the 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions
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Transport simulations of spin and chiral dynamics

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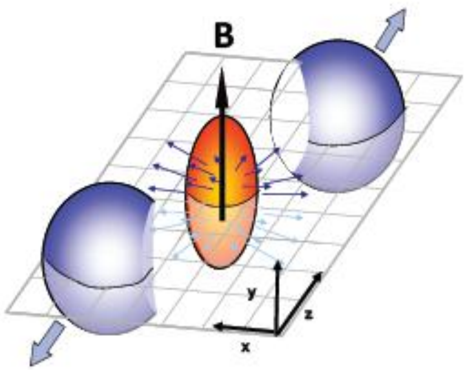
Outline

- Spin and chiral equations of motion
- Spin and chiral dynamics in a box (reminder of Wen-Hao's talk)
  - CME&CSE
  - CMW
- Chiral dynamics in relativistic heavy-ion collisions
  - Space-time evolution of the magnetic field
  - $v_2(\pi^-)-v_2(\pi^+) \sim A_{ch}$
  - Splitting of spin polarizations between Λ and Λbar

Equations of motion for massless particles

$$h = \pm\vec{\sigma}\cdot(\vec{p}-\vec{A}) = c\vec{\sigma}\cdot\vec{k}$$

$$\vec{B} = \nabla\times\vec{A}$$ **Under a vector potential**



using $$\vec{\sigma} \approx c\hat{k} - \frac{\hbar}{2k^2}\hat{k}\times\frac{d\hat{k}}{dt}$$

Adiabatic approximation

$c\vec{\sigma}\cdot\vec{k} = k$

E. van der Bijl and R.A. Duine, PRL (2011)
X.G. Huang, Scientific Report (2016)

Spin kinetic equations of motion (SEOM)

$$\frac{d\vec{r}}{dt} = c\vec{\sigma}$$

$$\frac{d\vec{k}}{dt} = c\vec{\sigma}\times\vec{B}$$

$$\frac{d\vec{\sigma}}{dt} = 2c\vec{k}\times\vec{\sigma}$$

chiral kinetic equations of motion (CEOM)

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

Phase-space volume changed

D. Xiao, J. Shi, and Q. Niu, PRL (2005)

$$d^3r d^3k/(2\pi\hbar)^3 \Rightarrow \sqrt{G} d^3r d^3k/(2\pi\hbar)^3$$

$$\langle A\rangle = \sum_i A_i\sqrt{G_i}/\sum_i\sqrt{G_i}$$

M.A. Stephenov and Y. Yin, PRL (2012)
J.W. Chen, S. Pu, Q. Wang, and X.N. Wang, PRL (2013)
D.T. Son and N. Tamamoto, PRD (2013)

CME, CSE, and CMW

4 types of particles： q=±1, c=±1

$$\mu_{qc} = q\mu + c\mu_5$$

Number density
$$\rho_{qc} = qN_c \int \frac{d^3k}{(2\pi\hbar)^3}\sqrt{G} f\left(\frac{k-\mu_{qc}}{T}\right),$$

Current density
$$\vec{J}_{qc} = N_c \int \frac{d^3k}{(2\pi\hbar)^3}\sqrt{G}\dot{\vec{r}} f\left(\frac{k-\mu_{qc}}{T}\right),$$

$$\rho_R = \rho_{q(+)c(+)} + \rho_{q(-)c(-)}$$
$$\rho_L = \rho_{q(+)c(-)} + \rho_{q(-)c(+)}$$
$$\vec{J}_R = \vec{J}_{q(+)c(+)} - \vec{J}_{q(-)c(-)}$$
$$\vec{J}_L = \vec{J}_{q(+)c(-)} - \vec{J}_{q(-)c(+)}$$

$$\rho = \rho_R + \rho_L, \quad \rho_5 = \rho_R - \rho_L$$
$$\vec{J} = \vec{J}_R + \vec{J}_L, \quad \vec{J}_5 = \vec{J}_R - \vec{J}_L$$

Isotropic Fermi-Dirac distribution f →

$$\vec{J} = \frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B},$$ Chiral magnetic effect (CME)

$$\vec{J}_5 = \frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}.$$ Chiral separation effect (CSE)

K.E. Kharzeev and H.-U. Yee, PRD (2011)

$$\mu/T \ll 1 \text{ and } \mu_5/T \ll 1,$$

$$\rho \approx \frac{N_c T^2}{3\hbar^3}\mu,$$
$$\rho_5 \approx \frac{N_c T^2}{3\hbar^3}\mu_5.$$

→

$$\vec{J}_{R/L} = \pm\frac{3\hbar e\vec{B}}{2\pi^2 T^2}\rho_{R/L}$$

$$\partial_t \rho_{R/L} + \nabla \cdot \vec{J}_{R/L} = 0$$

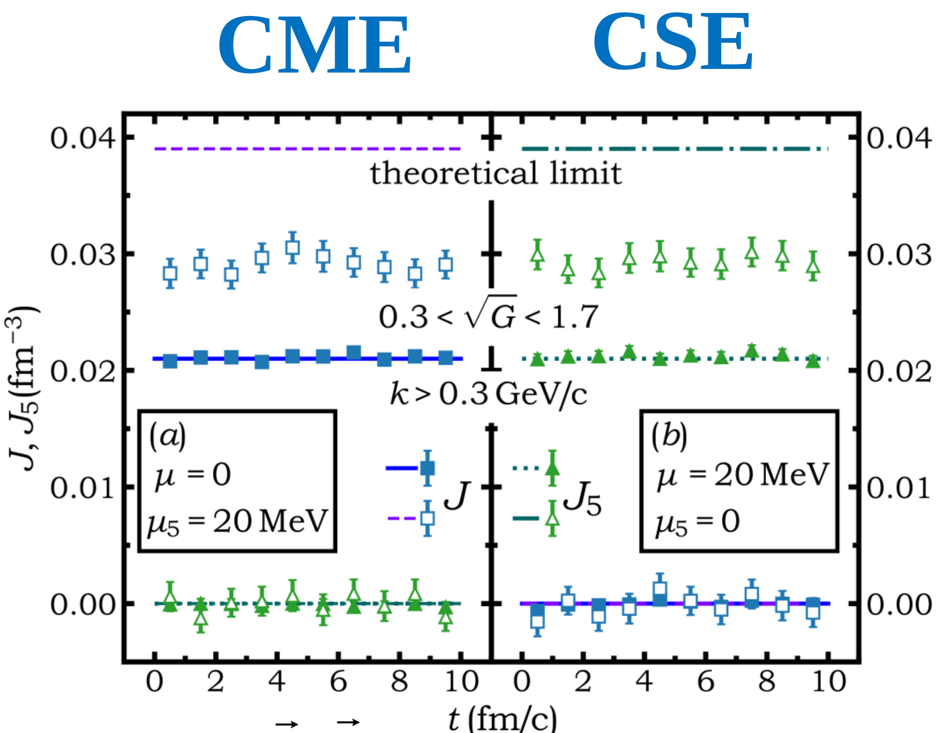
dissipation →

Chiral magnetic wave (CMW)

