



Chiral kinetic theory in curved spacetime

Yu-chen Liu
Fudan University

Collaborators: Lan-lan Gao , Kazuya Mameda , Xu-guang Huang

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Chiral magnetic effect and chiral vortical effect



- ▶ Chirality is the same as helicity for massless particles: the sign of the projection of the spin vector onto the momentum vector (picture from wikipedia)



- ▶ Chiral magnetic effect(CME)(Kharzeev, McLerran, Warringa, Fukushima 2008; Son, Zhitnitsky 2004)

$$\begin{aligned} \mathbf{J} &= \sigma \mathbf{B}, & \sigma &= \frac{1}{2\pi^2} \mu_5, \\ \mathbf{J}_5 &= \sigma_5 \mathbf{B}, & \sigma_5 &= \frac{1}{2\pi^2} \mu, \end{aligned} \quad (1)$$

with \mathbf{B} the magnetic field.

- ▶ Chiral vortical effect(CVE)(Erdmenger etal 2008; Barnerjee etal 2008, Son, Surowka 2009; Landsteiner etal 2011)

$$\begin{aligned} \mathbf{J} &= \xi \vec{\omega}, & \xi &= \frac{1}{\pi^2} \mu \mu_5, \\ \mathbf{J}_5 &= \xi_5 \vec{\omega}, & \xi_5 &= \frac{(\mu^2 + \mu_5^2)}{2\pi^2} + \frac{T^2}{6}, \end{aligned} \quad (2)$$

with $\vec{\omega}$ the fluid vorticity.

Rotating fluid vs. rotating frame



- ▶ Lorentz force \iff Coriolis force in rotating frame

In magnetic field, Lorentz force:

$$\mathbf{F} = e(\dot{\mathbf{x}} \times \mathbf{B})$$

In rotating frame, Coriolis force:

$$\mathbf{F} = 2\epsilon(\dot{\mathbf{x}} \times \boldsymbol{\omega}) + \mathbf{O}(\omega^2)$$

- ▶ Correspondence: $\mathbf{B} \iff 2\epsilon\vec{\omega}$ (Stephanov, Yin 2012)

$$\mathbf{J}_{CME} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \mathbf{B}(\mathbf{b} \cdot \hat{\mathbf{p}}) f \quad \iff \quad \mathbf{J}_{CVE} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} 2\epsilon\vec{\omega}(\mathbf{b} \cdot \hat{\mathbf{p}}) f \quad (3)$$

where $\mathbf{b} = \frac{\hat{\mathbf{p}}}{2|\mathbf{p}|^2}$ is the Berry curvature. This (+ antiparticle) indeed gives expected result.

A kinetic theory in curved spacetime would be helpful to study rotating frame and CVE.

Wigner operator and Horizontal lift



► Wigner operator

$$\hat{W}_{\alpha\beta}(x, p) \equiv \int \frac{\sqrt{-g}d^4y}{(2\pi)^4} e^{-ip \cdot y/\hbar} [\bar{\psi}(x) e^{1/2y \cdot \overleftarrow{\nabla}}]_{\beta} [e^{-1/2y \cdot \nabla} \psi(x)]_{\alpha}. \quad (4)$$

Where the derivative $\overleftarrow{\nabla}_{\mu}$ (∇_{μ}) acting to the left(right).

- We emphasize that x in equation (4) is the coordinate of point(P) in curved spacetime, and y is vector in the tangent space of point P, and p is vector in cotangent space of P.
- Horizontal lifted covariant derivatives (Winter. 1985; Calzetta, Habib, Hu. 1988; Fonarev. 1994)

$$\nabla_{\mu} \equiv D_{\mu} - \Gamma_{\mu\nu}^{\lambda} y^{\nu} \frac{\partial}{\partial y^{\lambda}} + \Gamma_{\mu\nu}^{\lambda} p_{\lambda} \frac{\partial}{\partial p_{\nu}} \underbrace{+ \Gamma_{\mu} + \frac{i}{\hbar} A_{\mu}}_{\text{connection for spinor}}, \quad (5)$$

