Chiral and Charged Pion Condensate in Magnetic Field with Rotation

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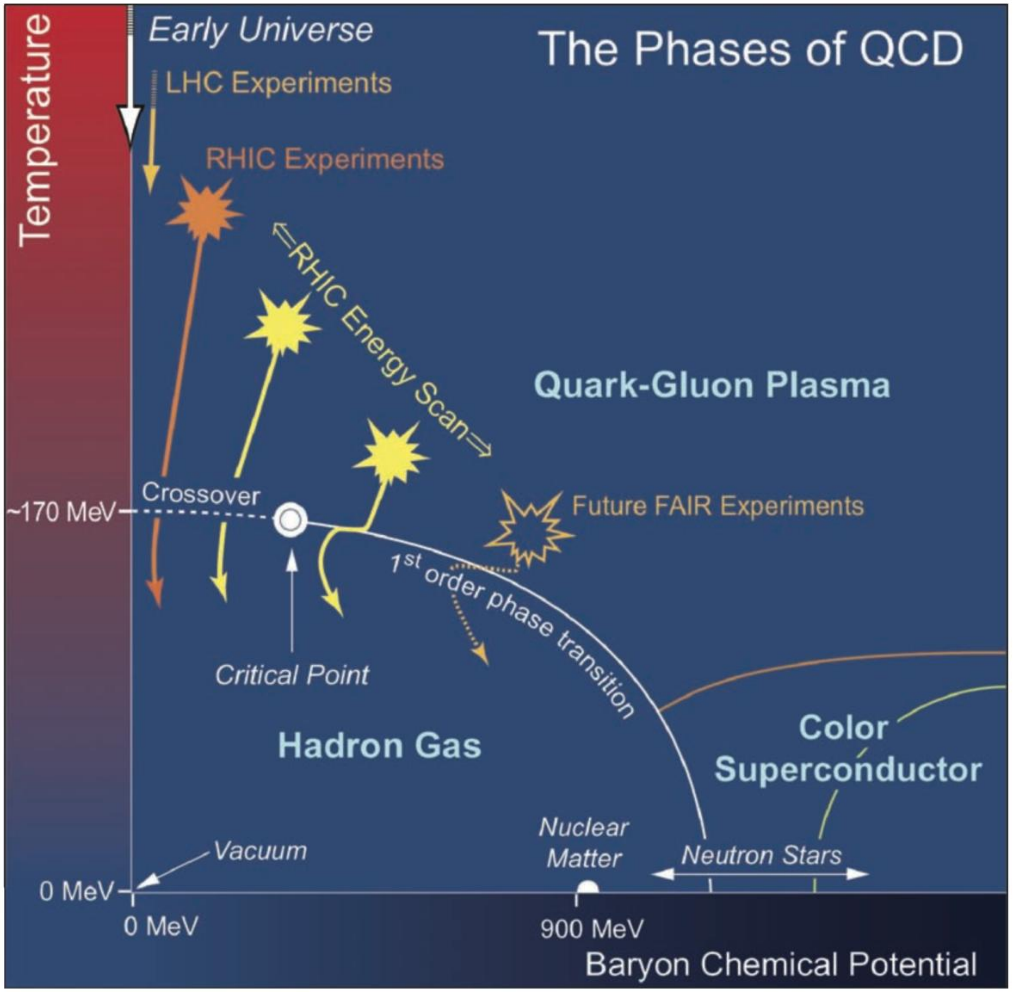
H-L. C, K.Fukushima, X.G.Huang, K. Mameda,
Phys. Rev. D 93, 104052 (2016), arXiv: 1512.08974;
Phys. Rev. D 96, 054032 (2017), arXiv: 1707.09130
H-L. C, X.G.Huang, K. Mameda, in progress

Outline

- Motivation
- Rotating frame & NJL model
- Rotational magnetic inhibition
- Charged pion condensate in rotating frame

$\boldsymbol{T}-\boldsymbol{\mu}$ phase diagram of QCD



- Other Backgrounds:

