Mass Correction to CKT

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on behalf of

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2019.04.08

BACKGROUND

QCD topology & magnetic field

Chiral Magnetic Effect

Quantum Transport

QCD topology & magnetic field

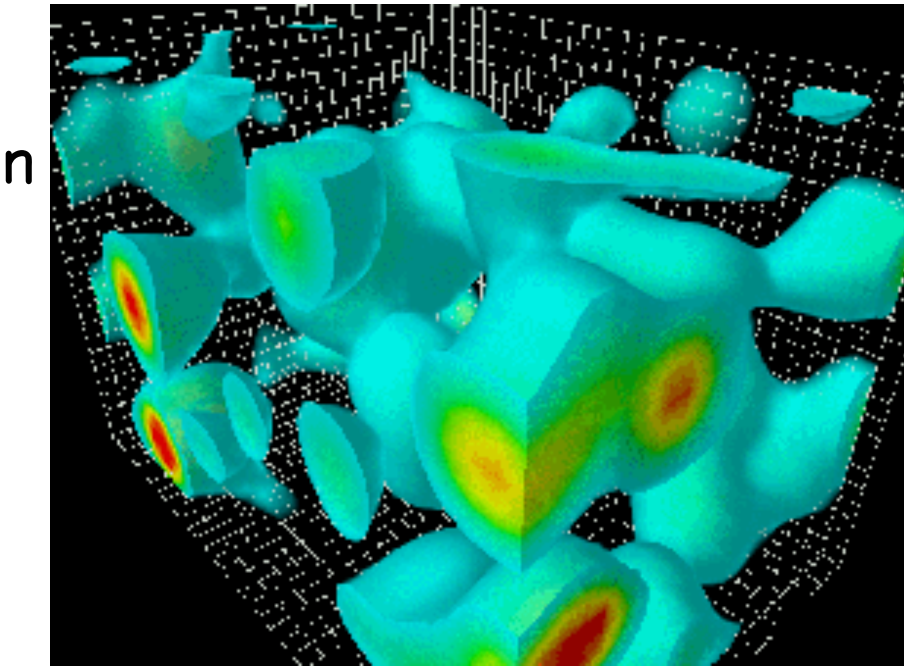
- **Mysterious QCD**

  Confinement, Chiral symmetry breaking, Hadronization

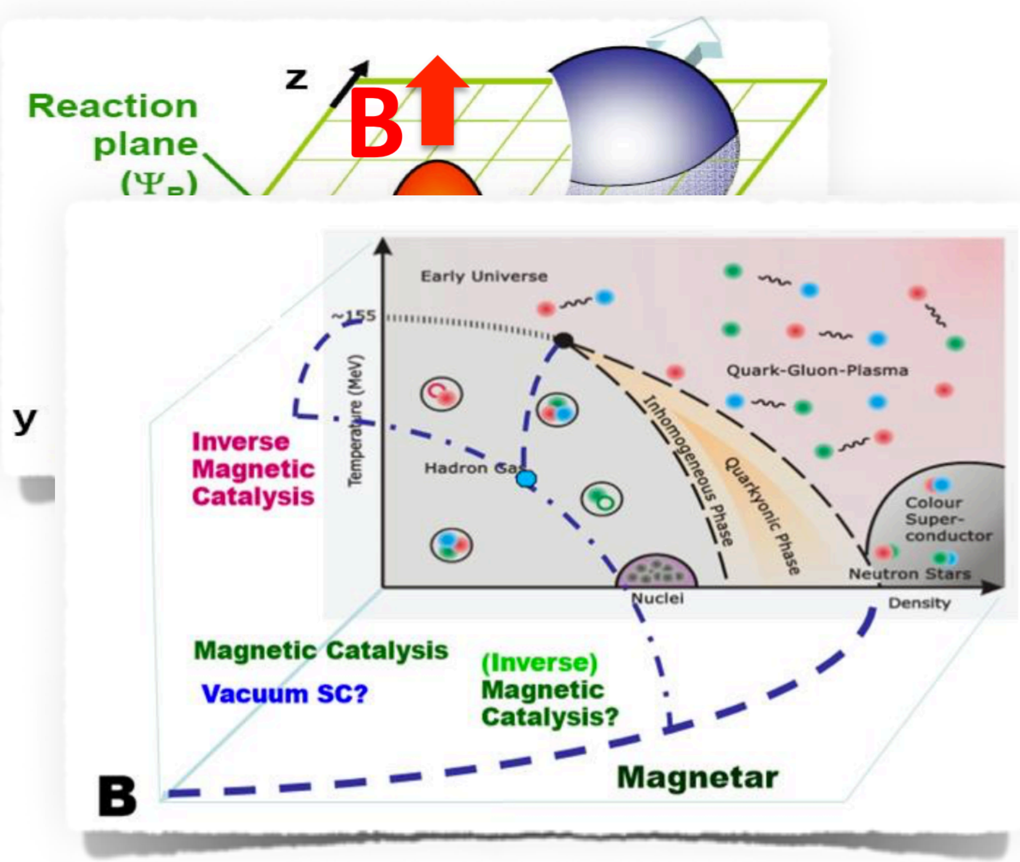
  Topologically nontrivial gluon field configurations

