A microscopic model for global polarization from particle collisions

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with

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Outline

- **Introduction**
- **Theoretical models on particle polarization: [Spin-orbit coupling, Statistical-hydro, Kinetic]**
- **A microscopic model for global polarization through spin-orbit couplings in particle scatterings, a non-equilibrium model**
- **Summary**

Introduction

Global OAM and Magnetic field in HIC

- **Huge global orbital angular momenta are produced**

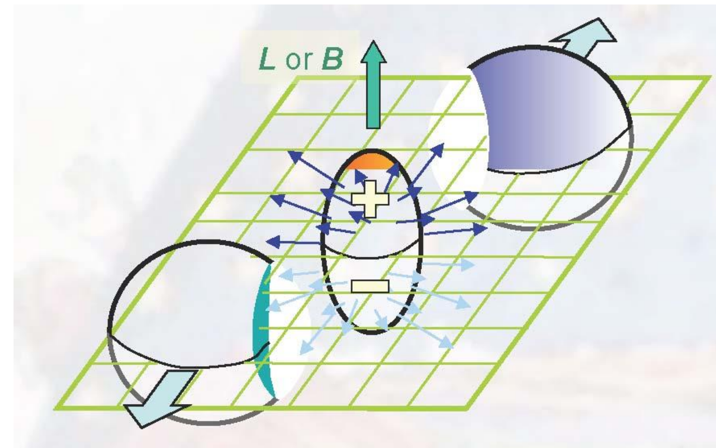
$$L \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$B \sim m_\pi^2 \sim 10^{18}\ \text{Gauss}$$

- **How do orbital angular momenta be transferred to the matter created?**

- **Any way to measure angular momentum?**



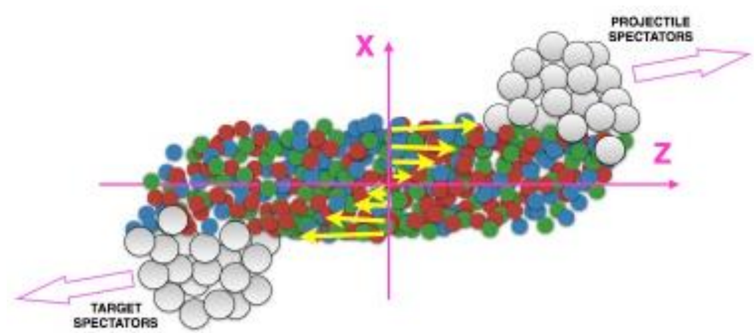


Figure taken from
Becattini et al, 1610.02506

Rotation vs Polarization

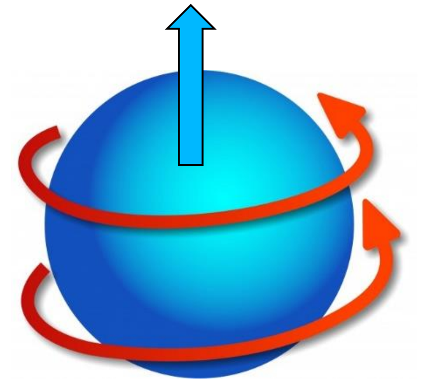
- **Barnett effect: rotation to polarization**

  uncharged object in rotation

  → spontaneous magnetization

  → polarization (spin-orbital coupling)

  [Barnett, Rev.Mod.Phys.7,129(1935)]



- **Einstein-de Haas Effect: polarization to rotation**

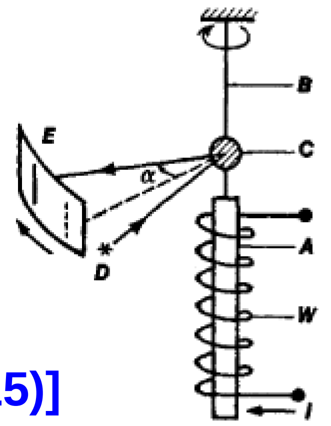
  magnetic field (impulse)

  → polarization of electrons

  → ΔL_electron

  → ΔL_mechanical = - ΔL_electron

[Einstein, de Haas, DPG Verhandlungen 17, 152(1915)]



Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

- **Polarizations of Λ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

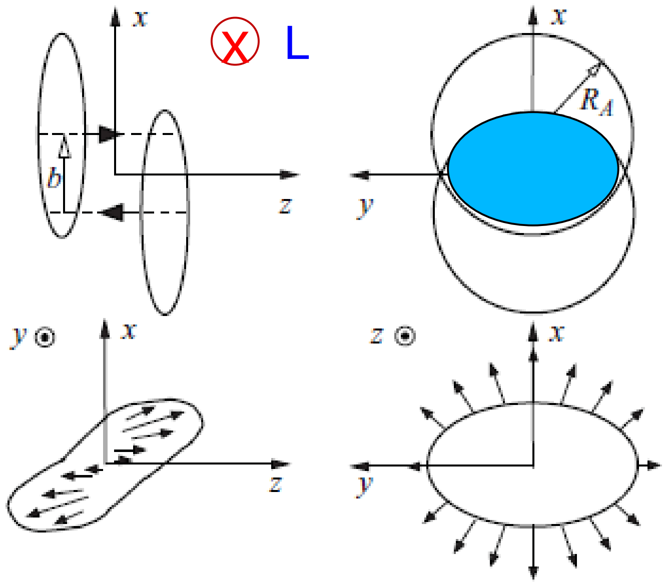
- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

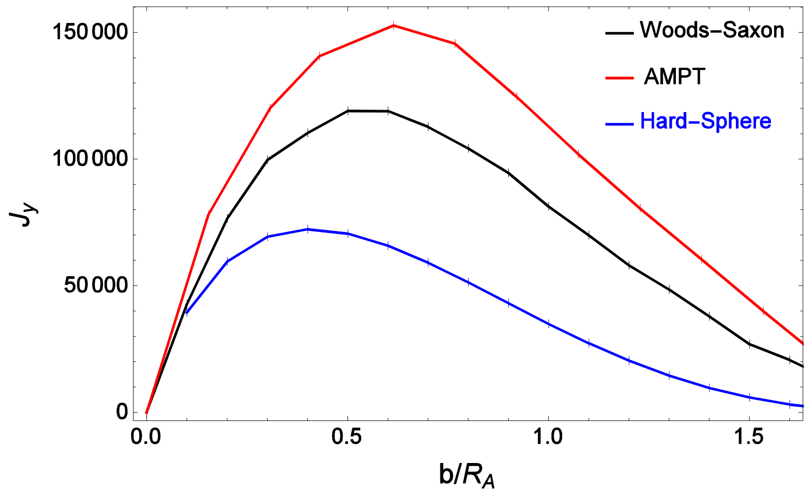
- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

Global OAM in HIC

- **Non-central collisions produce global orbital angular momentum**

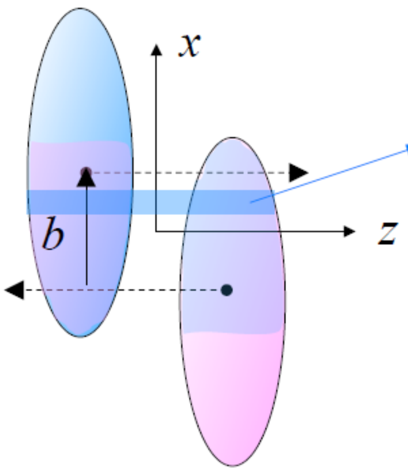




$$L_y = -p_{\rm in} \int x dx \left(\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx} \right)$$