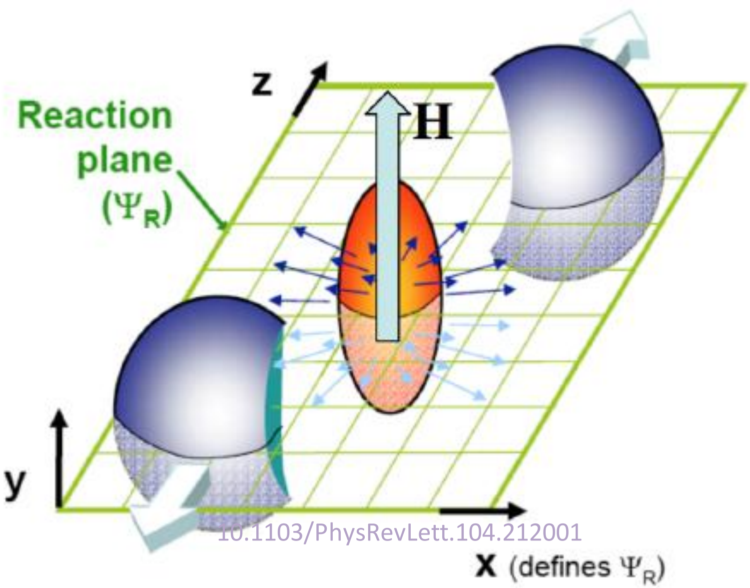
  Isospin chemical potential

  Magnetic Field

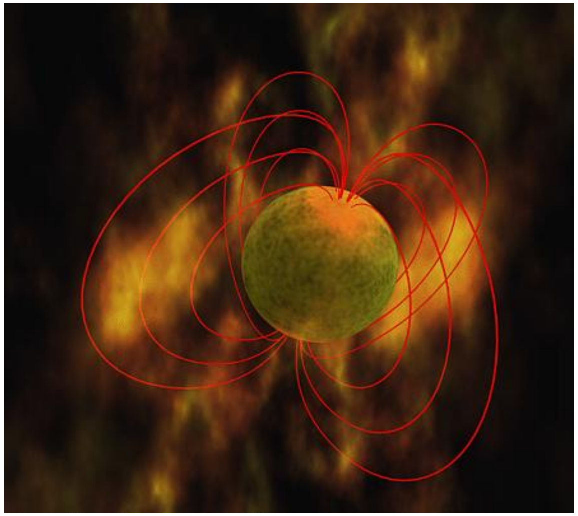
  Rotation

  Gravity

QCD in strong magnetic fields with rotation



$$R\Omega \sim 10^{-1}$$



Rotating Frame

- K-G equation

$$\frac{1}{\sqrt{-g}}D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu\phi)+m^2\phi=0$$

$$g_{\mu\nu}=\begin{pmatrix}1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0\\ y\Omega & -1 & 0 & 0\\ -x\Omega & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

- Dirac equation

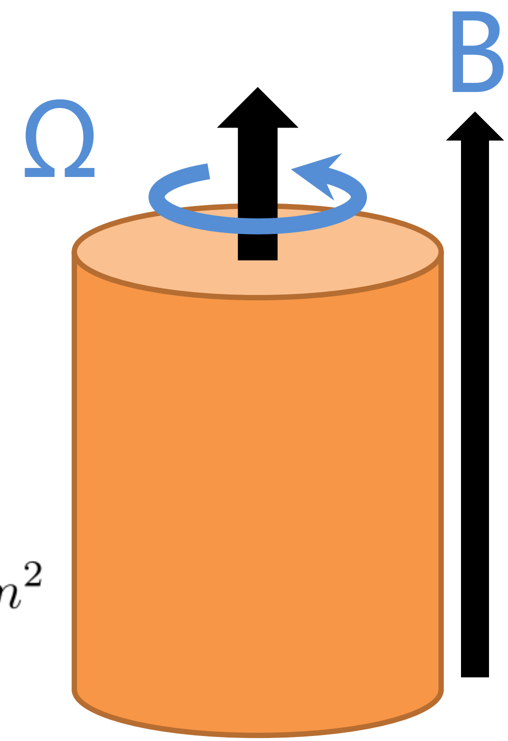
$$[ie_i^\mu\gamma^i(\partial_\mu+iqA_\mu+\Gamma_\mu)-m]\psi(x)=0$$