  θ -vacuum , non-perturbative

  UA(1) anomaly, Strong CP problem



- **Magnetic field in heavy ion collision**



- Graphene, Dirac (Weyl) semimental 10^5 Gauss

  Compact stars 10^{10}~ 10^{16} Gauss

  Heavy ion collision 10^{18}~ 10^{19}Gauss (eB ~ $6m_\pi^2$)

- Particle production

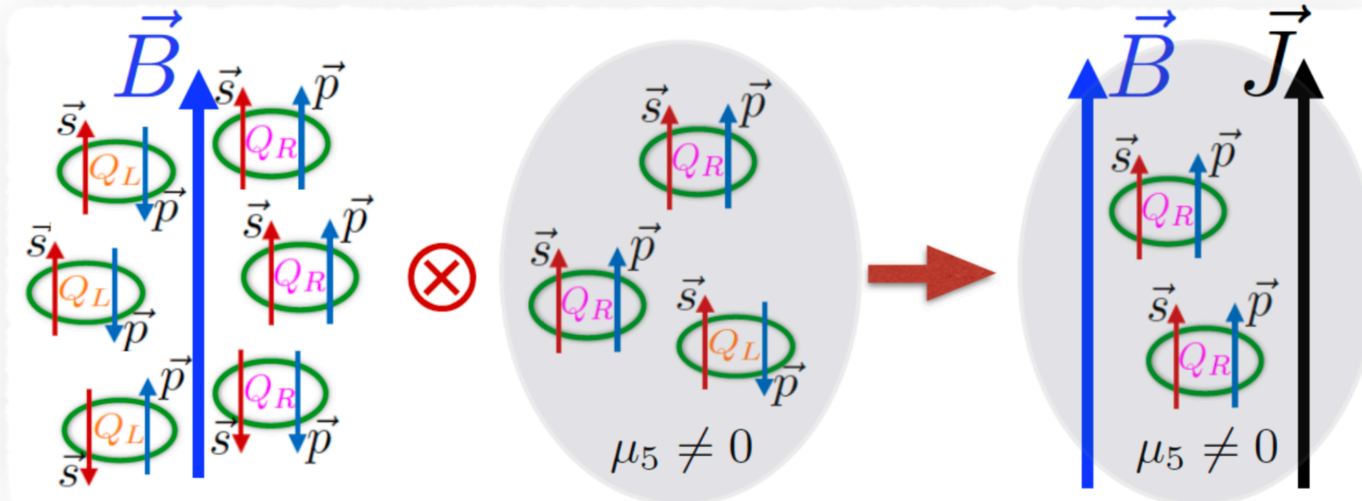
  Phase structure (MC, IMC)

  Transport property (CME, CMW, CSE...)

