Relativistic Quantum Kinetic Theory for Massive Fermions & Spin Effects

Jian-Hua Gao (高建华)

Shandong University at Weihai

ArXiv:1902.06510 JHG and Z.T. Liang

The 5th Workshop on Chirality, Vorticity and Magnetic field in heavy ion collisions

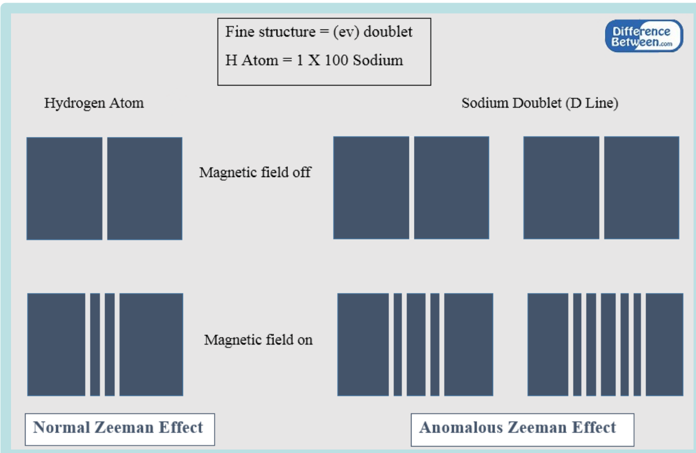
Tsinghua University， Beijing， China， April 8 – April 12， 2019

Outline

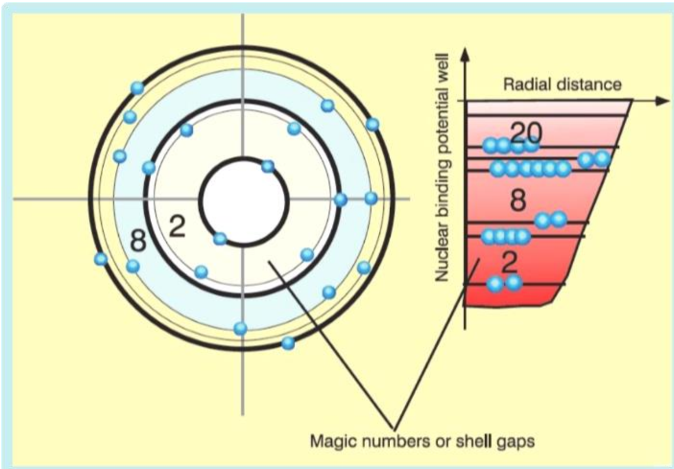
- Introduction
- Relativistic Quantum Kinetic Theory for Massive Fermions
- Spin Effects from Relativistic Quantum Kinetic Theory
- Summary