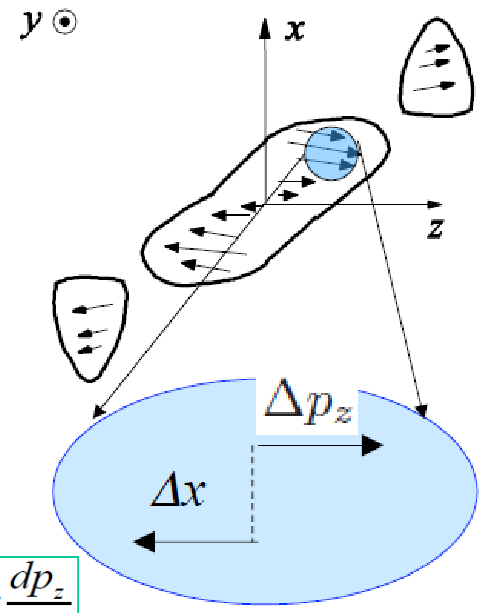
Liang & Wang, PRL 94, 102301(2005); PLB 629, 20(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Huang, Huovinen, Wang,PRC 84,054910(2011); Jiang, Lin, Liao, PRC 94,044910(2016); Deng,Huang, PRC 93,064907(2016); many others ……

Global OAM in HIC



$$\frac{dN_{part}^{P,T}}{dx}$$

Number of participant nucleons per unit x in projectile or target



Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2c(s)} \frac{\frac{dN_{\rm part}^{P}}{dx} - \frac{dN_{\rm part}^{T}}{dx}}{\frac{dN_{\rm part}^{P}}{dx} + \frac{dN_{\rm part}^{T}}{dx}}$$

$$\Delta p_z = \Delta x \frac{dp_z}{dx}$$

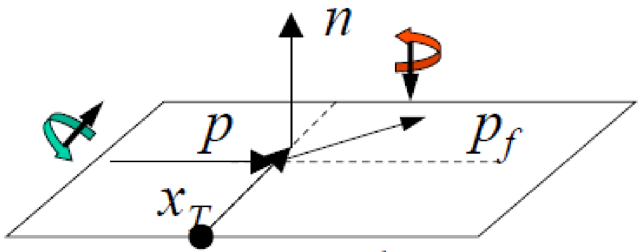
$$L_y = -\Delta x \Delta p_z = -(\Delta x)^2 \frac{dp_z}{dx} \sim \beta \omega_y$$

Liang & Wang (2005); Gao, et al. (2008); Betz, Gyulassy, Torrieri (2007); Becattini, Piccinini, Rizzo (2008); Jiang, Lin, Liao (2016); Deng, Huang (2016); many others ……

Spin-orbital coupling model

Quark scatterings in potential

- Quark scatterings at small angle in static potential with screening mass
- Unpolarized and polarized cross sections



$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \vec{n}\cdot(\vec{x}_T \times \vec{p})$$

$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$\mu \sim T\sqrt{\alpha_S}$

Polarization vector   OAM   Spin-Orbit coupling

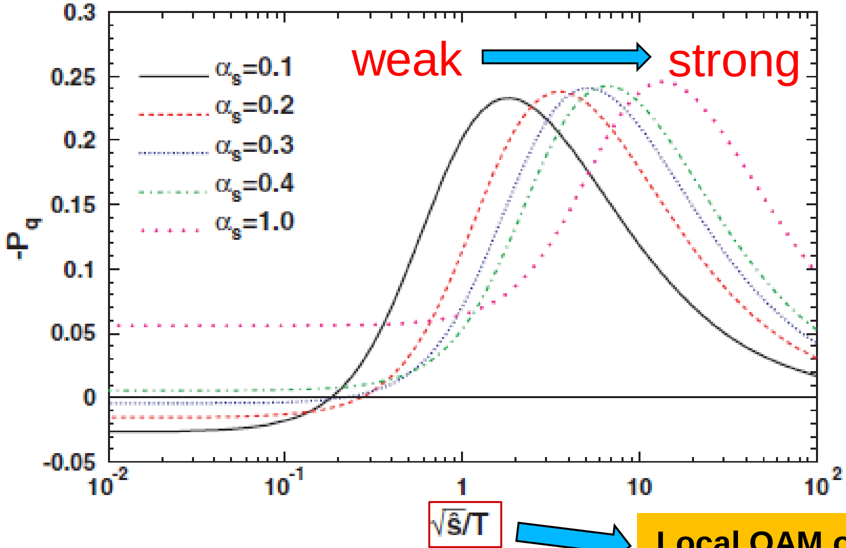
- Polarization for small angle scattering and $m_q \gg p, \mu$

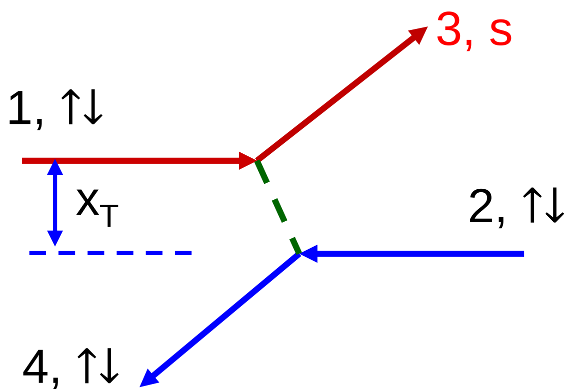
$$P_q \approx -\pi\frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

Quark-quark scattering

- **Beyond small angle approximation with HTL gluon progagator**





Local OAM or vorticity

$$L \sim \langle x \rangle p \sim \frac{p}{\mu} \sim \beta\omega$$

Quark polarization as functions of the square root of parton-parton scattering energy over T [≈ local OAM or vorticity] which increases with α_S

Liang, Wang, PRL 94, 102301(2005); PLB 629, 20(2005);
Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008)

Statistical-hydro model

Covariant form of quantum statistical physics (local equilibrium)

- To obtain covariant form in local equilibrium, we use principle of maximal entropy with conservation of total energy-momentum and particle number,

$$d\Sigma_\mu = d\Sigma n_\mu$$

space-like hyper-surface
n^μ is time-like vector

$$\hat{\rho}_{\mathrm{LE}} = \frac{1}{Z}\exp\left[\int_\Sigma d\Sigma_\mu\left(-\hat{T}^{\mu\nu}\beta_\nu + \zeta\hat{j}^\mu\right)\right]$$

Zubarev (1979);
Weert (1982);
Becattini et al. (2012-2015);
Hayat, et al. (2015);
Floerchinger (2016)

- Given n^μ, one can determine β^μ and ζ by

$$n_\mu\left\langle\hat{T}^{\mu\nu}(x)\right\rangle_{\mathrm{LE}}(\beta^\alpha,\zeta) = n_\mu T^{\mu\nu}(x), \quad n_\mu\left\langle\hat{j}^\mu(x)\right\rangle_{\mathrm{LE}}(\beta^\alpha,\zeta) = n_\mu j^\mu(x)$$

Energy condition — Particle number condition

- where statistical average is defined by $\left\langle\hat{O}(x)\right\rangle_{\mathrm{TE}} = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{TE}}\hat{O}(x)\right]$

Global equilibrium and stationary conditions

- **Stationary conditions**

Becattini (2012); Becattini, Bucciantini, Grossi, Tinti (2015) Becattini, Grossi (2015)

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \qquad \partial_\mu \zeta = 0$$

Killing equation

Thermal vorticity tensor

Killing vector solution ⇒

$$\beta^\mu = b^\mu + \varpi^{\mu\nu} x_\nu$$

$$\varpi^{\mu\nu} = -\frac{1}{2}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu)$$

$$b^\mu = \frac{1}{T} u^\mu$$

Density operator at global equilibrium

$$\hat{\rho}_{\mathrm{GE}} = \frac{1}{Z} \exp\left[-\beta u_\nu \hat{P}^\nu + \frac{1}{2} \hat{J}^{\nu\rho} \varpi_{\nu\rho} + \zeta \hat{Q} \right]$$

Total particle number

4-momentum vector operator

Total angular momentum tensor (OAM+spin)