Chiral Magnetic Effect

- **Chiral Magnetic Effect (CME)**

$$\vec{J}_{\rm CME} = \frac{e^2}{2\pi^2}\mu_A \vec{B}$$



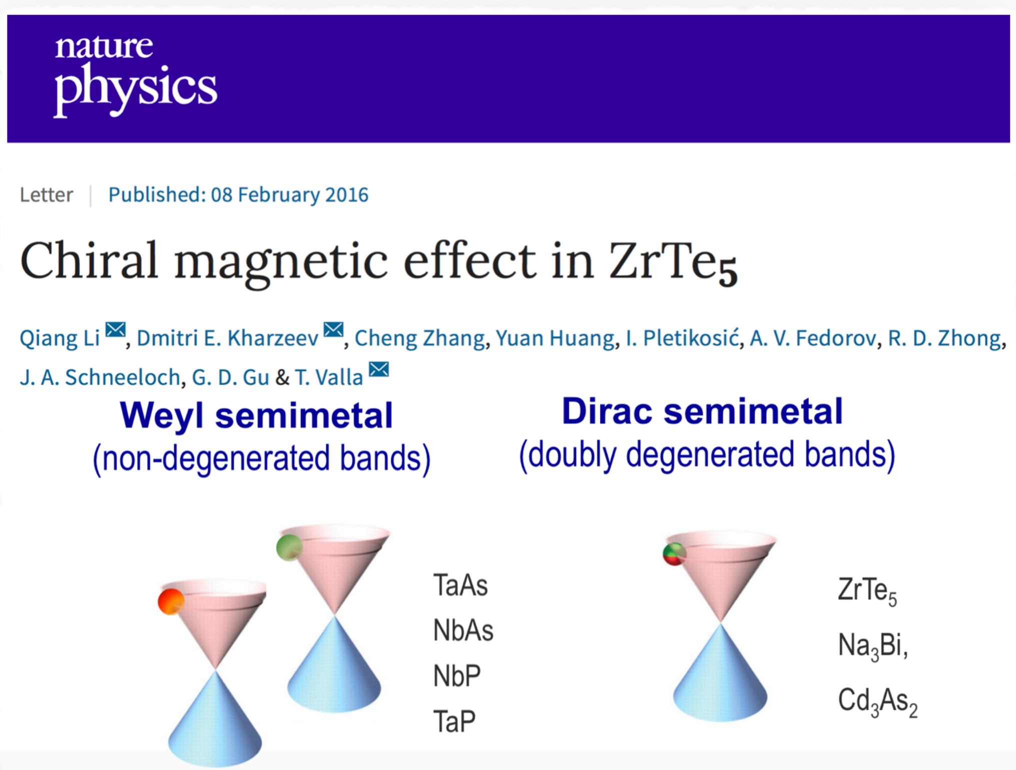
Electric current along B

Chiral charge density

T-even: non-dissipative, topological protected

- **Realize in systems with chiral fermion and B field**

Condensed mater system $\vec{E}\cdot\vec{B}$



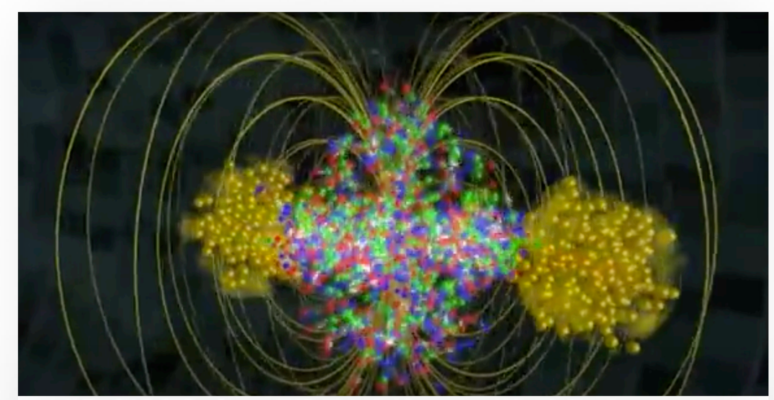
nature physics

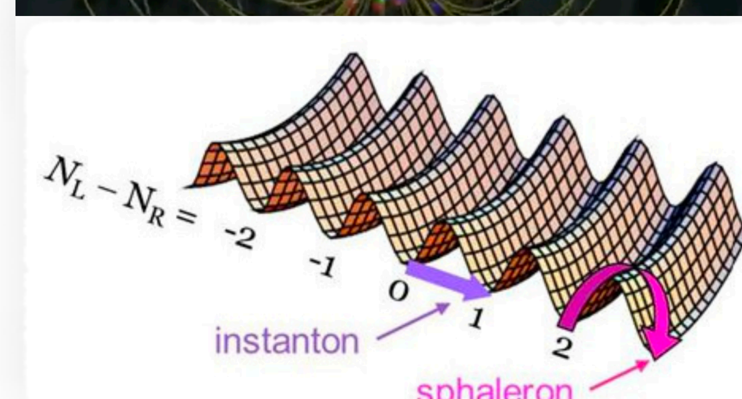
Letter | Published: 08 February 2016

Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla

Heavy Ion Collision $QCD \times QED$



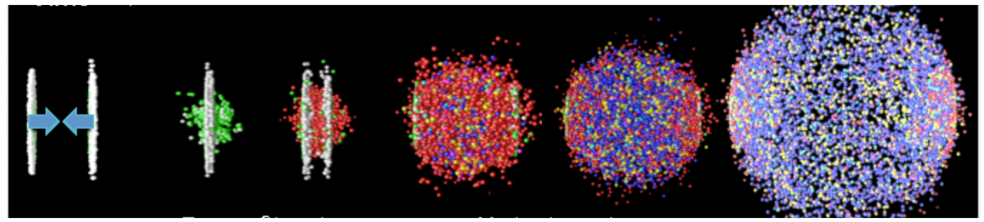


Quantum Transportation

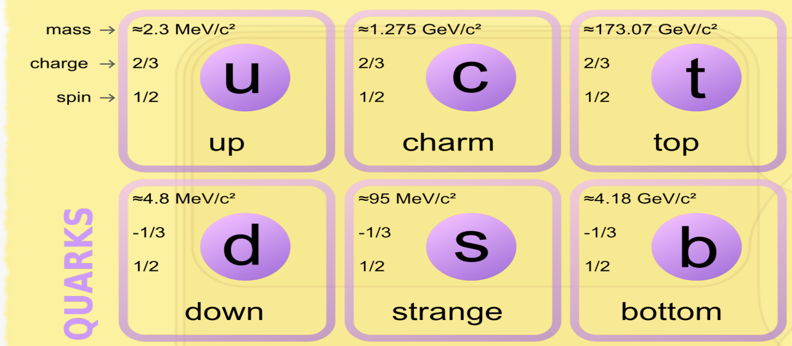
- **CME in heavy ion collision**

$$\mu_A = \partial_t \theta$$

non-equilibrium nature



Highly dynamical, inhomogeneous



Finite mass

- **Dynamical fluctuation of chiral imbalance**
- **Transpot** **local equilibrium** — Relativistic hydrodynamics & Triangle anomaly

  Non-equilibrium — Chiral Kinetic Theory & Berry curvature
- **Finite mass:** **Is there still CME when quark has mass ?**

Dissipation rate — for hydrodynamical modeling D.F.Hou and S.Lin, PhysRevD.98.054014 (2018)

Modify CKT framework — non-Abel berry curvature J.W.Chen, J.Y.Pang, S.Pu and Q.Wang, PhysRevD 89, 094003 (2014)

Covariant Wigner function J.H.Gao and Z.T.Liang, 1902.06510 [hep-ph]

N.Weickgenannt, X.L.Sheng, E.Speranza, Q.Wang, D.H.Rischke, 1902.06513.

K.Hattori, Y.Hidaka, D.L.Yang, 1903.01653 [hep-ph]

Equal-time Wigner function

Z.Y.Wang, X.Y.Guo, S.Z.Shi, P.F.Zhuang, 1903.03461 [hep-ph]

Equal-time Spin Components

Covariant to Equal-time

Equal-time equations

Constraint & Transport Equation

Covariant to Equal-time

- u, d quark moving in external EM field

