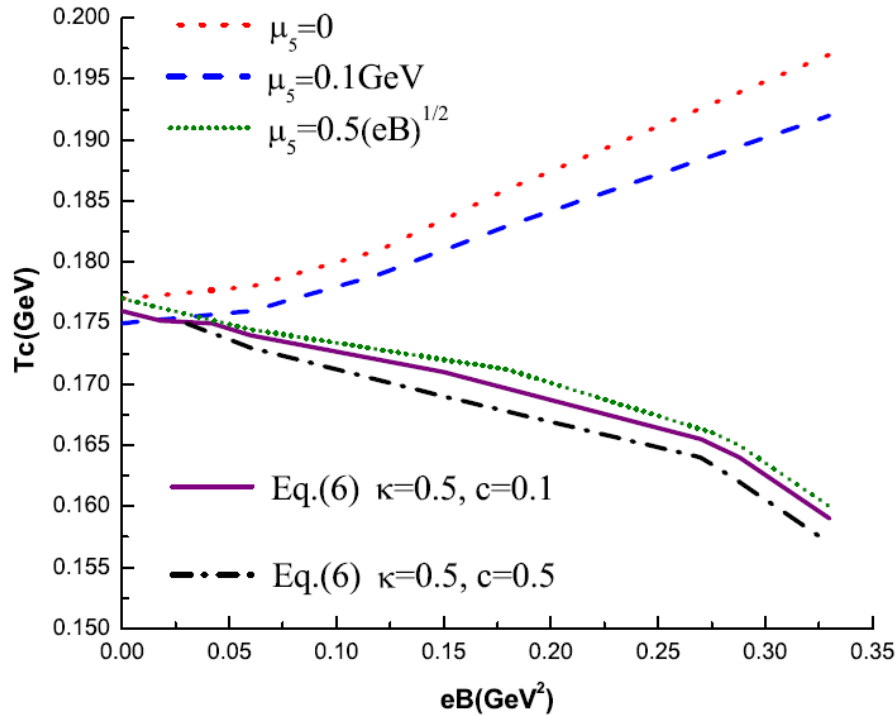
Debye mass for longitudinal gluons:

$$g(T + c\sqrt{eB})$$

Sphaleron transition rate:

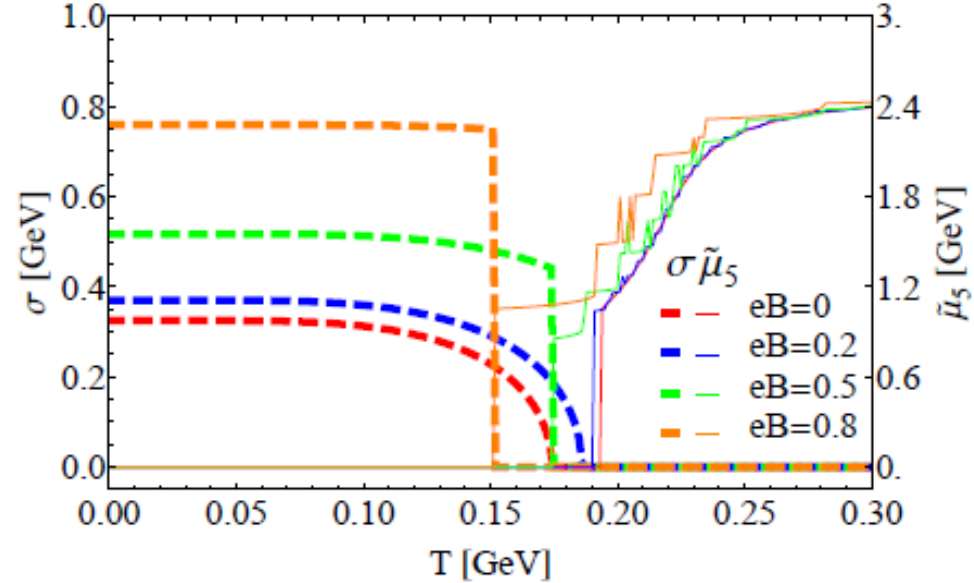
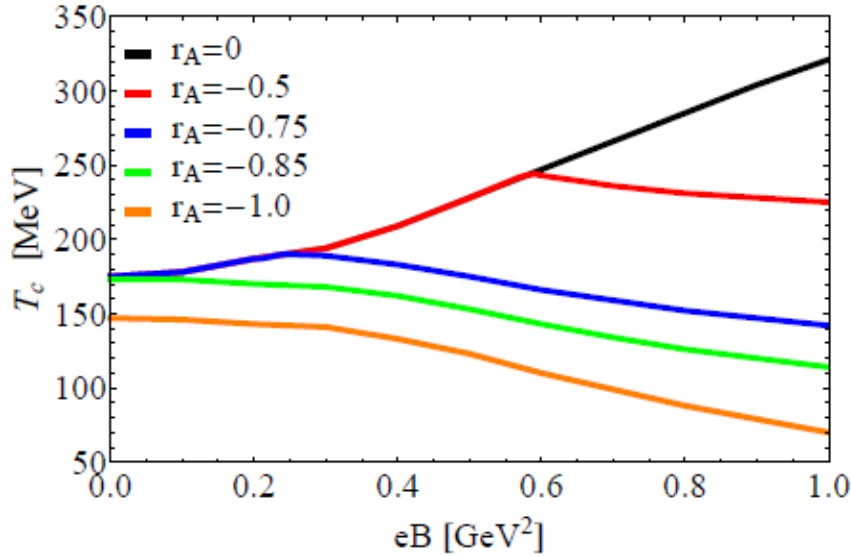
$$\Gamma_{ss} \sim (T^4 + c^2 e B T^2)$$

$$\mu_5 = \sqrt{3}\pi \left( \frac{320 N_f^2 \Gamma_{ss}}{T^2} - \frac{T^2}{3} \right)^{\frac{1}{2}}$$



Jingyi Chao, P.C. Chu, MH,  
arXiv:1305.1100, PRD88(2014)

# Inverse magnetic catalysis induced by chirality imbalance



**First order phase transition!**

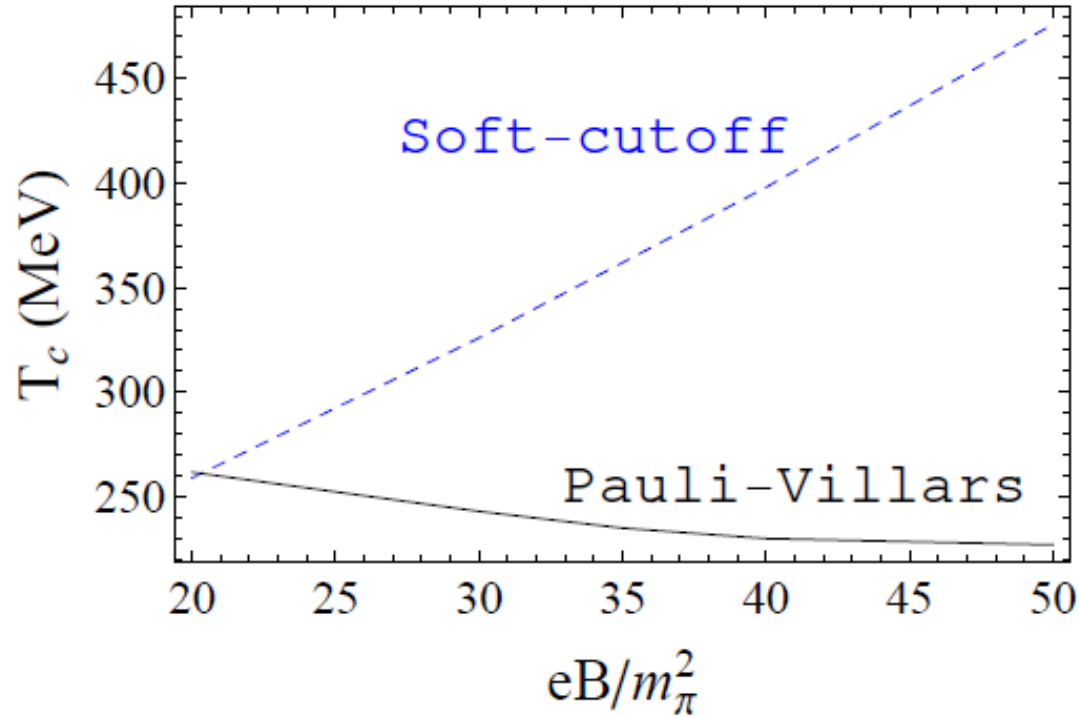
**Maybe more suitable for EW!**

$$n_5 > 0$$

$$n_5 < 0$$

Lang Yu, Hao Liu, MH, arXiv:1404.6969,  
PRD90,074009(2014); arXiv:1411.7552,PRD2015;  
arXiv:1511.03073, PRD2016