Spin in Physics

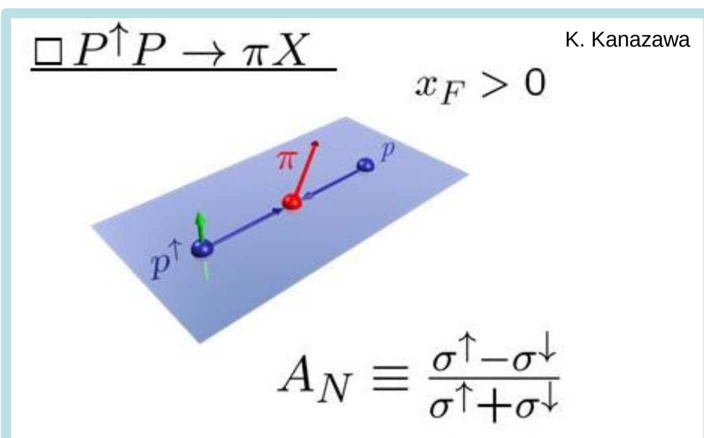
Anomalous Zeeman Effect



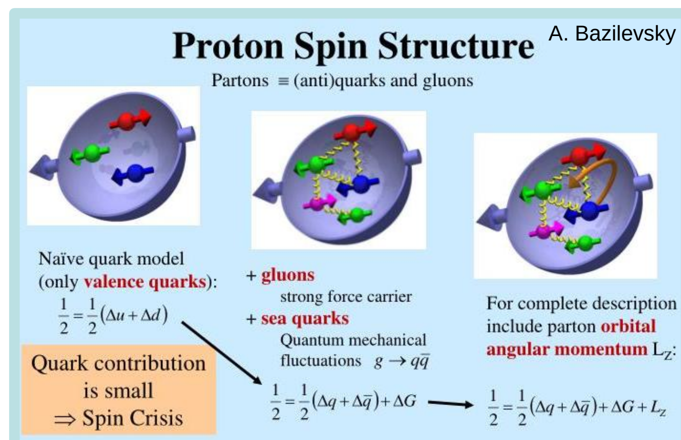
Nuclear Shell Model



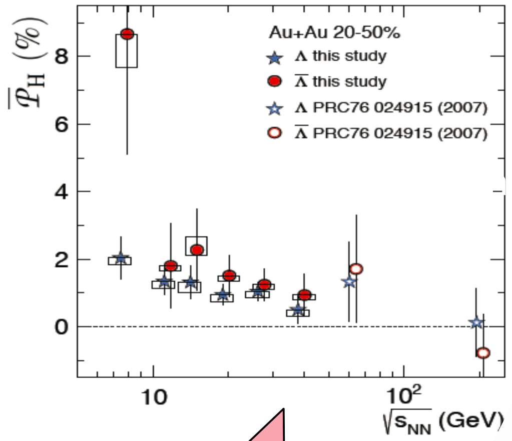
Single Spin Asymmetry



Proton Spin Crisis

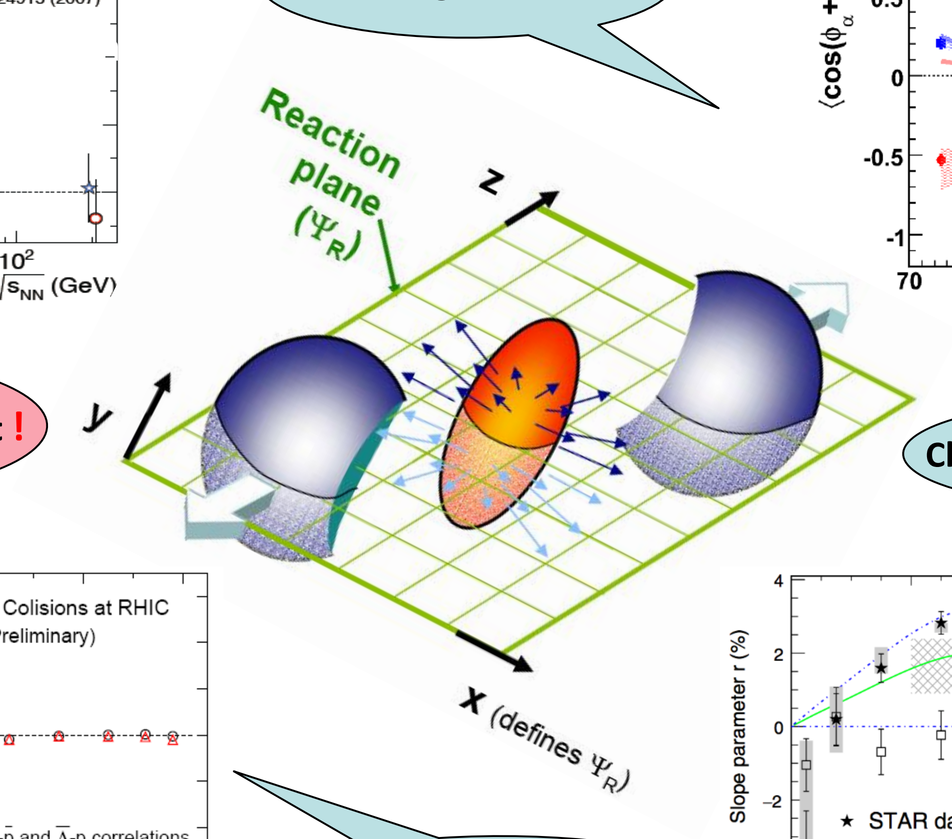


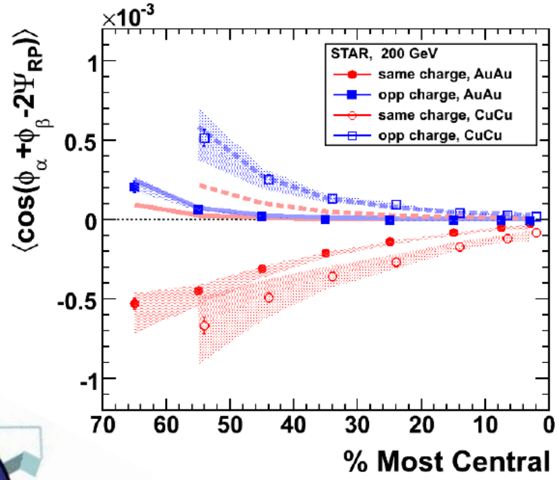
Spin in Relativistic HIC



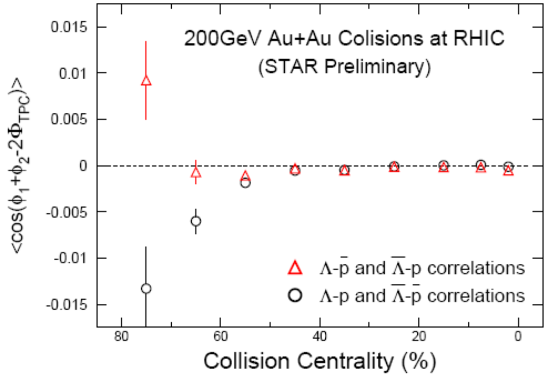
Global Polarization Effect !

Chiral Magnetic Effect ?