$$\left(\partial_t \pm \vec{v}_p \cdot \nabla - D_L \nabla^2\right)\rho_{R/L} = 0$$

$$v_p = \frac{3\hbar e B}{2\pi^2 T^2}$$

Box simulation of CME and CSE



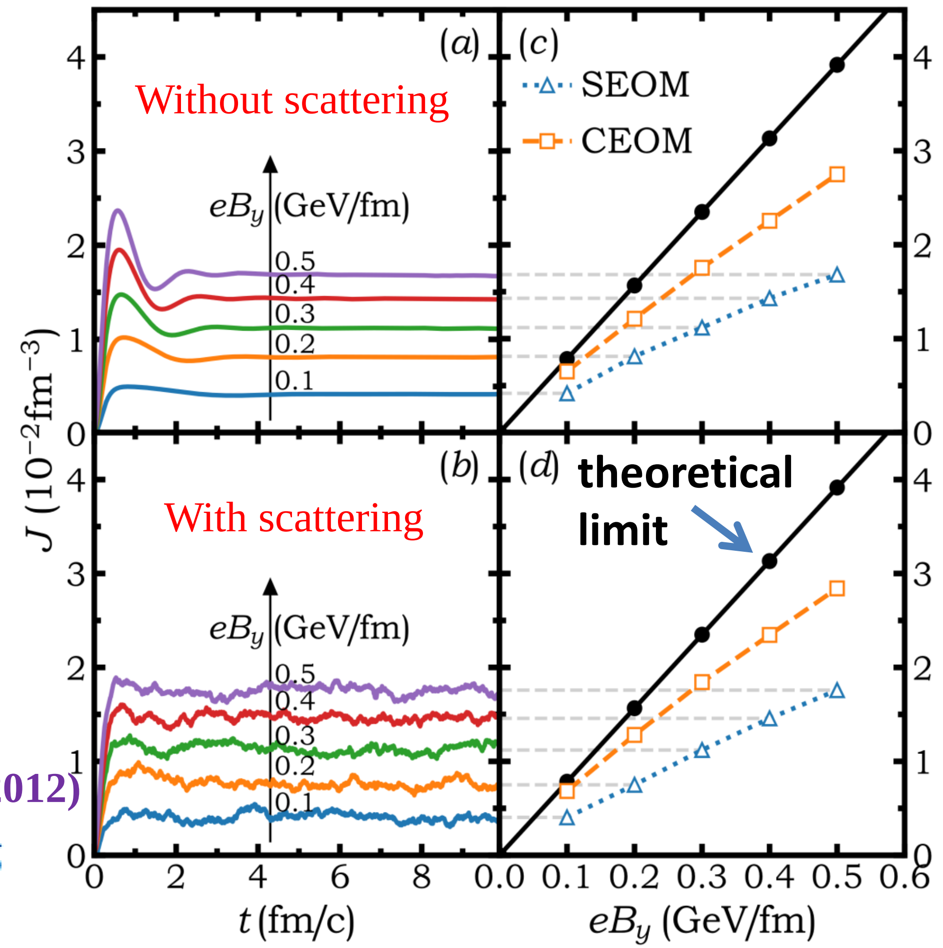
$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

Divergent at too small k

M.A. Stephenov and Y. Yin, PRL (2012)

$$\vec{J} \approx \alpha\frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B} \qquad \vec{J}_5 \approx \alpha\frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}$$

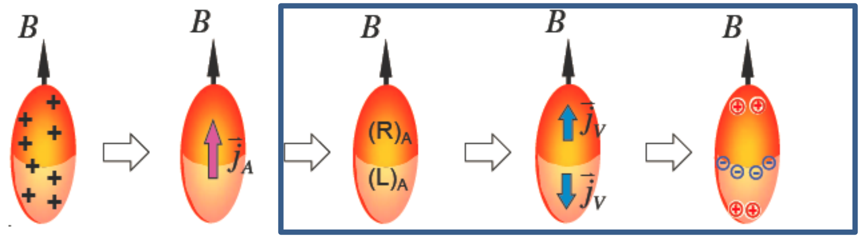
W.H. Zhou and JX, Phys. Rev. C 98, 044904 (2018)