# Inverse magnetic catalysis induced by neutral pions



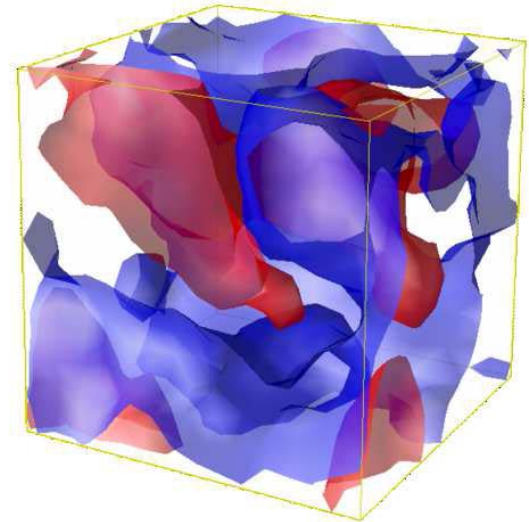
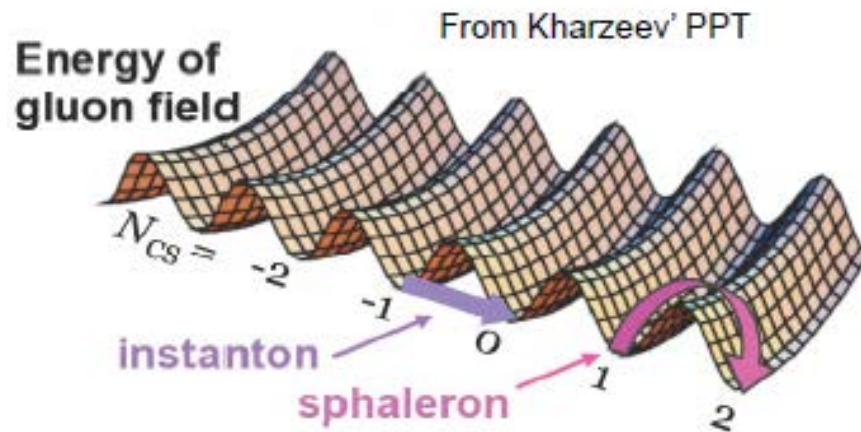
Shijun Mao, arXiv:1602.06503,PRD2016

**Massless neutral pion in chiral limit!**

# Inverse magnetic catalysis ...

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**More understanding is needed on nontrivial gluodynamics under B and T!**



**More discussion from Hengtong Ding's talk!**

## **II. Is there vacuum superconductor?**

# Point particles under B

•M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]

-Energy of relativistic particle in the external magnetic field B:

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - 2\text{sgn}(q)s_z + 1)|qB| + m^2$$

↙ nonnegative integer number ↘

the momentum along the external magnetic field

projection of spin on the direction of magnetic field

-Masses of  $\rho$  mesons and  $\pi$  in magnetic field:

$$m_{\pi^\pm}^2(B) = m_{\pi^\pm}^2 + eB \quad \text{becomes larger}$$

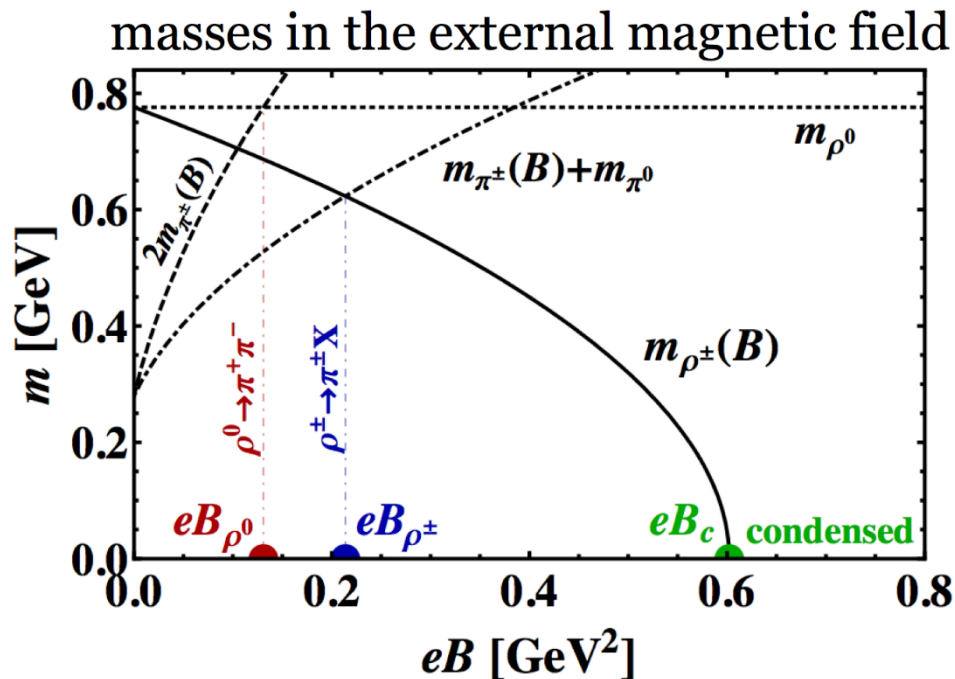
$$m_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - eB \quad \text{becomes lighter}$$

where  $m_{\rho^\pm} = 768\text{MeV}$  ,  $m_{\pi^\pm} = 140\text{ MeV}$

# Vacuum Superconductor

The charged rho becomes massless and condensate at a critical magnetic fields :  $eB_c = m_{\rho^\pm}^2$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]



The pions become heavier while the charged vector mesons become lighter in the external magnetic field

The  $\rho^\pm \rightarrow \pi^\pm\pi^0$  decay stops at a critical  $eB$



# Charged and neutral vector meson in NJL model

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i \not{D} - \hat{m})\psi + G_S [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2] \\ & - G_V [(\bar{\psi}\gamma^\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi)^2] \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \end{aligned}$$