- e_i^μ is vierbein

  Γ_μ is spin connection

- Dispersion relation

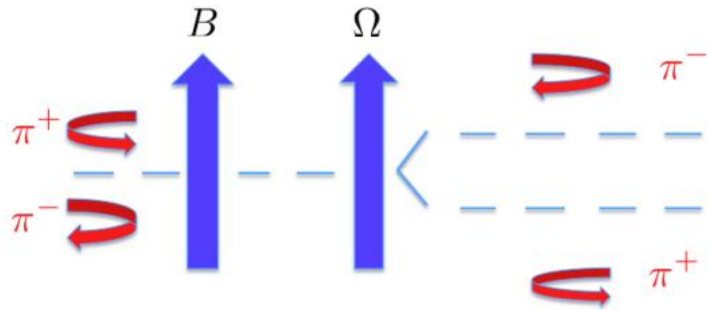
$$[E+\mathrm{sgn}(q)\Omega(l+s_z)]^2=p_z^2+(2\lambda+1-2s_z)eB+m^2$$



Pion as the DoF

- Dispersion relation

$$[E + \mathrm{sgn}(q)\Omega l]^2 = p_z^2 + (2\lambda + 1)eB + m^2$$



Y. Liu, I. Zahed, Phys.Rev.Lett. 120 (2018)

- $\Omega l \rightarrow \mu \rightarrow$ pion condensate

The Nambu-Jona-Lasinio (NJL) model

- The two flavor effective Lagrangian:

$$\mathcal{L}_{NJL} = \bar{\psi} i \gamma^{\mu} \nabla_{\mu} \psi - m_0 \bar{\psi}\psi + \mu_B \bar{\psi}\gamma^0 \psi + + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5 \vec{\tau}\psi)^2]$$

$$\nabla_{\mu} = \partial_{\mu} + iQA_{\mu} + \Gamma_{\mu}$$

- Chiral symmetry breaking
- Lack of confinement
- Nonrenormalizable field theory
- The results depend on the regularization scheme and on the UV cut-off that is used

Mean Field Approximation

- Effective action

Hard to diagonalize

$$\Gamma = -\int \mathrm{d}^4x \frac{\sigma^2 + \vec{\pi}^2}{2G} + \frac{1}{i} \ln \det(iD - i\gamma^5 \vec{\pi} \cdot \vec{\tau}),$$

$$iD = i\gamma^\mu \nabla_\mu - \sigma + \mu_B \gamma^0$$

- Magnetic field breaks SU(2) symmetry
- Perturbative expansion in π field

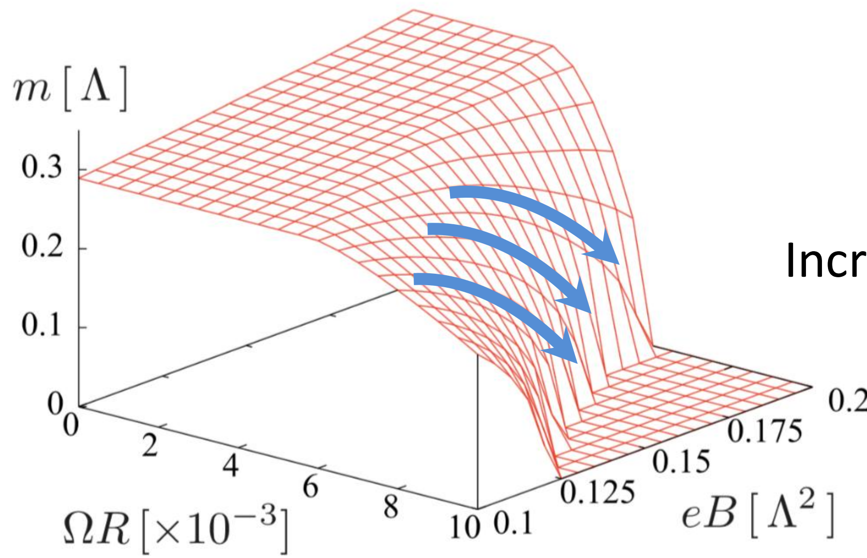
$$\Gamma = \Gamma^{(0)} + \Gamma^{(2)} + \dots,$$

$$\Gamma^{(0)} = -\int \mathrm{d}^4x \frac{\sigma^2}{2G} + \frac{1}{i} \ln \det iD,$$