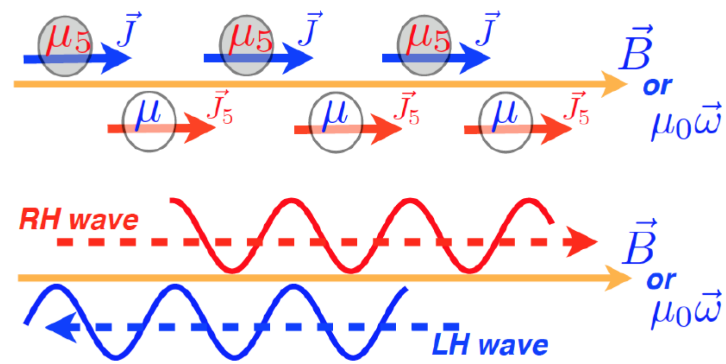


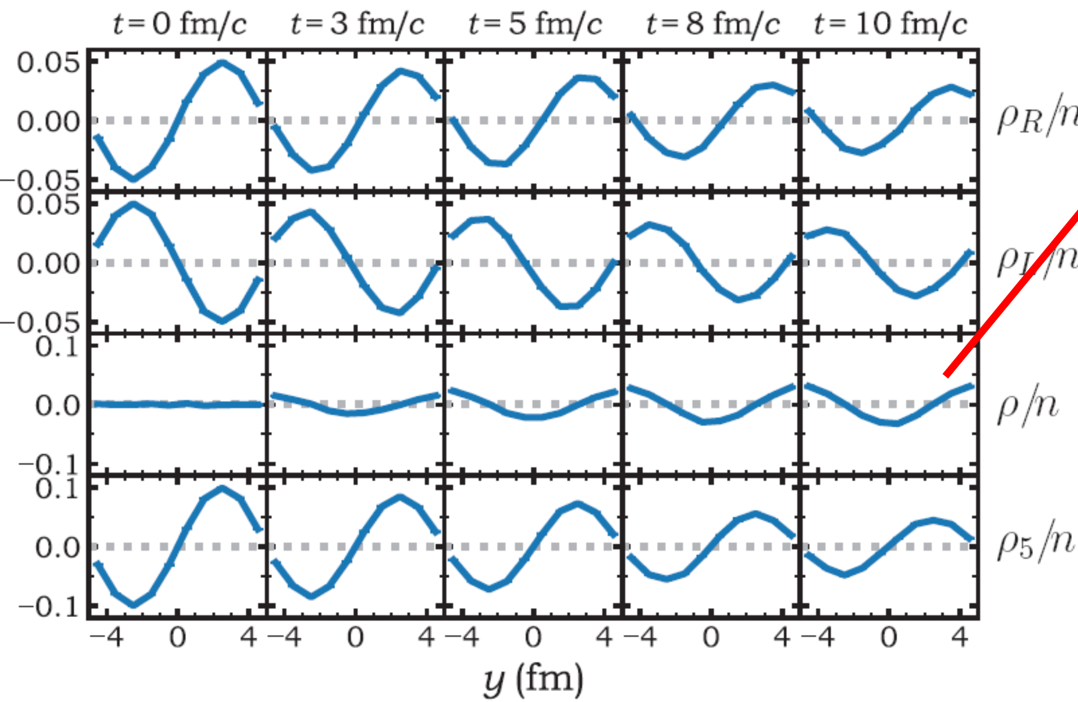
W.H. Zhou and JX, arXiv: 1904.01834 [nucl-th]

Box simulation of CMW with CEOM

$$j_A = \frac{N_c e}{2\pi^2}\mu_V B \qquad j_V = \frac{N_c e}{2\pi^2}\mu_A B$$

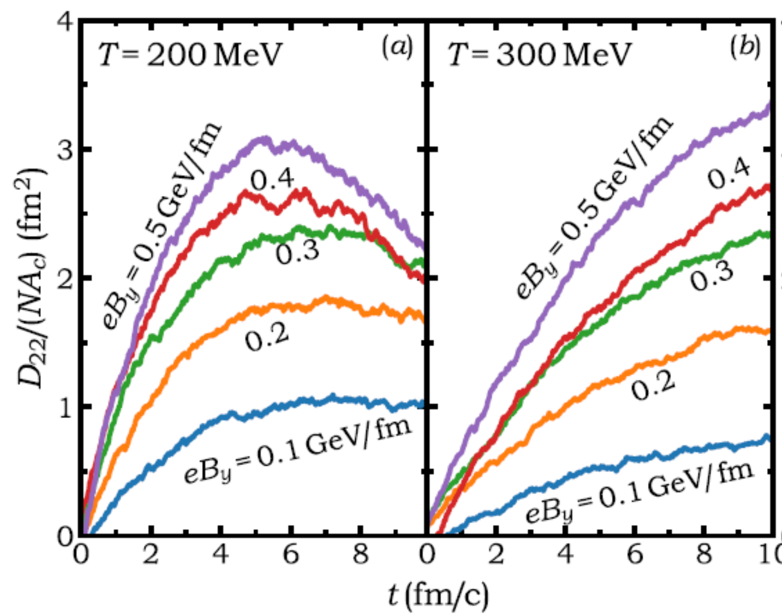






Electric quadruple moment

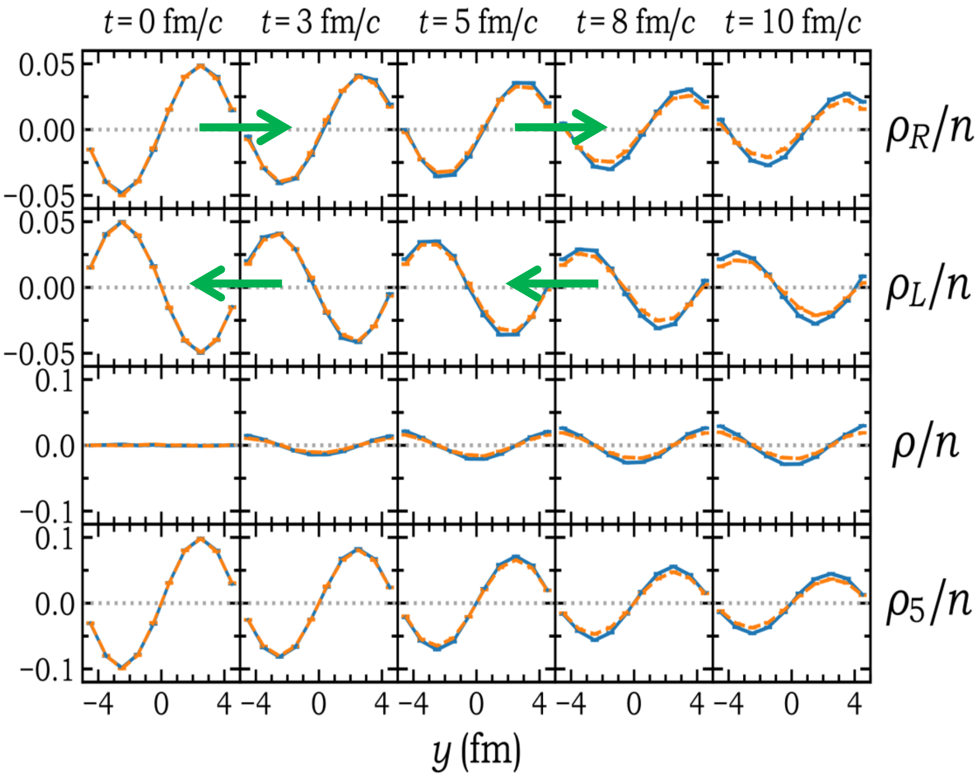
$$\mathcal{D}_{ij} = \int \rho(\vec{r})(3r_i r_j - r^2\delta_{ij})d^3r$$

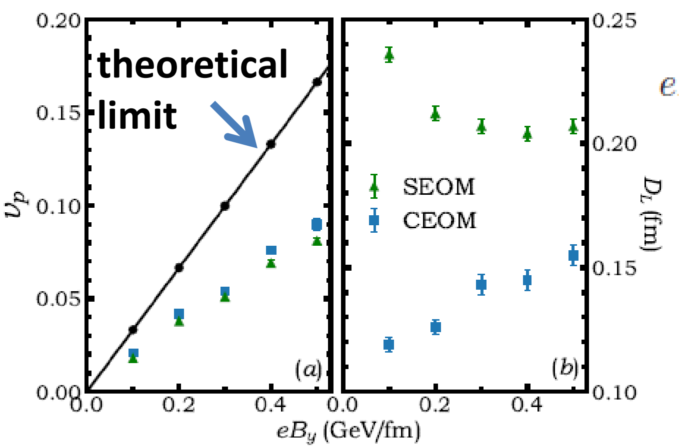


W.H. Zhou and JX, Phys. Rev. C 98, 044904 (2018)

