Chiral kinetic study of magnetic and vortical effects in HIC

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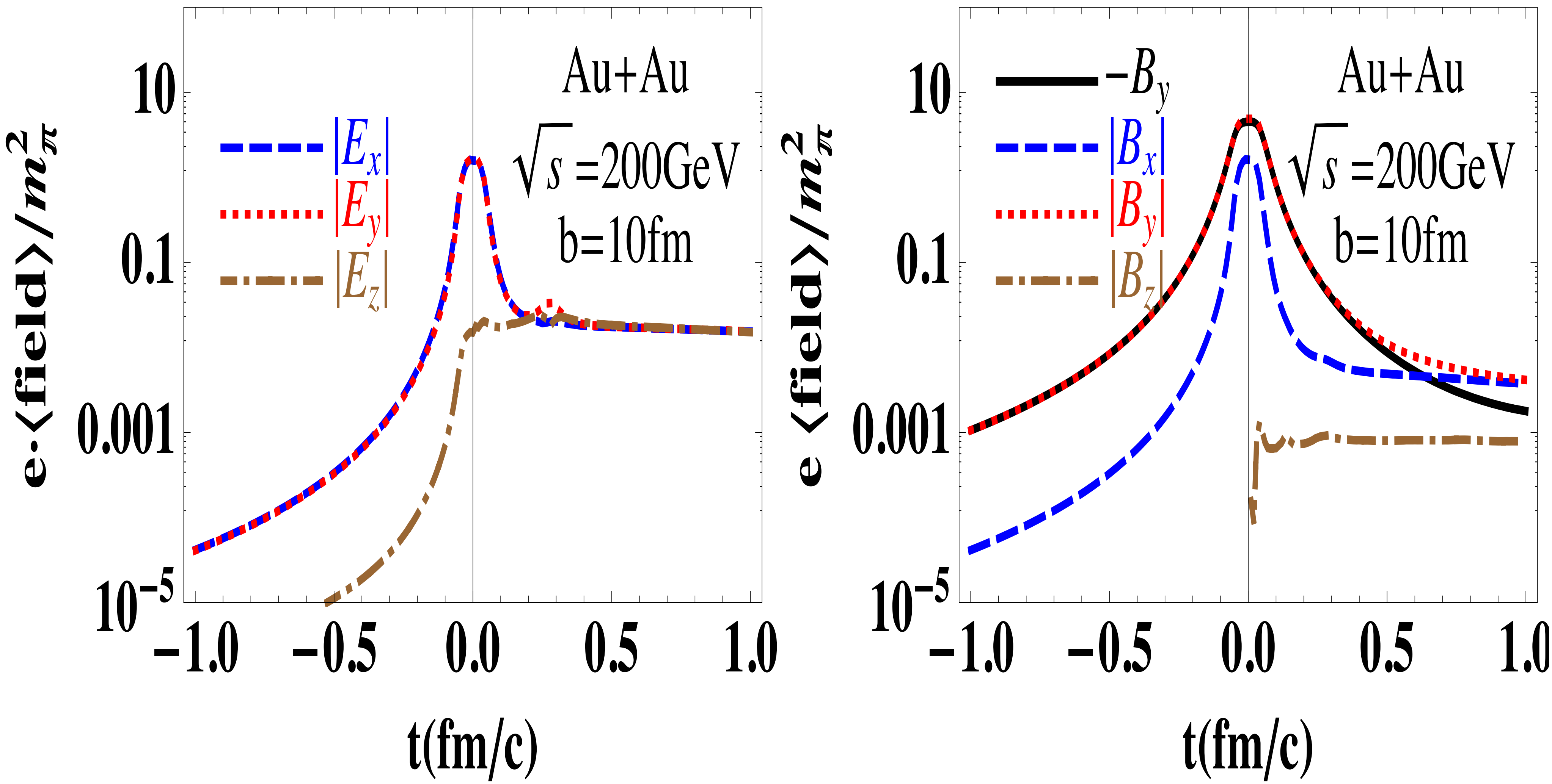
- ❑ Introduction
- ❑ Chiral transport model
- ❑ Chiral magnetic effect in isobaric collisions
- ❑ Spin polarization
- ❑ Chiral vortical effect
- ❑ Summary

Based on work [PRC 96, 024906 (2017); 98, 014911 (2018); 99, 011903(R) (2019); PLB 789, 228 (2019)] in collaboration with **Yifeng Sun**

Supported by US Department of Energy and the Welch Foundation

Introduction: Electromagnetic field in relativistic HIC

W. T. Deng and X. G. Huang, PRC 85, 044907 (2012)



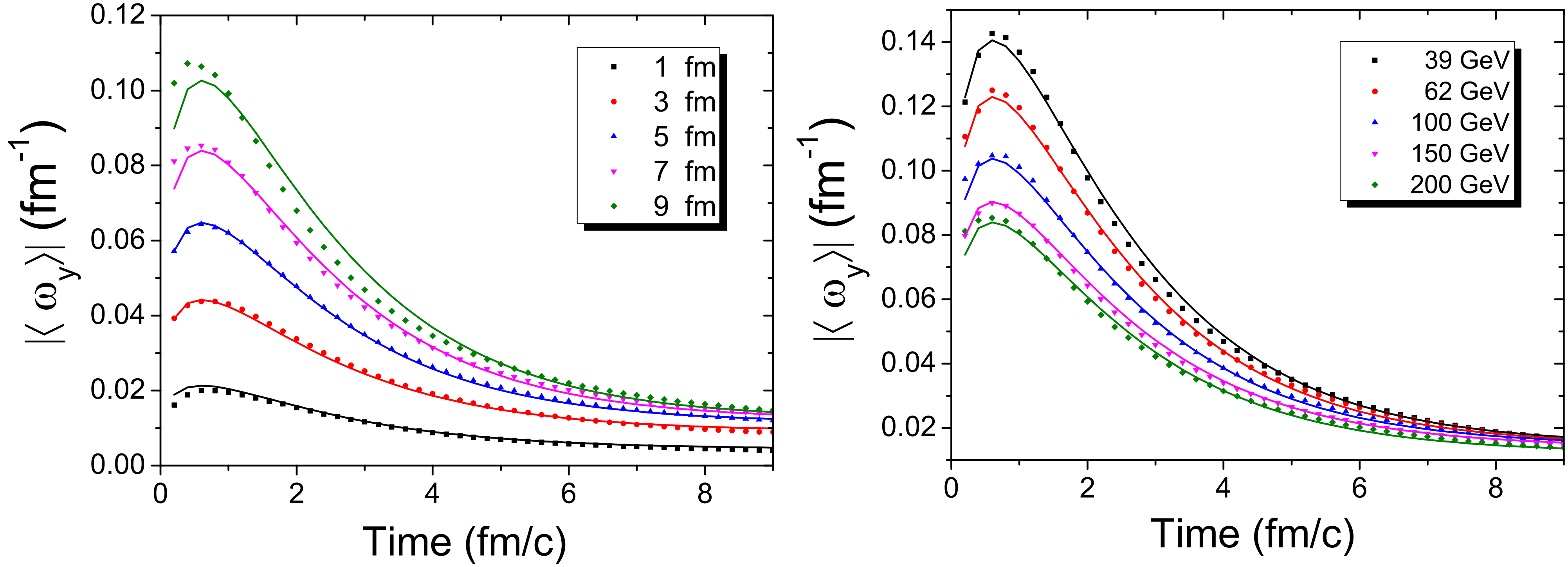
- Based on HIJING and similar results from AMPT.

Vorticity in relativistic heavy ion collisions

Jiang, Lin & Liao, PRC 94, 044910 (2016)

Based on AMPT

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{v}, \quad \langle \omega_y \rangle = \frac{\int d^3\vec{r}\,[\mathcal{W}(\vec{r})]\,\omega_y(\vec{r})}{\int d^3\vec{r}\,[\mathcal{W}(\vec{r})]}, \quad \mathcal{W}(\vec{r}) = \rho\epsilon(\vec{r})$$



- Average vorticity decreases with time, decreasing impact parameter, and increasing collision energy.

Chiral kinetic equations

- Path integral and Coriolis force: Stephanov & Yin, PRL 109, 162001 (2012)
- Covariant Wigner function: Chen, Pu, Wang & Wang, PRL 110, 262301 (2013)
- Equal time quantum transport, Guo & Zhuang, PRD 98, 016007 (2018)
- ………..

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0$$

$$\frac{dt}{d\tau} = 1 + \lambda Q\mathbf{b}\cdot\mathbf{B} + 4\lambda|\mathbf{p}|(\mathbf{b}\cdot\boldsymbol{\omega}), \quad \lambda = \pm 1 = \text{helicity}$$

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} + \lambda Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B} + \lambda Q(\mathbf{E}\times\mathbf{b}) + \lambda\frac{1}{|\mathbf{p}|}\boldsymbol{\omega}$$

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) + \lambda Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{b} - \lambda Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\mathbf{b} + 3\lambda Q(\mathbf{b}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}}$$

$$\text{Three}-\text{dimensional Berry curvature} \quad \mathbf{b} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

Modified quark scattering

- To ensure massless fermions to reach the equilibrium distribution

$$\sqrt{G} f\left(\frac{p-\mu}{T}\right), \quad \sqrt{G} = 1 + \lambda Q(\mathbf{b}\cdot\mathbf{B}) + 6\lambda|\mathbf{p}|(\mathbf{b}\cdot\boldsymbol{\omega})$$

the momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ of two colliding fermions after a collision are determined by momentum conservation with the probability

$$\sqrt{G(\mathbf{p}_3)G(\mathbf{p}_4)}$$

which is set to zero if negative and to a random number between (0,Λ), where Λ is a large number such as 25, if greater than one.

- For collision between a fermion and its antiparticle that have opposite helicities, their helicities can be flipped after the collision [Sun, Ko & Li, PRC 94, 045204 (2016); 95, 034909 (2017)].

