Hydrodynamization and magnetic field evolution in very early stages of HIC

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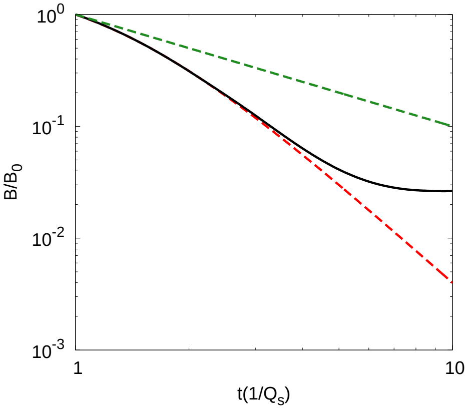
Work with Xu-Guang Huang

Highlights of the talk

- Goal: We solve dynamical evolution of $\vec{B}$ field in the early stages of heavy-ion collisions, by coupling $\vec{B}$ field evolution and the thermalization process of QGP. We consider the weakly coupled QGP scenario, namely, $\alpha_s \ll 1$.

- It is likely that there exists residual $\vec{B}$ field, with around a few % of initial field strength when hydro starts.



Motivation

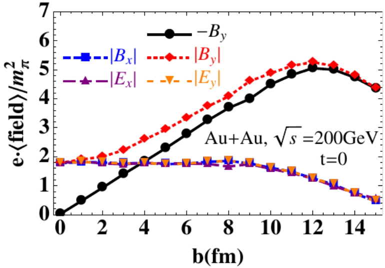
There are two important questions with respect to the study of high energy heavy-ion collisions, which have been studied so far in parallel in most cases,

- *Dynamical evolution of $\vec{B}$ field*: How does $\vec{B}$ field evolve in the presence of quark-gluon system, especially, when the quark-gluon system is far from local equilibrium?

- *Hydrodynamization*: How onset of hydro is realized in the quark-gluon system starting from far from local equilibrium?

Let's try to find overlaps in these two questions!

Dynamical evolution of $\vec{B}$ field



(Deng, Huang)

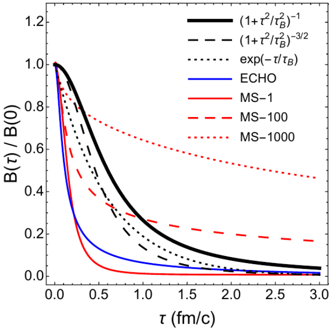


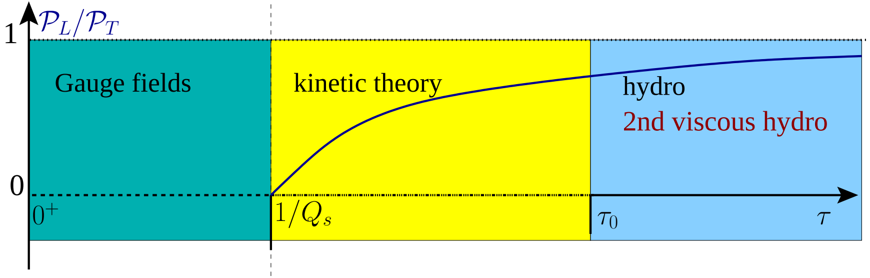
figure from 1711.02496 (Shi et. al.)

- Field strength for $t \leq t_{\rm col}$ is well-determined.
- Life-time is strongly affected by the properties of QGP.

very sensitive to early stages: out-of-equilibrium, σ,...