Box simulation of CMW with SEOM vs CEOM

dashed: SEOM   solid: CEOM

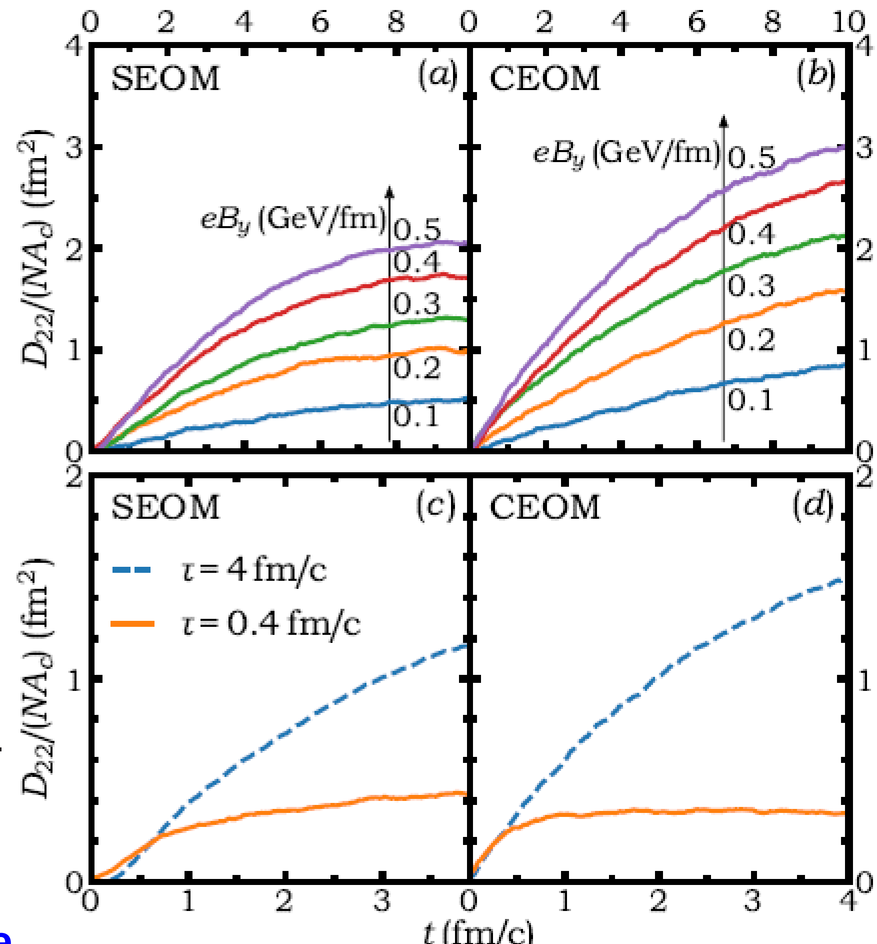




$$eB_y(t) = \frac{eB_y^0}{1+(t/\tau)^2}$$

Electric quadruple moment

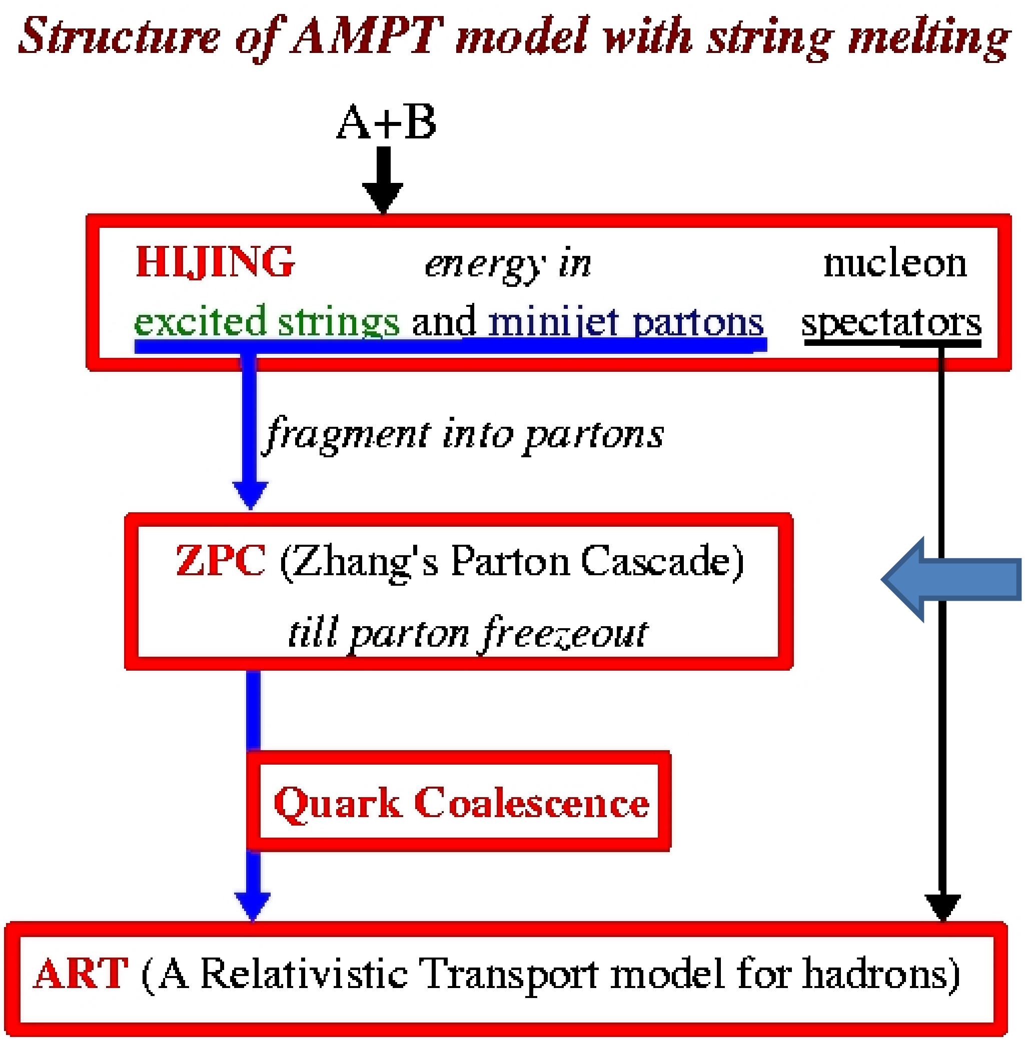
$$\mathcal{D}_{ij} = \int \rho(\vec{r})(3r_i r_j - r^2\delta_{ij})d^3r$$