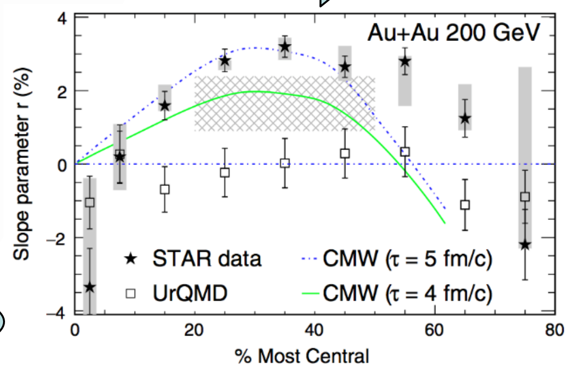




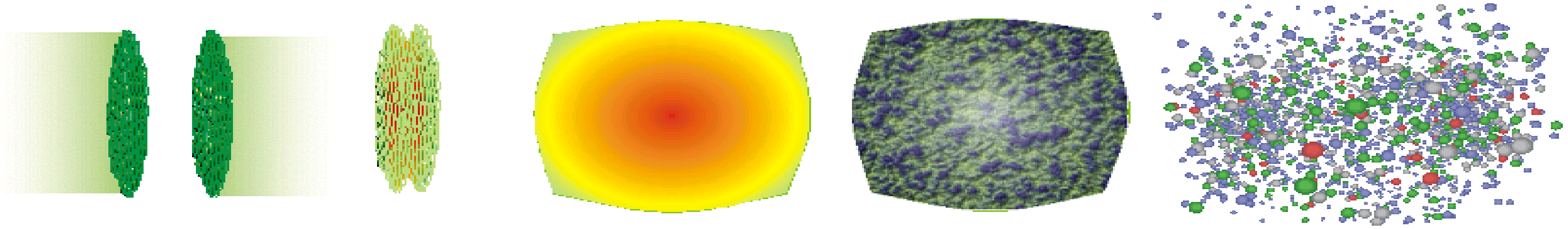
Chiral Magnetic Wave ?



Chiral Vortical Effect ?



Spin Evolution - Massless



- **Anomalous Hydrodynamics:**

Son & Surowka PRL2009; Kharzeev & Yee PRD2011; Yee & Yin PRC2014; Yin & Liao PLB 2016; Hongo, Y.Hirono & T. Hirono PLB2017; Gorbar, Rybalka & Shovkovy PRD2017 … … … …

- **Chiral Kinetic Theory:**

Stephanov & Yin PRL2012; Chen, Pu, Q. Wang & X. Wang PRL2013, Son & Yamamoto PRD2013; Manuel & Torres-Rincon PRD2014; Chen, Son & Stephanov PRL2015; Hidaka,Pu & Yang PRD 2017; Mueller & Venugopalan PRD2018; Huang, Shi, Jiang, Liao & Zhuang PRD2018; Gao, Liang, Q.Wang & X.Wang PRD2018; Lin & Shukla 1901.01528 … … … …

- **Relevant talks in this meeting:**

Apr. 8 AM: C. Ko; S. Shi

Apr. 9 AM: N. Mueller；Y. Yin

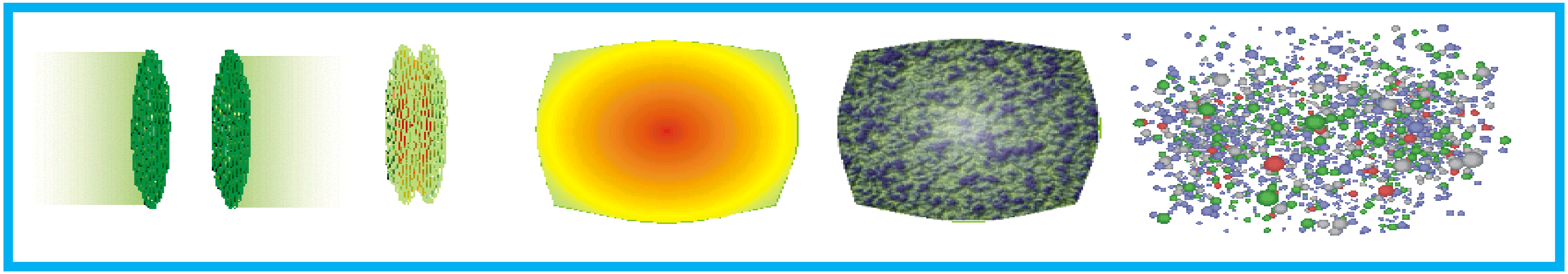
Apr.11 AM: S. Pu

Apr. 8 PM: X. Huang; Y. Liu, A. K. Shukla

Apr. 10 PM: J. Xu

Apr. 11 PM: Shu Lin

Spin Evolution - Massive



- Hydrodynamics with Spin:

Florkowski, Friman, Jaiswal & Speranza PRC2018; Becattini & Karpendo PRL2018;
Florkowski, Speranza & Becattini APPB2018;Becattini, Florkowski & Speranza PLB2019;
Hattori, Hongo, Huang, Matsuo & Taya 1901.06615 … … … …