Chiral magnetic effect in isobaric (Zr+Zr and Ru+Ru) collisions

Shi, Jiang, Lilleskov & Liao, Ann. Phys. 394, 50 (2018)

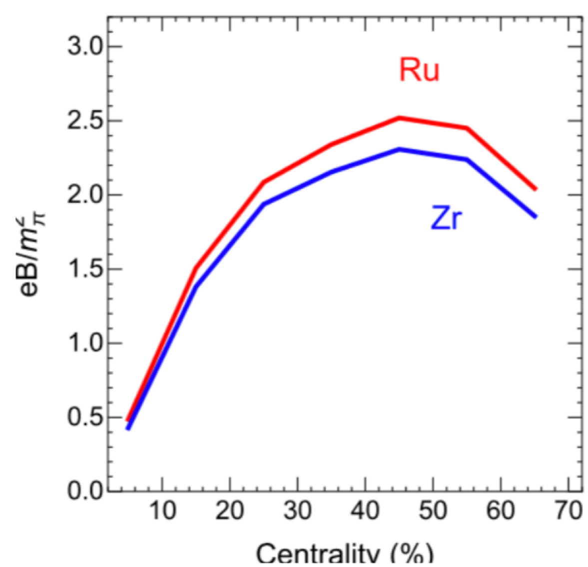
- Magnetic field can lead to electric current in the presence of non- vanishing axial charges → charge separation

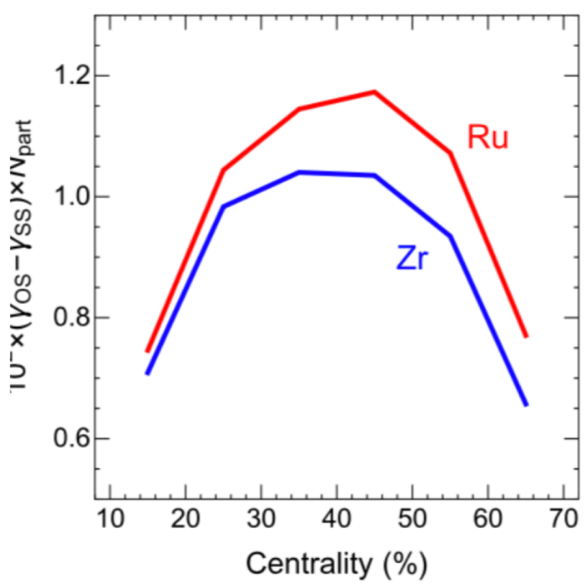
$$\mathbf{J} = \frac{N_c \mu_5 Q}{2\pi^2}\mathbf{B}$$

$$\boldsymbol{B} = \frac{B_0}{1+\tau^2/\tau_B^2}\hat{\boldsymbol{y}}$$

- Based on Anomalous-Viscous Fluid Dynamics (AVFD) with magnetic field parametrized by a Lorentzian function and having strength B_0 determined by spectator protons and $\tau_B = 0.6$ fm/*c*.

- Initial axial charges are estimated using the chirality imbalance arising from gluonic topological charge fluctuations in the early-stage glasma → $\frac{n_5}{s} \approx 0.12$ at 40-50% centrality.





$\gamma^{OS} - \gamma^{SS}$ correlator (Voloshin)

$$\gamma^{\mathrm{OS}} = \langle \cos(\phi^{+(-)} + \phi^{-(+)} - 2\Psi)\rangle$$
$$\gamma^{\mathrm{OS}} = \langle \cos(\phi^{+(-)} + \phi^{+(-)} - 2\Psi)\rangle$$

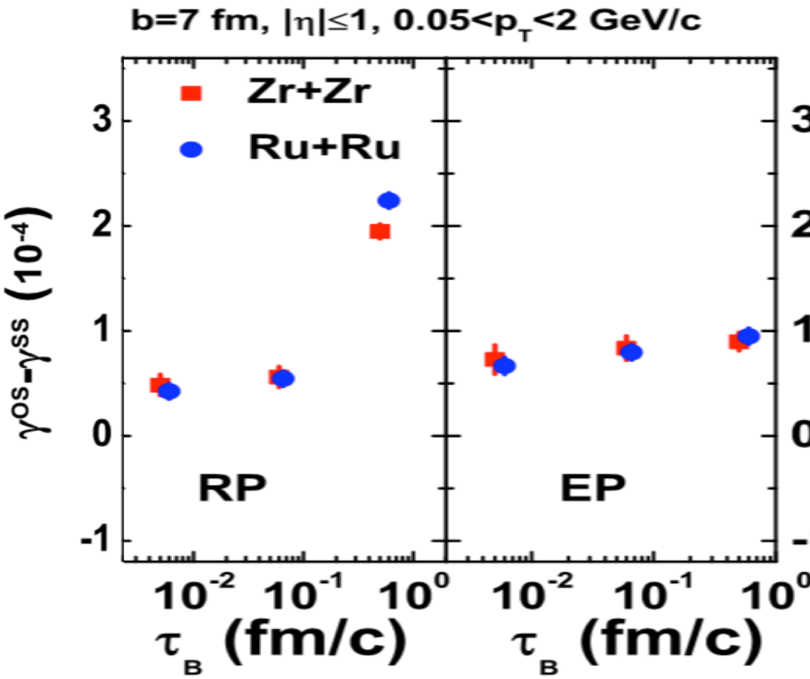
Chiral magnetic effect from chiral kinetic approach

Sun & Ko, PRC 98, 014911 (2018)

Probability for quark helicity $P_\lambda = \frac{1+\lambda p}{2}$

$$p = \frac{d\sqrt{\langle N_5^2 \rangle}/d\eta}{dN/d\eta} \approx 0.2$$

$\gamma^{OS} - \gamma^{SS}$ correlator



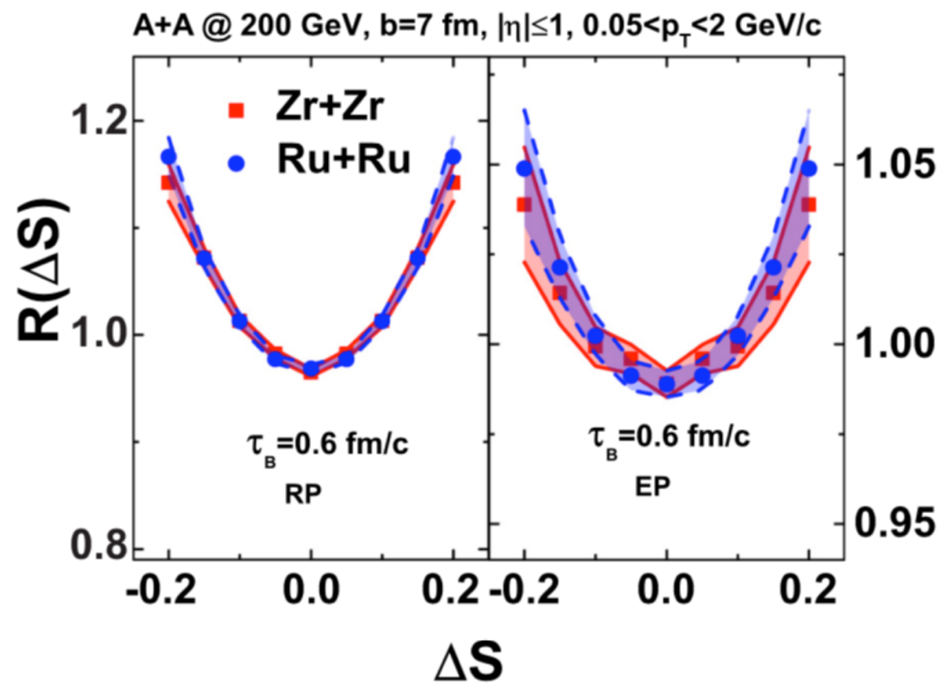
- Signal small with event plane

$R(\Delta S)$ correlator (Lacey)

$$R(\Delta S) = \frac{N_{\rm real}(\Delta S)/N_{\rm shuffle}(\Delta S)}{N^{\perp}_{\rm real}(\Delta S)/N^{\perp}_{\rm shuffle}(\Delta S)}$$

$$\Delta S = \frac{\sum_i \sin(\phi_i^+ - \Psi)}{N_p} - \frac{\sum_i \sin(\phi_i^- - \Psi)}{N_n}$$

$\perp$: $\Psi \to \Psi + \pi/2$

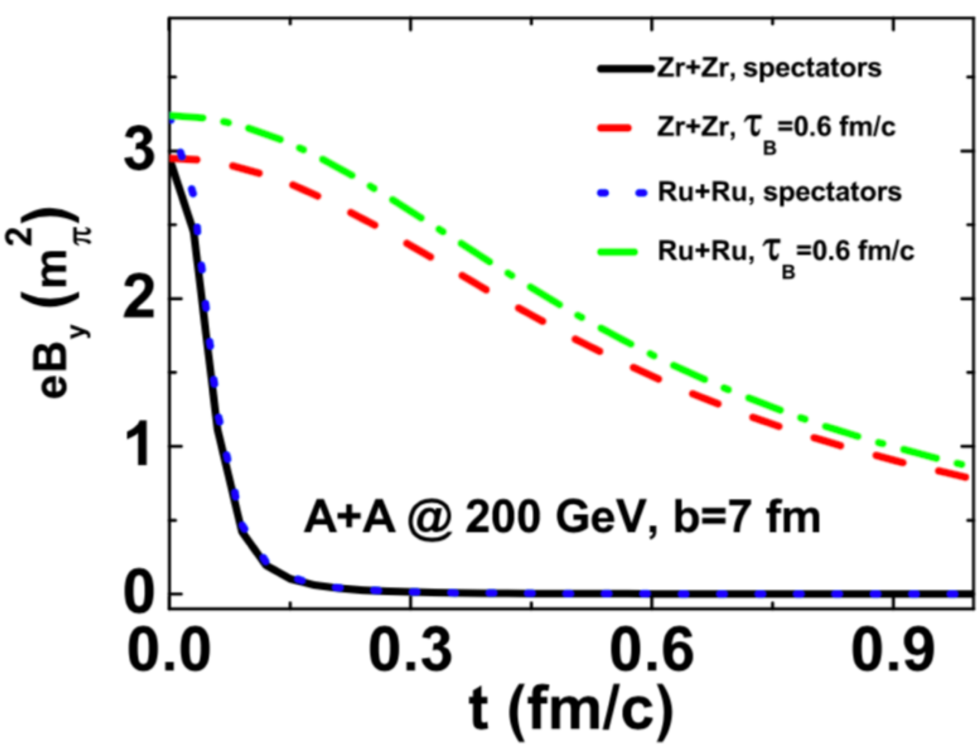


- Effect plausible for long-lived B

- Similar to that based on Anomalous-Viscous Fluid Dynamics (AVFD) [Shi et al., Ann. Phys. 394, 50 (2018); Magdy et al., PRC 98, 061902 (2018)].

