Magnetic Field in the Charged Subatomic Swirl

Xingyu Guo[1] Jinfeng Liao[2,1] Enke Wang[1]

[1] Institute of Quantum Matter, South China Normal University

[2] Physics Department and Center for Exploration of Energy and Matter, Indiana University

The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Based on the work: arXiv:1904.04704

Outline

1. Motivation
2. Model Calculations
3. Magnetic Field in Heavy Ion Collisions

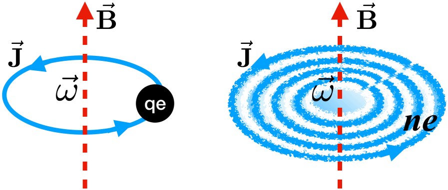
Motivation

What does the title mean?
In heavy ion collisions:

- Subatomic swirl? $\surd$

- Charged? $\surd$
- Magnetic field? $\surd$ but ...

Motivation



Charged rotating fluid should result in magnetic fields.

- Relation between B and ω?
- Beam energy / centrality dependence? Time evolution?
- Implications?

Single Particle: Classical

For a single classical charged particle with charge qe and mass m, moving with angular speed ω_0 on a circle of radius R_0
B field:

$$B_z(\rho) = B_0 \left[\frac{E\left(\frac{4\tilde{\rho}}{(1+\tilde{\rho})^2}\right)}{\pi(1-\tilde{\rho})} + \frac{K\left(\frac{4\tilde{\rho}}{(1+\tilde{\rho})^2}\right)}{\pi\left(1+\tilde{\rho}\right)} \right]$$

$$B_0 = B_z(\rho = 0) = \frac{qe\omega_0}{4\pi R_0}$$

Where $\tilde{\rho} = \frac{\rho}{R_0}$, $K(x)$ and $E(x)$ are complete elliptic integrals of first and second kind.

$$B \propto qe\omega_0$$

$$\Phi_B \sim B_0 \pi R_0^2 \propto L \sim mR_0^2\omega_0$$

Single Particle: Quantum Mechanics

For a quantum mechanical particle constrained on a 1D circle with angular momentum $L = k\hbar = mR_0^2\omega_0$, wave function $\phi = \frac{e^{ik\phi}}{\sqrt{2\pi}}$

$$B_0 = \frac{qek\hbar}{2\pi R_0^2} \propto qe\omega_0$$
$$\Phi_B \propto L$$

Charged Fluid

For an ideal fluid with charge density n,

$$J^\mu = neu^\mu.$$

Maxwell's equation:

$$\partial_\mu F^{\mu\nu} = J^\nu$$

Assuming n to be homogeneous or slow varying,

$$\begin{aligned} ne\omega^\mu &= ne\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_{[\rho} u_{\sigma]} \\ &= \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_{[\rho} J_{\sigma]} \\ &= \epsilon^{\mu\nu\rho\sigma} u_\nu \Box F_{\rho\sigma} \end{aligned}$$

Static case, local rest frame:

$$ne\vec{\omega} = \nabla^2 \vec{B}$$

Charged Fluid: Classical

For a general velocity profile $\mathbf{v} = v_0 F(\frac{\rho}{R_0})\hat{\phi}, \rho \leq R_0, \int_0^1 \mathrm{d}x F(x) x = \frac{1}{2}$,

$$\mathbf{L} = 2\pi \int_0^{R_0} \mathrm{d}\rho \rho n \rho v = 2\pi n v_0 R_0^3 \int_0^1 \mathrm{d}x F(x) x^2 \hat{z}.$$