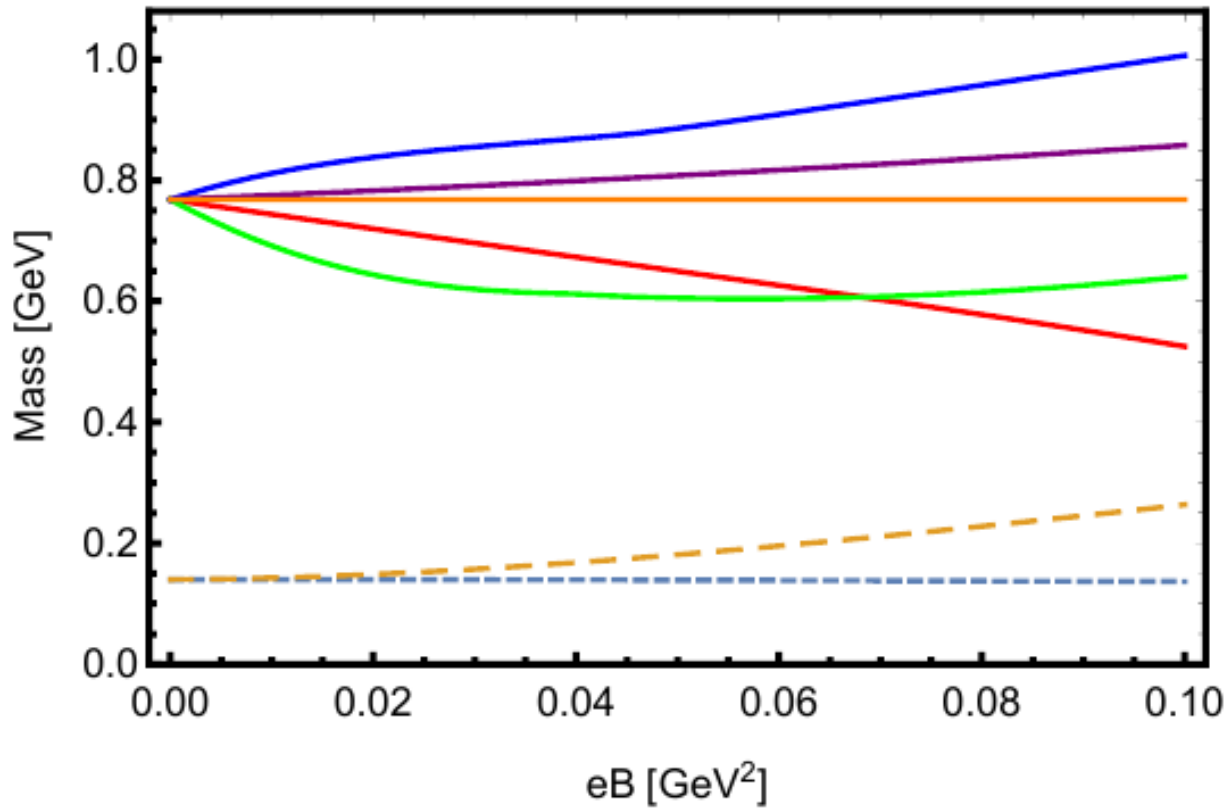
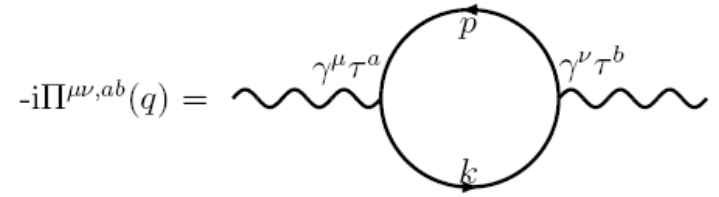
$$\begin{aligned} \text{wavy line} &= \text{wavy line} + \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line} + \\ & \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line} + \dots \\ &= \text{wavy line} + \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line} \end{aligned}$$

$$-i\Pi^{\mu\nu,ab}(q) = \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line}$$

The diagram shows a circle with two external wavy lines. The top vertex is labeled  $\gamma^\mu\tau^a$  and the bottom vertex is labeled  $\gamma^\nu\tau^b$ . The top arc of the circle is labeled  $\vec{p}$  and the bottom arc is labeled  $\vec{k}$ .

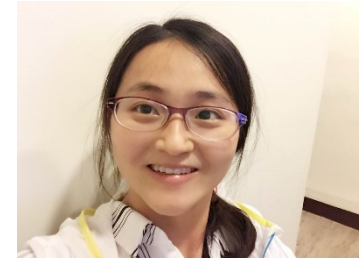
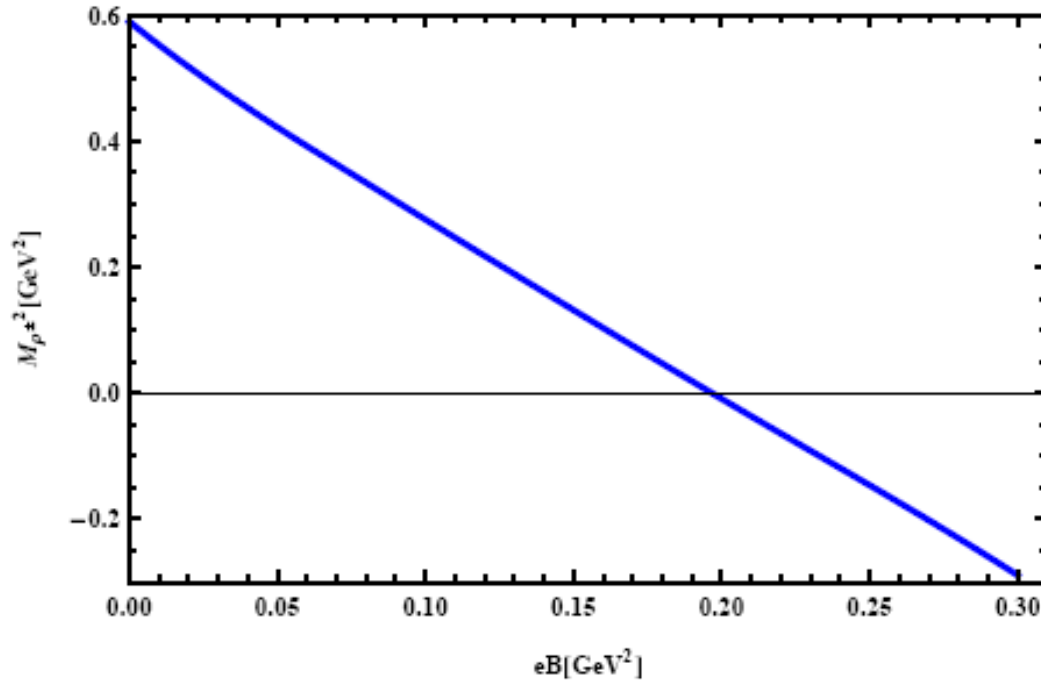
# Vector meson under B: spin decomposition!

$$\Pi_{\rho^0}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi_{\rho^0}^{11} & 0 & 0 \\ 0 & 0 & \Pi_{\rho^0}^{22} & 0 \\ 0 & 0 & 0 & \Pi_{\rho^0}^{33} \end{pmatrix} \quad \Pi_{\rho^\pm}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi^{11} & \Pi^{12} & 0 \\ 0 & \Pi^{21} & \Pi^{22} & 0 \\ 0 & 0 & 0 & \Pi^{33} \end{pmatrix}$$



- $\pi^0$
- $\pi^\pm$
- $\rho^+(s_z=1), \rho^-(s_z=-1)$
- $\rho^\pm(s_z=0)$
- $\rho^+(s_z=-1), \rho^-(s_z=+1)$
- $\rho^0(s_z=\pm 1)$
- $\rho^0(s_z=0)$