Spin and polarization

- **Spin (Pauli-Lubanski) pseudo-vector**

$$\hat{S}^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\hat{J}_{\nu\rho}\hat{P}_\sigma$$
$$S^\mu = \mathrm{Tr}(\hat{\rho}_{\mathrm{GE}}\hat{S}^\mu)$$
$$\Pi^\mu = \frac{1}{S}S^\mu$$

$$[\hat{S}^\mu, \hat{P}^\nu] = 0, \quad \hat{S}^\mu \hat{P}_\mu = 0$$
$$\hat{S}^\mu \hat{S}_\mu = -S(S+1)$$

properties of spin vector

spin density in phase space for spin ½-fermions

$$S^\mu(x,p) = -\frac{1}{8m}[1-n_F(x,p)]\epsilon^{\mu\rho\sigma\tau}p_\tau \varpi_{\rho\sigma}$$

particle number at freezeout

$$N = \int \frac{d^3p}{E_p}\int d\Sigma_\lambda p^\lambda n_F(x,p)$$

Spin polarization at freezeout hypersurface

$$S^\mu = \frac{1}{N}\int \frac{d^3p}{E_p}\int d\Sigma_\lambda p^\lambda n_F(x,p) S^\mu(x,p)$$

Becattini, et al., 1610.02506;
Karpenko, Becattini, 1610.04717

Kinetic model with Wigner function

- **Kinetic approach**
- **Classical kinetic approach: $f(t,x,p)$**
- **Quantum kinetic approach: $W(t,x,p)$**

Wigner functions for fermions in background EM field

- **The Wigner function for spin 1/2 fermions in constant EM field satisfies EOM, which can be solved perturbatively in Planck constant $\hbar$.**
- **Wigner function can be decomposed in 16 generators of Clifford algebra**

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

4x4 matrix — scalar, p-scalar, vector, axial-vector, tensor

$$j^\mu = \int d^4p\mathscr{V}^\mu, \qquad j_5^\mu = \int d^4p\mathscr{A}^\mu, \qquad T^{\mu\nu} = \int d^4p p^\mu\mathscr{V}^\nu$$

Heinz, Phys.Rev.Lett. 51, 351 (1983);
Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986);
Zhuang, Heinz, Annals Phys. 245, 311(1996).
Many others ......

Spin pseudo-vector component for massive spin-1/2 fermions

- **Spin pseudo-vector is given by axial vector component of WF (Pauli-Lubanski pseudo-vector)**

$$\mathscr{A}_\mu^{(0)} = -\frac{1}{2m}\epsilon_{\mu\beta\nu\sigma}p^\beta \underline{\mathscr{S}_{(0)}^{\nu\sigma}}$$

tensor component

$$A^{(0)}(x,p) \equiv \frac{2}{(2\pi\hbar)^3}\sum_{es} s\,\theta(e\,p^0) f_s^{(0)e}(x, e\,\mathbf{p})$$

- **where in the zeroth order**

$$\mathscr{A}_\mu^{(0)}(x,p) = m\, n_\mu^{(0)}(x,\mathbf{p})\delta(p^2-m^2)\underline{A^{(0)}(x,p)}$$

$$\mathscr{S}_{\mu\nu}^{(0)}(x,p) = m\, \Sigma_{\mu\nu}^{(0)}(x,\mathbf{p})\delta(p^2-m^2)\underline{A^{(0)}(x,p)}$$

Fang, Pang, QW, Wang, PRC 94,024904(2016);
Weickgenannt, Sheng, Speranza, QW, Rischke, 1902.06513;
Gao, Liang, 1902.06510;
Hattori, Hidaka, Yang, 1903.01653;
Wang, Guo, Shi, Zhuang, 1903.03461.

Talks by
Jian-Hua Gao, Xin-Li Sheng

Polarization (spin) vector in WF

- **Polarization at zeroth order is vanishing if we assume that the chemical potential for spin-up and spin-down fermions are equal.**

Fang, Pang, QW, Wang, PRC 94,024904 (2016);

QW, Nucl. Phys. A967, 225(2017)

- **Polarization vector at the first order**

$$\Pi^{\alpha}_{(1)} \approx \frac{1}{2m}\hbar\beta\int\frac{d^3p}{(2\pi)^3}\left\{[E_p\omega^{\alpha}+QB^{\alpha}]\frac{e^{\beta(E_p-\mu)}}{[e^{\beta(E_p-\mu)}+1]^2} + [E_p\omega^{\alpha}-QB^{\alpha}]\frac{e^{\beta(E_p+\mu)}}{[e^{\beta(E_p+\mu)}+1]^2}\right\}$$

susceptiblility

+/- → particle/antiparticle

- **Polarization spectra at freezeout HS**

$$E_p\frac{d\Pi^{\alpha}(p)}{d^3p} \approx \frac{\hbar}{2m}\beta\int d\Sigma_{\lambda}p^{\lambda}\left(\tilde{\Omega}^{\alpha\sigma}p_{\sigma}\pm Q\tilde{F}^{\alpha\sigma}u_{\sigma}\right)f^{\pm}_{\rm FD}(x,p)\left[1-f^{\pm}_{\rm FD}(x,p)\right]$$

vorticity | magnetic field | susceptiblility

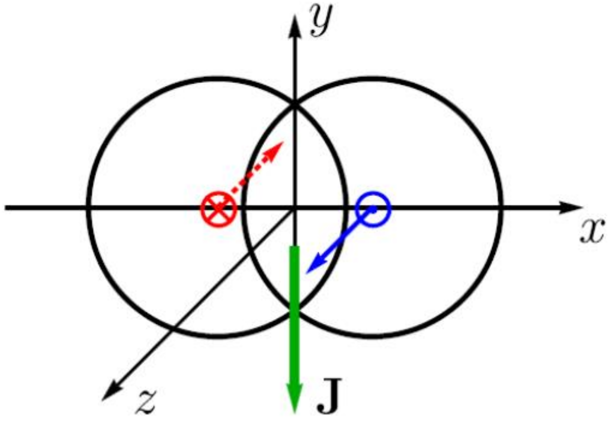
Comparison with data for global polarization

Global polarization of Λ from AMPT

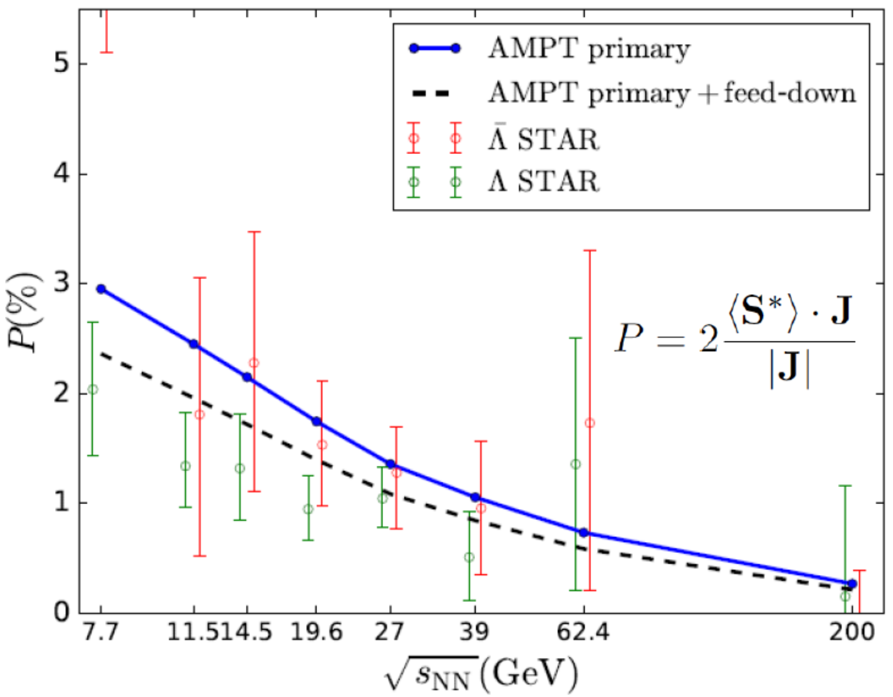
- **Polarization of Λ: average over events with |η|<1**

$$\langle \mathbf{S}^* \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathbf{S}^* (x, p)$$