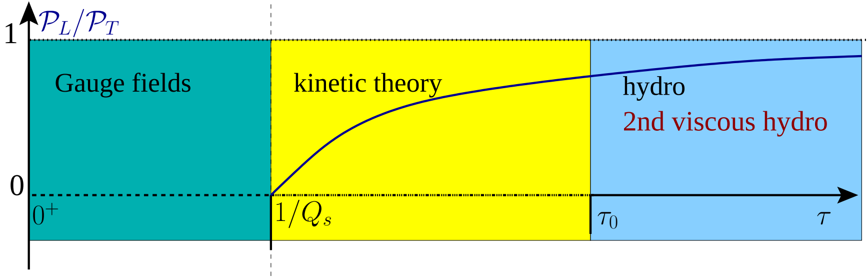
- Major source of theoretical uncertainties of CME, etc.

Hydrodynamization (weakly-coupled QGP)



- Can be described by kinetic theory, after $\tau \sim 1/Q_s$
  'bottom-up' scenario (Baier, Mueller, Schiff and Son)
- Isotropization ($\mathcal{P}_L/\mathcal{P}_T = 1$) $\Leftrightarrow$ onset of ideal hydro.
- Anisotropy ($\mathcal{P}_L \ll \mathcal{P}_T$) is expected at very early times.
  cf. solution of classical YM fields (Epelbaum, Gelis)

Hydrodynamization (weakly-coupled QGP)



- It takes very long time to reach isotropization, but viscous hydro can start very early with effective transport coefficients, especially, η/s is reduced effectively ("shear thinning fluid").
(Shuryak, Lublinsky, Romatschke, Blaizot, Yan,...)

- EB-field does not affect thermalization process: $\alpha_{\rm EM} \ll \alpha_s$

Coupled equations

$$\begin{cases} \partial_\mu A^{\mu\nu} = j^\nu \,, & \text{Maxwell} \\ \frac{1}{p^0}\left[p^\mu \partial_\mu + eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}\right] f_q = -\mathcal{C}[f_q]\,, & \text{Boltzmann} \end{cases}$$

$$\text{where } f_q = \overbrace{\bar{f}_q}^{\text{background}} + \overbrace{\delta f_q}^{\text{change due to EB}} \,,$$

$$j^\mu = e\sum_F Q_F s_F \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_p} p^\mu \underbrace{(f_q^F - f_{\bar{q}}^F)}_{g^F = \delta f_q^F - \delta f_{\bar{q}}^F}$$

* $g^F = \delta f_q^F - \delta f_{\bar{q}}^F$ difference in q and $\bar{q}$ due to coupling to EB.

* $\bar{f}_q = \bar{f}_{\bar{q}}$ because of QCD symmetry.

Simplification of the coupled equations

$$\frac{1}{p^0}p^\mu\partial_\mu g^F + \frac{1}{p^0}eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}[2\bar{f}_q + \delta f_q^F + \delta f_{\bar{q}}^F] = -\mathcal{C}[g^F]$$

$$\Rightarrow \quad \frac{1}{p^0}p^\mu\partial_\mu g^F + \underbrace{\frac{2}{p^0}eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}\bar{f}_q}_{\hat{\Gamma}[\bar{f}_q]} = \mathcal{O}(\delta f) \qquad (*)$$

- Assumptions and facts about Eq.(*):

  1 EB-field has little effect on thermalization process of QGP:

  $$|\delta f| \ll \bar{f}$$

  2 If $\bar{f}_q$ highly anisotropic, $\hat{\Gamma}[\bar{f}]$ is sizable $\leftrightarrow$ very early stages.

  3 Therefore, $$\frac{1}{p^0}p^\mu\partial_\mu g^F = -\hat{\Gamma}[\bar{f}] + \underbrace{\mathcal{O}(\delta f)}_{\text{neglected}}$$

Configuration background QGP and EB-field

- Background QGP with respect to Bjorken symmetry,

$$(t, z) \quad \Leftrightarrow \quad (\tau, \xi), \qquad \text{no dep. on } \mathbf{x}_\perp$$

$$\text{hence, } \bar{f}_q(t, z, \mathbf{p}) (= \bar{f}_{\bar{q}}(t, z, \mathbf{p})) \quad \Leftrightarrow \quad \bar{f}_q(\tau, \mathbf{p})$$

- EB-field does not obey Bjorken symmetry:

$$A^\mu(t, z) = (0, A^x(t, z), 0, 0) \quad \Rightarrow \quad \begin{cases} E_x = -\partial_t A^x \\ B_y = \partial_z A^x \end{cases}$$

We are able to study EB-field at $\vec{x}_\perp = 0$.