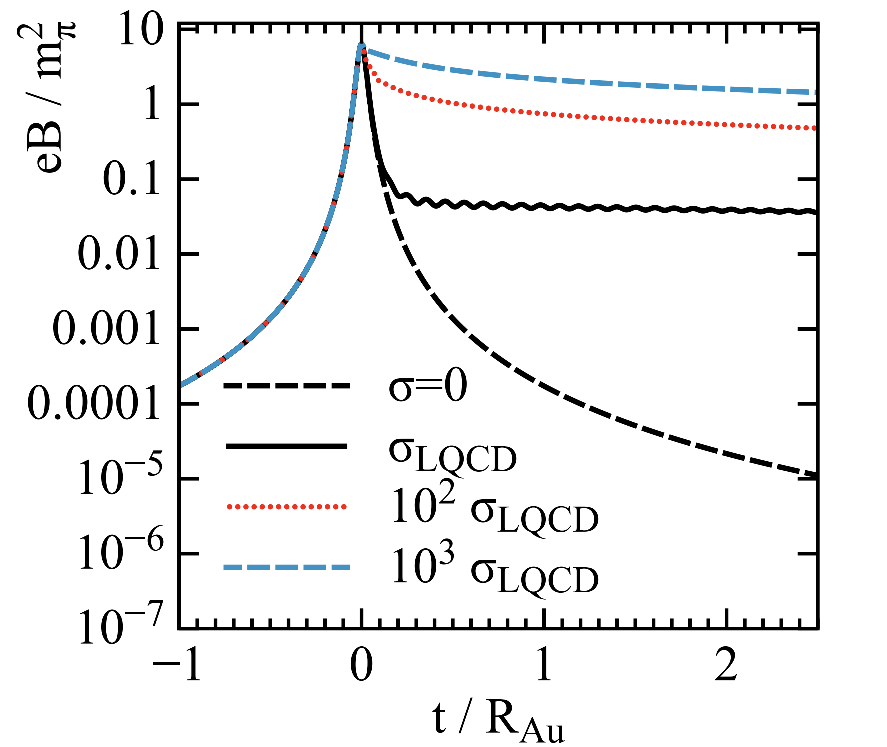
Is the magnetic field in HIC long-lived?

Shi et al., Ann. Phys. 394, 50 (2018)



$$\boldsymbol{B} = \frac{B_0}{1 + \tau^2/\tau_B^2}\hat{\boldsymbol{y}}$$

Mclerran & Skokov, NPA 929, 184 (2014)



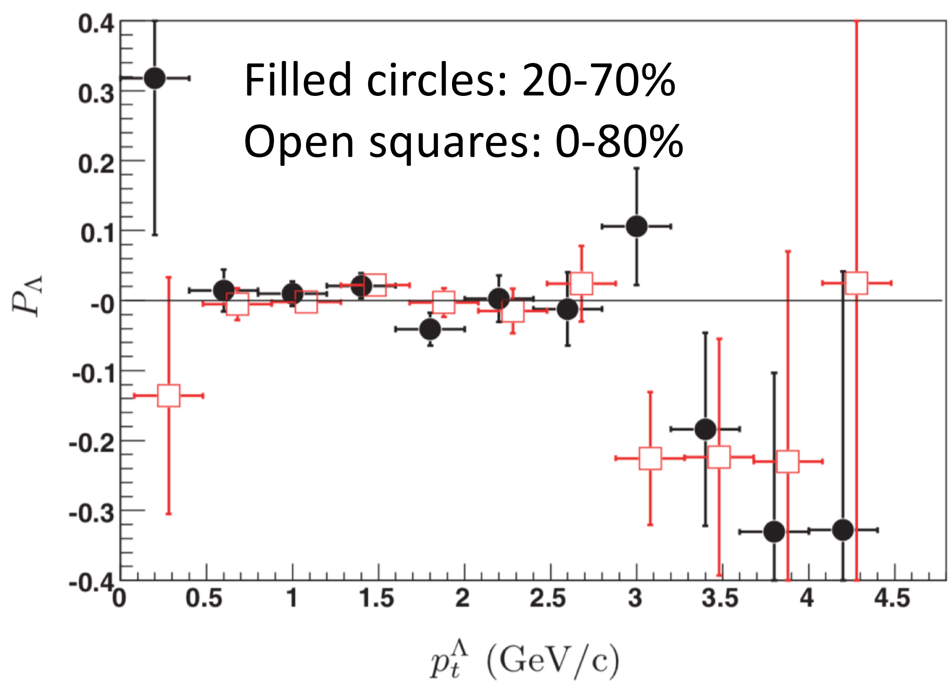
$$\sigma_{\rm Ohm}^{\rm LQCD} = (5.8 \pm 2.9)\frac{T}{T_0}\ \mathrm{MeV}$$

- Lifetime of magnetic field is long only if QGP is a perfect conductor.

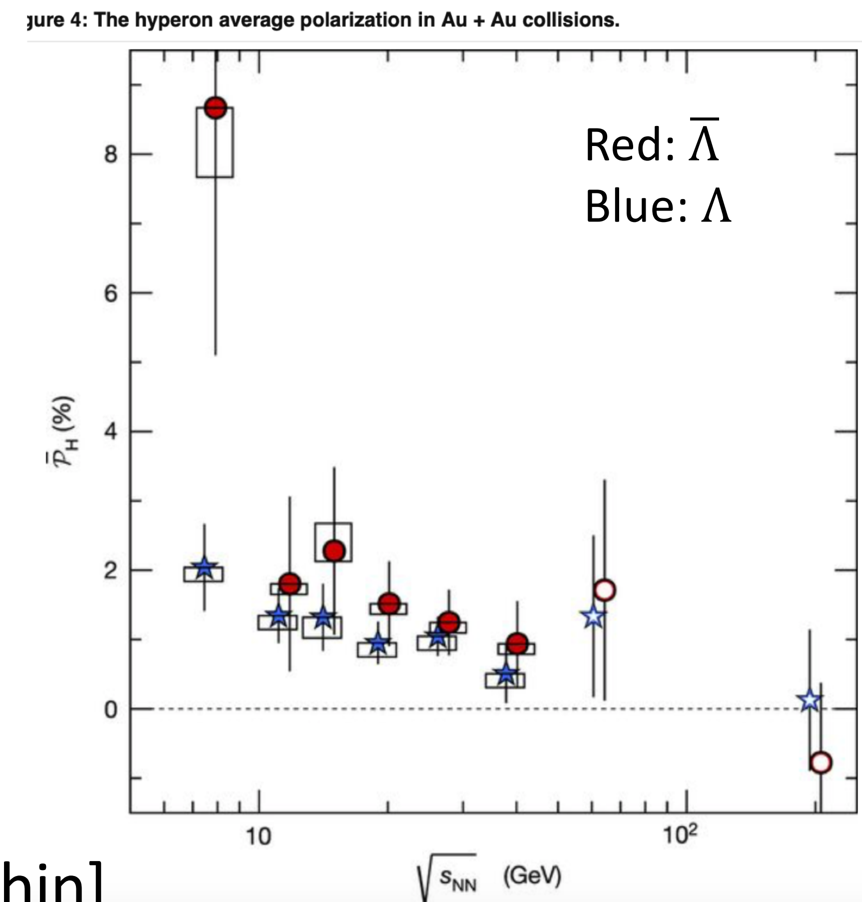
Λ polarization in relativistic heavy ion collisions

- First suggested by Z. T. Liang & X. N. Wang, PRL 94, 102301 (2005)

Abelev (STAR), PRC 76, 024915 (2007)

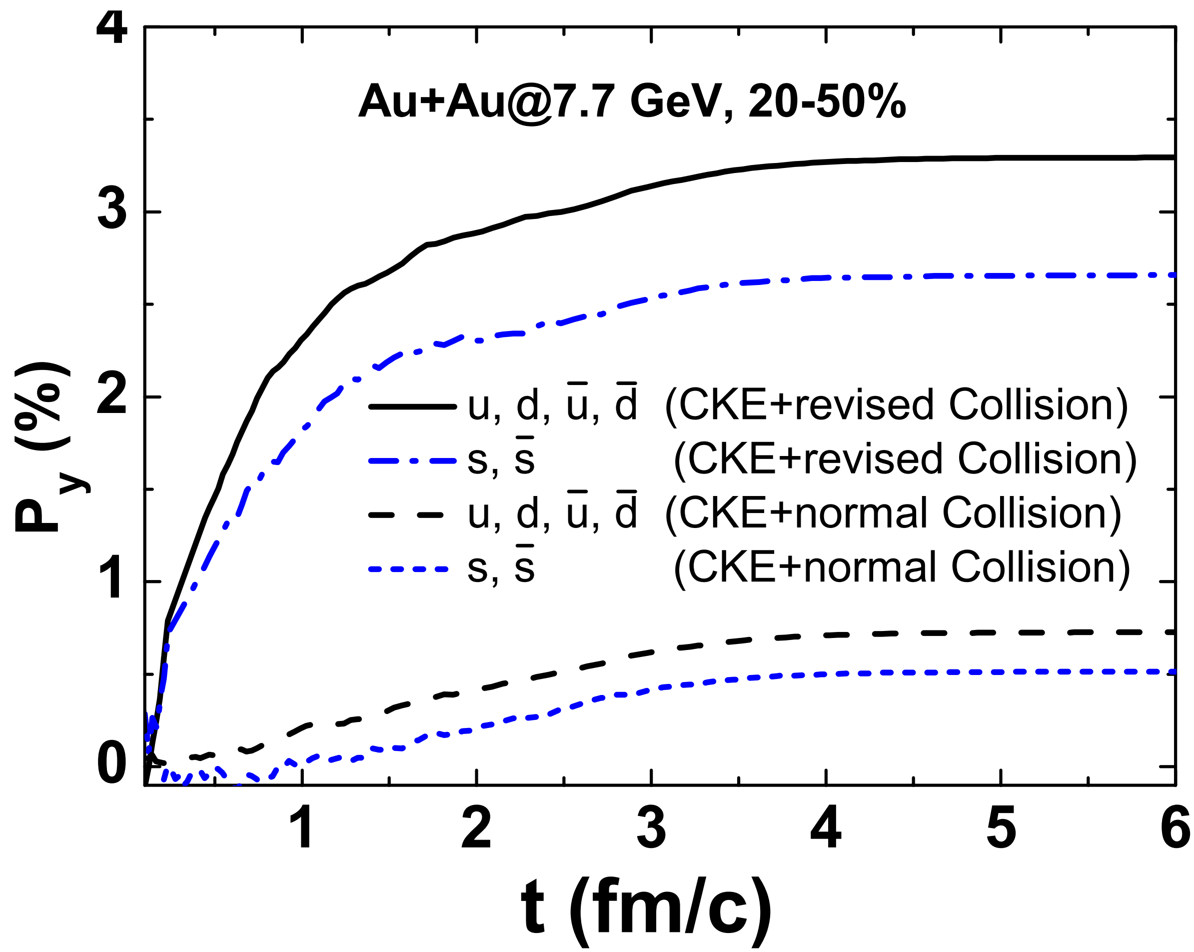


STAR Collaboration, Nature 548, 62 (2017)



- Studies based on fluid dynamics [Cernai, Becatiini, Karpenko, Voloshin] and transport models [Wang] assuming Λ in thermal equilibrium in the rotating fireball at kinetic freeze out of HIC all predict Λ polarizations comparable to the STAR data.