$$P = 2 \frac{\langle \mathbf{S}^* \rangle \cdot \mathbf{J}}{|\mathbf{J}|}$$



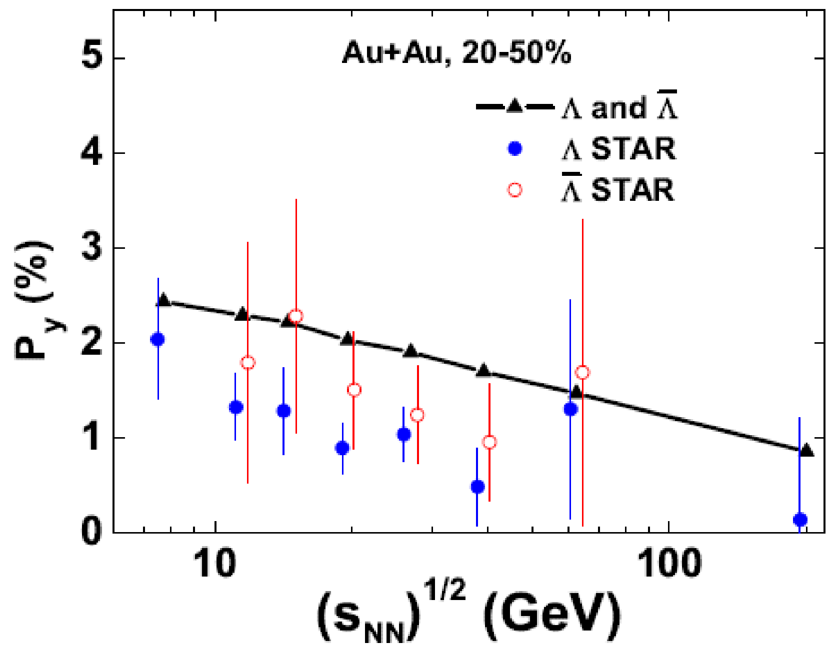
Au+Au, 20%-50%, with feed-down



Li, Pang, QW, Xia, PRC96,054908(2017)

Global polarization of Λ from Chiral Kinetic approach

- Chiral kinetic approach+ AMPT model
- Spin polarizations of quarks and antiquarks
- Quarks and antiquarks are converted to hadrons via the coalescence Model

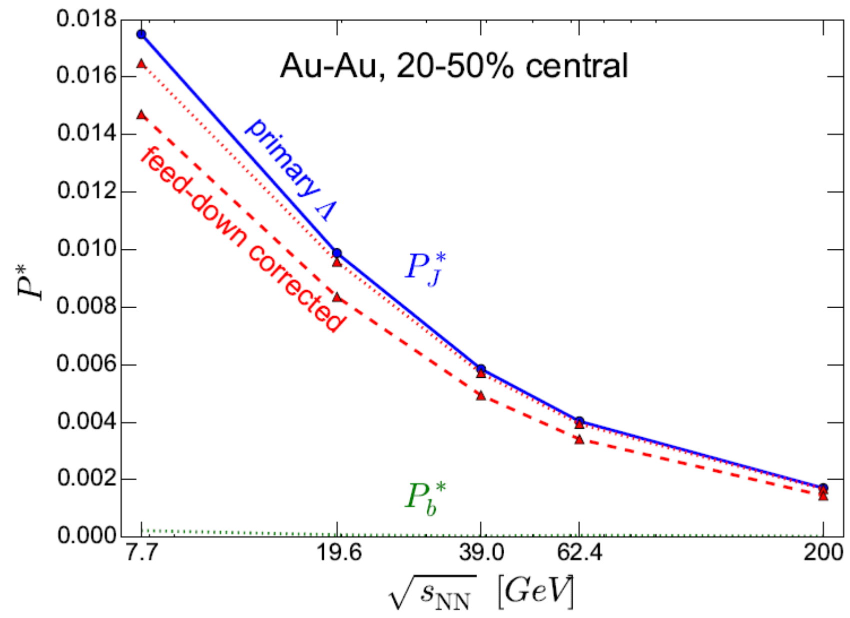


Sun, Ko, PRC96, 024906(2017)

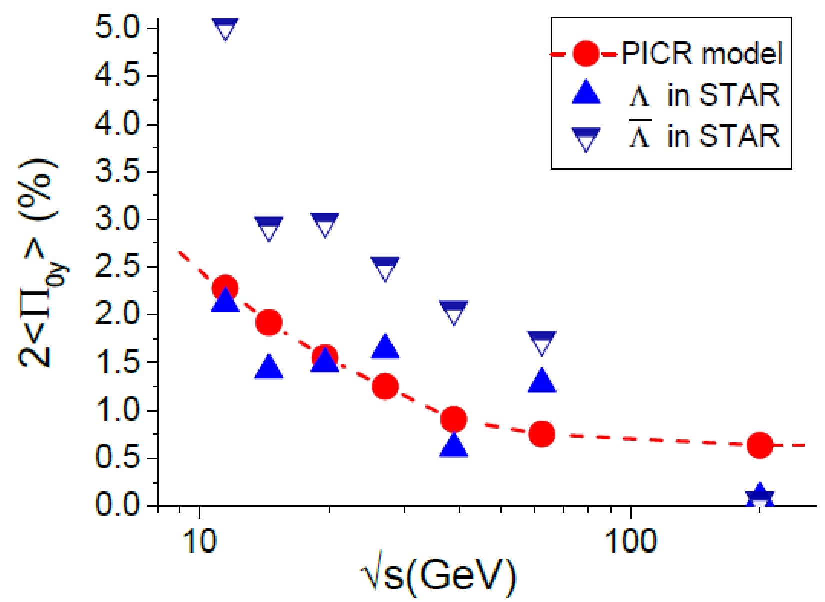
Chiral kinetic approach:

Son, Yamamoto, PRL 109 (2012) 181602;
Stephanov, Yin, PRL 109 (2012) 162001;
Chen, Pu, QW, Wang, PRL 110 (2013) 262301;
Mueller, Venugopalan, PRD 96 (2017) 016023.

Global polarization of Λ from other methods



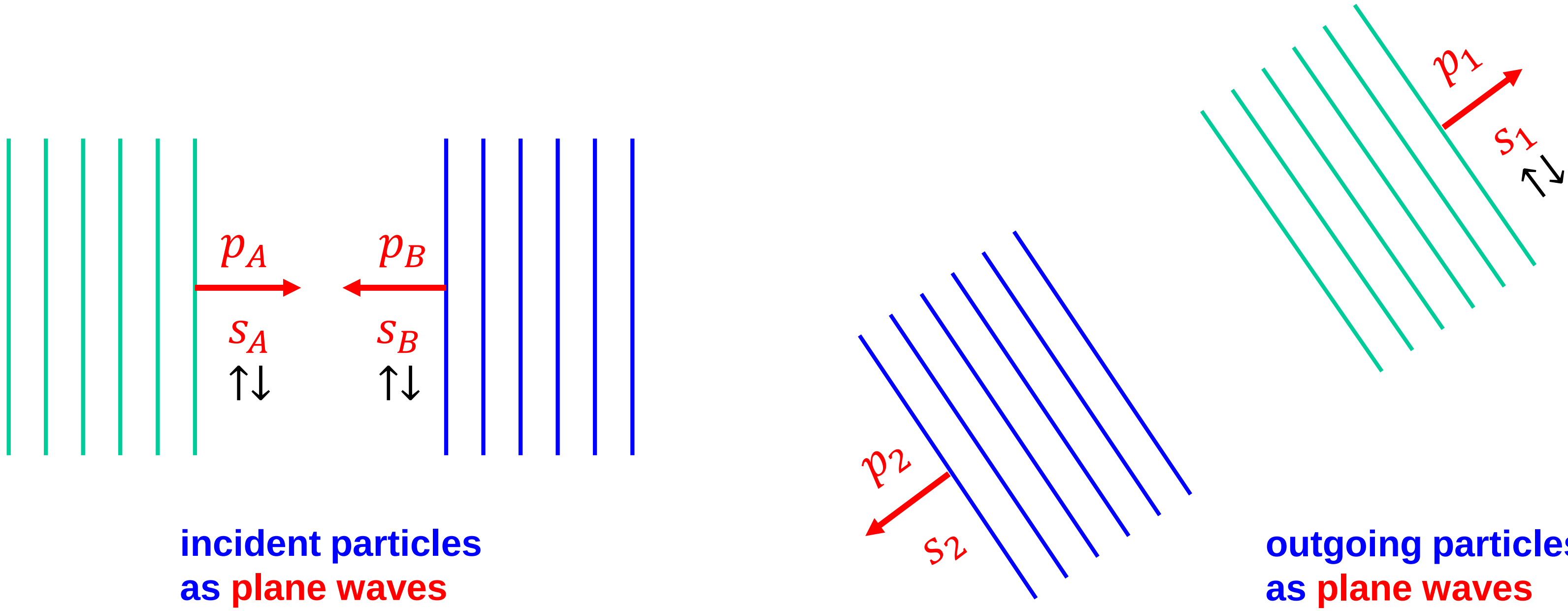
Karpenko, Becattini, EPJC 77,213(2017)
UrQMD + vHLLE hydro