Assumption: $\langle \pi^0 \rangle = 0$

$$\Gamma^{(2)} = -\int \mathrm{d}^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} \mathrm{Tr}[(iD)^{-1} \gamma^5 \vec{\pi} \cdot \vec{\tau}]^2,$$

Rotational Magnetic Inhibition



$$1/\Omega \geq R \gg 1/\sqrt{qB}$$

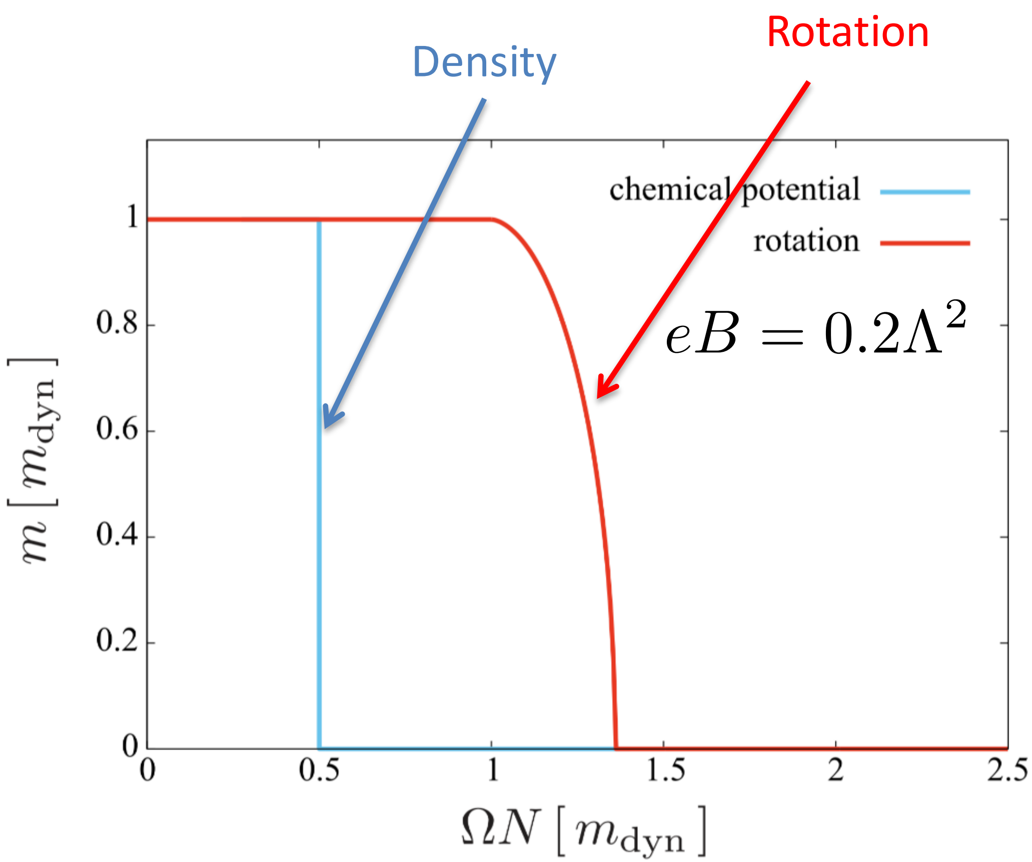
Increasing eB $\longrightarrow$ Decreasing m

HLC, et al, Phys. Rev. D 93, 104052 (2016)

- Dropping start around $\Omega N : \sqrt{eB}$
- For finite density system the inverse magnetic catalysis start around $\mu : \sqrt{eB}$ Preis, Rebhan, Schmitt (2012)

Rotational Magnetic Inhibition

- Second Order transition



Boundary effect (without rotation)