# Charged vector meson in vacuum



Hao Liu, Lang Yu, MH,  
arXiv:1408.1318, PRD2015

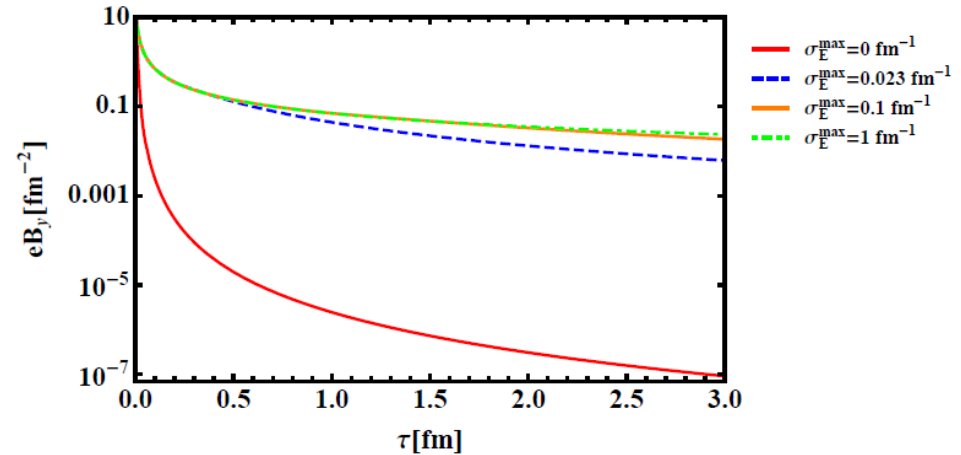
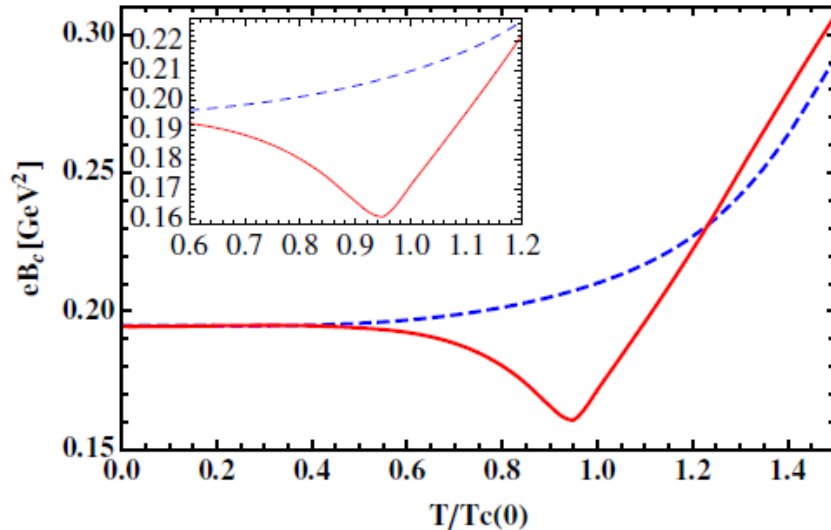
FIG. 4: The mass square of charged  $\rho^\pm$  with spin component  $s_z = \pm 1$  as a function of  $eB$ .

$$eB_c \simeq 0.2 \text{ GeV}^2 \approx 1/3 \quad eB_c = m_{\rho^\pm}^2 \quad !$$

**Quark polarization is important!**

# Charged vector meson at finite temperature

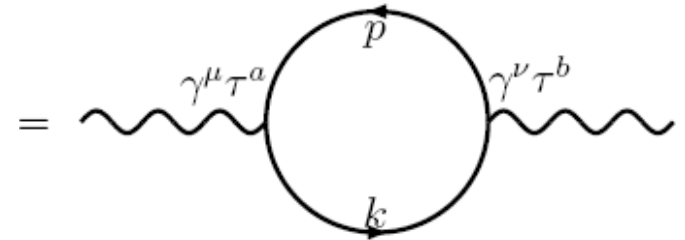
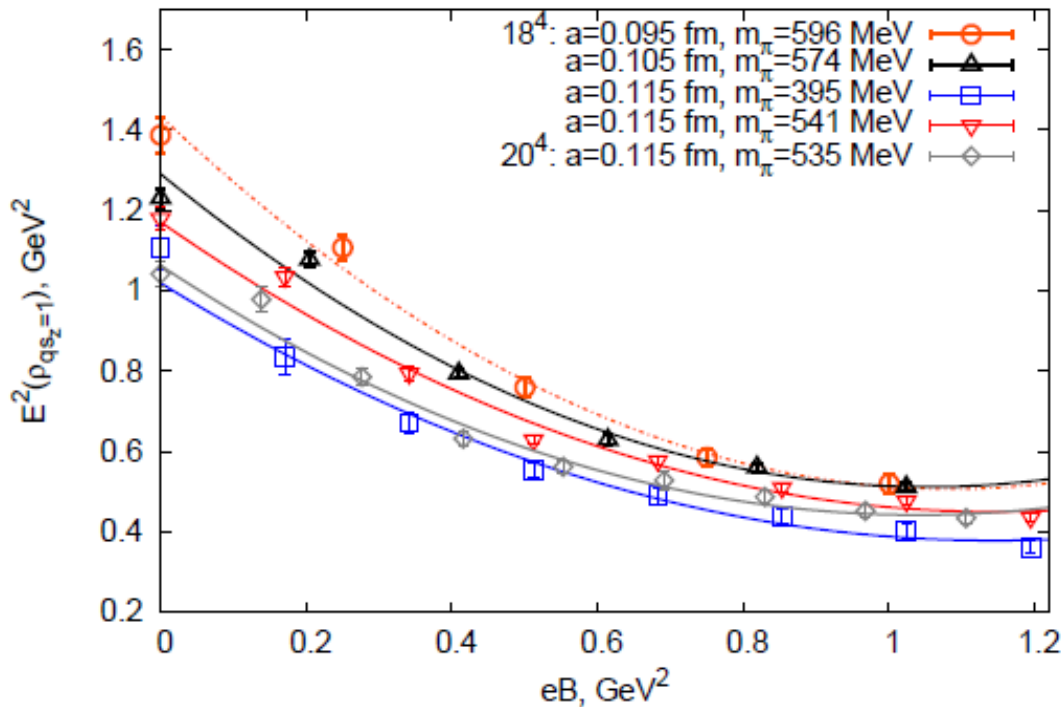
**High-T superconductor! A natural source for high conductivity and induces long lifetime for magnetic field!**



**Hao Liu, Lang Yu, MH, arXiv:1507.05809**

**Hao Liu, Lang Yu, M.Chernodub, MH, arXiv:1604.06662**

# Lattice QCD on charged vector meson under B



**Pion mass (quark mass) is still too heavy!**

**Looking forward to physical pion mass case!**

E.V. Luschevskaya<sup>a,b</sup> O.E. Solovjeva<sup>a</sup> O.V. Teryaev<sup>c,d</sup>

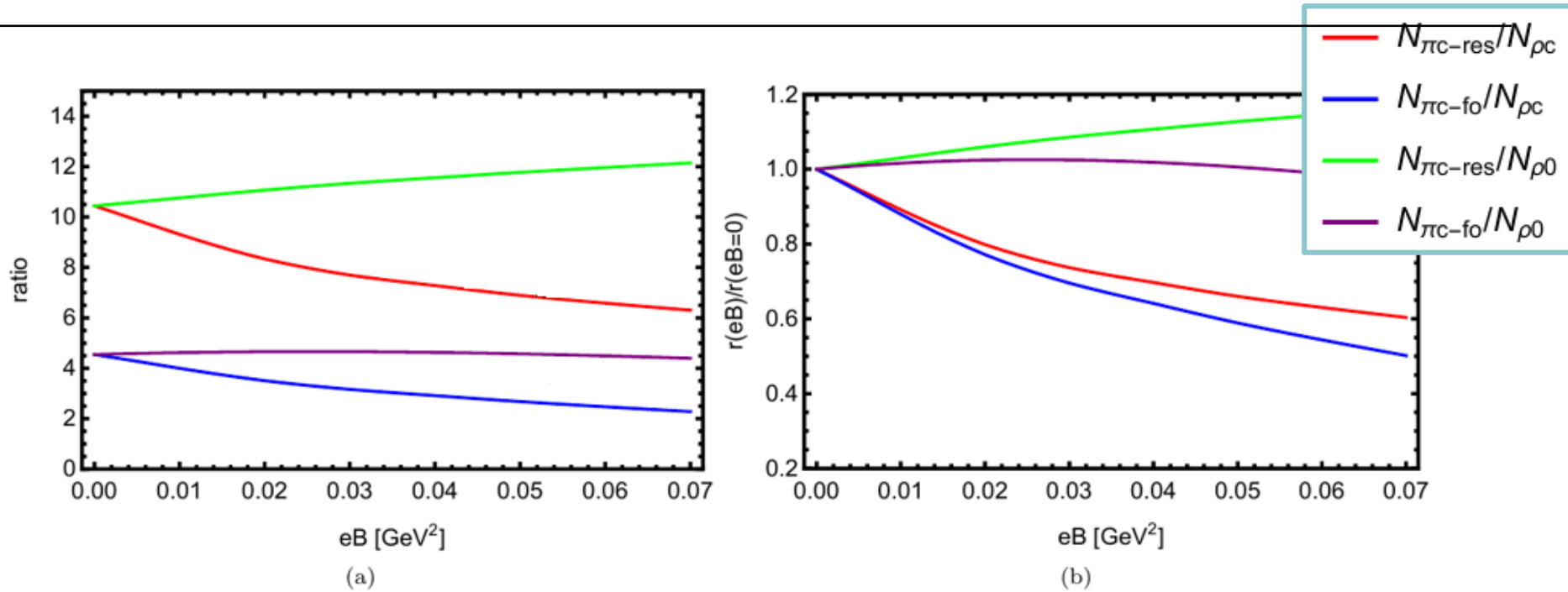
E.V. Luschevskaya<sup>a,b</sup> O.V. Teryaev<sup>c,a</sup> D.Yu. Golubkov<sup>a</sup> O.V. Solovjeva<sup>a</sup>