$$\mathcal{L} = \bar{\psi}\Big(i\gamma^\mu(\partial_\mu + ieA_\mu) - m\Big)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- Covariant equation of Wigner operator

$$\left[\gamma^\mu\left(\Pi_\mu + \frac{1}{2}i\hbar D_\mu\right) - m\right]\hat{W}_4(x,p) = 0$$

$$D_\mu(x,p) = \partial_\mu - e\int_{-1/2}^{1/2} ds F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

$$\Pi_\mu(x,p) = p_\mu - ie\hbar\int_{-1/2}^{1/2} dss F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

Covariant to equal-time

Mix between 0th & 1st moment

$\int dp_0 W(x,p)\gamma^0$ — **Off-shell** $\int dp_0\, p_0 W(x,p)\gamma^0$

Agrees only in classical limit

- Physical components are equal-time components

$f(x,\vec{p}) \to W(x,\vec{p})$

Equal-time transport equation $\int dp_0 W(x,p)\gamma^0$

Equal-time constraint equation $\int dp_0\, p_0 W(x,p)\gamma^0$

- Spin decomposition — 16 components

$$\mathcal{W} = \frac{1}{4}\left[f_0 + \gamma_5 f_1 - i\gamma_0\gamma_5 f_2 + \gamma_0 f_3 + \gamma_5\gamma_0\gamma\cdot\mathbf{g}_0 + \gamma_0\gamma\cdot\mathbf{g}_1 - i\gamma\cdot\mathbf{g}_2 - \gamma_5\gamma\cdot\mathbf{g}_3\right]$$

Semi-classical Expansion

$$
\begin{aligned}
\hbar(D_t f_0 + \mathbf{D}\cdot\mathbf{g}_1) &= 0\\
\hbar(D_t f_1 + \mathbf{D}\cdot\mathbf{g}_0) &= 0\\
\hbar(D_t \mathbf{g}_0 + \mathbf{D} f_1) - 2\mathbf{\Pi}\times\mathbf{g}_1 &= 0\\
\hbar(D_t \mathbf{g}_1 + \mathbf{D} f_0) - 2\mathbf{\Pi}\times\mathbf{g}_0 &= 0\\
\hbar D_t f_2 - 2\mathbf{\Pi}\cdot\mathbf{g}_3 &= 0\\
\hbar D_t f_3 - 2\mathbf{\Pi}\cdot\mathbf{g}_2 &= 0\\
\hbar(D_t \mathbf{g}_2 - \mathbf{D}\times\mathbf{g}_3) + 2\mathbf{\Pi} f_3 &= 0\\
\hbar(D_t \mathbf{g}_3 + \mathbf{D}\times\mathbf{g}_2) + 2\mathbf{\Pi} f_2 &= 0
\end{aligned}
$$