- Kinetic Theory with Spin:

Vasak, Gyulassy, Elze AP1987; Zhuang, Heinz AP1996; Fang, Pang, Q.Wang & X. Wang PRC2016;
Florkowski, Kumar, Ryblewski PRC2018; Weickgenannt, Sheng, Speranza & Wang 1902.06513;
Gao & Liang 1902.06510; Hattori, Hidaka & Yang 1903.01653; Wang, Guo, Shi & Zhuang 1903.03461 … … … …

- Relevant talks in this meeting:

Apr. 8 PM: A. Huang; Z. Wang; X. Sheng Apr. 9 AM: Y. Yin, L. Yang;
Apr. 11 AM: R. Ryblewski; H. Taya Apr. 12 AM: E. Speranza; A.Pazos; A. Kumar; M. Matsuo

Wigner functions

Wigner matrix elements for spin-1/2 fermion in Abelian gauge field:

$$\boxed{W_{\alpha\beta}(x,p) = \left\langle : \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta\left(x+\frac{y}{2}\right) \underbrace{U\left(x+\frac{y}{2}, x-\frac{y}{2}\right)}_{\text{gauge link}} \psi_\alpha\left(x-\frac{y}{2}\right) : \right\rangle}$$

16 independent Wigner functions:

$$\boxed{W = \frac{1}{4}\left[\underbrace{\mathscr{F}}_{\text{scalar}} + i\gamma^5 \underbrace{\mathscr{P}}_{\text{pseudo}} + \gamma^\mu \underbrace{\mathscr{V}_\mu}_{\text{vector}} + \gamma^5\gamma^\mu \underbrace{\mathscr{A}_\mu}_{\text{axial}} + \frac{1}{2}\sigma^{\mu\nu} \underbrace{\mathscr{S}_{\mu\nu}}_{\text{tensor}}\right]}$$

Physical meaning of different Wigner functions:

- $\mathscr{F}$ Mass density, particle number distribution function
- $\mathscr{A}_\mu$ space components: spin density, spin polarization vector
- $\mathscr{V}_\mu$ Charge density and current density, current vector
- $\mathscr{S}_{\mu\nu}$ space components: magnetic moment density
- $\mathscr{P}$ Pseudo scalar density

Vasak, Gyulassy, Elze, Annals Phys. 1987; Bialynicki-Birula, Gornicki, Rafelski PRD 1991

Wigner equations

Wigner equations in background field at $O(\hbar)$: $\nabla^\mu \equiv \partial_x^\mu - F^{\mu\nu}\partial_\nu^p$

16 Wigner functions

32 Wigner equations

$$
\begin{aligned}
\nabla^\mu \mathscr{V}_\mu &= 0 & m\mathscr{F} &= p^\mu \mathscr{V}_\mu \\
p^\mu \mathscr{A}_\mu &= 0 & m\mathscr{P} &= -\frac{1}{2}\nabla^\mu \mathscr{A}_\mu \\
\frac{1}{2}\nabla_\mu \mathscr{F} - p^\nu \mathscr{S}_{\mu\nu} &= 0 & m\mathscr{V}_\mu &= p_\mu \mathscr{F} + \frac{1}{2}\nabla^\nu \mathscr{S}_{\mu\nu} \\
p_\mu \mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu \mathscr{S}^{\rho\sigma} &= 0 & m\mathscr{A}_\mu &= \frac{1}{2}\nabla_\mu \mathscr{P} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}p^\nu \mathscr{S}^{\rho\sigma} \\
(p_\mu \mathscr{V}_\nu - p_\nu \mathscr{V}_\mu) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho \mathscr{A}^\sigma &= 0 & m\mathscr{S}_{\mu\nu} &= \frac{1}{2}(\nabla_\mu \mathscr{V}_\nu - \nabla_\nu \mathscr{V}_\mu) - \epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma
\end{aligned}
$$

Choose $\mathscr{F}$ and $\mathscr{A}^\mu$ as the independent fundamental components

Eleven of 32 provide the expressions of other components:

$$
\begin{aligned}
\mathscr{P} &= -\frac{\hbar}{2m}\nabla^\mu \mathscr{A}_\mu \\
\mathscr{V}_\mu &= \frac{1}{m}p_\mu \mathscr{F} - \frac{\hbar}{2m^2}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu p^\rho \mathscr{A}^\sigma \\
\mathscr{S}_{\mu\nu} &= -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma + \frac{\hbar}{2m^2}(\nabla_\mu p_\nu - \nabla_\nu p_\mu)\mathscr{F}
\end{aligned}
$$

Vasak, Gyulassy, Elze Annals Phys. 1987； ArXiv:1902.06510 JHG and Liang

Transport or Constraint Equations

Five of 32 lead to coupled transport equation for $\mathscr{F}$ and $\mathscr{A}^\mu$:

$$\boxed{\begin{aligned} p\cdot\nabla\mathscr{F} &= \frac{\hbar}{2m}p^\mu(\partial^x_\lambda\tilde{F}_{\mu\nu})\partial^\lambda_p\mathscr{A}^\nu \\ p\cdot\nabla\mathscr{A}_\mu &= F_{\mu\nu}\mathscr{A}^\nu + \frac{\hbar}{2m}p^\nu(\partial^x_\lambda\tilde{F}_{\mu\nu})\partial^\lambda_p\mathscr{F} \end{aligned}}$$