R.A.Ishkuvatov<sup>a,b</sup>

arXiv:1608.03472, 1811.02344

# Meson mass $M(B) \rightarrow$ Meson production number $N(B)$

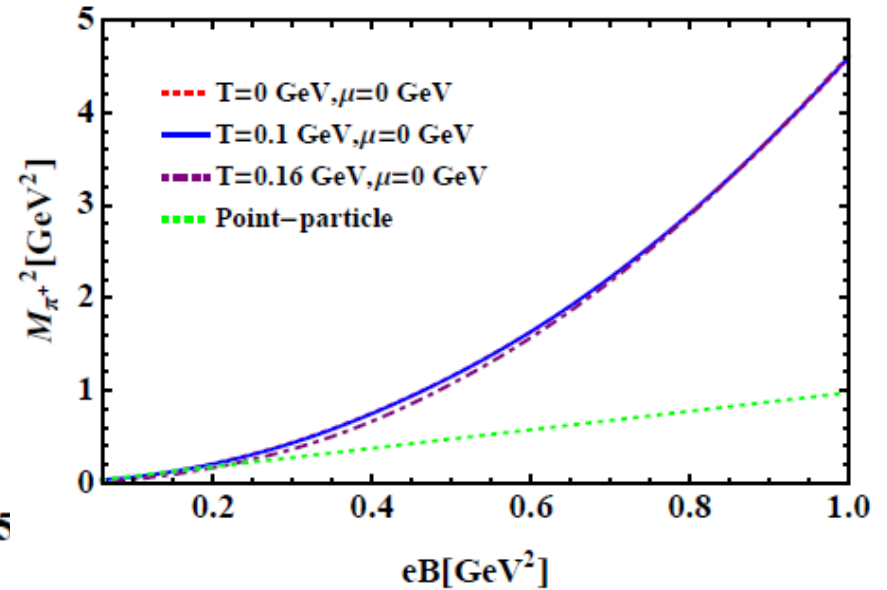
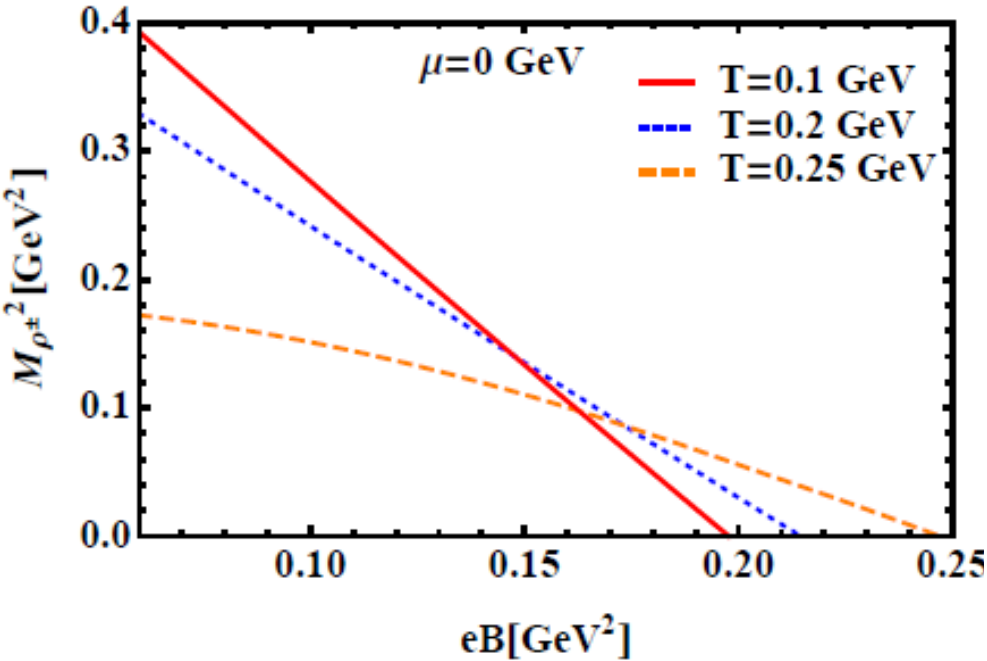
$\rightarrow$  Measurement of  $B$  at freeze-out



Magnetic field influences the ratio of  $N(\text{charged pion})/N(\text{charged rho})$  significantly!

**Kun Xu**, Hui Zhang, Shuzhe Shi, Xinyang Wang,  
Jinfeng Liao, Defu Hou, MH, to appear!

# Meson mass $M(B,T) \rightarrow$ Meson production number $N(B,T)$ $\rightarrow$ Measurement of B at freeze-out (T)



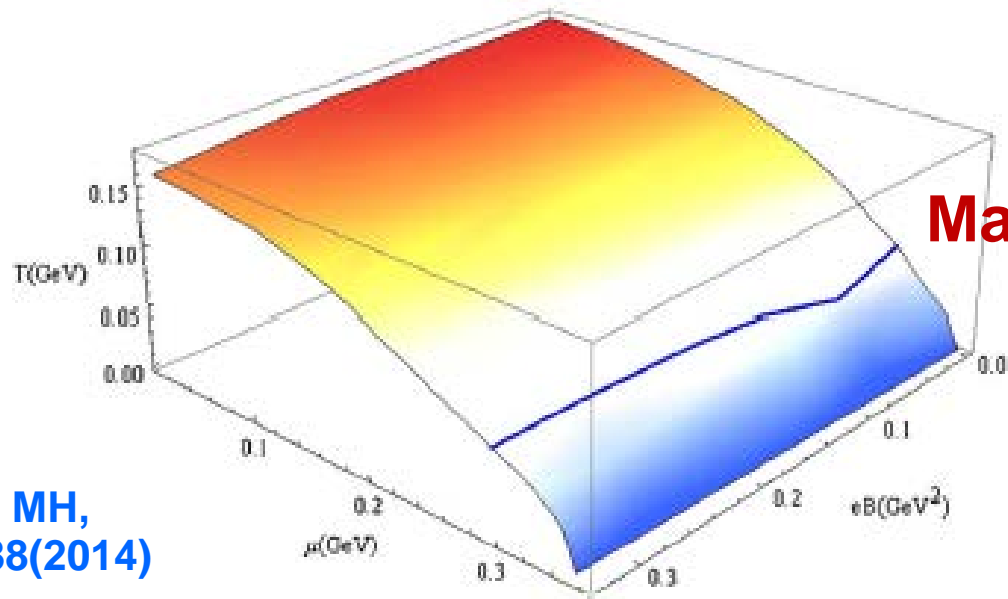
Magnetic field influences the ratio of  $N(\text{charged pion})/N(\text{charged rho})$  **more** significantly!