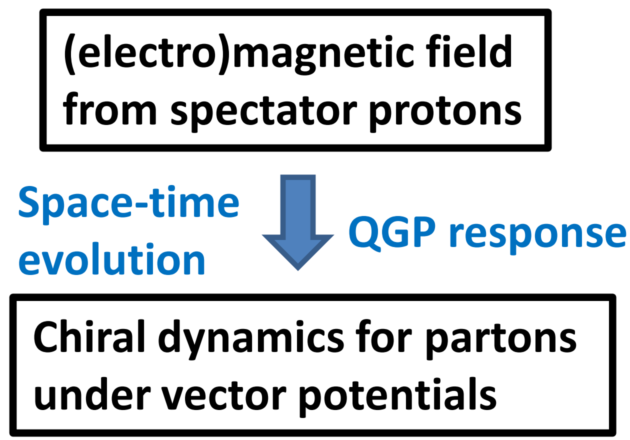


SEOM is less sensitive to the fast decay of eBy compared with CEOM.

W.H. Zhou and JX, arXiv: 1904.01834 [nucl-th]

An extended AMPT with chiral dynamics





$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B} + c\frac{\hbar}{2k^3}\vec{E}\times\vec{k}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B} + c\frac{\hbar\vec{k}}{2k^3}(\vec{E}\cdot\vec{B}) + \vec{E}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

To study spin polarization and CMW,
so far we only consider B but neglect E.

EOM under effective and real EB field

Lagrangian with vector potential $$\mathcal{L} = \bar{\psi}\gamma_\mu(i\partial^\mu - Q\,A^\mu_{ext} - \frac{2}{3}G_V\langle\bar{\psi}\gamma^\mu\psi\rangle)\psi$$

Vector density/current $$\langle\bar{\psi}\gamma^\mu\psi\rangle = 2N_c\sum_{i=u,d,s}\int\frac{d^3k}{(2\pi)^3E_i}k^\mu(f_i - \bar{f}_i)$$

Single-particle Hamiltonian $$H = c\vec{\sigma}\cdot\vec{k} + A_0$$

$$A_0 = b_i g_V \rho_0 + q_i e\varphi$$

$$\vec{A} = b_i g_V \vec{\rho} + q_i e\vec{A}_m$$

Effective EB **real EB**

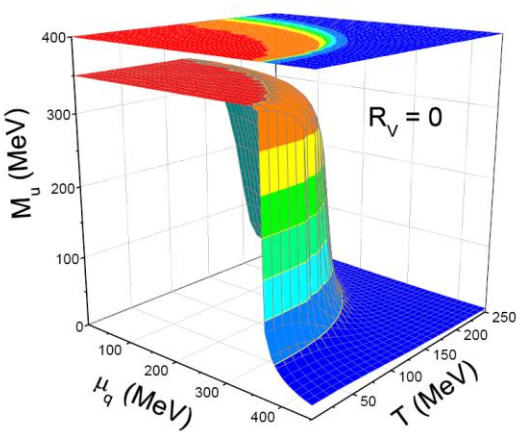
EOM
$$\dot{\vec{r}} = c\vec{\sigma},$$
$$\dot{\vec{k}} = c\vec{\sigma}\times\vec{B} + \vec{E},$$
$$\dot{\vec{\sigma}} = 2c\vec{k}\times\vec{\sigma},$$

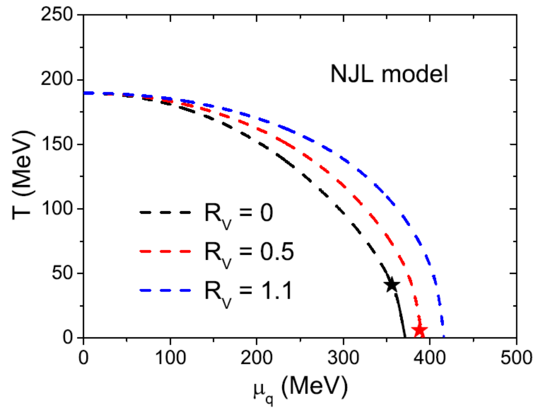
⇓

$$\sqrt{G}\dot{\vec{r}} = \hat{k} + \frac{c\hbar}{2k^2}\vec{B} + \frac{c\hbar}{2k^3}\vec{E}\times\vec{k},$$
$$\sqrt{G}\dot{\vec{k}} = \hat{k}\times\vec{B} + \frac{c\hbar\vec{k}}{2k^3}(\vec{E}\cdot\vec{B}) + \vec{E},$$

Nambu-Jona-Lasinio model

$$R_V = G_V/G$$





QGP response to the magnetic field

In vacuum (Liénard-Wiechert potential): $$A_1(\boldsymbol{r},t) = \frac{\gamma e v \hat{z}}{4\pi} \frac{1}{\sqrt{b^2 + \gamma^2 (vt - z)^2}}$$

$$j = ev\hat{z}\delta(z - vt)\delta(\boldsymbol{b}).$$

With QGP response: $$\nabla^2 \boldsymbol{A}_2(\boldsymbol{r},t) = \partial_t^2 \boldsymbol{A}_2(\boldsymbol{r},t) + \sigma \partial_t \boldsymbol{A}_2(\boldsymbol{r},t) - \boldsymbol{j}(\boldsymbol{r},t),$$

Initial condition

$$\boldsymbol{A}_2(\boldsymbol{r},t_0) = \boldsymbol{A}_1(\boldsymbol{r},t_0)$$

$$\partial_t \boldsymbol{A}_2(\boldsymbol{r},t_0) = \partial_t \boldsymbol{A}_1(\boldsymbol{r},t_0)$$

t_0 is the time when QGP is produced

Ultrarelativistic limit ⇓

$$\boldsymbol{A}_2(\boldsymbol{r},t) = \frac{\hat{z}e}{4\sigma(z/v)} \frac{\exp\left\{-\frac{b^2}{4[\lambda(t)-\lambda(z/v)]}\right\}}{4[\lambda(t) - \lambda(z/v)]}\theta(tv - z)\theta(z - vt_0)$$

Valence magnetic field

Start from t=t_0

$$+ \frac{\gamma e v \hat{z}}{4\pi}\int_0^\infty dk_\perp J_0(k_\perp b) e^{-k_\perp^2 \lambda(t) - k_\perp \gamma |z - vt_0|}$$