Five of 32 modify on-shell conditions :

$$\boxed{\begin{aligned} \left(p^2-m^2\right)\mathscr{F} &= -\frac{\hbar}{m}p^\mu\tilde{F}_{\mu\nu}\mathscr{A}^\nu \\ \left(p^2-m^2\right)\mathscr{A}_\mu &= -\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathscr{F} \end{aligned}}$$

One of 32 provide a subsidiary condition: $\boxed{p_\mu\mathscr{A}^\mu = 0}$

All the rest **10** of the 32 Wigner equations are satisfied automatically !

4 independent Wigner functions, **1** is $\mathscr{F}$, **3** are from 4-vector $\mathscr{A}^\mu$	**satisfy**	**4** on-shell conditions **4** transport equations

Vasak, Gyulassy, Elze Annals Phys. 1987； ArXiv:1902.06510 JHG and Liang

Solve the constraint equations

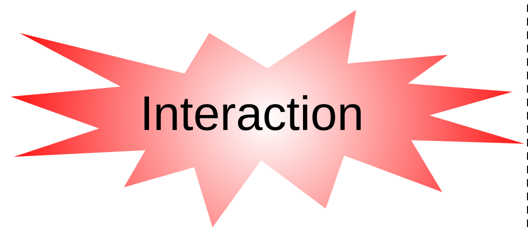
Solve the modified on-shell conditions :

$$\boxed{\begin{aligned}\left(p^2-m^2\right)\mathscr{F} &= -\frac{\hbar}{m}p^{\mu}\tilde{F}_{\mu\nu}\mathscr{A}^{\nu},\\ \left(p^2-m^2\right)\mathscr{A}_{\mu} &= -\frac{\hbar}{m}p^{\nu}\tilde{F}_{\mu\nu}\mathscr{F},\end{aligned}} \Longrightarrow \boxed{\begin{aligned}\mathscr{F} &= \delta\left(p^2-m^2\right)\mathcal{F}+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^{\mu}\mathcal{A}^{\nu}\delta'\left(p^2-m^2\right)\\ \mathscr{A}_{\mu} &= \delta\left(p^2-m^2\right)\mathcal{A}_{\mu}+\frac{\hbar}{m}p^{\nu}\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\end{aligned}}$$

Introduce $\mathcal{F}$ and $\mathcal{A}^{\mu}$ as new independent Wigner functions.

Convenient to deal with transient EM field:

$$\begin{aligned}\mathscr{F}^{i} &= \delta\left(p^2-m^2\right)\mathcal{F}^{i}\\ \mathscr{A}^{i}_{\mu} &= \delta\left(p^2-m^2\right)\mathcal{A}^{i}_{\mu}\end{aligned}$$



$$\begin{aligned}\mathscr{F}^{f} &= \delta\left(p^2-m^2\right)\mathcal{F}^{f}\\ \mathscr{A}^{f}_{\mu} &= \delta\left(p^2-m^2\right)\mathcal{A}^{f}_{\mu}\end{aligned}$$

$t < t_i, \quad F^{\mu\nu} = 0$ $\qquad$ $t_i < t < t_f, \quad F^{\mu\nu} \neq 0$ $\qquad$ $t > t_f, \quad F^{\mu\nu} = 0$

$t = t_i$ $\qquad$ $t = t_f$

Unintegrated Kinetic Equations

Substitute the solution into the transport equations

$$\boxed{\begin{aligned}\mathscr{F} &= \delta\left(p^2-m^2\right)\mathcal{F}+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^{\mu}\mathcal{A}^{\nu}\delta'\left(p^2-m^2\right)\\ \mathscr{A}_{\mu} &= \delta\left(p^2-m^2\right)\mathcal{A}_{\mu}+\frac{\hbar}{m}p^{\nu}\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\end{aligned}}$$

$$\boxed{\begin{aligned}p\cdot\nabla\mathscr{F} &= \frac{\hbar}{2m}p^{\mu}(\partial_{\lambda}^{x}\tilde{F}_{\mu\nu})\partial_{p}^{\lambda}\mathscr{A}^{\nu},\\ p\cdot\nabla\mathscr{A}_{\mu} &= F_{\mu\nu}\mathscr{A}^{\nu}+\frac{\hbar}{2m}p^{\nu}(\partial_{\lambda}^{x}\tilde{F}_{\mu\nu})\partial_{p}^{\lambda}\mathscr{F}.\end{aligned}}$$