# **III. Phase structure under B revisited**

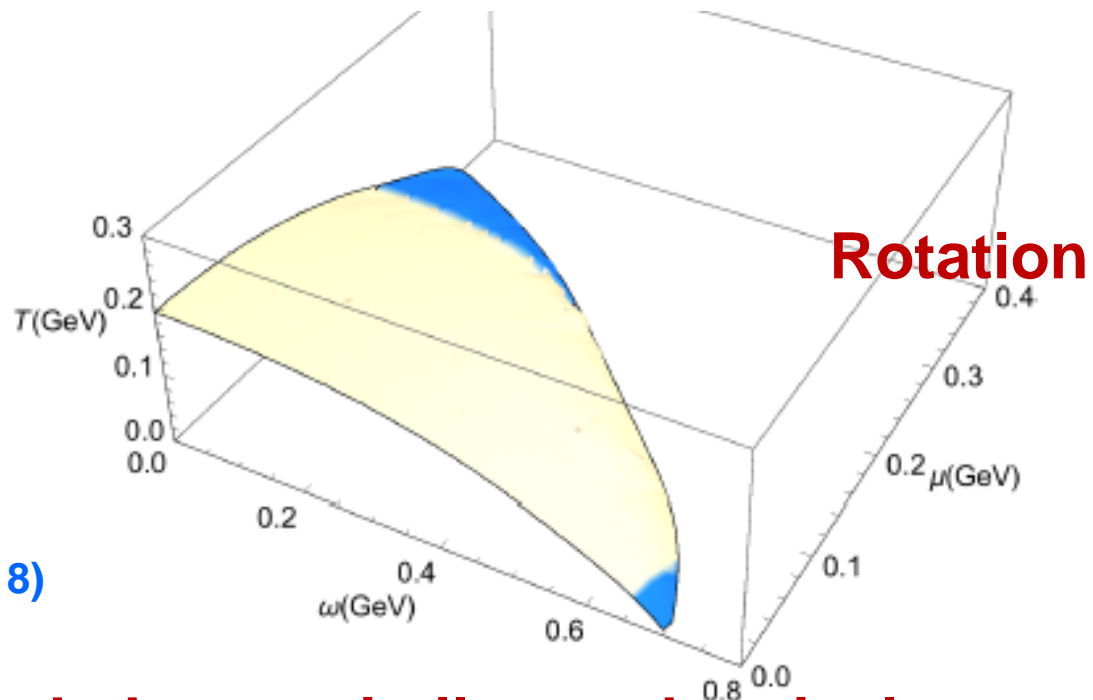




**Jingyi Chao, P.C. Chu, MH,**  
arXiv:1305.1100, PRD88(2014)

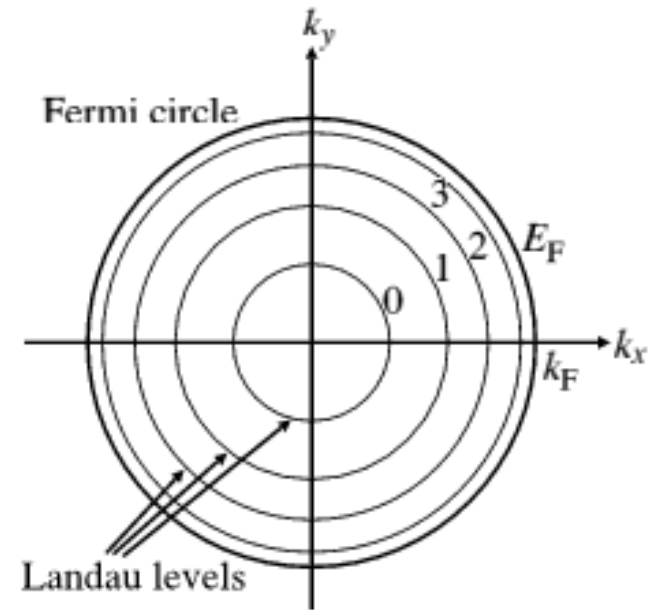
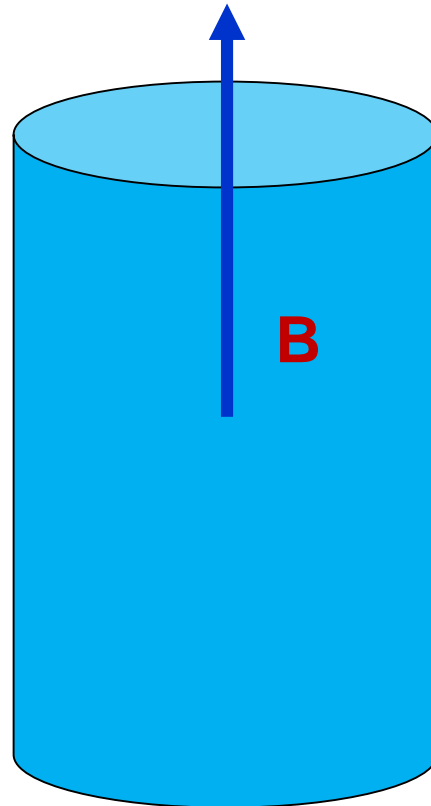
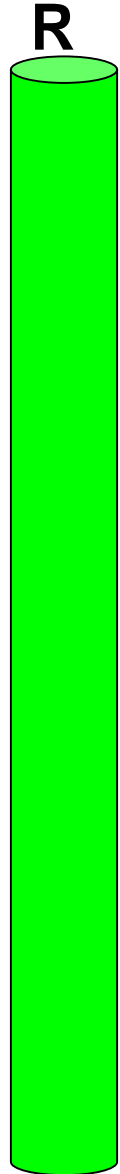