From 4+4
To 16+16

$$
\begin{aligned}
\int dp_0 p_0 V_0 - \mathbf{\Pi}\cdot\mathbf{g}_1 + \Pi_0 f_0 &= 0\\
\int dp_0 p_0 A_0 + \mathbf{\Pi}\cdot\mathbf{g}_0 - \Pi_0 f_1 &= 0\\
\int dp_0 p_0 \mathbf{A} + \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_1 + \mathbf{\Pi} f_1 - \Pi_0\mathbf{g}_0 &= 0\\
\int dp_0 p_0 \mathbf{V} - \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_0 + \mathbf{\Pi} f_0 - \Pi_0\mathbf{g}_1 &= 0\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_3 + \Pi_0 f_2 &= 0\\
\int dp_0 p_0 F - \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_2 + \Pi_0 f_3 &= 0\\
\int dp_0 p_0 S^{0i}\mathbf{e}_i - \frac{1}{2}\hbar\mathbf{D} f_3 + \mathbf{\Pi}\times\mathbf{g}_3 - \Pi_0\mathbf{g}_2 &= 0\\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\mathbf{e}_i - \hbar\mathbf{D} f_2 + 2\mathbf{\Pi}\times\mathbf{g}_2 + 2\Pi_0\mathbf{g}_3 &= 0
\end{aligned}
$$

- **Equal-time equations**

1) Two groups coupled by mass

2) Field strength instead of A_μ

3) Equal-time components $\{f_i, \mathbf{g}_i\}$ coupled to 1st moment $\int dp_0 p_0 \Gamma_a(x,p)$

4) Only with on-shell condition $p_0 = \pm E_p$, moments reduce to $\pm E_p\{f_i, \mathbf{g}_i\}$

5) With off-shell effect, all moments are independent, and form a hierarchy.

- **Expand all 16 constraint + 16 transport equation by order of $\hbar$**

On-shell (quasi-particle)
— Classical transport equation @ 0th

Quantum effects
— Quantum transport equation @ 0th+1st

Semi-classical Expansion

$$\begin{aligned}
\hbar(D_t f_0 + \mathbf{D}\cdot\mathbf{g}_1) &= 0\\
\hbar(D_t f_1 + \mathbf{D}\cdot\mathbf{g}_0) &= -2mf_2\\
\hbar(D_t \mathbf{g}_0 + \mathbf{D} f_1) - 2\mathbf{\Pi}\times\mathbf{g}_1 &= 0\\
\hbar(D_t \mathbf{g}_1 + \mathbf{D} f_0) - 2\mathbf{\Pi}\times\mathbf{g}_0 &= -2m\mathbf{g}_2\\
\hbar D_t f_2 - 2\mathbf{\Pi}\cdot\mathbf{g}_3 &= 2mf_1\\
\hbar D_t f_3 - 2\mathbf{\Pi}\cdot\mathbf{g}_2 &= 0\\
\hbar(D_t \mathbf{g}_2 - \mathbf{D}\times\mathbf{g}_3) + 2\mathbf{\Pi} f_3 &= 2m\mathbf{g}_1\\
\hbar(D_t \mathbf{g}_3 + \mathbf{D}\times\mathbf{g}_2) + 2\mathbf{\Pi} f_2 &= 0
\end{aligned}$$

From 4+4
To 16+16

$$\begin{aligned}
\int dp_0 p_0 V_0 - \mathbf{\Pi}\cdot\mathbf{g}_1 + \Pi_0 f_0 &= mf_3\\
\int dp_0 p_0 A_0 + \mathbf{\Pi}\cdot\mathbf{g}_0 - \Pi_0 f_1 &= 0\\
\int dp_0 p_0 \mathbf{A} + \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_1 + \mathbf{\Pi} f_1 - \Pi_0\mathbf{g}_0 &= -m\mathbf{g}_3\\
\int dp_0 p_0 \mathbf{V} - \frac{1}{2}\hbar\mathbf{D}\times\mathbf{g}_0 + \mathbf{\Pi} f_0 - \Pi_0\mathbf{g}_1 &= 0\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_3 + \Pi_0 f_2 &= 0\\
\int dp_0 p_0 F - \frac{1}{2}\hbar\mathbf{D}\cdot\mathbf{g}_2 + \Pi_0 f_3 &= mf_0\\
\int dp_0 p_0 S^{0i}\mathbf{e}_i - \frac{1}{2}\hbar\mathbf{D} f_3 + \mathbf{\Pi}\times\mathbf{g}_3 - \Pi_0\mathbf{g}_2 &= 0\\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\mathbf{e}_i - \hbar\mathbf{D} f_2 + 2\mathbf{\Pi}\times\mathbf{g}_2 + 2\Pi_0\mathbf{g}_3 &= 2m\mathbf{g}_0
\end{aligned}$$

- **Equal-time equations**

1) Two groups coupled by mass

2) Field strength instead of A_μ

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4) Only with on-shell condition $p_0 = \pm E_p$, moments reduce to $\pm E_p\{f_i, \mathbf{g}_i\}$

5) With off-shell effect, all moments are independent, and form a hierarchy.