$$\begin{aligned}p\cdot\nabla\left[\mathcal{F}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^{\mu}\mathcal{A}^{\nu}\delta'\left(p^2-m^2\right)\right] &= \frac{\hbar}{2m}(\partial_{\lambda}^{x}\tilde{F}_{\mu\nu})\partial_{p}^{\lambda}\left[p^{\mu}\mathcal{A}^{\nu}\delta\left(p^2-m^2\right)\right],\\ p\cdot\nabla\left[\mathcal{A}_{\mu}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p^{\nu}\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)\right] &= F_{\mu\nu}\left[\mathcal{A}^{\nu}\delta\left(p^2-m^2\right)+\frac{\hbar}{m}p_{\lambda}\tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2-m^2\right)\right]\\ &\quad+\frac{\hbar}{2m}(\partial_{\lambda}^{x}\tilde{F}_{\mu\nu})\partial_{p}^{\lambda}\left[p^{\nu}\mathcal{F}\delta\left(p^2-m^2\right)\right].\end{aligned}$$

$$\boxed{p_{\mu}\mathcal{A}^{\mu}\,\delta\left(p^2-m^2\right) = 0}$$

Unintegrated kinetic equations:

Manifest Lorentz Covariance !

Singular Dirac delta function !

Integrated Kinetic Equations

Particle： integrate p_0 from 0 to $+\infty$ $\overline{\text{Particle}}$： integrate p_0 from $-\infty$ to 0

Integrated kinetic equations in **4**-vector form:

$$\boxed{\begin{aligned} p\cdot\nabla\mathcal{F} &= -\frac{\hbar\, p^\mu}{2mE_p^2}\left[\tilde{F}_{\mu\nu}\bar{p}^\lambda\nabla_\lambda - E_p^2(\bar{\partial}_x^\lambda\tilde{F}_{\mu\nu})\bar{\partial}_\lambda^p\right]\mathcal{A}^\nu \\ p\cdot\nabla\mathcal{A}_\mu &= F_{\mu\nu}\mathcal{A}^\nu - \frac{\hbar\, p^\nu}{2mE_p^2}\left[\tilde{F}_{\mu\nu}\bar{p}^\lambda\nabla_\lambda - E_p^2(\bar{\partial}_x^\lambda\tilde{F}_{\mu\nu})\bar{\partial}_\lambda^p\right]\mathcal{F} \\ p^\mu\cdot\mathcal{A}_\mu &= 0 \end{aligned}}$$

$$\begin{aligned} p &= (E_p,\vec{p}) & \bar{p} &= (0,\vec{p}) \\ \bar{\partial}_\mu^x &= (0,\vec{\nabla}_x) & \bar{\partial}_\mu^p &= (0,\vec{\nabla}_p) \\ \nabla^\mu &= \nabla_x^\mu - F^{\mu\nu}\bar{\partial}_\nu^p \\ E_p &= \sqrt{\vec{p}^2+m^2} \end{aligned}$$

Integrated kinetic equations in **3**-vector form:

$$\boxed{\begin{aligned} \left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\mathcal{F} &= -\frac{\hbar}{2mE_p}\left[(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)-(\vec{B}\cdot\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\vec{v}\right]\cdot\vec{\mathcal{A}} \\ \left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\vec{\mathcal{A}} &= \vec{B}\times\vec{\mathcal{A}}-\vec{E}(\vec{v}\cdot\vec{\mathcal{A}})-\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F} \end{aligned}}$$

$$\vec{v}=\vec{p}/E_p,\quad \nabla_t=\partial_t+\vec{E}\cdot\vec{\nabla}_p,\quad \vec{\nabla}=\vec{\nabla}_x+\vec{B}\times\vec{\nabla}_p,$$

$$\boxed{\mathcal{A}^0=\vec{v}\cdot\vec{\mathcal{A}}}$$

Simplified Version

Vector current and energy-momentum tensor at $O(\hbar)$:

$$j^\mu = \int d^4p\, \mathscr{V}^\mu, \qquad T^{\mu\nu} = \int d^4p\, p^\nu \mathscr{V}^\mu$$

$$\mathscr{V}_\mu = \frac{1}{m} p_\mu \mathscr{F} - \frac{\hbar}{2m^2} \epsilon_{\mu\nu\rho\sigma} \nabla^\nu p^\rho \mathscr{A}^\sigma,$$

Covariant unintegrated kinetic equations for $\mathcal{A}^\mu$ at $O(1)$:

$$p \cdot \nabla \left[\mathcal{A}_\mu \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \cancel{p^\nu \tilde{F}_{\mu\nu} \mathcal{F} \delta'\left(p^2 - m^2\right)} \right] = F_{\mu\nu} \left[\mathcal{A}^\nu \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \cancel{p_\lambda \tilde{F}^{\nu\lambda} \mathcal{F} \delta'\left(p^2 - m^2\right)} \right] + \frac{\hbar}{2m} (\partial^x_\lambda \cancel{\tilde{F}_{\mu\nu}) \partial^\lambda_p \left[p^\nu \mathcal{F} \delta\left(p^2 - m^2\right) \right]}.$$

$$p \cdot \nabla \left[\mathcal{A}_\mu \delta\left(p^2 - m^2\right) \right] = F_{\mu\nu} \mathcal{A}^\nu \delta\left(p^2 - m^2\right)$$