**Xinyang Wang, Minghua,**  
**Zhibin Li, MH,**  
arXiv:1808.01931, PRD88(2018)



**Angular velocity behaves similar to chemical potential!**

# Strong magnetic field vs small Size



$$R \sim 1/\sqrt{eB}$$

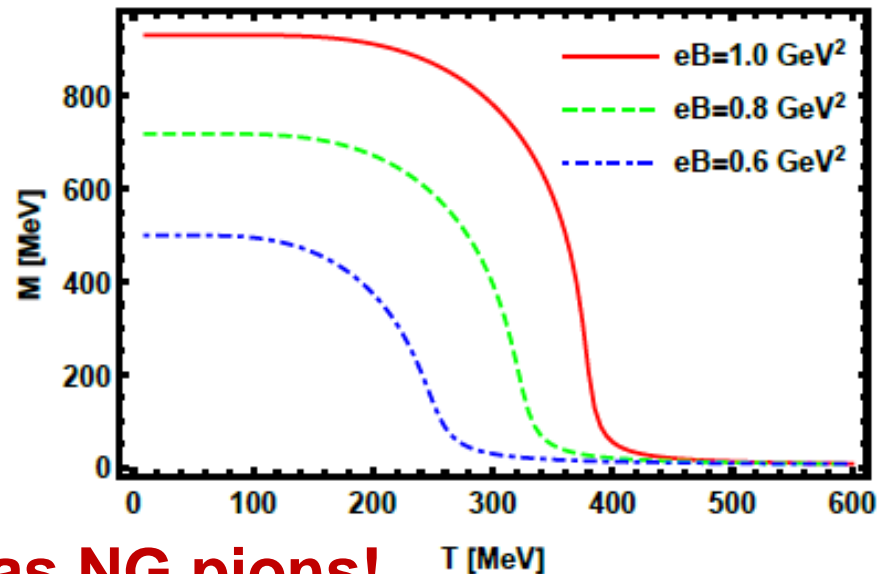
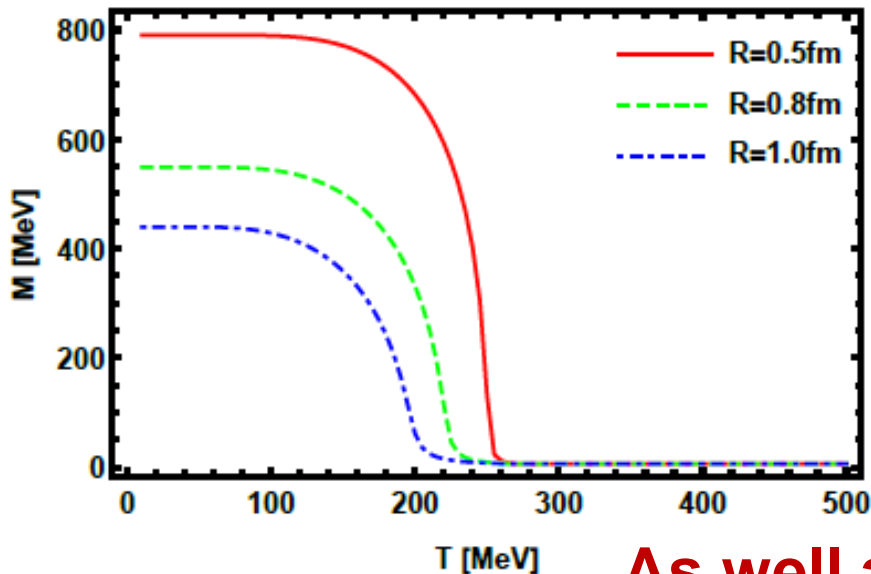
# Small size catalysis vs magnetic catalysis



$$\Omega = \frac{(M - m_0)^2}{4G} - \frac{2N_c N_f}{R^2} \int \frac{dp_z}{2\pi} \left\{ E + 2T \ln(1 + e^{-\frac{E}{T}}) \right\}$$

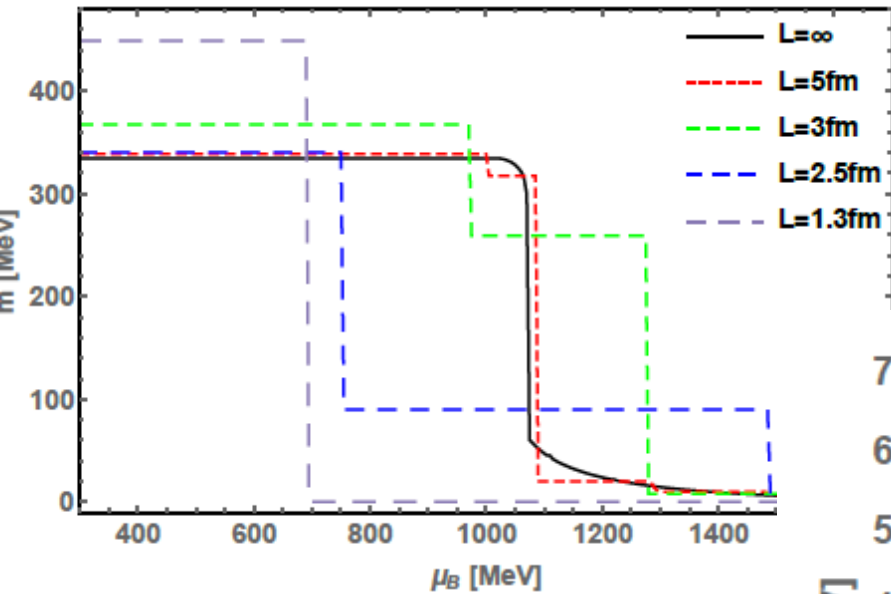
$$R \sim 1/\sqrt{eB}$$

$$\Omega = \frac{(M - m_0)^2}{4G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{dp_z}{2\pi} \left\{ E + 2T \ln(1 + e^{-\frac{E}{T}}) \right\}$$

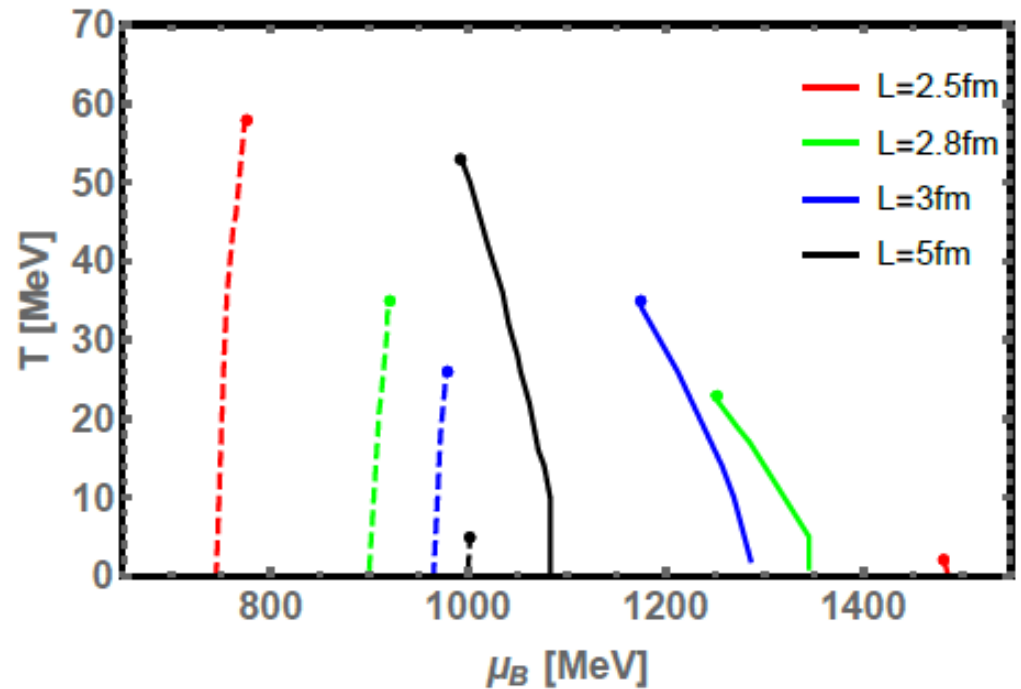


As well as NG pions!

# Quantized 1<sup>st</sup>-order phase transition and two sets of CEP!

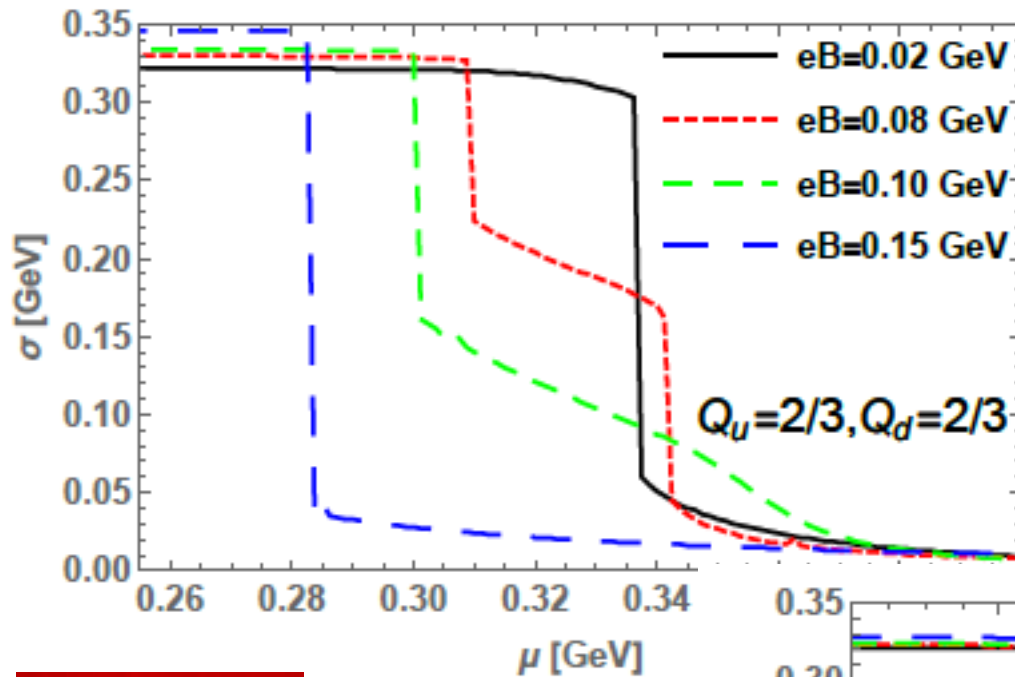


Zero mode contribution dominant at small size!