- **Expand all 16 constraint + 16 transport equation by order of $\hbar$**

On-shell (quasi-particle)
— Classical transport equation @ 0th

Quantum effects
— Quantum transport equation @ 0th+1st

Constraint Equations

$$
\begin{aligned}
f_1^{(0)\pm} &= \pm\frac{\mathbf{p}}{E_p}\cdot\mathbf{g}_0^{(0)\pm} \\
f_2^{(0)\pm} &= 0 \\
f_3^{(0)\pm} &= \pm\frac{m}{E_p}f_0^{(0)\pm} \\
\mathbf{g}_1^{(0)\pm} &= \pm\frac{\mathbf{p}}{E_p}f_0^{(0)\pm} \\
\mathbf{g}_2^{(0)\pm} &= \frac{\mathbf{p}\times\mathbf{g}_0^{(0)\pm}}{m} \\
\mathbf{g}_3^{(0)\pm} &= \mp\frac{E_p^2\mathbf{g}_0^{(0)\pm}-(\mathbf{p}\cdot\mathbf{g}_0^{(0)\pm})\mathbf{p}}{mE_p}
\end{aligned}
$$

1). **classical**: two independent components

$f_0^{(0)}$: number density $\mathbf{g}_0^{(0)}$: spin density

Constraint Equations

$$
\begin{aligned}
f_1^{(0)\pm} &= \\
f_2^{(0)\pm} &= \\
f_3^{(0)\pm} &= \\
\mathbf{g}_1^{(0)\pm} &= \quad X\left[f_0^{(0)}, \mathbf{g}_0^{(0)}\right] \\
\mathbf{g}_2^{(0)\pm} &= \\
\mathbf{g}_3^{(0)\pm} &=
\end{aligned}
$$

1). **classical**: two independent components

$f_0^{(0)}$: number density $\quad$ $\mathbf{g}_0^{(0)}$: spin density

Constraint Equations

$$f_1^{(0)\pm} = \quad f_2^{(0)\pm} = \quad f_3^{(0)\pm} = \quad \mathbf{g}_1^{(0)\pm} = \quad \mathbf{g}_2^{(0)\pm} = \quad \mathbf{g}_3^{(0)\pm} = \quad X\left[f_0^{(0)}, \mathbf{g}_0^{(0)}\right]$$

1). **classical**: two independent components

$f_0^{(0)}$: number density $\mathbf{g}_0^{(0)}$: spin density

2). $f_0^{(1)}$ & $\mathbf{g}_0^{(1)}$ are still independent components @ **first order**

3). other components depends on $f_0^{(1)}$ & $\mathbf{g}_0^{(1)}$, $f_0^{(0)}$ & $\mathbf{g}_0^{(0)}$,

off-shell effects related to EM field

$$f_1^{(1)\pm} = \pm\frac{\mathbf{p}\cdot\mathbf{g}_0^{(1)\pm}}{E_p} \pm\frac{\mathbf{p}\cdot\mathbf{B}}{2E_p^3}f_0^{(0)\pm}$$

$$f_2^{(1)\pm} = 0 - \frac{\mathbf{D}^{(0)}\cdot\mathbf{g}_0^{(0)\pm}}{2m} + \frac{\mathbf{p}\cdot(\mathbf{p}\cdot\mathbf{D}^{(0)})\mathbf{g}_0^{(0)\pm}}{2mE_p^2} - \frac{(\mathbf{B}\times\mathbf{p})\cdot\mathbf{g}_0^{(0)\pm}}{mE_p^2} \mp \frac{\mathbf{E}\cdot\mathbf{g}_0^{(0)\pm}}{2mE_p}$$

$$f_3^{(1)\pm} = \pm\frac{mf_0^{(1)}}{E_p} \mp\frac{(\mathbf{p}\times\mathbf{D}^{(0)})\cdot\mathbf{g}_0^{(0)\pm}}{2mE_p} + \frac{\mathbf{p}\cdot(\mathbf{E}\times\mathbf{g}_0^{(0)\pm})}{2mE_p^2} \mp \frac{\mathbf{B}\cdot\mathbf{g}_0^{(0)\pm}}{2mE_p} \mp \frac{(\mathbf{B}\cdot\mathbf{p})(\mathbf{p}\cdot\mathbf{g}_0^{(0)\pm})}{2mE_p^3}$$