+

$$p_\mu \mathcal{A}^\mu \delta\left(p^2 - m^2\right) = 0$$

Inserting the solved $\mathcal{A}^\mu$ into the transport equation for $\mathcal{F}$

$$p \cdot \nabla \left[\mathcal{F} \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \tilde{F}_{\mu\nu} p^\mu \mathcal{A}^\nu \delta'\left(p^2 - m^2\right) \right] = \frac{\hbar}{2m} (\partial^x_\lambda \tilde{F}_{\mu\nu}) \partial^\lambda_p \left[p^\mu \mathcal{A}^\nu \delta\left(p^2 - m^2\right) \right]$$

Simplified Version

Define:

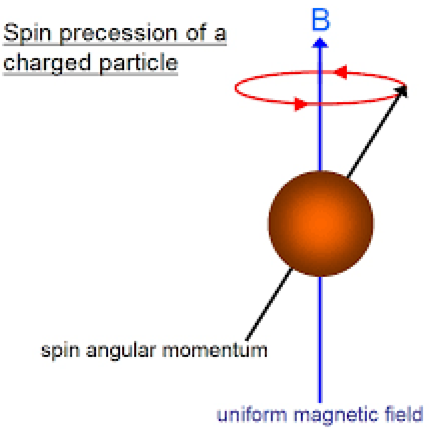
$$\frac{\mathcal{A}_\mu}{\mathcal{F}} = P s_\mu \qquad s^2 = -1, \quad p \cdot s = 0$$

P : Spin polarization magnitude

s^μ : Spin polarization direction

Decoupled equations for P and s_μ :

$$p \cdot \nabla \left[P \delta \left(p^2 - m^2 \right) \right] = 0,$$
$$p \cdot \nabla \left[s_\mu \delta \left(p^2 - m^2 \right) \right] = F_{\mu\nu} s^\nu \delta \left(p^2 - m^2 \right).$$



Rewrite the transport equations for $\mathcal{F}$ as:

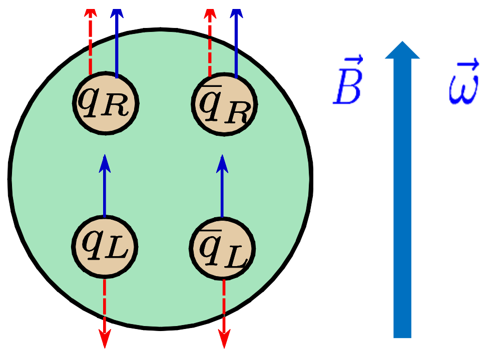
$$p \cdot \nabla \left[\mathcal{F} \delta \left(p^2 - m^2 - 2E_p \Delta E \right) \right] = \frac{\hbar}{2m} (\partial_\lambda^x \tilde{F}^{\rho\sigma}) \partial_p^\lambda \left[p_\rho s_\sigma P \mathcal{F} \delta \left(p^2 - m^2 - 2E_p \Delta E \right) \right]$$

The effective interaction energy:

$$\Delta E = -\frac{\hbar P}{2mE_p} \tilde{F}^{\rho\sigma} p_\rho s_\sigma$$

CSE with mass correction

Chiral separation effect :



$$j_5^\mu = \frac{\mu}{2\pi^2}B^\mu + \frac{1}{2\pi^2}\left(\frac{\pi^2 T^2}{3} + \mu^2 + \mu_5^2\right)\omega^\mu$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$

$$\mathcal{A}_\mu = 0, \quad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

$\beta_\mu = u_\mu / T$

$\Omega_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu$

$$\mathscr{A}_\mu = \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right) \quad \rightarrow \quad j_5^\mu = \int d^4 p \mathscr{A}^\mu = \sigma B^\mu$$

$$\rightarrow \quad \sigma = \frac{\hbar}{2\pi^2}\int_0^\infty dp\left(n_+ - n_-\right), \quad n_\pm = \frac{1}{e^{(E_p\mp\mu)/T}+1}$$

Lin & Yang PRD2018

Chiral limit: $\sigma|_{m=0} = \frac{\hbar\mu}{2\pi^2}$ Zero temperature limit: $\sigma|_{T\to 0} = \frac{\hbar\sqrt{\mu^2 - m^2}}{2\pi^2}$

Quantum magnetization effect

Wigner function associated to spin magnetic moment density:

$$\mathscr{S}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^{\rho}\mathscr{A}^{\sigma} + \frac{\hbar}{2m^2}\left(\nabla_{\mu}p_{\nu} - \nabla_{\nu}p_{\mu}\right)\mathscr{F}$$

Spin magnetic moment vector:

$$M_{\mu} = \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}u^{\nu}\int d^4p\,\mathscr{S}^{\alpha\beta}$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$:

$$\mathcal{A}_{\mu} = 0 \qquad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