Global quark polarization from chiral kinetic approach



Sun & Ko, PRC 96, 024906 (2017)

- Quark polarization of about 2.5% is generated early in time due to decrease of vorticity field with time.

- Obtained with AMPT initial conditions and including the effect of finite strange quark mass by replacing in the chiral kinetic equation

$$\hat{\mathbf{p}}, p \text{ and } \mathbf{b} = \frac{\hat{\mathbf{p}}}{2p^3} \text{ by } \frac{\mathbf{p}}{E_p}, E_p \text{ and } \frac{\hat{\mathbf{p}}}{2E_p^2}$$

Lambda polarization

$$\frac{dN_\Lambda}{d^3\mathbf{P}_\Lambda} = g_C g_S \int d^3\mathbf{x}_1 d^3\mathbf{p}_1 d^3\mathbf{x}_2 d^3\mathbf{p}_2 d^3\mathbf{x}_3 d^3\mathbf{p}_3 f_{q_1}(\mathbf{x}_1,\mathbf{p}_1) f_{q_2}(\mathbf{x}_2,\mathbf{p}_2) f_{q_3}(\mathbf{x}_3,\mathbf{p}_3)$$
$$\times W_\Lambda(\mathbf{y}_1,\mathbf{k}_1;\mathbf{y}_2,\mathbf{k}_2)\delta^{(3)}(\mathbf{P}_\Lambda - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$

Lambda Wigner function

$$W_{\Lambda,n_1,n_2}(\mathbf{y}_1,\mathbf{k}_1;\mathbf{y}_2,\mathbf{k}_2) = 8^2 e^{\frac{-\mathbf{y}_1^2}{\sigma_1^2} - \frac{\mathbf{y}_2^2}{\sigma_2^2} - \mathbf{k}_1^2\sigma_1^2 - \mathbf{k}_2^2\sigma_2^2}$$

with

$$\mathbf{y}_1 = \frac{\mathbf{x}_1' - \mathbf{x}_2'}{\sqrt{2}}, \quad \mathbf{k}_1 = \sqrt{2}\frac{m_2\mathbf{p}_1' - m_1\mathbf{p}_2'}{m_1+m_2}$$
$$\mathbf{y}_2 = \sqrt{\frac{2}{3}}\left(\frac{m_1\mathbf{x}_1' + m_2\mathbf{x}_2'}{m_1+m_2} - \mathbf{x}_3'\right), \quad \mathbf{k}_2 = \sqrt{\frac{3}{2}}\frac{m_3(\mathbf{p}_1'+\mathbf{p}_2') - (m_1+m_2)\mathbf{p}_3'}{m_1+m_2+m_3}$$

Primed coordinates and momenta refer to baryon rest frame after earlier produced quarks are propagated to the time of the latest produced one.

Color and spin statistical factors for polarized Λ

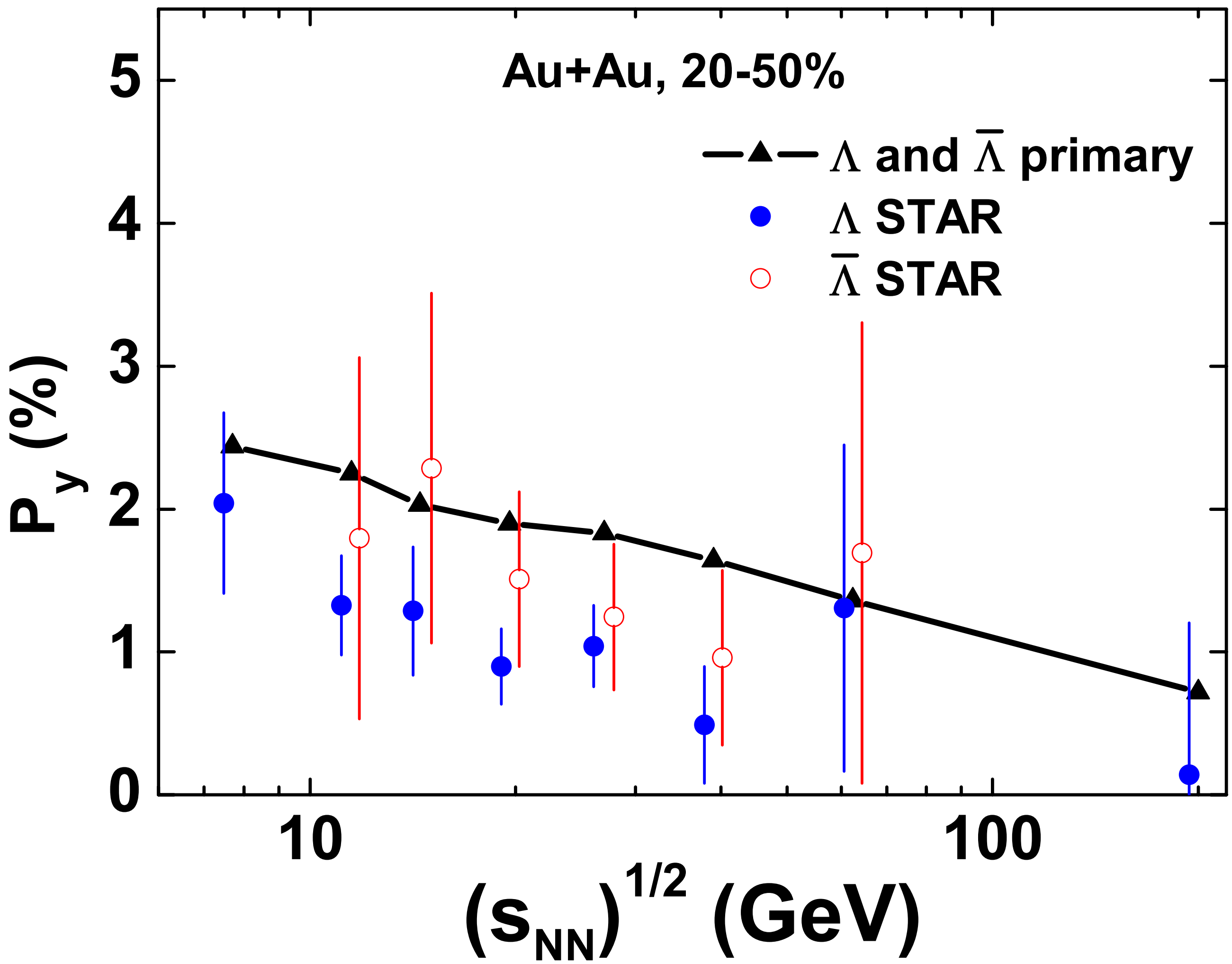
- Statistical factor for 3 colored quarks or antiquarks to form a colorless Λ or anti-Λ: g_C=1/27

- Statistic factor for 3 spin-1/2 quarks to form a polarized spin ½ Λ: Λ spin determined by the spin of s-quark, so u and d quarks form a spin singlet

$$\begin{aligned} g_S &= |(1/\sqrt{2})(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)[\cos(\theta_1/2)\cos(\theta_2/2)|\uparrow\uparrow\rangle \\ &\quad + \cos(\theta_1/2)\sin(\theta_2/2)e^{i\phi_2}|\uparrow\downarrow\rangle \\ &\quad + \sin(\theta_1/2)\cos(\theta_2/2)e^{i\phi_1}|\downarrow\uparrow\rangle \\ &\quad + \sin(\theta_1/2)\sin(\theta_2/2)e^{i(\phi_1+\phi_2)}]|\downarrow\downarrow\rangle|^2 \\ &= \frac{1}{4}(1 - \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)) \\ &= \frac{1}{4}(1 - \lambda_1\lambda_2\hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_2) \end{aligned}$$

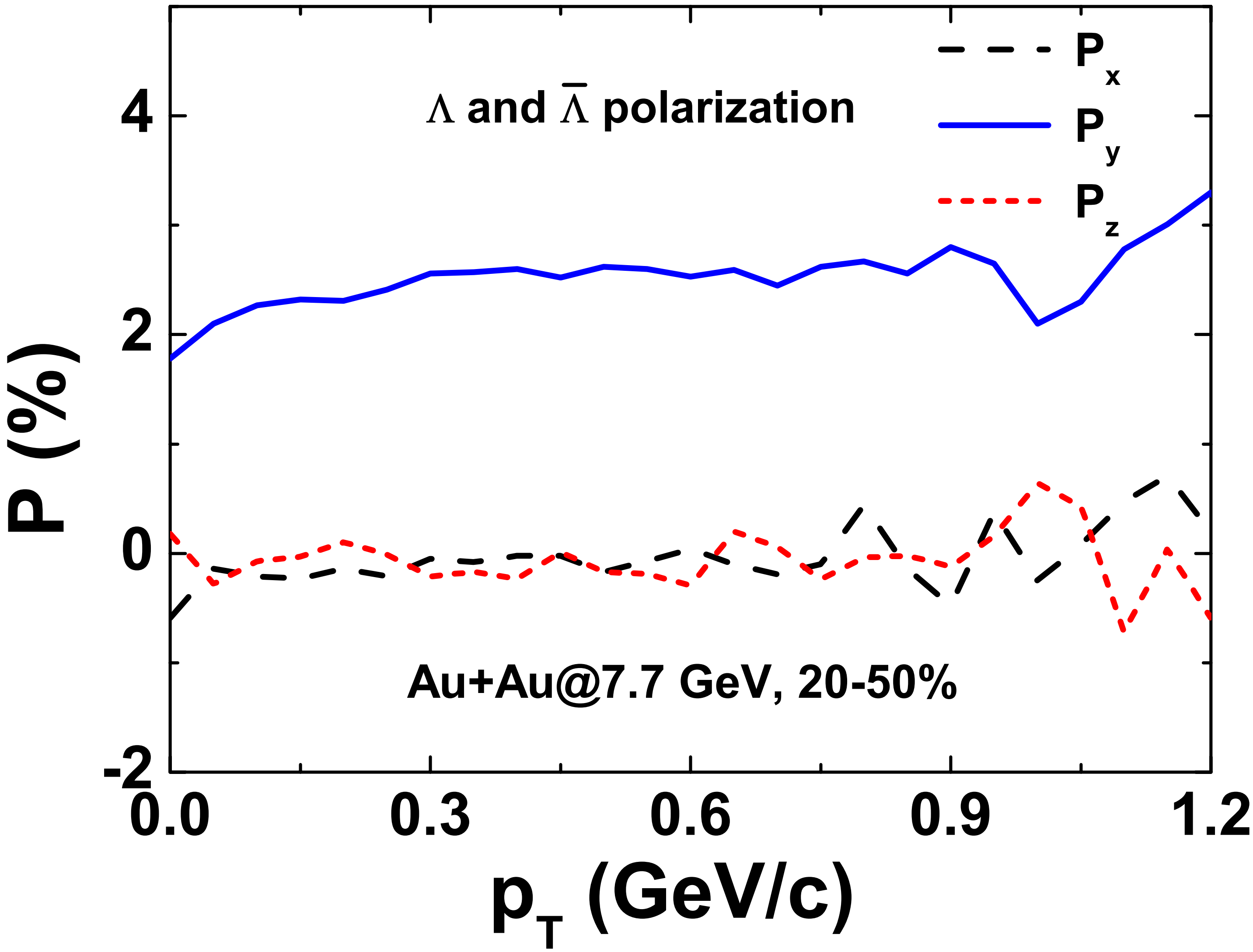
where $\theta_1(\theta_2)$ and $\phi_1(\phi_2)$ are polar angles of u(d) quark.