Initial magnetic field

Start from t=0

$$\lambda(t) = \int_{t_0}^{t} \frac{dt'}{\sigma(t')}$$

K. Tuchin, PRC (2016)

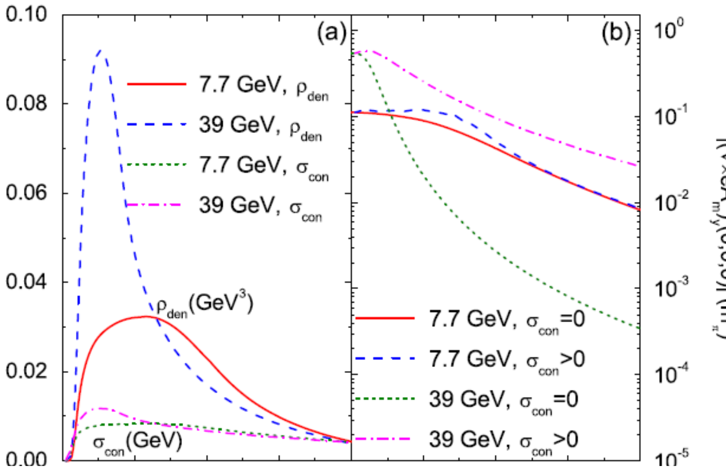
Space-time evolution of the magnetic field

In vacuum (σ_{con}=0):

$$\vec{A}_m(t,\vec{r}) = \frac{e}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n\cdot\vec{R}_n}$$

In QGP (σ_{con}>0):

$$\vec{A}^e_m = \frac{\hat{z}e}{4\sigma_{con}[(z-z_0)/v]} \times \frac{\exp\left\{\frac{-b^2}{4\{\lambda(t)-\lambda[-(z-z_0)/v]\}}\right\}}{4\{\lambda(t)-\lambda[-(z-z_0)/v]\}} \times \theta[vt-(z-z_0)]\theta[(z-z_0)-vt_0] + \frac{\hat{z}ev\gamma}{4\pi}\int_0^{+\infty} dk_\perp J_0(k_\perp b)\exp[-k_\perp^2\lambda(t) - k_\perp\gamma|(z-z_0)-vt_0|]$$

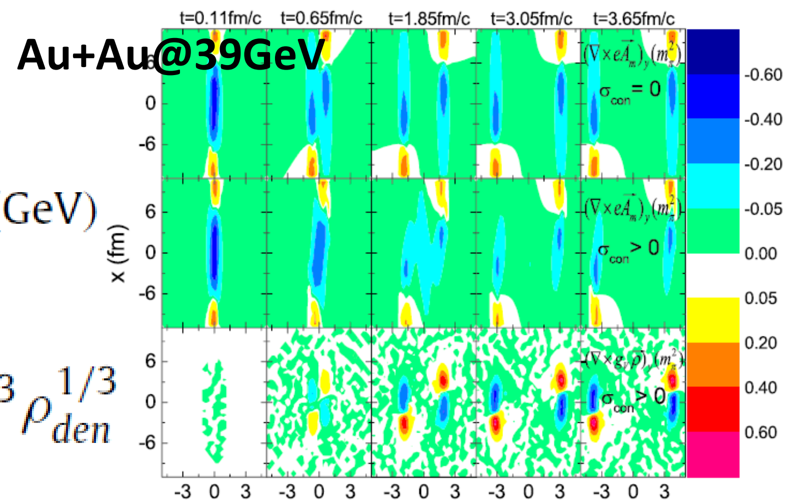


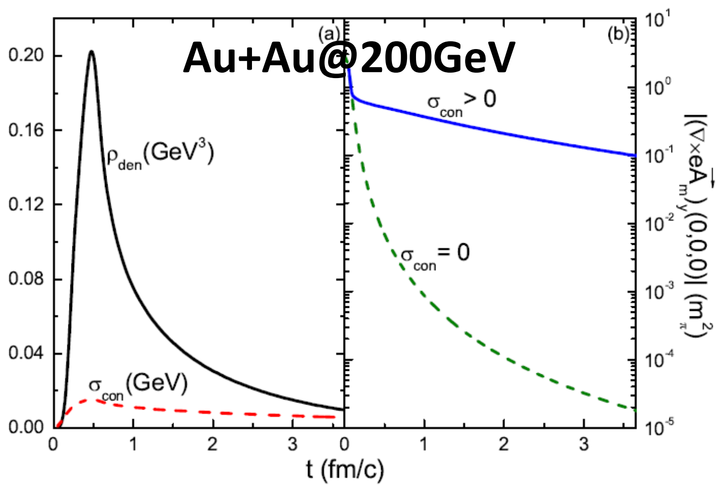
LQCD result:

$$\sigma_{con} = 0.0058\frac{T}{T_c}(\text{GeV})$$

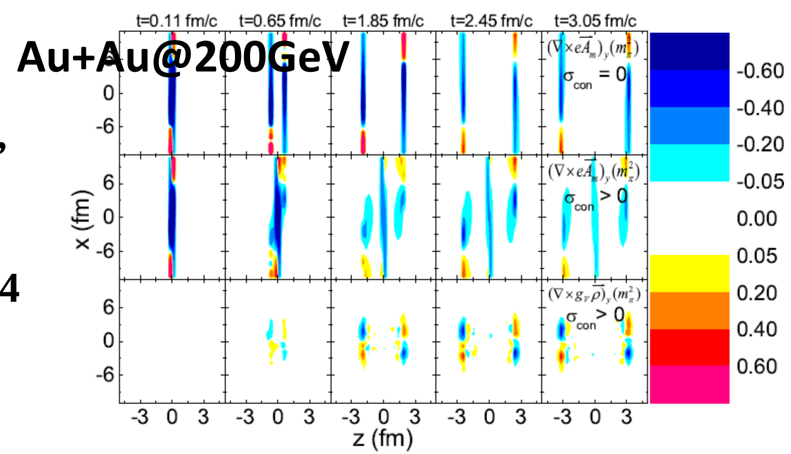
assume

$$T \approx (\pi^2/24)^{1/3}\rho_{den}^{1/3}$$

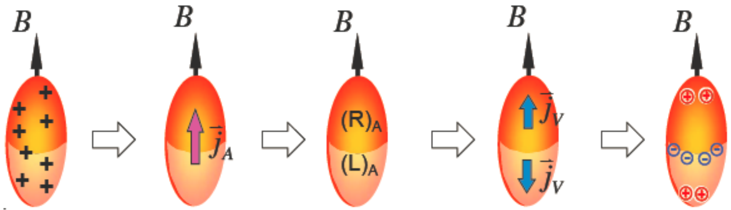


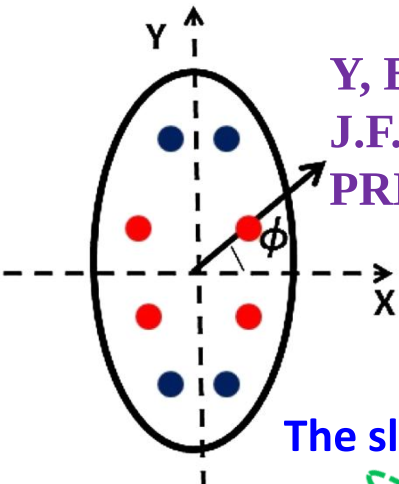


Z.Z. Han and JX, Phys. Lett. B 786, 255 (2018) ; arXiv: 1904.03544 [nucl-th]