Quantum magnetization effect

$$M_{\mu} = \hbar\kappa B_{\mu} - \frac{\hbar\rho}{m}\omega_{\mu}$$

Susceptibility:

$$\kappa = \frac{m}{2\pi^2}\int\frac{dp}{E_p}\left(n_{+} + n_{-}\right)$$

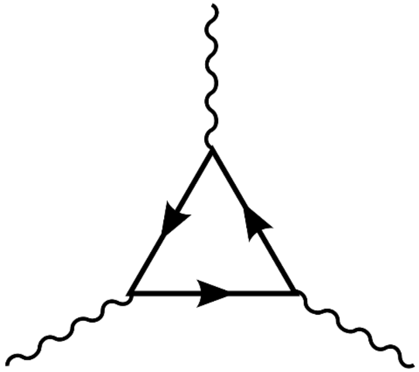
Charge density:

$$\rho = \frac{1}{\pi^2}\int dp\,p^2\left(n_{+} - n_{-}\right)$$

Chiral Anomaly

Chiral anomaly:

$$\boxed{\partial_\mu j_5^\mu = -2m j_5 - \frac{e^2}{2\pi^2} E\cdot B}$$



Pseudo scalar Wigner function:

$$\boxed{\mathscr{P} = -\frac{\hbar}{2m}\nabla^\mu \mathscr{A}_\mu}$$

$$\boxed{\mathscr{A}_\mu = \delta\left(p^2 - m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m} p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)}$$

Integrate over momentum:

$$\boxed{j_5^\mu = \int d^4 p \mathscr{A}^\mu \qquad j_5 = \int d^4 p \mathscr{P}}$$

$$\boxed{\partial_\mu j_5^\mu = -\frac{2m}{\hbar} j_5 + \hbar C E\cdot B}$$

$$\boxed{C = \frac{1}{2m}\int d^4 p\, \partial^\lambda \left[\mathcal{F}\partial_\lambda \delta(p^2-m^2)\right]}$$

Two specific solutions

Free vacuum solution :

$$\boxed{\mathcal{A}^{\mu} = 0, \qquad \mathcal{F} = \frac{m}{4\pi^3}}$$

No suppression at $p \to \infty$ and large momentum dominates

Global equilibrium solution at chiral limit:

$$\boxed{\mathcal{A}_{\mu} = 0 \qquad \frac{\mathcal{F}}{m} = \frac{1}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]}$$

$e^{-p/T}$ suppression at $p \to \infty$ and small momentum dominates

$$\boxed{\partial_{\mu} j_5^{\mu} = -2mj_5 - \frac{e^2}{2\pi^2} E\cdot B}$$

More general solution: $\begin{cases} \text{more subtle regularization} \\ \text{more tricky integration} \end{cases}$

Global Polarization Generation

Transient EM field process:

$$\mathscr{A}_\mu = \delta\left(p^2-m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)$$

$$\mathscr{F}^i = \delta\left(p^2-m^2\right)\mathcal{F}^i \neq 0$$

$$\mathscr{A}^i_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^i_\mu = 0$$

Interaction

$$\mathscr{A}^f_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^f_\mu = ?$$

$t < t_i, \quad F^{\mu\nu} = 0$ | $t_i < t < t_f, \quad F^{\mu\nu} \neq 0$ | $t > t_f, \quad F^{\mu\nu} = 0$

$t = t_i$ | $t = t_f$

Evolution equation for spin polarization vector up to $O(1)$:

$$\boxed{\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{A} = \vec{B}\times\vec{A} - \vec{E}(\vec{v}\cdot\vec{A})} \xrightarrow[\vec{A}(t_i)=0]{\text{near } t_i} \boxed{\frac{\partial\vec{A}}{\partial t} = 0}$$

No way to generate the polarization from a zero initial value !

Global Polarization Generation

Evolution equation for spin vector up to $O(\hbar)$:

$$\boxed{\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{A} = \vec{B}\times\vec{A} - \vec{E}(\vec{v}\cdot\vec{A}) - \frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}}$$

near t_i $\Downarrow$ $\vec{A}(t_i) = 0$

$$\boxed{\frac{\partial\vec{A}}{\partial t} = -\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}(t_i)}$$

Polarization seed: EM field + inhomogeneous $\mathcal{F}(t_i)$

Turn off the background EM field: $\boxed{\frac{\partial\vec{A}}{\partial t} = 0 \text{ ?}}$

Self-consistent background EM field: Vasak, Gyulassy, Elze, Annals Phys. 1987

$$\boxed{\partial_\mu F^{\mu\nu} = j^\nu} \longrightarrow \boxed{\partial_\lambda\partial^\lambda F_{\mu\nu} = (\partial_\mu j_\nu - \partial_\nu j_\mu)}$$

Global polarization: **Vorticity → EM field → polarization**

Production by scattering : Q. Wang's Talk

Summary

- Relativistic quantum kinetic theory for particle with spin-1/2 up to first order in $\hbar$ is derived from the Wigner function formalism.

- Different spin effects such as chiral separate effect, quantum magnetization effect, chiral anomaly, and global polarization can arise from it automatically.

Thanks for your attention!