Kun Xu, M.H., arXiv:1903.08416, 1904.1154

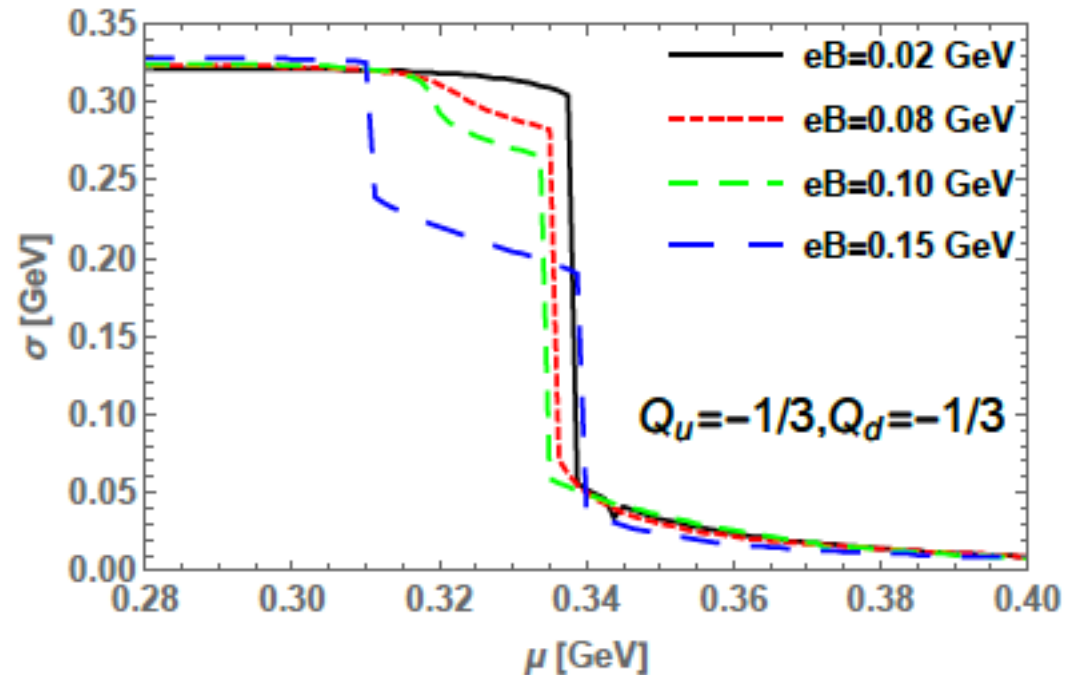
# Phase transition under B revisited!



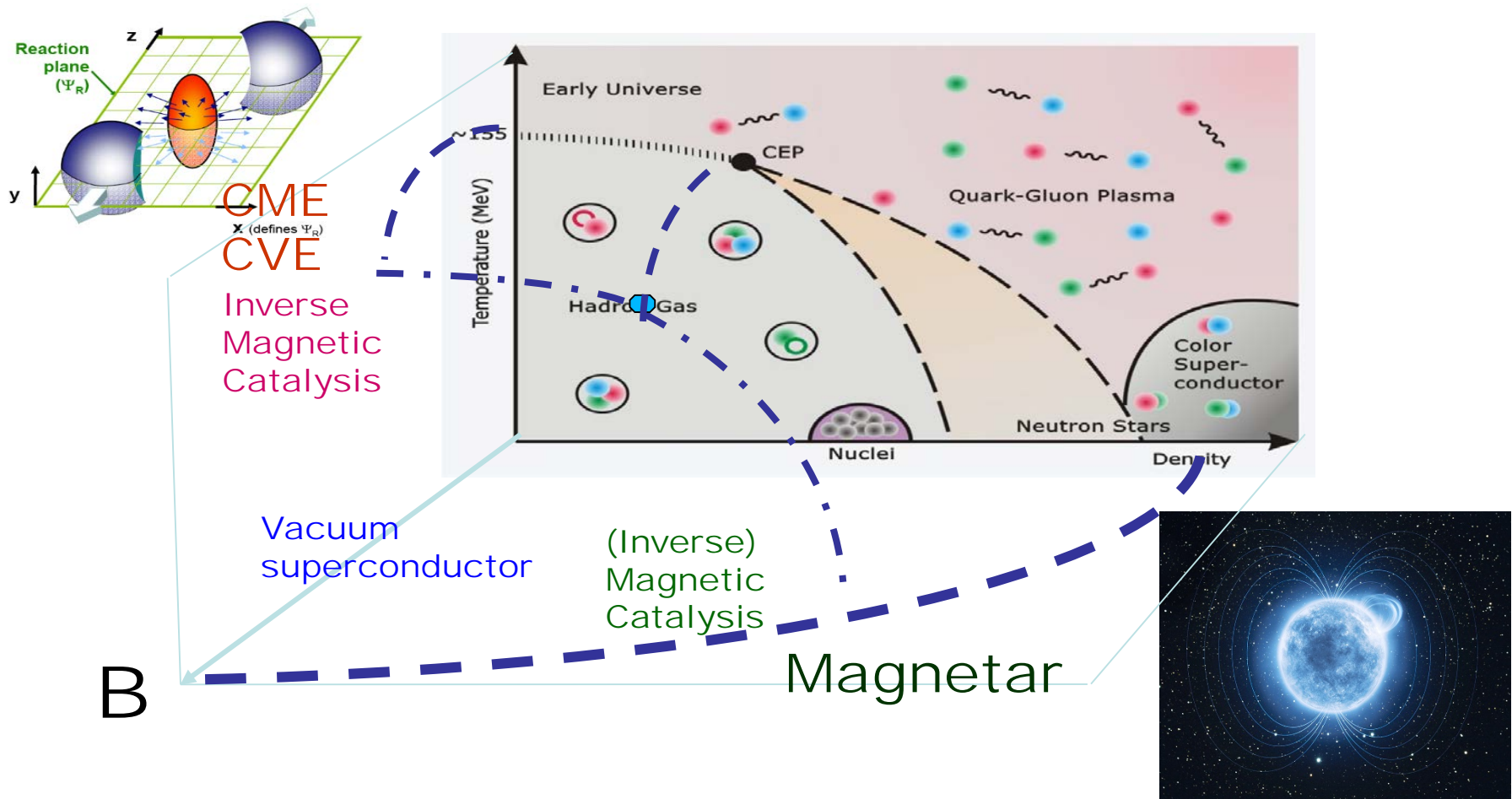
**Quantized 1<sup>st</sup>-order  
phase transition  
and two sets of CEP  
for each flavor under B!**



**Kun Xu, M.H., in progress!**



# IV. Summary



Still lots of open questions remain !  
 No much conclusive statement we can make!

**Thanks for your attention!**