Define 'average vorticity' by:

$$\frac{\pi}{2} n R_0^4 \bar{\omega} = \mathbf{L}$$

$$\bar{\omega} = \frac{4 v_0}{R_0} \int_0^1 \mathrm{d}x F(x) x^2 \hat{z}$$

Charged Fluid: Classical

$$\mathbf{B} = nev_0 R_0 \int_{\frac{rho}{R_0}}^{1} \mathrm{d}x F(x) \hat{z}$$

$$\bar{\mathbf{B}} = \frac{\int_0^{R_0} \mathrm{d}\rho\rho\mathbf{B}(\rho)}{\int_0^{R_0} \mathrm{d}\rho\rho} \hat{z}$$

$$= nev_0 R_0 \int_0^1 \mathrm{d}x F(x) x^2 \hat{z}$$

$$e\bar{\mathbf{B}} = \frac{e^2}{4\pi} n A \bar{\omega}$$

A is the transverse area of the vortex.

$$A = \pi R_0^2$$

The ratio is independent of $F(x)$!

Magnetic Field in Heavy Ion Collisions

$$e\bar{\mathbf{B}} = \frac{e^2}{4\pi} n A \bar{\omega}$$

Take $(20-50)\%$ centrality of Au-Au collision in $(10-200)$GeV at RHIC.

- AMPT simulations give the evolution of vorticity.
- Charge density at freeze-out can be extracted either from AMPT or using thermal freeze-out models. Its time evolution is $n_Q \sim 1/\tau^c$.
- We take $A = \pi R_0^2, R_0 = 4fm$.

Estimated Magnetic Field

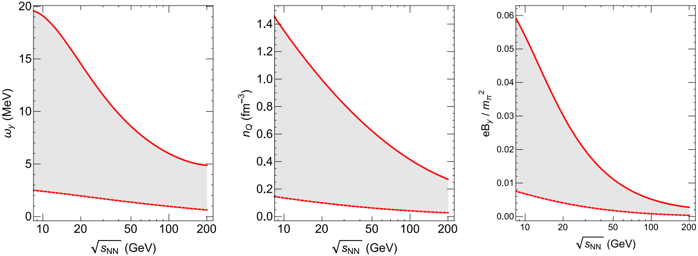


Figure: The vorticity ω_y (left), charge density n_Q (middle) and magnetic field $e\bar{B}$ (right) as functions of collisional beam energy $\sqrt{s_{NN}}$. The upper limits are for $\tau = 0.5\mathrm{fm}$ and lower limits for $\tau = 5\mathrm{fm}$.

Λ Polarization Splitting

With such an magnetic field, one expects the splitting of Λ and $\bar{\Lambda}$ polarization

$$\Delta P \equiv P_{\bar{\Lambda}} - P_{\Lambda} \simeq \frac{2|\mu_{\Lambda}|\bar{B}}{T_{fo}}$$

we use $|\mu_{\Lambda}| = 0.613\mu_N = \frac{0.613e}{2M_N}, M_N = 938MeV, T_{fo} = 155MeV$

Λ Polarization Splitting

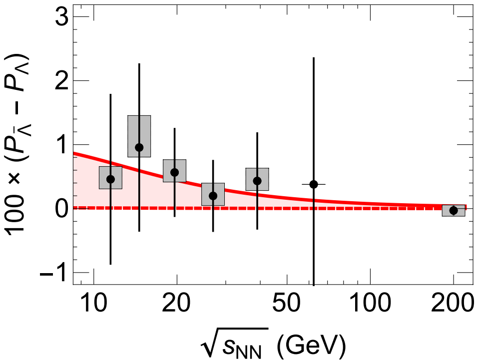


Figure: The induced polarization difference between hyperons and anti-hyperons, $\Delta P = P_{\bar{\Lambda}} - P_{\Lambda}$ as a function of collisional beam energy $\sqrt{s_{NN}}$, in comparison with STAR data[L. Adamczyk *et al.* [STAR Collaboration],Nature **548**, 62 (2017))].

Summary

- We demonstrated the relation between the average vorticity of a certain fluid and the average B field it carries.
- Using this relation, we estimated this magnetic field produced in HICs and its possible effect on Λ polarization.

- Outlook
  - More quantitative modeling.
  - More application, eg. CME: $eB\tau \simeq (5 \sim 60)\mathrm{MeV}$.