$$\mathbf{g}_1^{(1)\pm} = \pm\frac{\mathbf{p}}{E_p}f_0^{(1)} \pm\frac{1}{2E_p}\mathbf{D}^{(0)}\times\mathbf{g}_0^{(0)} + \frac{\mathbf{E}}{2E_p^2}\times\mathbf{g}_0^{(0)\pm} \pm \frac{\mathbf{B}(\mathbf{p}\cdot\mathbf{g}_0^{(0)\pm})}{2E_p^3}$$

$$\mathbf{g}_2^{(1)\pm} = \frac{\mathbf{p}\times\mathbf{g}_0^{(1)\pm}}{m} \pm\left(\frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{E})}{2mE_p^3} - \frac{\mathbf{E}}{2mE_p}\right)f_0^{(0)\pm} + \frac{\mathbf{p}}{2mE_p^2}\mathbf{p}\cdot\mathbf{D}^{(0)}f_0^{(0)\pm} - \frac{1}{2m}\mathbf{D}^{(0)}f_0^{(0)\pm}$$

$$\mathbf{g}_3^{(1)\pm} = \mp\frac{E_p^2\mathbf{g}_0^{(1)\pm} - (\mathbf{p}\cdot\mathbf{g}_0^{(1)\pm})\mathbf{p}}{mE_p} + \left(\frac{\mathbf{E}\times\mathbf{p}}{2mE_p^2} \mp \frac{m\mathbf{B}}{2E_p^3}\right)f_0^{(0)\pm} \mp \frac{1}{2mE_p}\mathbf{p}\times\mathbf{D}^{(0)}f_0^{(0)\pm}$$

Constraint Equations

$$f_1^{(0)\pm} = \quad f_2^{(0)\pm} = \quad f_3^{(0)\pm} = \quad \mathbf{g}_1^{(0)\pm} = \quad \mathbf{g}_2^{(0)\pm} = \quad \mathbf{g}_3^{(0)\pm} = \quad X\left[f_0^{(0)}, \mathbf{g}_0^{(0)}\right]$$

1). **classical**: two independent components

$f_0^{(0)}$: number density $\quad \mathbf{g}_0^{(0)}$: spin density

2). $f_0^{(1)}$ & $\mathbf{g}_0^{(1)}$ are still independent components @ **first order**

3). other components depends on $f_0^{(1)}$ & $\mathbf{g}_0^{(1)}$, $f_0^{(0)}$ & $\mathbf{g}_0^{(0)}$,

off-shell effects related to EM field

$$f_1^{(1)\pm} = \quad f_2^{(1)\pm} = \quad f_3^{(1)\pm} = \quad \mathbf{g}_1^{(1)\pm} = \quad \mathbf{g}_2^{(1)\pm} = \quad \mathbf{g}_3^{(1)\pm} = \quad X\left[f_0^{(1)}, \mathbf{g}_0^{(1)}\right] \quad Y\left[\mathbf{E}, \mathbf{B}, f_0^{(0)}, \mathbf{g}_0^{(0)}\right]$$

Transport Equations

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p}\cdot\mathbf{D}^{(0)}\right) f_0^{(0)\pm} = 0$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p}\cdot\mathbf{D}^{(0)}\right) \mathbf{g}_0^{(0)\pm} = \frac{1}{E_p^2}\left[\mathbf{p}\times\left(\mathbf{E}\times\mathbf{g}_0^{(0)\pm}\right) \mp E_p\mathbf{B}\times\mathbf{g}_0^{(0)\pm}\right]$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p}\cdot\mathbf{D}^{(0)}\right) f_0^{(1)\pm} = \frac{\mathbf{E}}{2E_p^2}\cdot\mathbf{D}^{(0)}\times\mathbf{g}_0^{(0)\pm} \mp \frac{1}{2E_p^3}\mathbf{B}\cdot(\mathbf{p}\cdot\mathbf{D}^{(0)})\mathbf{g}_0^{(0)\pm} + \frac{\mathbf{B}\times\mathbf{p}}{E_p^4}\cdot\mathbf{E}\times\mathbf{g}_0^{(0)\pm}$$

$$\begin{aligned}\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p}\cdot\mathbf{D}^{(0)}\right) \mathbf{g}_0^{(1)\pm} = {} & \frac{1}{E_p^2}\left[\mathbf{p}\times\left(\mathbf{E}\times\mathbf{g}_0^{(1)\pm}\right) \mp E_p\mathbf{B}\times\mathbf{g}_0^{(1)\pm}\right] \\ & \mp\left(\frac{\mathbf{B}}{2E_p^3} \pm \frac{\mathbf{E}\times\mathbf{p}}{2E_p^4}\right)\mathbf{p}\cdot\mathbf{D}^{(0)} f_0^{(0)\pm} \mp \left(\frac{(\mathbf{p}\cdot\mathbf{E})(\mathbf{E}\times\mathbf{p})}{E_p^5} \pm \frac{\mathbf{p}\times(\mathbf{B}\times\mathbf{E})}{2E_p^4}\right) f_0^{(0)\pm}\end{aligned}$$