$$\overleftarrow{\nabla}_{\mu} \equiv \overleftarrow{D}_{\mu} - \overleftarrow{\frac{\partial}{\partial y^{\lambda}}} \Gamma_{\mu\nu}^{\lambda} y^{\nu} + \overleftarrow{\frac{\partial}{\partial p_{\nu}}} \Gamma_{\mu\nu}^{\lambda} p_{\lambda} - \Gamma_{\mu} - \frac{i}{\hbar} A_{\mu}, \quad (6)$$

where D_{μ} is the usual covariant derivative operator, A_{μ} is gauge field, $\Gamma_{\mu} \equiv -\frac{i}{4} \omega_{\mu}^{ab} \sigma_{ab}$ is spin connection with $\sigma_{ab} = \frac{i}{2} [\gamma_a, \gamma_b]$ and ω_{μ}^{ab} the vierbein connection.

- Vierbein: $e^a = e_{\mu}^a \partial^{\mu}$.

Dynamic equation for Wigner function



- ▶ Dirac equation

$$\gamma^\mu \nabla_\mu \psi = 0 = \bar{\psi} \overleftarrow{\nabla}_\mu \gamma^\mu. \quad (7)$$

- ▶ Dynamic equation

$$\begin{aligned} \gamma^\mu \left(p_\mu + \frac{i\hbar}{2} \nabla_\mu \right) W &= i\hbar \gamma^\mu \int d^4 y \sqrt{-g} e^{-ip \cdot y / \hbar} \\ &\times \left[\bar{\psi}(x, y/2) \otimes (-\mathcal{H}_\mu + \mathcal{G}_\mu) \psi(x, -y/2) \right. \\ &\left. - \bar{\psi}(x, y/2) \overleftarrow{\mathcal{G}}_\mu \otimes \psi(x, -y/2) \right] \end{aligned} \quad (8)$$

- ▶ This equation is exact.

- ▶ $\bar{\psi}(x, y/2) \equiv \bar{\psi}(x) e^{1/2y \cdot \overleftarrow{\nabla}}$, $\psi(x, -y/2) \equiv e^{-1/2y \cdot \nabla} \psi(x)$.

- ▶ Operators

$$\begin{aligned} \mathcal{H}_\mu \psi(x, y) &= -\frac{iy^\nu}{\hbar} \sum_{n=0}^{\infty} \frac{[\mathcal{C}(y \cdot \nabla)]^n}{(n+1)!} G_{\mu\nu} \psi(x, y), \\ \mathcal{G}_\mu \psi(x, y) &= -\frac{iy^\nu}{2\hbar} \sum_{n=0}^{\infty} \frac{[\mathcal{C}(y \cdot \nabla)]^n}{(n+2)!} G_{\mu\nu} \psi(x, y), \end{aligned} \quad (9)$$

where $\mathcal{C}(A)B \equiv [A, B]$ represents a commutator and $G_{\mu\nu} \equiv -i\hbar[\nabla_\mu, \nabla_\nu]$.

Dynamic equation for Wigner function



Up to $O(\hbar^2)$ order

$$\begin{aligned} \gamma^\mu \left(\Pi_\mu + \frac{i\hbar}{2} \Delta_\mu \right) \hat{W} &= \frac{i\hbar^2}{32} \gamma^\mu R_{\mu\alpha\rho\sigma} \left[\partial_\rho^\alpha \hat{W}, \sigma^{\mu\nu} \right] \\ &\quad - \frac{\hbar^3}{8 \times 4!} (\nabla_\beta R_{\mu\alpha\rho\sigma}) \gamma^\mu \left[\partial_\rho^\alpha \partial_\rho^\beta \hat{W}, \sigma^{\rho\sigma} \right] \end{aligned} \quad (10)$$