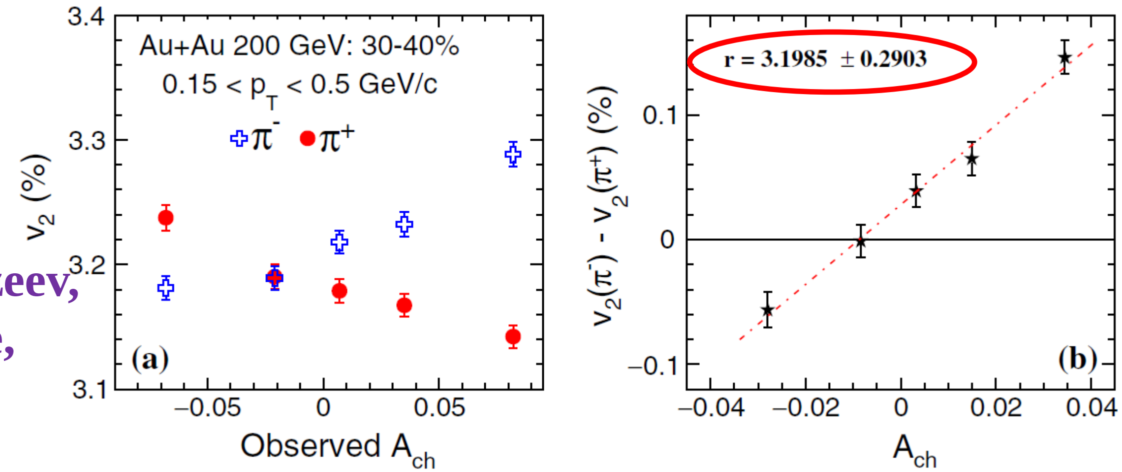


CMW and $v_2(\pi^-)$-$v_2(\pi^+) \sim A_{ch}$





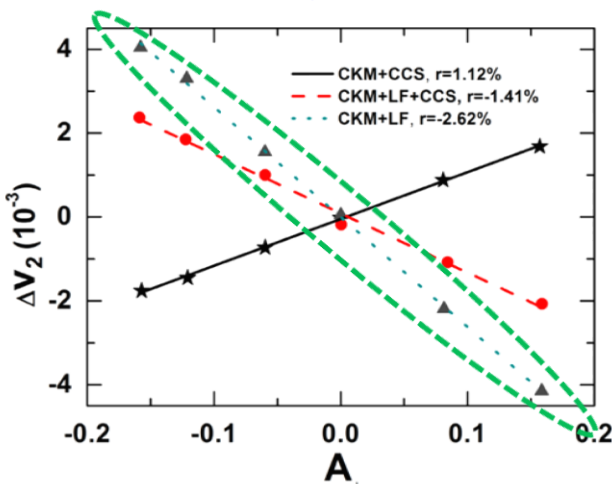
Y, Burnier, D.E. Kharzeev, J.F. Liao, and H.U. Yee, PRL (2011)



STAR, PRL (2015)

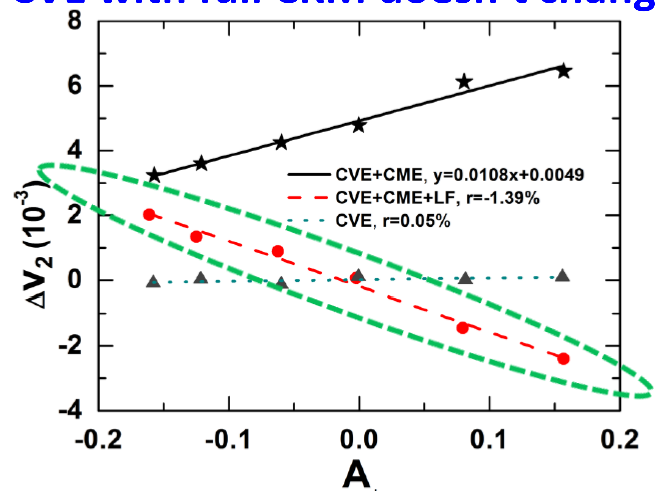
$$A_\pm \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

The slope is negative with full CKM.



Y.F. Sun and C.M. Ko, PRC (2016)

CVE with full CKM doesn't change the slope.



Y.F. Sun and C.M. Ko, PRC (2017)