Collision energy dependence of Λ polarization



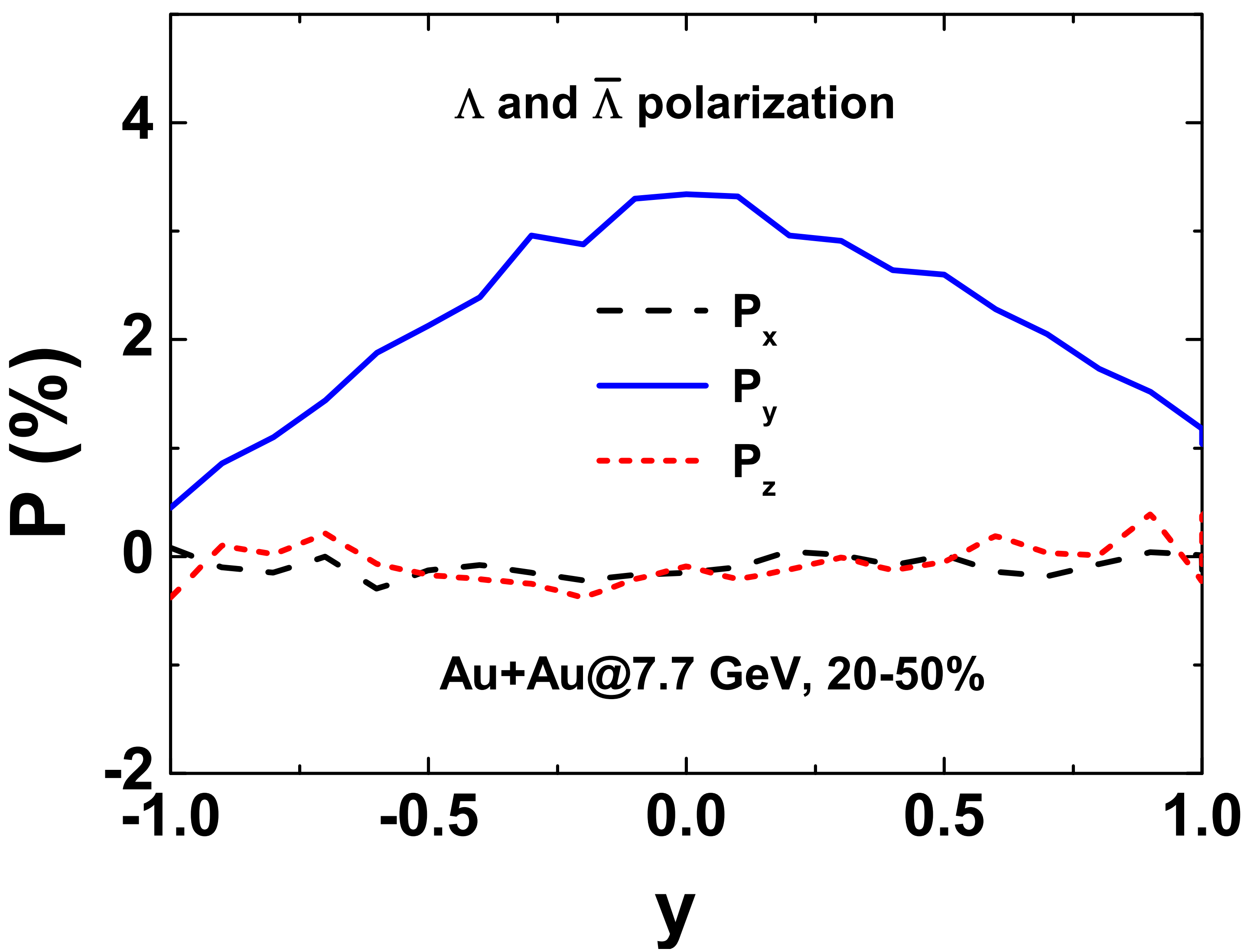
- Consistent with data that Λ polarization decreases with collision energy due to decreasing vorticity field with increasing energy.

Transverse momentum dependence of Λ polarization



- Λ polarization is perpendicular to reaction plane, has similar magnitude as that of the s quark, and is independent of p_T.

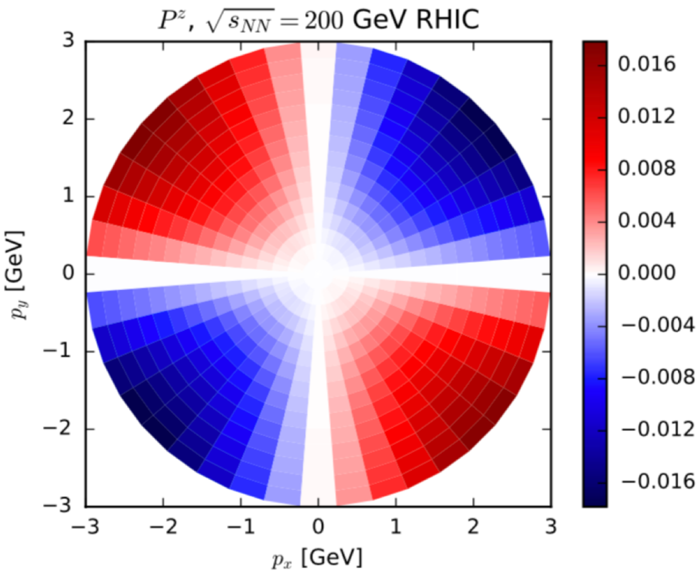
Rapidity dependence of Λ polarization



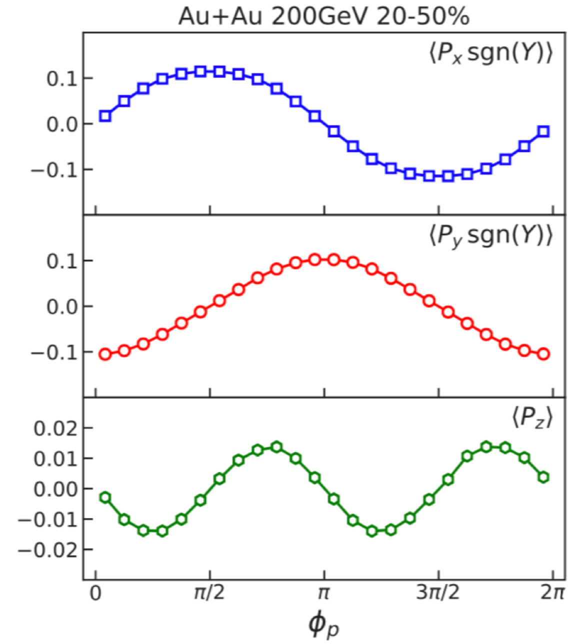
- Λ polarization peaks at midrapidity due to decreasing number of scattering with increasing rapidity.

Longitudinal spin polarization

Becattini & Karpeno, PRL 120, 012302 (2018)



Xia, Li, Tang & Wang, PRC 98, 024905 (2018)

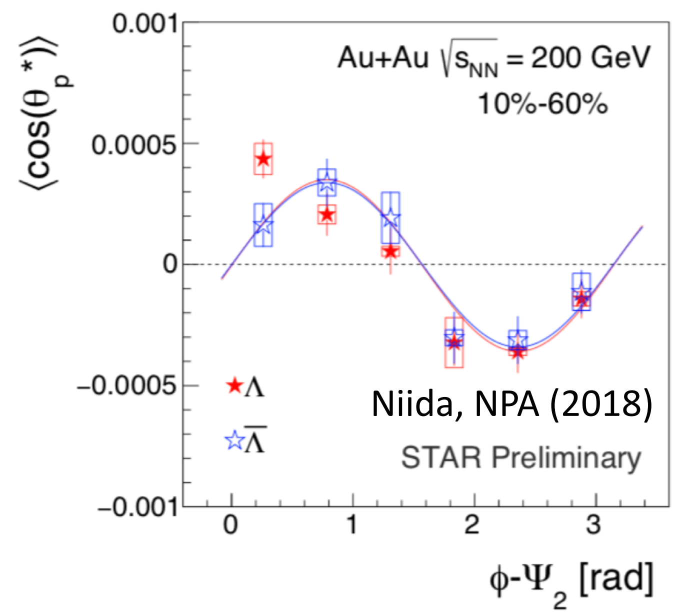


In terms of thermal vorticity

$$\bar{\omega}_{\mu\nu} = \frac{1}{2}(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu), \quad \beta = \frac{u}{T}$$

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \times \frac{\int_\Sigma d\Sigma_\lambda p^\lambda \bar{\omega}_{\rho\sigma} n_F (1 - n_F)}{\int_\Sigma d\Sigma_\lambda p^\lambda n_F}$$