with

$$\begin{aligned} \Pi_\mu &= p_\mu - \frac{\hbar^2}{12} (\nabla_\rho F_{\mu\nu}) \partial_\rho^\nu \partial_\rho^\rho + \frac{\hbar^2}{24} R^\rho{}_{\sigma\mu\nu} \partial_\rho^\sigma \partial_\rho^\nu p_\rho + \frac{\hbar^2}{4} R_{\mu\nu} \partial_\rho^\nu, \\ \Delta_\mu &= \nabla_\mu - F_{\mu\lambda} \partial_\rho^\lambda - \frac{\hbar^2}{12} (\nabla_\rho R_{\mu\nu}) \partial_\rho^\rho \partial_\rho^\nu - \frac{\hbar^2}{24} (\nabla_\lambda R^\rho{}_{\sigma\mu\nu}) \partial_\rho^\nu \partial_\rho^\sigma \partial_\rho^\lambda p_\rho \\ &\quad + \frac{\hbar^2}{8} R^\rho{}_{\sigma\mu\nu} \partial_\rho^\nu \partial_\rho^\sigma D_\rho + \frac{\hbar^2}{24} (\nabla_\alpha \nabla_\beta F_{\mu\nu} + 2R^\rho{}_{\alpha\mu\nu} F_{\beta\rho}) \partial_\rho^\nu \partial_\rho^\alpha \partial_\rho^\beta, \end{aligned} \quad (11)$$

where $R^\mu{}_{\nu\rho\sigma}$ is Riemann curvature and $R_{\mu\nu}$ is Ricci tensor.



► kinetic equation

$$0 = \delta(p^2 - \hbar F_{\alpha\beta} \Sigma^{\alpha\beta}) \left[p_\mu \Delta^\mu f_r - \frac{\hbar}{p \cdot n} \tilde{F}_{\mu\nu} n^\nu \Delta^\mu f_r + \underbrace{\hbar \Delta^\mu (\Sigma_{\mu\nu} \Delta^\nu f_r)}_{\text{spin-curvature coupling}} \right] \quad (12)$$

- $\Delta_\mu = \nabla_\mu - F_{\mu\nu} \partial_p^\nu$;
- n^μ is a frame choosing vector;
- $\Sigma_{\mu\nu} \equiv \frac{1}{2p \cdot n} \epsilon_{\mu\nu\rho\sigma} p^\rho n^\sigma$ is spin operator.

► Current

$$J_I^\mu = 4\pi \int \frac{d^4 p}{\sqrt{-g}(2\pi)^4} \delta(p^2 - \frac{\hbar p^\sigma B_\sigma}{p \cdot n}) \left[p_\mu f_r - \hbar \Sigma_{\mu\nu} \left(\frac{E^\nu}{p \cdot n} - \nabla^\nu \right) f_r \right], \quad (13)$$

where $E_\mu = n^\nu F_{\mu\nu}$ and $B_\mu = n^\nu \tilde{F}_{\mu\nu}$.

Abelian gauge field in Minkowski space



- ▶ kinetic equation with $n^\mu = (1, 0, 0, 0)$

$$0 = \left\{ \left(1 + \frac{\hbar(\mathbf{B} \cdot \mathbf{p})}{2|\mathbf{p}|^3} \right) \partial_t + \left(\mathbf{v} + \frac{\hbar \mathbf{B}}{2|\mathbf{p}|^2} + \frac{\hbar}{2|\mathbf{p}|^3} [(\mathbf{E} - \nabla \epsilon_p) \times \mathbf{p}] \right) \cdot \nabla \right. \\ \left. + (\mathbf{E} - \nabla \epsilon_p) \cdot \nabla_{\mathbf{p}} + \mathbf{v} \times \mathbf{B} \cdot \nabla_{\mathbf{p}} + \frac{\hbar}{2|\mathbf{p}|^3} [(\mathbf{E} - \nabla \epsilon_p) \cdot \mathbf{B}] \mathbf{p} \cdot \nabla_{\mathbf{p}} \right\} f_r \quad (14)$$

where $\epsilon_p \equiv p_0 = |\mathbf{p}| - \frac{\hbar \mathbf{B} \cdot \mathbf{p}}{2|\mathbf{p}|^2}$ and $\mathbf{v} \equiv \frac{\partial \epsilon_p}{\partial \mathbf{p}}$. (Son, Yamamoto. 2013; Hidaka, Pu, Yang. 2017; Huang, Shi, Jiang, et al. 2018.)