- Boundary condition

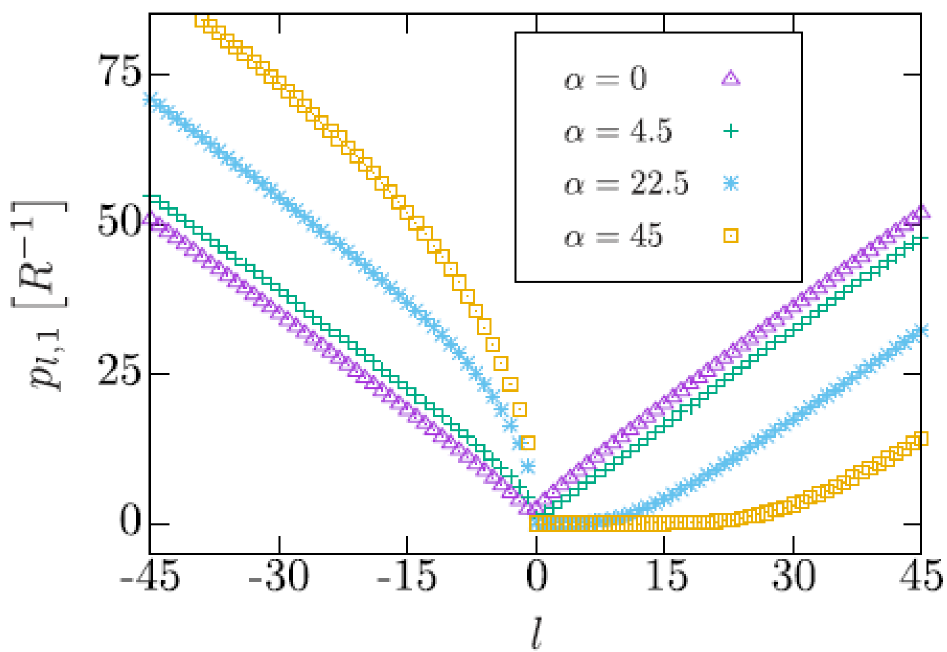
HLC, et al, Phys. Rev. D 96, 054032 (2017)

$$R\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta\, \bar{\psi}\gamma^r\psi\Big|_{r=R} = 0$$