Background QGP: Boltzmann & 2-2 scatterings

$$D_t f^a_{\mathbf{p}} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f^a_{\mathbf{p}} = \mathcal{C}[f^a_{\mathbf{p}}]$$

for very early stages, dominated by $2 \leftrightarrow 2$ scatterings in QCD

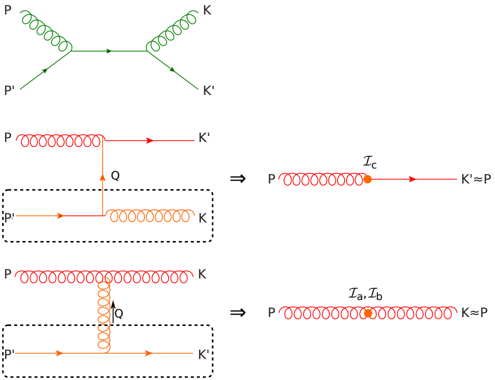
$$\begin{aligned}
\mathcal{C}[f^a_{\mathbf{p}}] =& \frac{1}{2E_p \nu_a} \sum_{b,c,d} \frac{1}{s_{cd}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} \frac{d^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}} \frac{d^3\mathbf{k}'}{(2\pi)^3 2E_{\mathbf{k}'}} \\
& \times (2\pi)^4 \delta^{(4)}(P + P' - K - K') |\mathcal{M}^{ab}_{cd}|^2 \\
& \times \left[f^c_{\mathbf{k}} f^d_{\mathbf{k}'} (1 + \epsilon_a f^a_{\mathbf{p}})(1 + \epsilon_b f^b_{\mathbf{p}'}) - f^a_{\mathbf{p}} f^b_{\mathbf{p}'} (1 + \epsilon_c f^c_{\mathbf{k}})(1 + \epsilon_d f^d_{\mathbf{k}'}) \right] ,
\end{aligned}$$

where $|\mathcal{M}|^2 \ni gg \leftrightarrow q\bar{q},\ gq \leftrightarrow gq,\ g\bar{q} \leftrightarrow g\bar{q},\ gg \leftrightarrow gg$

Diffusion approximation

small angle collisions dominate in $|\mathcal{M}|^2$:

Landau, Muller, ...



under which collision term reduces to $\mathcal{C}[f] \longrightarrow \nabla \cdot \mathcal{J} + \mathcal{S}$

More details in 1402.5049 (Blaizot, Wu, Yan), also 1703.01372 (Tanji, Venugopalan)

Initial conditions, parameters, etc.

- Background EB-field from two colliding nuclei,

$$A^x \sim \left[\frac{\tilde{z} + \tilde{v}\tilde{t}}{(\tilde{b}^2/4 + \gamma^2(\tilde{z} + \tilde{v}\tilde{t})^2)^{1/2}} + \frac{\tilde{z} - \tilde{v}\tilde{t}}{(\tilde{b}^2/4 + \gamma^2(\tilde{z} - \tilde{v}\tilde{t})^2)^{1/2}} \right]$$

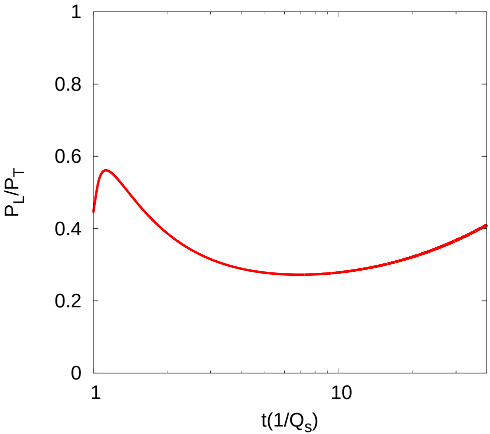
- Background QGP: CGC inspired initial quark distribution,