Xie,Wang,Csernai,PRC 95,031901(2017)
PICR hydro

A microscopic model for global polarization through spin-orbit couplings in particle scatterings

Collisions of particles as plane waves



incident particles
as plane waves

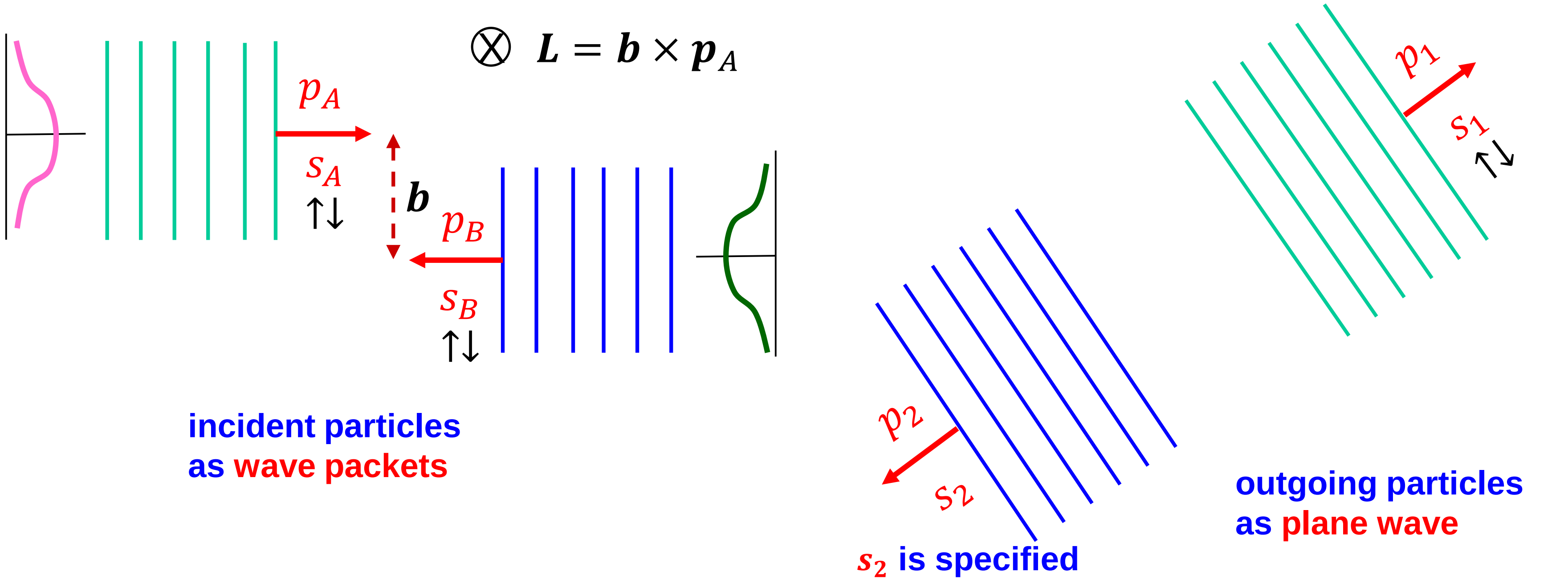
s_2 is specified

outgoing particles
as plane waves

Particle collisions as plane waves:
since there is no preferable position for particles, so there is no OAM and polarization

$$\langle \hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}} \rangle = 0 \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Incident particles as wave packets

- **Wave packets for incident particles $i = A, B$ located in phase space (x, p)**

$$|\phi_i(x_i, p_i)\rangle_{\rm in} = \int \frac{d^3 k_i}{(2\pi)^3} \frac{1}{\sqrt{2E_{i,k}}} \phi_i(\mathbf{k}_i - \mathbf{p}_i) e^{-i\mathbf{k}_i\cdot\mathbf{x}_i} |\mathbf{k}_i\rangle_{\rm in}$$

WP as Wigner function ($|\phi_i(x_i, p_i)\rangle_{\rm in}$) WP amplitude ($\phi_i(\mathbf{k}_i - \mathbf{p}_i)$) phase factor ($e^{-i\mathbf{k}_i\cdot\mathbf{x}_i}$) plane wave ($|\mathbf{k}_i\rangle_{\rm in}$)

- **Gaussian form of the wave packet amplitude in p-space**

$$\phi_i(\mathbf{k}_i - \mathbf{p}_i) = \frac{(8\pi)^{3/4}}{\alpha_i^{3/2}} \exp\left[-\frac{(\mathbf{k}_i - \mathbf{p}_i)^2}{\alpha_i^2}\right]$$

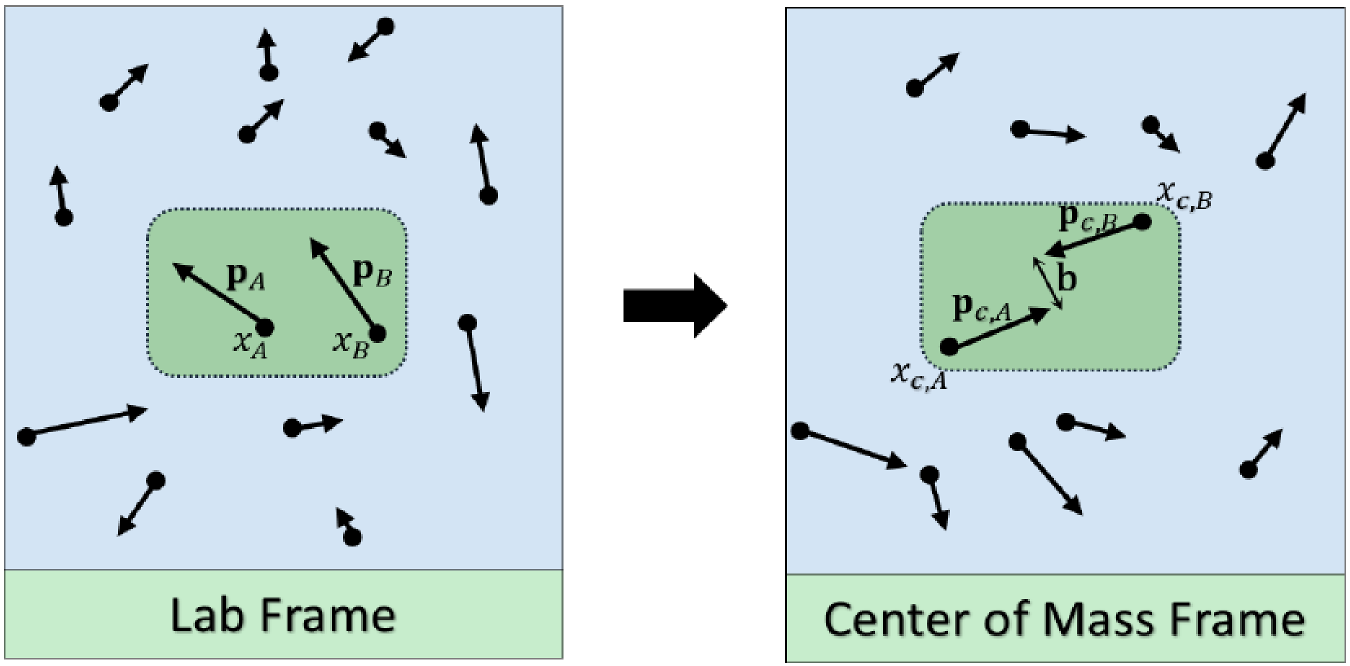
central momentum ($\mathbf{p}_i$) Gaussian width (α_i)