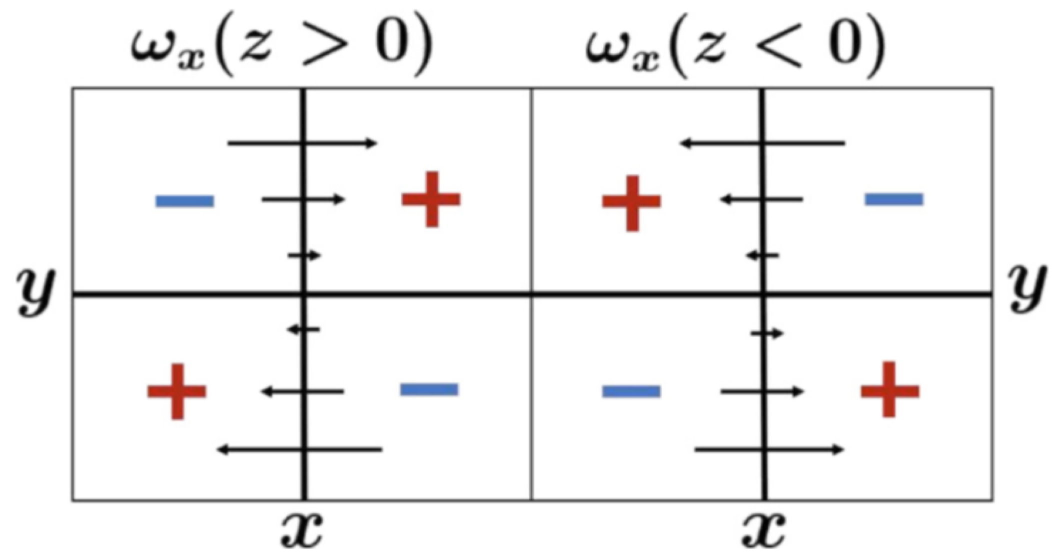
$$S^z(\mathbf{p}_T, y = 0) \approx \frac{dT}{d\tau}\frac{1}{mT} v_2(p_T) \sin 2\phi$$



Longitudinal spin polarization

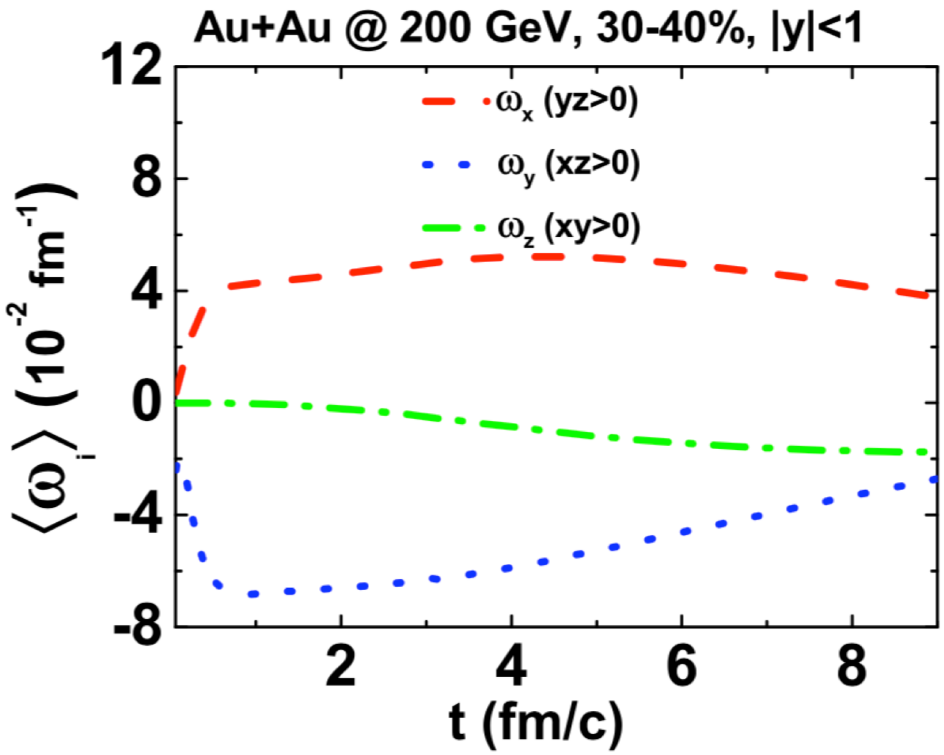
Sun & Ko, PRC 99, 011903 (R) (2019)

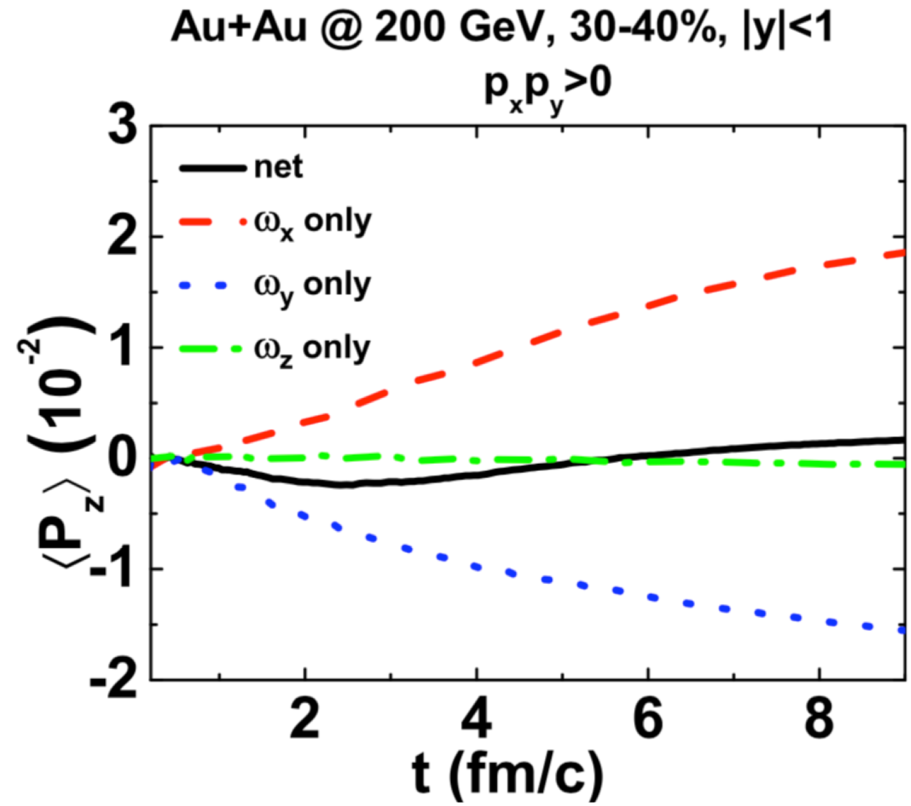
- Effect of transverse vorticity $\boldsymbol{\omega}_\perp = \frac{1}{2}\partial_z v_\perp(r,z)\mathbf{e}_\phi$



- In-plane component of vorticity (ω_x) is clockwise for z>0 and counterclockwise for z<0.
- Leading to quadrupole structure in the axial charge distribution and thus a similar quadrupole structure in the longitudinal spin polarization.
- Out-of-plane component of vorticity (ω_y) has opposite sign, which partially cancels that due to ω_x because of anisotropic flow.

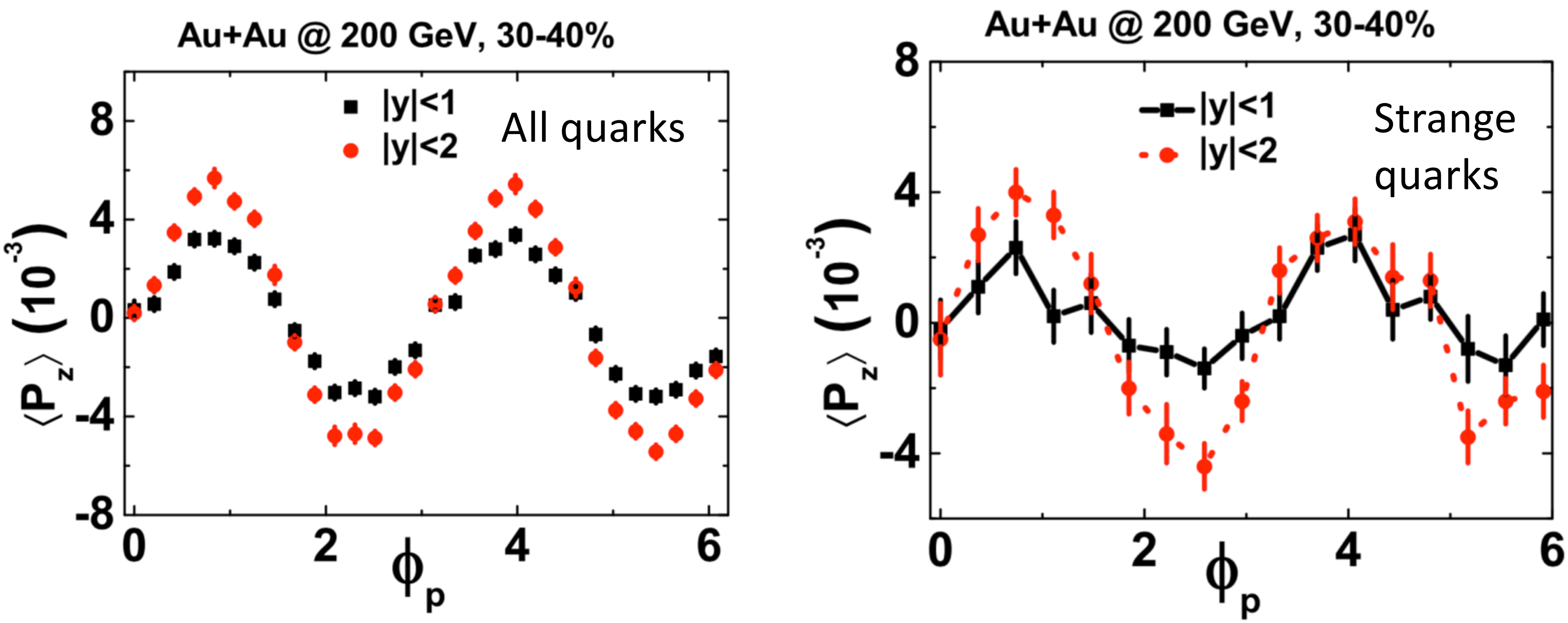
- Chiral kinetic transport study using AMPT initial conditions with random axial charge distribution and isotropic parton scattering cross section of 10 mb.