- ▶ current

$$J^0 = \int d^3 \mathbf{p} \left(1 + \frac{\hbar(\mathbf{B} \cdot \mathbf{p})}{2|\mathbf{p}|^3} \right) f_r, \\ \mathbf{J} = \int d^3 \mathbf{p} \left(\mathbf{v} + \frac{\hbar \mathbf{B}}{2|\mathbf{p}|^2} + \frac{\hbar}{2|\mathbf{p}|^3} \mathbf{E} \times \mathbf{p} - \frac{\hbar}{2|\mathbf{p}|^3} \epsilon_p \mathbf{p} \times \nabla \right) f_r \quad (15)$$

Equilibrium state without gauge field

Kinetic equation



$$0 = \delta(p^2) \nabla^\mu \left(p_\mu f_r + \hbar \Sigma_{\mu\nu} \nabla^\nu f_r \right), \quad (16)$$

We consider equilibrium distribution $f_r(g)$, with g a linear combination of collisional conserved quantities.

- ▶ Conserved quantities are particle number, momentum and angular momentum

$$g = \alpha_R(x) + \beta^\mu(x) p_\mu + \hbar \Sigma_{\mu\nu} \gamma^{\mu\nu} \quad (17)$$

- ▶ We obtain the constraints

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = \phi(x) g_{\mu\nu}, \quad (18)$$

$$\gamma_{\mu\nu} - \frac{1}{2} \nabla_\mu \beta_\nu = 0, \quad (19)$$

$$\nabla_\mu \alpha_R = 0. \quad (20)$$

- ▶ One can identify $\alpha_R = \beta \mu_R$ and $\beta^\mu = \beta u^\mu$.
- ▶ For $\phi = \text{constant}$, we have $\beta^\mu = \beta_{\text{con}}^\mu + \underbrace{\omega^{\mu\nu} x_\nu}_{\text{rotation}} + \underbrace{\phi x^\mu}_{\text{expansion}}$.



Rotating fluid in Minkowski space

- ▶ $\epsilon_p \equiv p_0 = |\mathbf{p}|$ and $\mathbf{v} \equiv \frac{\partial \epsilon_p}{\partial \mathbf{p}}$.
- ▶ kinetic equation with $n^\mu = u^\mu = (1, \boldsymbol{\Omega} \times \mathbf{x})$ the velocity vector of fluid

$$0 = \sqrt{G} \left[\partial_t + \left(\mathbf{v} + \frac{\hbar \boldsymbol{\Omega}}{2|\mathbf{p}|} - \frac{\hbar \mathbf{p} \cdot \boldsymbol{\Omega}}{2|\mathbf{p}|^3} \mathbf{p} \right) \cdot \nabla \right] f_r, \quad (21)$$

with $\boldsymbol{\Omega}$ the angular velocity of fluid and $\sqrt{G} \equiv (1 + \hbar \frac{\mathbf{p} \cdot \boldsymbol{\Omega}}{|\mathbf{p}|^2})$.

- ▶ Single particle EOM

$$\sqrt{G} \dot{\mathbf{x}} = \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{p}} + \frac{\hbar \boldsymbol{\Omega}}{|\mathbf{p}|}, \quad \sqrt{G} \dot{\mathbf{p}} = 0, \quad (22)$$

where $\tilde{\epsilon}_{\mathbf{p}} \equiv \epsilon_p - \frac{\hbar}{2} \hat{\mathbf{p}} \cdot \boldsymbol{\Omega} = |\mathbf{p}| - \frac{\hbar}{2} \hat{\mathbf{p}} \cdot \boldsymbol{\Omega}$.

- ▶ Equilibrium distribution function

$$f_r^{eq} = f_r(\mu + |\mathbf{p}| - \mathbf{p} \cdot (\boldsymbol{\Omega} \times \mathbf{x}) - \frac{\hbar}{2|\mathbf{p}|} \mathbf{p} \cdot \boldsymbol{\Omega}) \quad (23)$$

- ▶ The CVE current (Chen, Son, Stephanov, et al. 2014)

$$\mathbf{J} = \int d^3 \mathbf{p} \left\{ \mathbf{v} - \hbar \frac{\mathbf{p}}{2|\mathbf{p}|^2} \times \nabla \right\} f_r. \quad (24)$$