- **Outgoing particles are momentum states in plane waves**

$$|p_1\rangle, |p_2\rangle$$

Peskin, Schroeder (1995)

Collisions of particles at different space-time points



(1) Momentum distributions depend on $u^{\alpha}(x)$ in Lab frame

(2) Collisions of momentum states at one space-time point does not contain information about gradient of $u^{\alpha}(x)$

(3) The gradient of $u^{\alpha}(x)$ can only be probed by collisions of particles at different space-time points

Collisions of particles at different space-time points

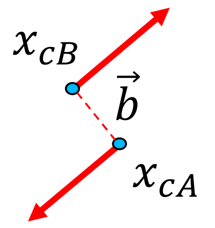
- Two incident particles at $x_A = (t_A, \mathbf{x}_A)$ and $x_B = (t_B, \mathbf{x}_B)$
- We have

$$\boxed{\mathbf{x}_A \neq \mathbf{x}_B}$$

$$t_A = t_B \;\Longrightarrow\; t_{c,A} \neq t_{c,B}$$

$$t_A \neq t_B \;\Longrightarrow\; t_{c,A} = t_{c,B}$$

$$\boxed{\mathbf{x}_{c,A} \neq \mathbf{x}_{c,B}}$$



CM frame

- We impose the causality condition in CM frame for scattering of particles at two different space-time points (the time interval and longitudinal distance of two space-time points should be small enough for scattering to take place)

$$\Delta t_c = t_{c,A} - t_{c,B} = 0$$

$$\Delta x_{c,L} = \hat{\mathbf{p}}_{c,A} \cdot (\mathbf{x}_{c,A} - \mathbf{x}_{c,B}) = 0$$

Collisions of particles at different space-time points

- **Collision rate of two particles at two space-time points in CMS**

$$
\begin{aligned}
R_{AB\to 12} \;=\; & \int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2} \\
& \times \frac{1}{C_{AB}} \int d^4x_A d^4x_B \delta(\Delta t)\delta(\Delta x_L) \\
& \times f_A(x_A,p_A) f_B(x_B,p_B) G_1 G_2 |v_A - v_B| \\
& \times (2E_A)(2E_B) |{}_{\text{out}}\langle p_1 p_2 | \phi_A(x_A,p_A)\phi_B(x_B,p_B)\rangle_{\text{in}}|^2
\end{aligned}
$$

$C_{AB} \equiv \int d^4X = t_X \Omega_{\text{int}}$

equal time and L-distance

distributions for incident particles At two points

scattering amplitude

- **We carry out integral over x_A and x_B**

$$
\begin{aligned}
I \;=\; & \int d^4x_A d^4x_B \delta(\Delta t)\delta(\Delta x_L) f_A(x_A,p_A) f_B(x_B,p_B) \\
& \times \exp\left(-i\mathbf{k}_A\cdot\mathbf{x}_A - i\mathbf{k}_B\cdot\mathbf{x}_B + i\mathbf{k}'_A\cdot\mathbf{x}_A + i\mathbf{k}'_B\cdot\mathbf{x}_B\right) \\
\approx\; & \int d^4X d^2\mathbf{b} f_A\left(X + \frac{y_T}{2}, p_A\right) f_B\left(X - \frac{y_T}{2}, p_B\right) \\
& \times \exp\left[i(\mathbf{k}'_A - \mathbf{k}_A)\cdot\mathbf{b}\right]
\end{aligned}
$$

$$X = \frac{1}{2}(x_A + x_B)$$
$$y = x_A - x_B.$$

$\vec{b} = \vec{x}_A - \vec{x}_B$

phase from impact parameter

all variables are defined in CMS but we suppress index 'c' for simplicity

Polarization of spin-1/2 particles from scatterings (general formula)

- **Polarization from particle scatterings $A + B \to 1 + 2$ at different space-time points**

all variables are defined in CMS index 'c'

$$
\begin{array}{ccc}
(s_A, p_A) + (s_B, p_B) & \rightarrow & (s_1, p_1) + (s_2, p_2) \\
\textit{wave packets} & & \textit{plane waves} \\
\textit{sum over } (s_A, s_B, s_1) & & s_2 \textit{ is open}
\end{array}
$$

$$
\begin{aligned}
\frac{d^4 \mathrm{P}_{AB\to 12}(X)}{dX^4} = & \frac{1}{(2\pi)^4} \int \frac{d^3 p_{c,A}}{(2\pi)^3 2E_{c,A}} \frac{d^3 p_{c,B}}{(2\pi)^3 2E_{c,B}} \frac{d^3 p_{c,1}}{(2\pi)^3 2E_{c,1}} \frac{d^3 p_{c,2}}{(2\pi)^3 2E_{c,2}} \\
& \times |v_{c,A} - v_{c,B}| G_1 G_2 \int \underline{d^3 k_{c,A} d^3 k_{c,B} d^3 k'_{c,A} d^3 k'_{c,B}} \\
& \times \phi_A(\mathbf{k}_{c,A} - \mathbf{p}_{c,A}) \phi_B(\mathbf{k}_{c,B} - \mathbf{p}_{c,B}) \phi^*_A(\mathbf{k}'_{c,A} - \mathbf{p}_{c,A}) \phi^*_B(\mathbf{k}'_{c,B} - \mathbf{p}_{c,B}) \\
& \times \delta^{(4)}(k'_{c,A} + k'_{c,B} - p_{c,1} - p_{c,2}) \delta^{(4)}(k_{c,A} + k_{c,B} - p_{c,1} - p_{c,2}) \\
& \times \int d^2 \mathbf{b}_c f_A\left(X_c + \frac{y_{c,T}}{2}, p_A\right) f_B\left(X_c - \frac{y_{c,T}}{2}, p_B\right) \exp\left[i(\mathbf{k}'_{c,A} - \mathbf{k}_{c,A}) \cdot \mathbf{b}_c\right] \\
& \times \sum_{s_A, s_B, s_1, s_2} \boxed{2 s_2 \mathbf{n}_c} \mathcal{M}\left(\{s_A, k_{c,A}; s_B, k_{c,B}\} \rightarrow \{s_1, p_{c,1}; s_2, p_{c,2}\}\right) \\
& \times \mathcal{M}^*\left(\{s_A, k'_{c,A}; s_B, k'_{c,B}\} \rightarrow \{s_1, p_{c,1}; s_2, p_{c,2}\}\right)
\end{aligned}
$$

wave packet momenta

wave packet

distributions for incident particles displaced by b

phase factor

scattering amplitude

polarization direction
$\vec{n}_c = \vec{b} \times \vec{p}_{cA}$

scattering amplitude

Application: quark polarization in 22 parton scatterings in QGP (locally thermalized in p)