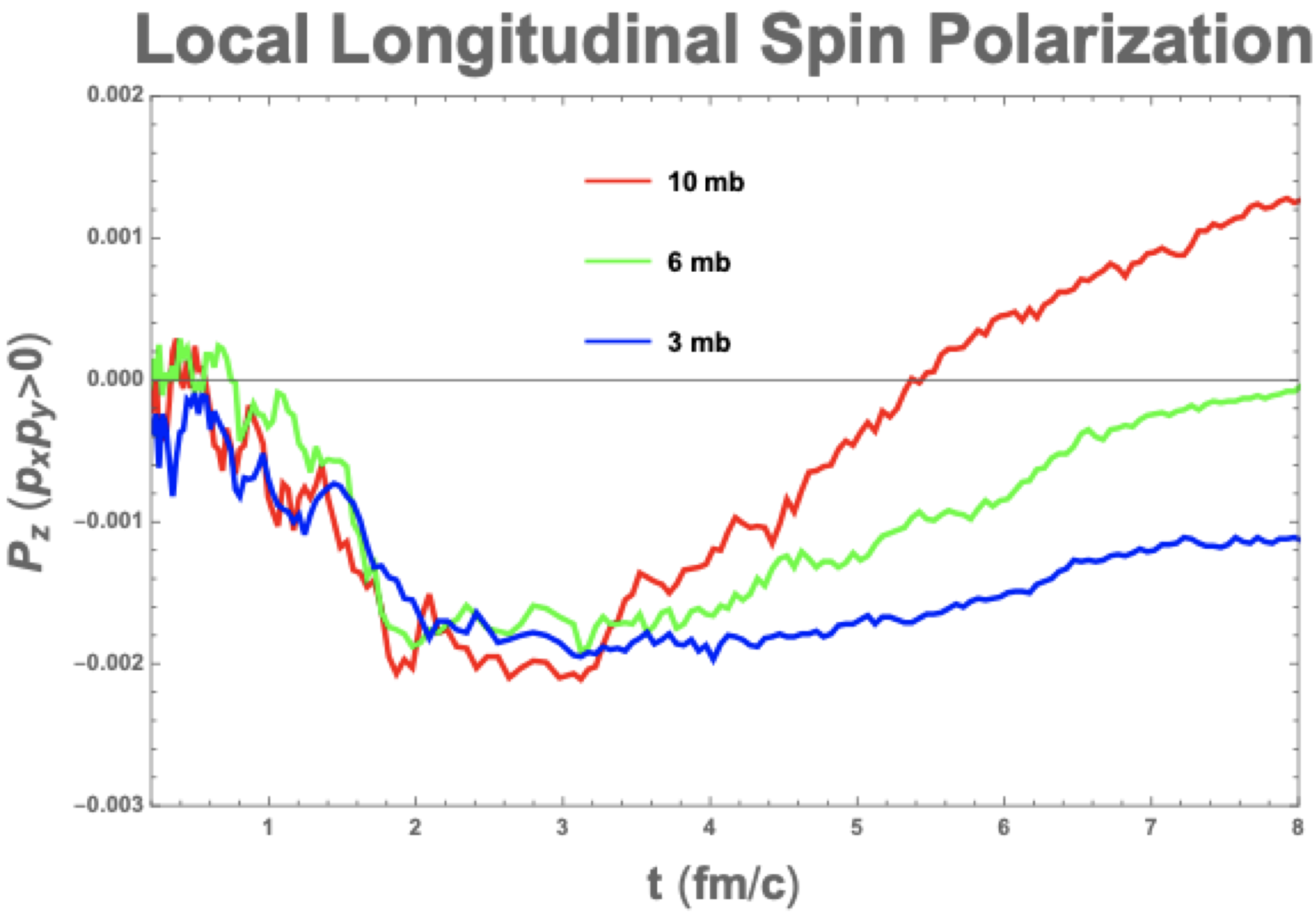
- In the first and third quadrants, $\omega_x > 0$ and $\omega_y, \omega_z < 0$.
- Longitudinal spin polarization (P$_z$) is positive, negative and zero including only ω_x, ω_y and ω_z, with a final small positive value.

- Azimuthal angle dependence of longitudinal spin polarization



- Similar to that from preliminary STAR data [Niida, NPA (2018)].
- Opposite sign from those based on thermal-vorticity [Becattini & Karpenko, PRL 120, 012302 (2018); Xia et al., PRC 98, 024905 (2018)].

- Parton scattering cross section dependence of longitudinal spin polarization



- Sign depends on the magnitude of parton scattering cross section and changes with reduced cross section.

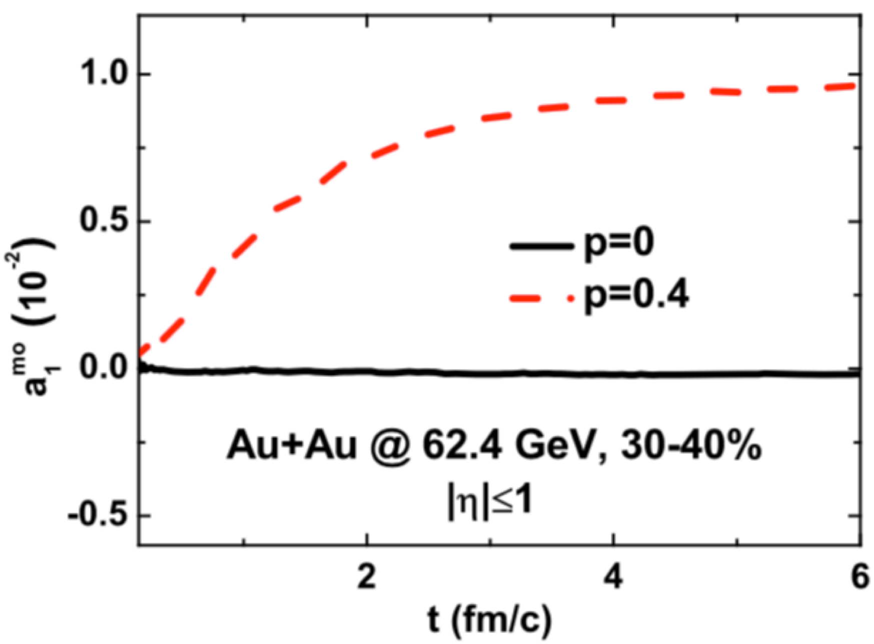
Multiplicity up-down asymmetry

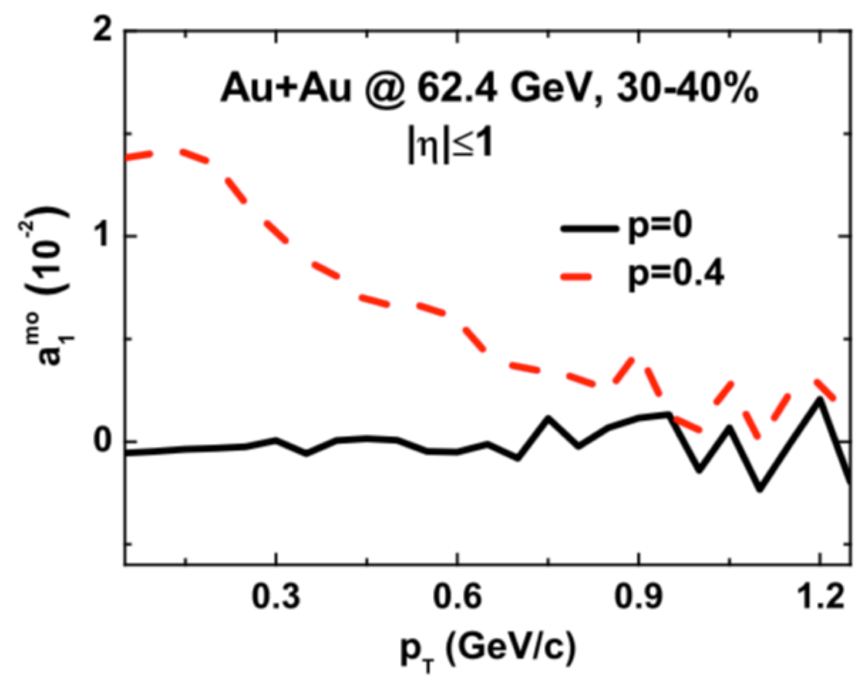
Sun & Ko, PLB 789, 228 (2019)

In the presence of both magnetic and vorticity fields and with net axial charge density, azimuthal angle distribution of charged particles

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_2 \cos(2\phi - 2\Psi_{\rm RP}) + 2(a_{\rm CVE} \pm a_{\rm CME}) \sin(\phi - \Psi_{\rm RP})$$

Considering both positively and negatively charged particles leads to a multiplicity up-down asymmetry $a_1^{mo} = \langle \sin\phi \rangle = a_{CVE}$