Rotating frame with angular velocity Ω



▶ Metric

$$g_{00} = 1 - (\Omega \times \mathbf{x})^2, \quad g_{0i} = g_{i0} = -(\Omega \times \mathbf{x})^i, \quad g_{ij} = -\delta_{ij}. \quad (25)$$

$$g^{00} = 1, \quad g^{0i} = g^{i0} = -(\Omega \times \mathbf{x})^i, \quad g^{ij} = -\delta^{ij} + (\Omega \times \mathbf{x})^i (\Omega \times \mathbf{x})^j. \quad (26)$$

▶ Nonzero connections

$$\Gamma^i_{00} = [\Omega \times (\Omega \times \mathbf{x})]^i, \quad \Gamma^i_{0j} = \Gamma^i_{j0} = \epsilon^{ijl} \Omega^l. \quad (27)$$

▶ Riemann curvature is identically zero in rotating frame.

Setup

▶ Momentum

$$\mathbf{p} \equiv -(p_1, p_2, p_3) \quad (28)$$

▶ Energy

$$\epsilon_{\mathbf{p}} \equiv p_0 = |\mathbf{p}| - (\Omega \times \mathbf{x}) \cdot \mathbf{p}, \quad p^0 = g^{0\nu} p_\nu = g^{00} p_0 + g^{0i} p_i = |\mathbf{p}|. \quad (29)$$

▶ Effective velocity

$$\mathbf{v} \equiv \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} = \frac{\mathbf{p}}{|\mathbf{p}|} - (\Omega \times \mathbf{x}) \quad (30)$$

Co-rotating observer and Inertial fluid



- ▶ Kinetic equation with $n^\mu = u^\mu = (1, -(\boldsymbol{\Omega} \times \mathbf{x}))^\mu$

$$0 = \left\{ \partial_t + \mathbf{v} \cdot \nabla + \mathbf{p} \times \boldsymbol{\Omega} \frac{\partial}{\partial \mathbf{p}} \right\} f_r. \quad (31)$$

- ▶ EOM

$$\begin{aligned} \dot{\mathbf{x}} &= \hat{\mathbf{p}} - (\boldsymbol{\Omega} \times \mathbf{x}) = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \\ \dot{\mathbf{p}} &= \mathbf{p} \times \boldsymbol{\Omega} = -\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}} \end{aligned} \quad (32)$$

- ▶ Current

$$J^0 = \int d^3 \mathbf{p} f_r \quad (33)$$

$$\mathbf{J} = \int d^3 \mathbf{p} \left(\mathbf{v} f_r - \frac{\hbar \mathbf{p}}{2|\mathbf{p}|^2} \times \nabla f_r \right) \quad (34)$$

- ▶ Equilibrium distribution function

$$f_r = f_r(\mu + |\mathbf{p}|) \quad (35)$$

we find there is **no CVE**.

Co-rotating observer and Co-rotating fluid



- ▶ kinetic equation with $n^\mu = u^\mu = (1, 0, 0, 0)$

$$0 = \left\{ \sqrt{G} \partial_t + \sqrt{G} \left[\mathbf{v} + \hbar \left(\frac{\boldsymbol{\Omega}}{2|\mathbf{p}|} - \frac{(\mathbf{p} \cdot \boldsymbol{\Omega}) \mathbf{p}}{2|\mathbf{p}|^3} \right) \right] \cdot \nabla + \mathbf{p} \times \boldsymbol{\Omega} \cdot \frac{\partial}{\partial \mathbf{p}} \right\} f_r. \quad (36)$$

with $\sqrt{G} \equiv (1 + \hbar \frac{\mathbf{p} \cdot \boldsymbol{\Omega}}{|\mathbf{p}|^2})$.