Romatschke, Strickland

$$f_q(t = t_0, z = 0, \mathbf{p}) f_0^q \Theta \left(1 - \frac{\sqrt{p_z^2 \xi^2 + p_\perp^2}}{Q_s} \right)$$

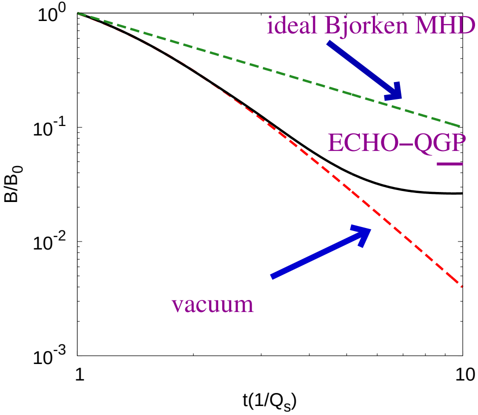
  * We have approximately $\alpha_s \sim 0.2$.
  * We take $f_0^q = 1$ as an optimistic initialization. ($f_0^q = 0$?)
    cf.1601.03576 (Gelfand, Hebenstreit, Berges)
  * $\xi > 1$ introduces anisotropy.
  * Saturation scale Q_s is the only dimensional scale, e.g., $\tilde{z} = zQ_s$

Backgound QGP evolution



- Gluon population taken: mid-central LHC PbPb 2.76TeV.
- System is away from equilibrium as expected.

B-field evolution ($z = 0$)



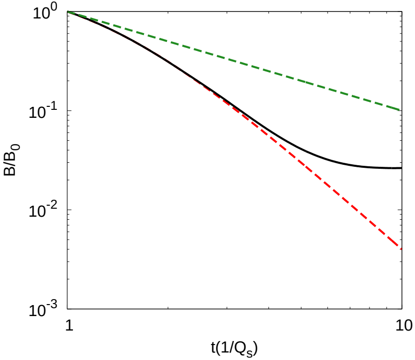
- Ideal Bjorken MHD: $\sigma \to \infty$ and $B(\tau) \sim 1/\tau$

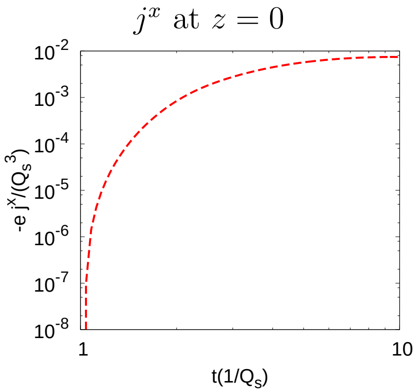
  PLB 750(2015)45-52(V.Roy et. al)

- ECHO-QGP: finite σ and 1+2D expansion.

  EPJC 2016 76:659(G.Inghirami, et.al)

B-field evolution ($z = 0$)





- Induced current

$$j^x \sim \int_{\mathbf{p}} g^F \sim \int dt \int_{\mathbf{p}} \hat{\Gamma}[\bar{f}] \sim \int dt \underbrace{n_q}_{\text{quark number density}} \times \underbrace{\int_{\mathbf{p}} \cos\theta \bar{f}}_{\text{anisotropy}}$$

- Induced B $= B - B_{\text{vcuum}} \sim \int dt j^x$

Summary and discussions

• We have solved the coupled evolution of QGP and EB-field.