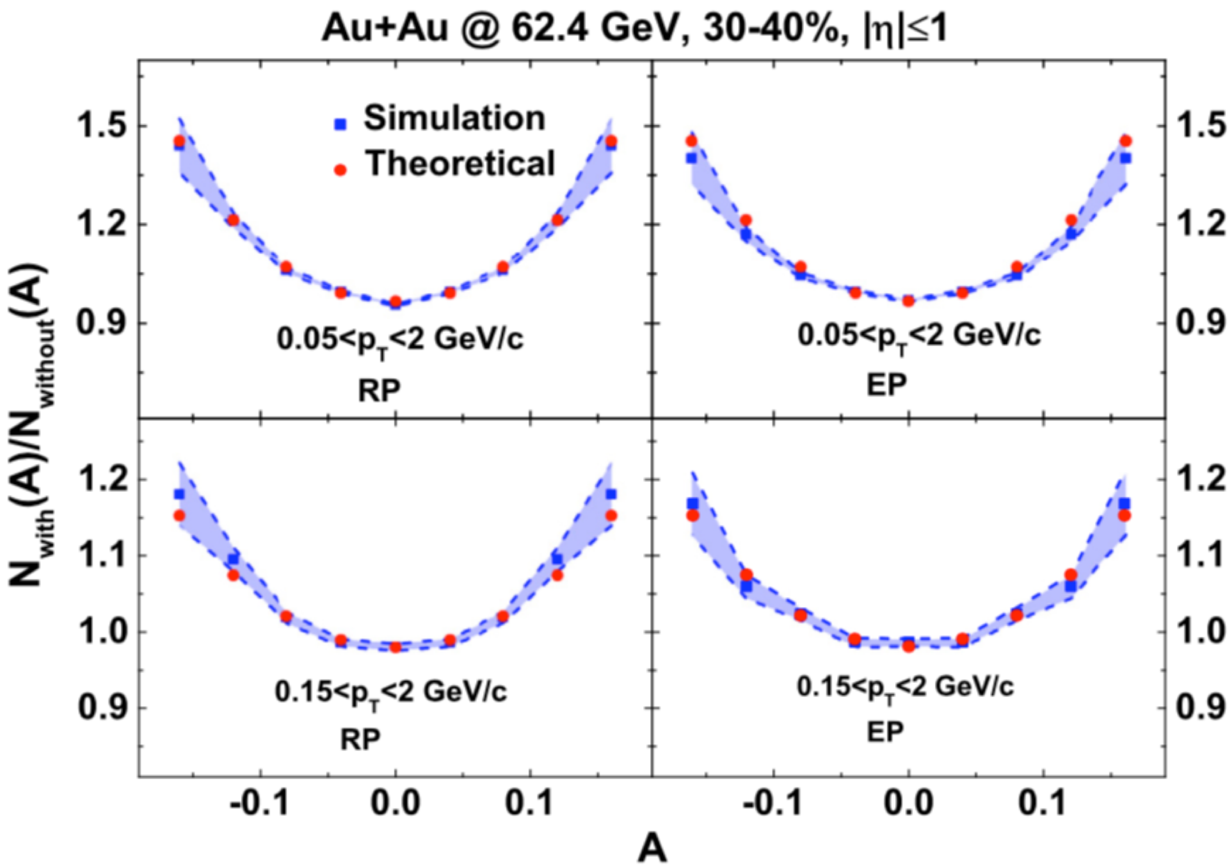




- Multiplicity up-down asymmetry event distribution

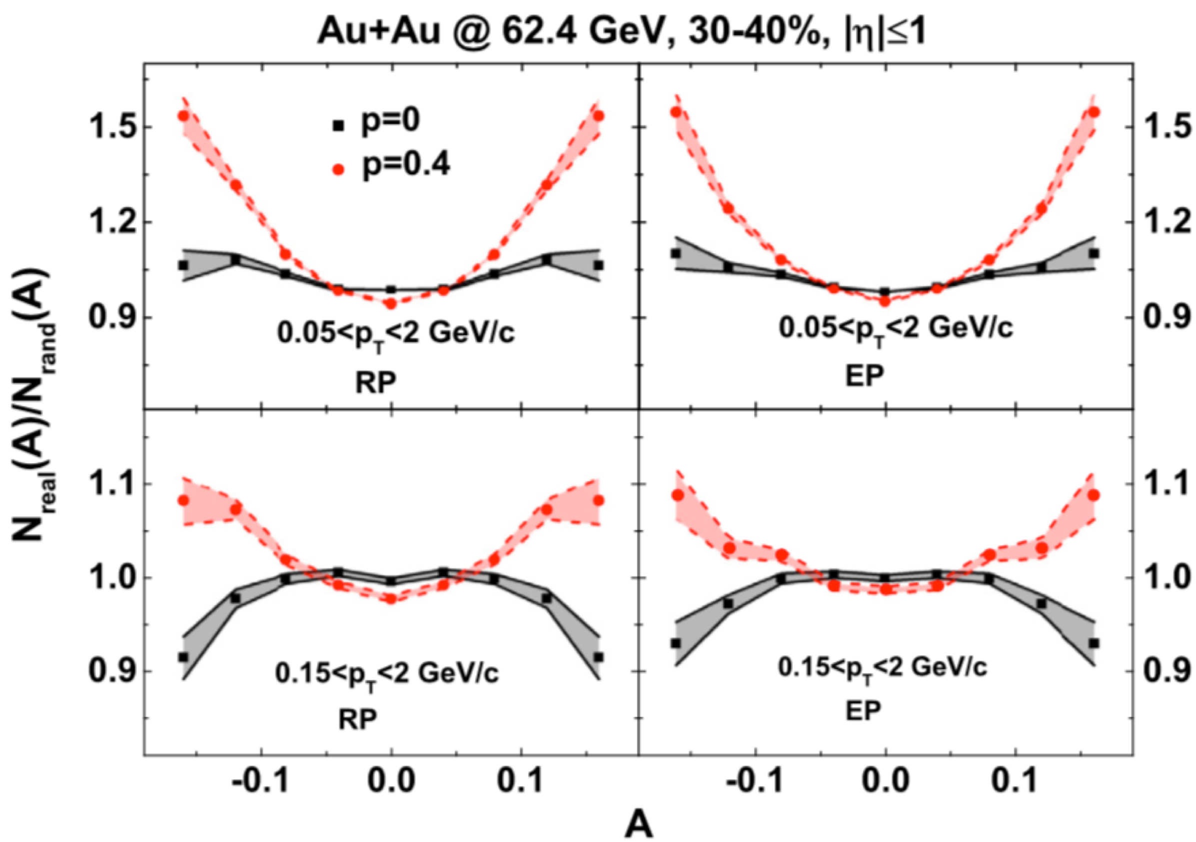
$$A = \frac{N_U - N_D}{N_U + N_D}$$

N_U and N_D are numbers of particles with momenta pointing towards upper and lower hemispheres of a collision



It has a concave shape that vanishes at A=0 and has a width equal to $\frac{4a_{CVE}}{\pi}$ as expected for a large number of uncorrelated particles in an event.

- Ratio of multiplicity up-down asymmetry distribution to that of random distribution

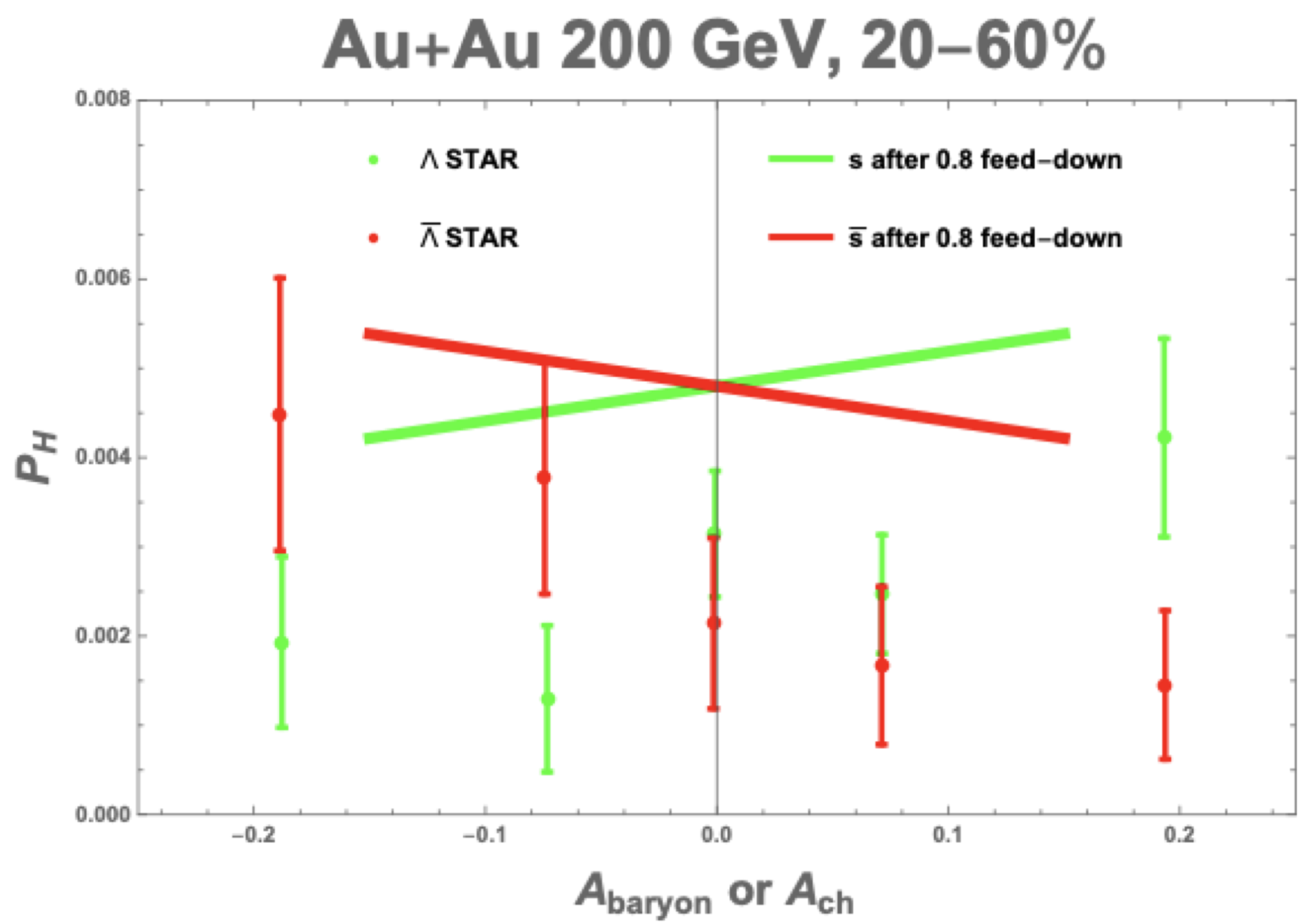


$$p = \frac{d\sqrt{\langle N_5^2 \rangle}/d\eta}{dN/d\eta}$$

- Width of the distribution is sensitive to the value of axial charge fluctuations in quark matter.
- Chiral separation effect due to the triangular anomaly [Son & Surowka, PRL 103, 191601 (2009)]?

Charge asymmetry dependence of Λ and $\bar{\Lambda}$ polarization

Including quark vector potential at finite baryon density, which is repulsive and attractive for quarks and antiquarks, respectively, leads to positive and negative slopes in the charge asymmetry dependence of Λ and $\bar{\Lambda}$ polarization, consistent with observations by STAR if [PRC 98, 014910 (2018)] baryon asymmetry is correlated with charge asymmetry (?). Sun & Ko, preliminary



Summary

- Magnetic field generated in non-central relativistic heavy ion collisions is large but short-lived, while the **vorticity field lasts longer**.
- In the presence of long-lived magnetic field chiral transport study shows that there is a difference in the chiral magnetic effect (charge separation) in collisions of isobaric nuclei Zr+Zr and Ru+Ru.
- Realistic vorticity field does lead to finite quark and thus **Λ global polarization** with magnitude comparable to experimental values.
- **Longitudinal spin polarization is affected by transverse components of vorticity field** and can lead to the correct azimuthal angle dependence observed in experiments if parton scattering cross section is sufficient large.
- Large vorticity field generated in noncentral HIC can lead to an **up-down asymmetry** of produced particles if **axial charge fluctuations** are present in the produced quark matter.
- **Quark vector potential** affects differently the baryon-asymmetry dependence of quark and antiquark polarizations.