- ▶ Single particle EOM

$$\begin{aligned} \sqrt{G} \dot{\mathbf{x}} &= \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{p}} + \frac{\hbar \boldsymbol{\Omega}}{|\mathbf{p}|}, \\ \sqrt{G} \dot{\mathbf{p}} &= \mathbf{p} \times \boldsymbol{\Omega}, \end{aligned} \quad (37)$$

with $\tilde{\epsilon}_{\mathbf{p}} \equiv \epsilon_{\mathbf{p}} - \frac{\hbar}{2} \hat{\mathbf{p}} \cdot \boldsymbol{\Omega} = |\mathbf{p}| - \mathbf{p} \cdot (\boldsymbol{\Omega} \times \mathbf{x}) - \frac{\hbar}{2} \hat{\mathbf{p}} \cdot \boldsymbol{\Omega}$

- ▶ Current

$$\mathbf{J} = \int d^3 \mathbf{p} \left(\mathbf{v} f_r - \frac{\hbar \mathbf{p}}{2|\mathbf{p}|^2} \times \nabla f_r \right) \quad (38)$$

- ▶ Equilibrium state

$$f_r^{eq} = f_r(\mu + |\mathbf{p}| - \mathbf{p} \cdot (\boldsymbol{\Omega} \times \mathbf{x}) - \frac{\hbar}{2} \hat{\mathbf{p}} \cdot \boldsymbol{\Omega}), \quad (39)$$

it induces CVE.

Co-rotating observer and Co-rotating fluid



One can find the Coriolis force

$$|\mathbf{p}|\ddot{\mathbf{x}} = 2\mathbf{p} \times \boldsymbol{\Omega}. \quad (40)$$

Redefining the three components momentum $\mathbf{k} = \mathbf{p} - |\mathbf{p}|\boldsymbol{\Omega} \times \mathbf{x}$

- ▶ Kinetic equation with $\sqrt{G} = (1 + \hbar \frac{\mathbf{k} \cdot \boldsymbol{\Omega}}{|\mathbf{k}|^2})$.

$$0 = \left\{ \sqrt{G} \partial_t + \sqrt{G} \left[\hat{\mathbf{k}} + \hbar \left(\frac{\boldsymbol{\Omega}}{2|\mathbf{k}|} - \frac{(\mathbf{k} \cdot \boldsymbol{\Omega})\mathbf{k}}{2|\mathbf{k}|^3} \right) \right] \cdot \nabla + 2\mathbf{k} \times \boldsymbol{\Omega} \cdot \frac{\partial}{\partial \mathbf{k}} \right\} f_r \quad (41)$$

- ▶ Single particle EOM

$$\begin{aligned} \sqrt{G} \dot{\mathbf{x}} &= \frac{\partial \tilde{\epsilon}_k}{\partial \mathbf{k}} + \frac{\hbar \boldsymbol{\Omega}}{|\mathbf{k}|}, \\ \sqrt{G} \dot{\mathbf{k}} &= 2\mathbf{k} \times \boldsymbol{\Omega}, \end{aligned} \quad (42)$$

$$\text{with } \tilde{\epsilon}_k = |\mathbf{k}| - \frac{\hbar \boldsymbol{\Omega} \cdot \mathbf{k}}{2|\mathbf{k}|}$$

Compare with magnetic field case

$$\begin{aligned} \sqrt{G_B} \dot{\mathbf{x}} &= \frac{\partial \epsilon_k}{\partial \mathbf{k}} + \frac{\hbar \mathbf{B}}{2|\mathbf{k}|^2} \\ \sqrt{G_B} \dot{\mathbf{k}} &= \hat{\mathbf{k}} \times \mathbf{B} \end{aligned} \quad (43)$$

$$\text{with } \sqrt{G_B} = \left(1 + \frac{\hbar(\mathbf{B} \cdot \mathbf{k})}{2|\mathbf{k}|^3} \right) \text{ and } \epsilon_k \equiv |\mathbf{k}| - \frac{\hbar \mathbf{B} \cdot \mathbf{k}}{2|\mathbf{k}|^2}$$

Correspondence: $\mathbf{B} \iff 2|\mathbf{k}|\boldsymbol{\Omega}$.

Summary and outlook



Summary

- ▶ We have derived covariant chiral kinetic theory up to $O(\hbar)$ order in curved spacetime.
- ▶ We found that CVE is due to rotating of the fluid in inertial frame, and a rotating frame itself does not generate CVE.
- ▶ We have derived the correspondence between magnetic field and vorticity.

Outlook

- ▶ Gravitational anomaly.
- ▶ Quantum correction to the collision term.
- ▶ Expanding fluid.
- ▶ Spin dynamics.

Thank you!