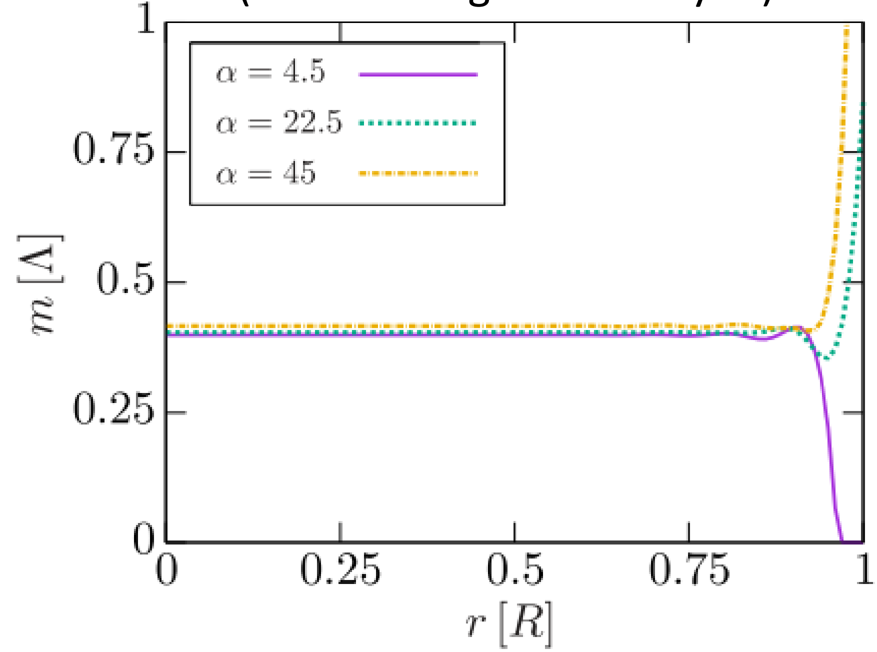
$$\alpha \equiv \frac{1}{2}eBR^2$$

Dispersion relation

$$p_{l,1} = \sqrt{2eB\lambda}$$

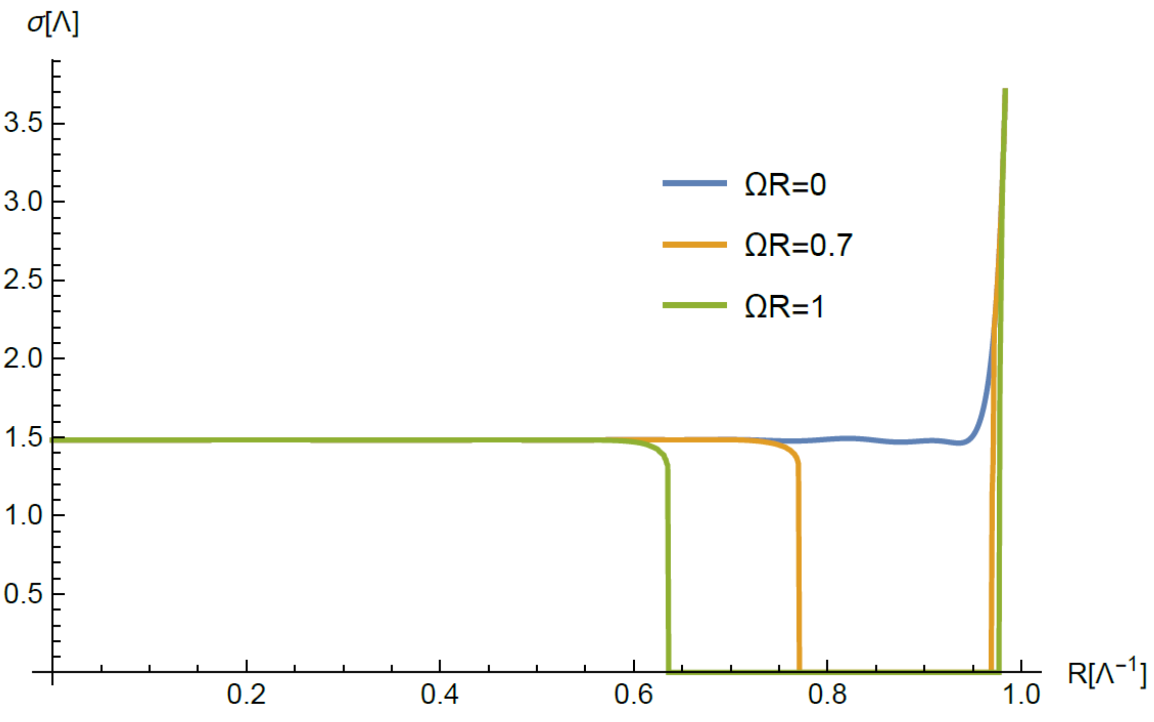


Chiral condensate
(Surface magnetic catalysis)



Rotational Magnetic Inhibition(with BC)

- Rotational Magnetic Inhibition & Surface magnetic catalysis



Schwinger Phase

Gauge independent

Schwinger phase

$$\Gamma^{(2)} = -\int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} Tr[(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1}\gamma^5 \vec{\pi}\cdot\vec{\tau}]^2$$

- LLL $\sum_{l=0}^{\infty} \frac{1}{l!} \mathrm{e}^{il(\Delta\theta+\Omega\Delta t)} (\frac{1}{2} qBr_1 r_2)^l \mathrm{e}^{-\frac{1}{4}qB(r_2^2+r_1^2)}$

$$= \exp[i\frac{1}{2}qBr_1r_2\sin(\Delta\theta+\Omega\Delta t) - \frac{1}{4}qB(r_2^2 - 2r_1r_2\cos(\Delta\theta+\Omega\Delta t)+r_1^2)]$$

$$= \exp[\boxed{-iq\int_{x_1}^{x_2} A_\mu \mathrm{d}z^\mu} - \frac{1}{4}qBC_\perp^2].$$

- In curved spacetime: geodesic line $\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\sigma^2} + \Gamma^\mu_{\nu\rho}\frac{\mathrm{d}x^\nu}{\mathrm{d}\sigma}\frac{\mathrm{d}x^\rho}{\mathrm{d}\sigma} = 0$

$$-iq\int_{x_1}^{x_2} A_\mu(z)dz^\mu = i\frac{1}{2}qBr_1r_2\sin(\Delta\theta+\Omega\Delta t)$$

- In flat spacetime:straight line

$$-iq\int_{x_1'}^{x_2'} A'_\mu(z)dz^\mu = i\frac{1}{2}qBr_1'r_2'\sin\Delta\theta'$$

$$C_\perp'^2 = (x_2'-x_1')^2 + (y_2'-y_1')^2$$

Charged Pion Condensate

$$\Gamma^{(2)} = -\int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} Tr[(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1}\gamma^5 \vec{\pi}\cdot\vec{\tau}]^2$$