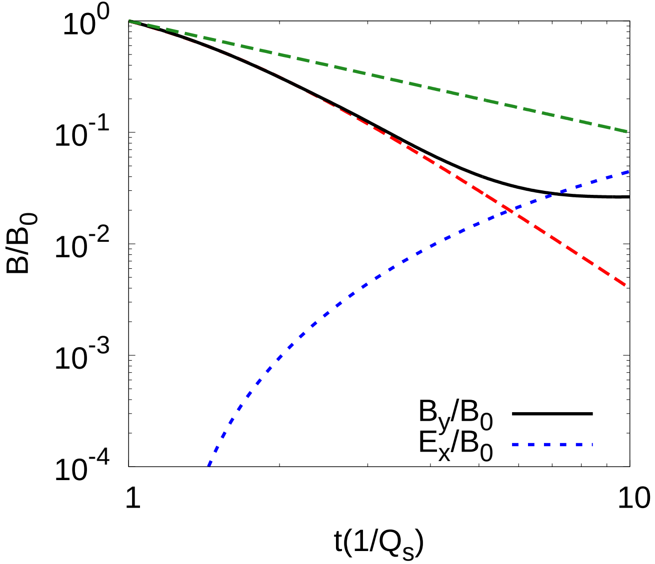
• The early stages of QGP is far from equilibrium with sizable pressure anisotropy, which plays an significant role in the dynamical evolution of EB-field at very early stages. (Implication of out-of-equilibrium effect on σ?)

• A small fraction (a few %) of initial B-field could be left in QGP medium, depending on initial quark population, initial pressure anisotropy.

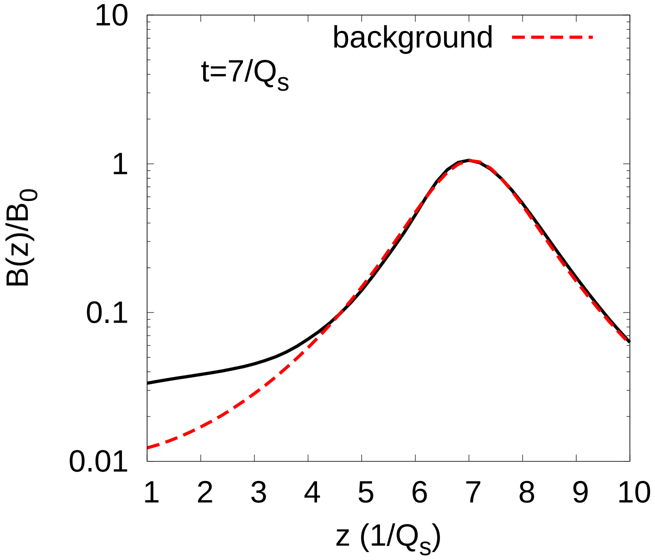
• Stay tuned for future updates!

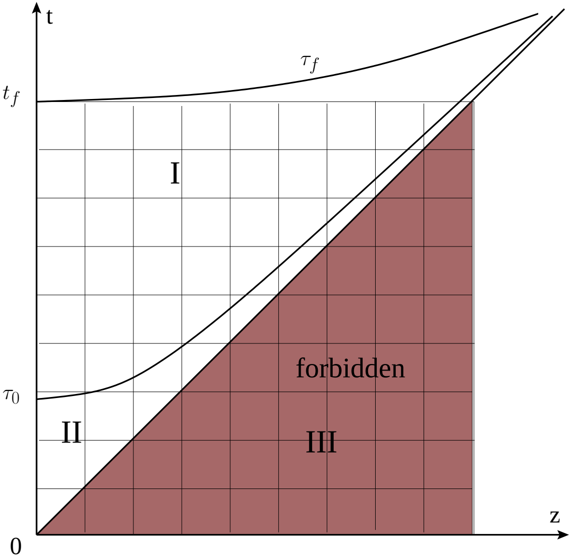
Back-up slides

EB-field evolution



B-field distribution

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τ_f
t_f
I
forbidden
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II
III
z
0