- **Classical: decoupled** — No self-interaction — collisionless

  Effective collision terms from spin interaction with EM field
- **Quantum level: coupled** — Fist order part — same structure

  Effective collision terms from spin interaction with EM field

  Number density modified by spin interaction with EM
- **Can be solved order by order**

Transport Equations

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) f_0^{(0)\pm} = 0$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) \mathbf{g}_0^{(0)\pm} = I\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(0)\pm}\right]$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) f_0^{(1)\pm} = 0 + J\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(0)\pm}\right]$$

$$\left(D_t^{(0)} \pm \frac{\mathbf{p}}{E_p} \cdot \mathbf{D}^{(0)}\right) \mathbf{g}_0^{(1)\pm} = I\left[\mathbf{E}, \mathbf{B}, \mathbf{g}_0^{(1)\pm}\right] + K\left[\mathbf{E}, \mathbf{B}, f_0^{(0)\pm}\right]$$

- **Classical: decoupled** — No self-interaction — collisionless
  Effective collision terms from spin interaction with EM field
- **Quantum level: coupled** — Fist order part — same structure
  Effective collision terms from spin interaction with EM field
  Number density modified by spin interaction with EM
- **Can be solved order by order**

Chiral Components

Chiral component

Solution for CME

Chiral components

- **m=0: Axial and Vector still coupled!** Decouple by introducing chiral component!

$$J^\mu_\chi = \frac{1}{2}(V^\mu - \chi A^\mu)$$

$$f_\chi = f_0 + \chi f_1$$
$$\vec{g}_\chi = \vec{g}_1 + \chi \vec{g}_0 = G[f_\chi]$$

- **m: All coupled!** Yet still introduce the same combination.

$$\vec{g}_\chi = \vec{g}_1 + \chi \vec{g}_0 = F[f_\chi, m\vec{g}_3]$$

- **Small m limit:** Taylor expansion, keep to the first order of m

$$\partial_t f^\pm_\chi + \dot{\mathbf{x}} \cdot \nabla f^\pm_\chi + \dot{\mathbf{p}} \cdot \nabla_p f^\pm_\chi = \chi m \frac{F_1[\mathbf{E}, \mathbf{g}_3^\pm]}{\sqrt{G}} + \hbar m \frac{F_2[\mathbf{E}, \mathbf{B}, \mathbf{g}_3^{(0)\pm}]}{\sqrt{G}}$$

$$\dot{\vec{x}} = \frac{1}{\sqrt{G}}\left(\vec{v}_p + \hbar(\vec{v}_p \cdot \vec{b})\vec{B} + \hbar \vec{E} \times \vec{b}\right)$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}}\left(\vec{v}_p \times \vec{B} + \vec{E} + \hbar(\vec{E} \cdot \vec{B})\vec{b}\right)$$

$$\vec{b} = \chi \frac{\hat{p}}{2p^2}$$

1. Same structure + effective collision (m, E, B, spin)
2. E : mass effect @ classical + quantum
3. **B : mass effect @ quantum** **solvable**

CME & Solution

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = m\hbar \frac{1}{2\sqrt{G} p^4} (\vec{p} \cdot \nabla)(\vec{B} \cdot \vec{g}_3^{(0)\pm})$$

EoM

$$\dot{\vec{x}} = \frac{\hat{p}}{\sqrt{G}} \left(1 + 2Q\hbar \vec{b} \cdot \vec{B}\right)$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}} Q\, \hat{p} \times \vec{B}$$

Berry curvature $\vec{b} = \chi \frac{\hat{p}}{2p^2}$

Phase space factor $\sqrt{G} = 1 + Q\hbar \vec{b} \cdot \vec{B}$

- **Small mass (high T):** same structure + effective collisions
- **Analytical solvable:** first solve collision, then transport
- **Mass correction is small — quantum level**

$$\frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm(\vec{x}, \vec{p}, t)$$

1st order, inhomogeneous

CME & Solution

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = \frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm(\vec{x}, \vec{p}, t)$$

EoM

$$\dot{\vec{x}} = \frac{\hat{p}}{\sqrt{G}} \left(1 + 2Q\hbar\vec{b} \cdot \vec{B}\right)$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}} Q\, \hat{p} \times \vec{B}$$

Berry curvature $\vec{b} = \chi \frac{\hat{p}}{2p^2}$

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$$\frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm(\vec{x}, \vec{p}, t)$$

1st order, inhomogeneous

SUMMARY

1. Finite mass: Transport equation of spin components

   Quantum off-shell effects

   Numerically solvable

2. Small mass: Mass correction to CME is small

   Analyticalyl solvable

On going: solving mass in transport equation…

Mass Correction to CKT

Thank you !

2019.04.08