- Ansatz

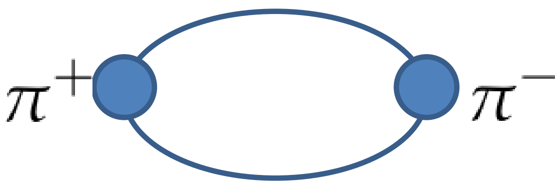
Gauge dependent part: Wilson line

$$\pi^+(x')\pi^-(x) = \mathrm{e}^{ie\int_{x'}^{x} A_\mu \mathrm{d}z^\mu}\,\tilde{\pi}^+\tilde{\pi}^-,$$

Constant for simplicity

- 2nd order thermodynamic potential

$$V_{eff}^{(2)} = C^{(2)}\tilde{\pi}^+\tilde{\pi}^-$$



- If $C^{(2)} < 0$ $\longrightarrow$ Pion condensate

Integral path 1

$$\pi^+(x')\pi^-(x) = \mathrm{e}^{ie\int_{x'}^{x} A_\mu \mathrm{d}z^\mu}\tilde{\pi}^+\tilde{\pi}^-,$$

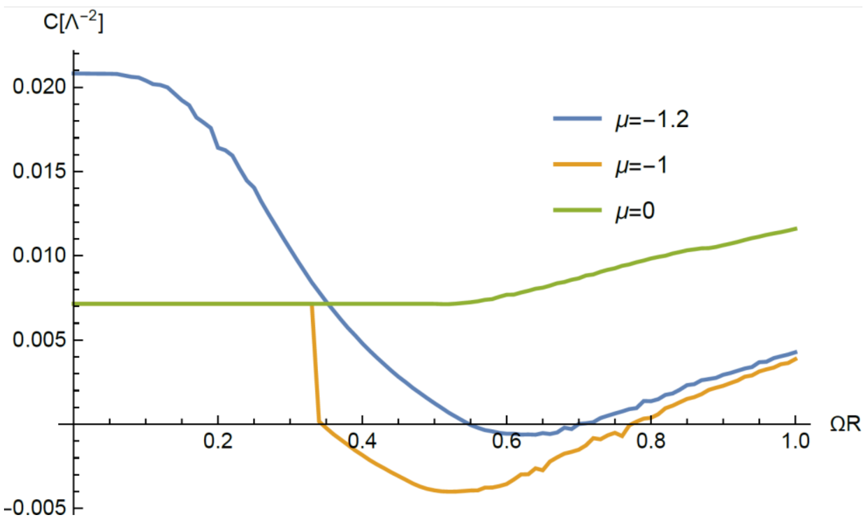
- Integrate along geodesic

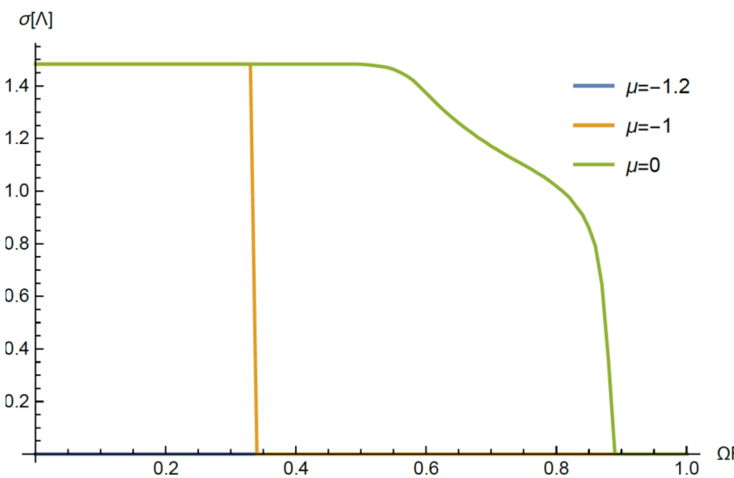
$$ie\int_{x_1}^{x_2} A_\mu dz^\mu = -i\frac{1}{2}eBr_1r_2\sin(\Delta\theta + \Omega\Delta t)$$

- Exactly cancel the Schwinger phase from the quark propagator
- Unfortunately, no pion condensate($C^{(2)} > 0$)