- **Asumptions:**

  (1) local equilibrium in momentum but not in spin

  (2) $f(x,p)$ depends on x^μ through $f(x,p) = f[\beta(x) p \cdot u(x)]$

  (3) All 22 scatterings with at least one quark the in final state

- **Expansion of $f_A(x_{cA}, p_{cA}) f_B(x_{cB}, p_{cB})$ in small $y_{c,T} = (0, \vec{b})$**

$$
\begin{aligned}
& f_A\left(X_c + \frac{y_{c,T}}{2}, p_{c,A}\right) f_B\left(X_c - \frac{y_{c,T}}{2}, p_{c,B}\right) \\
= \; & f_A\left(X_c, p_{c,A}\right) f_B\left(X_c, p_{c,B}\right) + \frac{1}{2} y^\mu_{c,T} \frac{\partial(\beta u_{c,\rho})}{\partial X^\nu_c} \\
& \times \left[p^\rho_{c,A} f_B\left(X_c, p_{c,B}\right) \frac{d f_A\left(X_c, p_{c,A}\right)}{d(\beta u_c \cdot p_{c,A})} - p^\rho_{c,B} f_A\left(X_c, p_{c,A}\right) \frac{d f_B\left(X_c, p_{c,B}\right)}{d(\beta u_c \cdot p_{c,B})} \right]
\end{aligned}
$$

$$
\begin{aligned}
y^\mu_{c,T} p^\rho_{c,A} \frac{\partial(\beta u_\rho)}{\partial X^\mu_c} = \; & -\frac{1}{2} y^{[\mu}_{c,T} p^{\rho]}_{c,A} \omega^{(c)}_{\mu\rho} \\
& + \frac{1}{4} y^{\{\mu}_{c,T} p^{\rho\}}_{c,A} \left[\frac{\partial(\beta u_{c,\rho})}{\partial X^\mu_c} + \frac{\partial(\beta u_{c,\mu})}{\partial X^\rho_c} \right]
\end{aligned}
$$

local OAM
L-ω coupling

non-zero

Quark polarization rate

- **Quark polarization per unit volume: 10D + 6D integration**

$$\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} = \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B} \times |v_{c,A}-v_{c,B}| [\Lambda^{-1}]^\nu_{\ j} \mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^h_{c,A} \times f_A(X,p_A) f_B(X,p_B)(p^\rho_A - p^\rho_B)\Theta_{jk}(\mathbf{p}_{c,A}) \equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\mathbf{W}^{\rho\nu}$$

6D integral

Lorentz boost

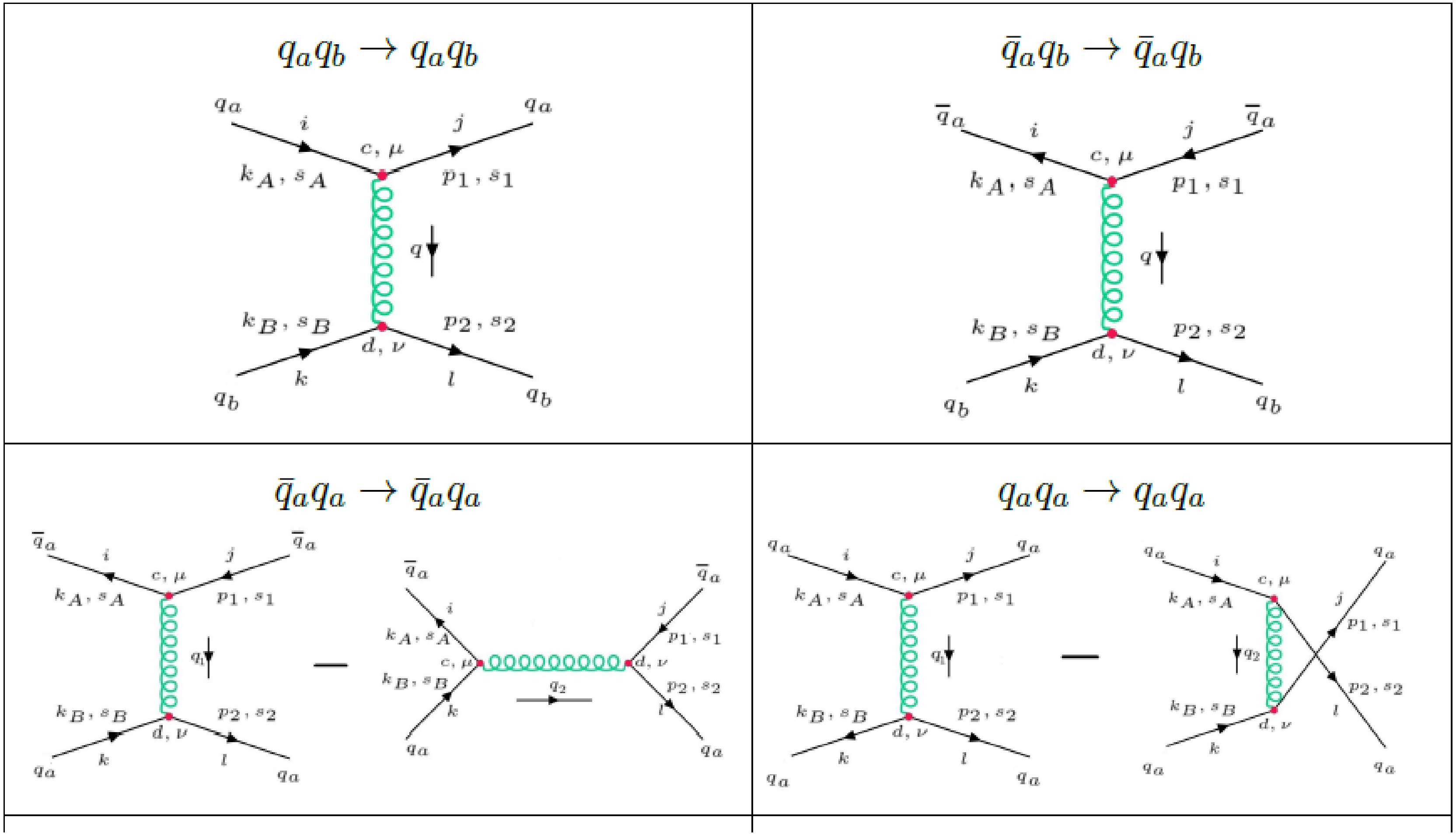
10D integral

16D integral !!

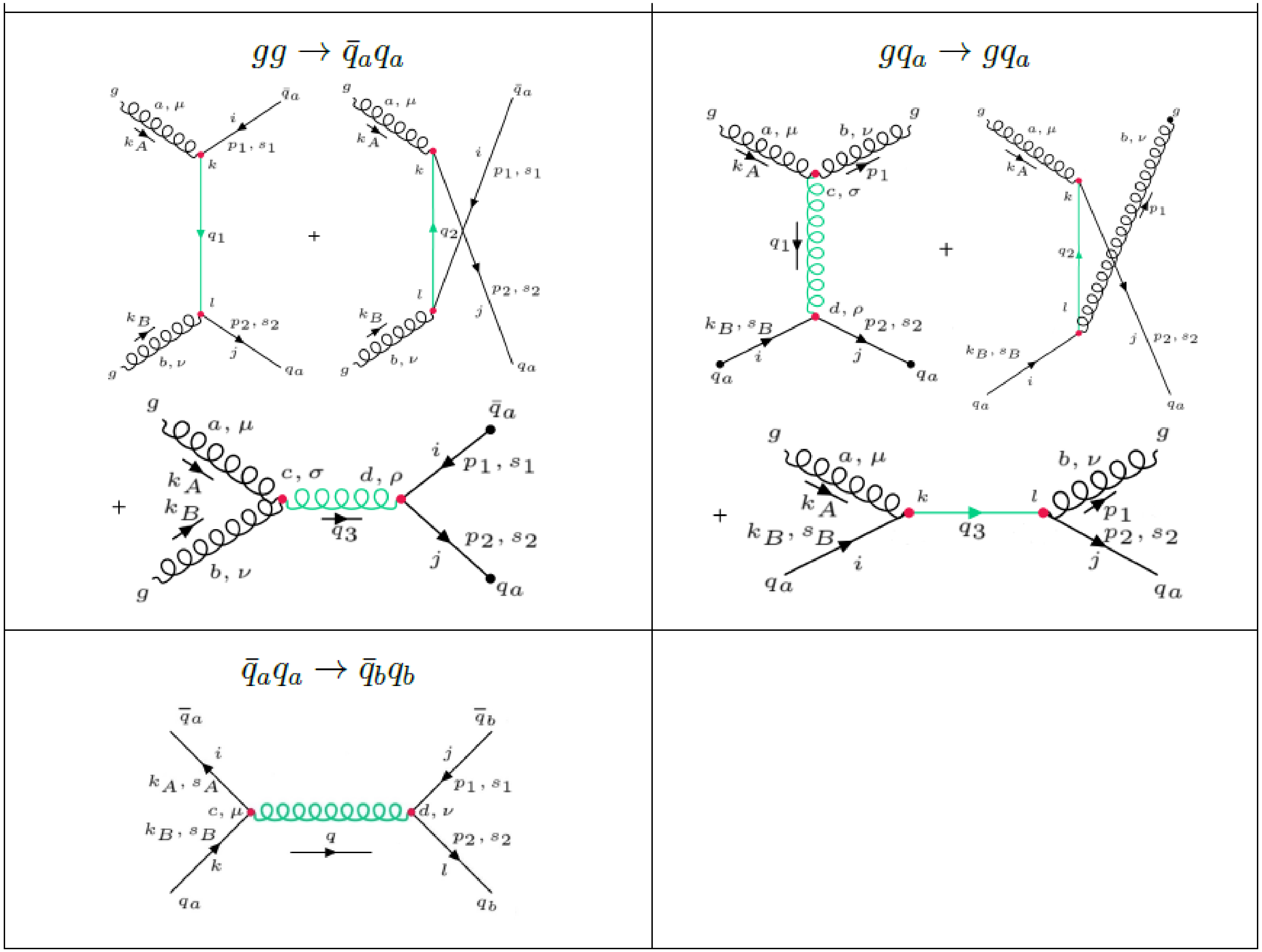
- **Numerical challenge !!! We use newly developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs [Hong-zhong Wu, Junjie Zhang, Long-gang Pang, QW, arXiv:1902.07916.]**
- **Another challenge: there are more than 5000 terms in polarized amplitude squared**

$$I_M^{q_a q_b\to q_a q_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\})\mathcal{M}^*(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\})$$

All 22 parton scaterings for quark polarization



All 22 parton scaterings for quark polarization



Numerical results for quark polarization

- **Numerical results show $W^{\rho\nu}$ has anti-symmetric structure**

$$\mathbf{W}^{\rho\nu} = W\epsilon^{0\rho\nu j}\mathbf{e}_j \quad \Rightarrow \quad \mathbf{W}^{\rho\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & W\mathbf{e}_z & -W\mathbf{e}_y \\ 0 & -W\mathbf{e}_z & 0 & W\mathbf{e}_x \\ 0 & W\mathbf{e}_y & -W\mathbf{e}_x & 0 \end{pmatrix}$$

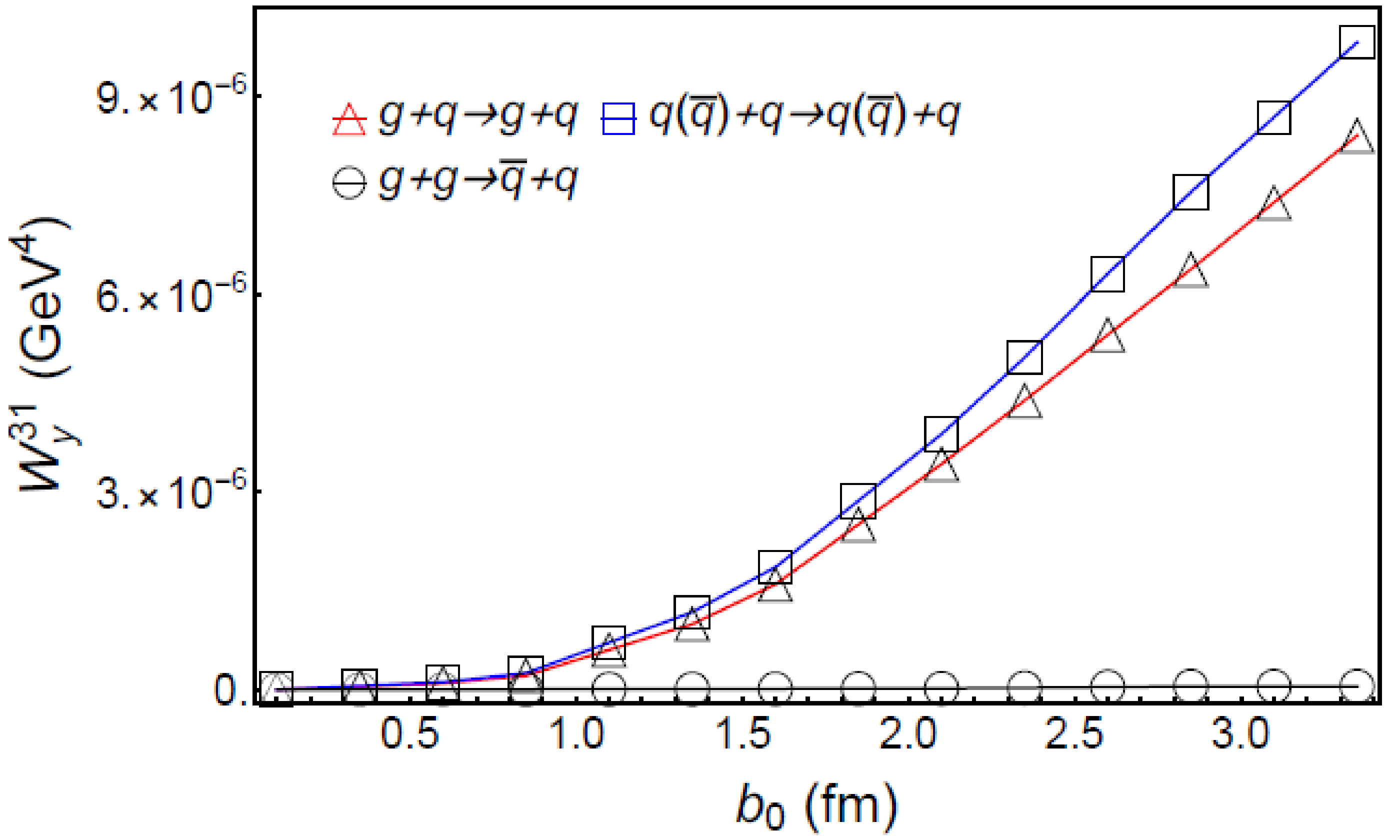
$$\begin{aligned} \frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} &= \epsilon^{0j\rho\nu}\frac{\partial(\beta u_\rho)}{\partial X^\nu}W\mathbf{e}_j = 2\epsilon_{jkl}\omega_{kl}W\mathbf{e}_j \\ &= 2W\nabla\times(\beta\mathbf{u}) \end{aligned}$$

$$\omega_{\rho\nu} = -(1/2)[\partial_\rho^X(\beta u_\nu) - \partial_\nu^X(\beta u_\rho)]$$

$$\omega_{kl} = (1/2)[\nabla_k(\beta\mathbf{u}_l) - \nabla_l(\beta\mathbf{u}_k)]$$

Polarization is given by the vorticity
up to a coefficient W
W can be calculated numerically

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4 \mathrm{P}_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

Summary

- A microscopic model for the polarization through the spin-orbit coupling in particle collisions.
- It is based on scatterings of particles as wave packets, an effective method to deal with particle scatterings at specified impact parameters.
- The spin-vorticity coupling naturally emerges from the spin-orbit one encoded in polarized scattering amplitudes of collisional integrals.
- The polarization is then the consequence of particle collisions in a non-equilibrium state of spins.
- Applications: high energy HIC (parton collisions), low energy HIC (NN collisions)