Integral path 1

- Include chemical potential

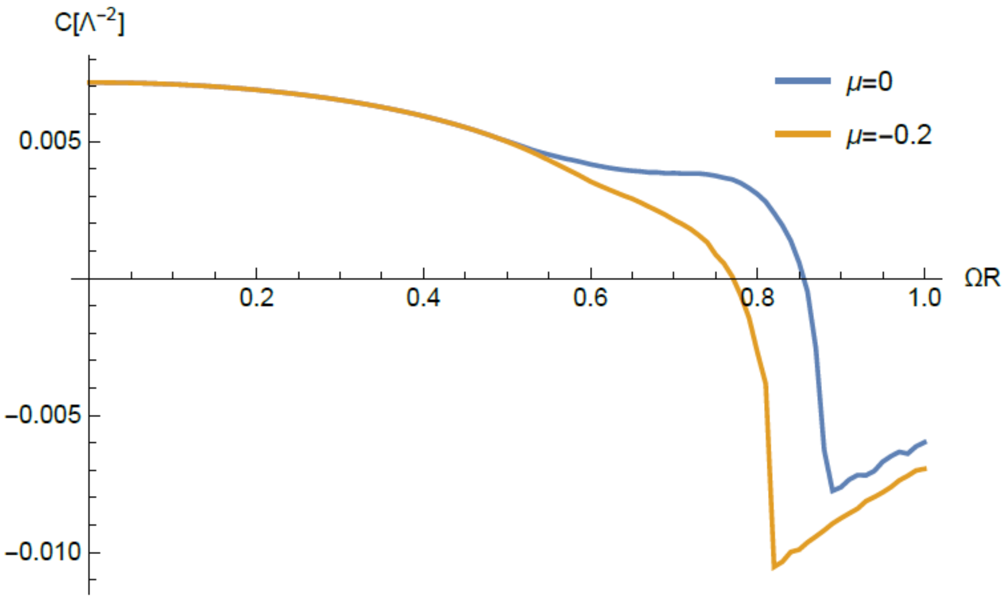




Integral path 2

$$(t_1, x_1, y_1, z_1) \to (t_1, 0, 0, z_2) \to (t_2, 0, 0, z_2) \to (t_2, x_1, y_1, z_2) \to (t_2, x_2, y_2, z_2)$$

- The same form as the Schwinger phase in flat spacetime $$ie \int_{x_1}^{x_2} A_\mu dz^\mu = -i\frac{1}{2} eBr_1 r_2 \sin \Delta\theta$$



Comparison

$$C^{(2)} = \frac{1}{2G} + \frac{q_u B|q_d B|}{S} \sum_{p_z \lambda l} \sum_{\lambda' l'} \sum_{a,b=\pm} G(a\varepsilon_u, b\varepsilon_d)$$

- Path 1

$$G(\varepsilon_u, \varepsilon_d) = \frac{1}{8\varepsilon_u \varepsilon_d} \frac{1}{\varepsilon_u - \varepsilon_d} (g(\varepsilon_u) - g(\varepsilon_d))$$

$$g(x) = \{2I_1[p_z^2 - x^2 + m^2] - 4\sqrt{q_u B|q_d B|}\,\mathrm{sgn}(jj') I_2 \sqrt{\lambda\lambda'}\} \tanh \frac{\beta(x - \Omega j - \mu_B)}{2};$$

- Path 2

$$G(\varepsilon_u, \varepsilon_d) = \frac{1}{8\varepsilon_u \varepsilon_d} \frac{1}{\varepsilon_u - \Omega J - \varepsilon_d} (g(\varepsilon_u - \Omega j) - g(\varepsilon_d + \Omega j'))$$

$$g(x) = \{2I_1[p_z^2 - (x + \Omega j)(x - \Omega j') + m^2] - 4\sqrt{q_u B|q_d B|}\,sgn(jj') I_2 \sqrt{\lambda\lambda'}\} \tanh \frac{\beta(x - \mu_B)}{2};$$

- With path 2, $\Omega J = \Omega(j_u + j_d)$ effectively behaves as an isospin chemical potential

Conclusion

- Rotational magnetic inhibition
- In certain region, rotation can induce pion condensate
- Analogy between rotation and(isospin) chemical potential?

Thank you very much!