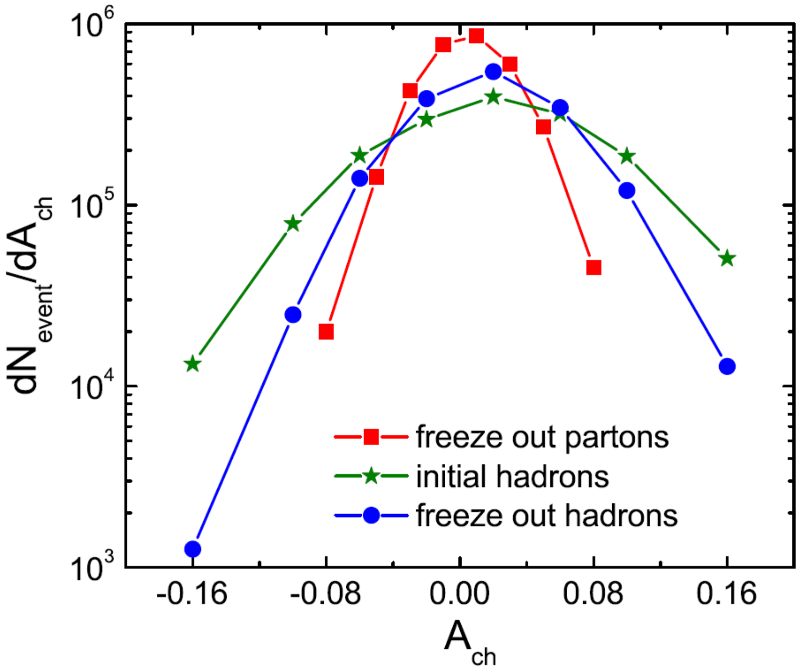
$v_2(\bar{u})$-$v_2(u)$ ~ A_{ch}

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B}$$ LF

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

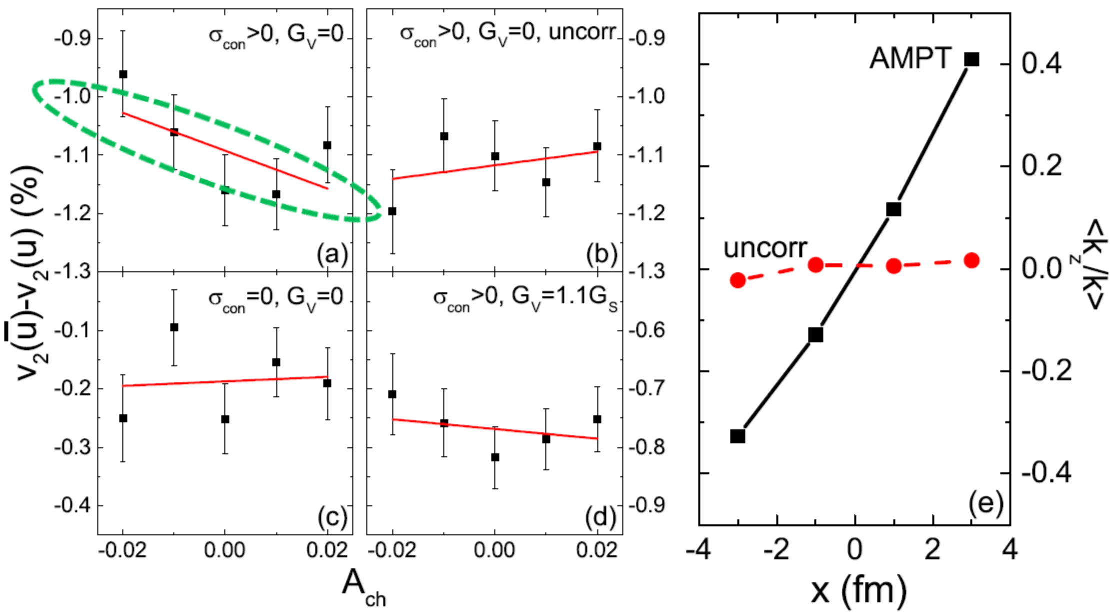
A_{ch} event distributions are different at different stages



$$A_{ch} = \sum_n q_n / \sum_n |q_n|$$

Related to charge chemical potential

Linear fit around A_{ch}~0



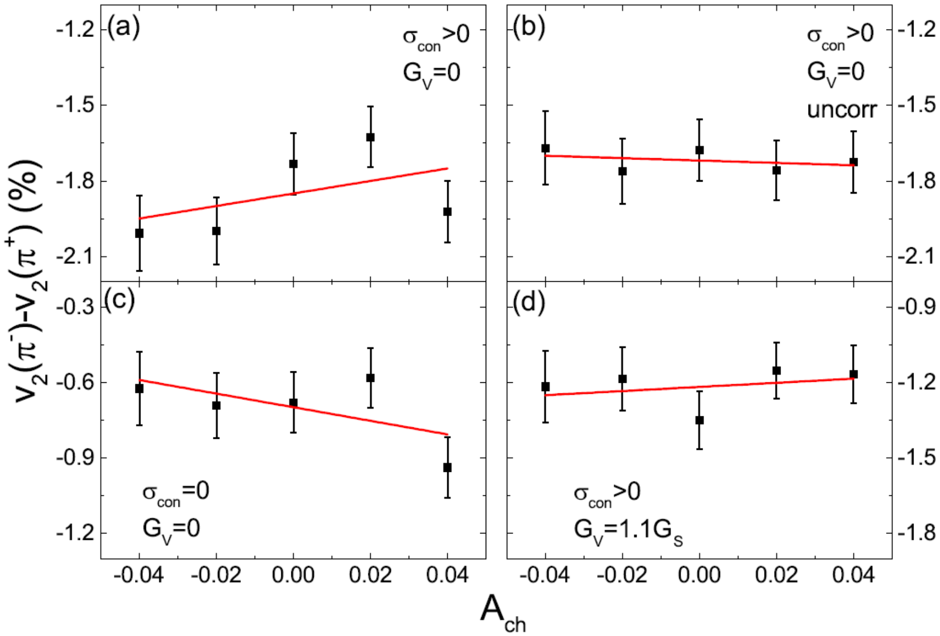
Negative slope due to the Lorentz force originated from the initial <k_z/k>~x correlation

Slope also affected by σ_{con} and G_V

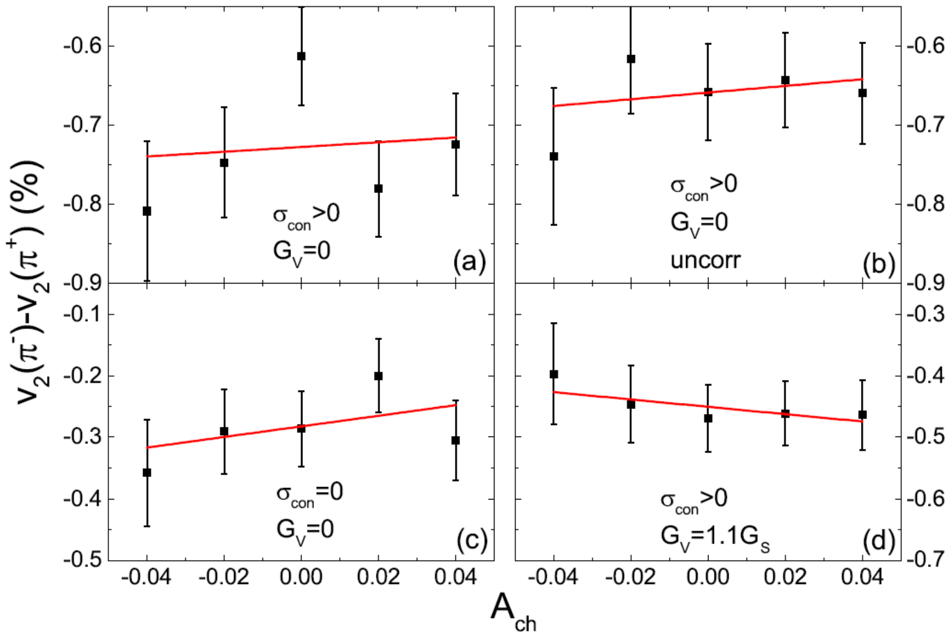
Z.Z. Han and JX, arXiv: 1904.03544 [nucl-th]

$v_2(\pi^-)-v_2(\pi^+) \sim A_{ch}$

Initial hadrons right after hadronization



Freeze-out hadrons



	$\sigma_{con} > 0,\ G_V = 0$	$\sigma_{con} > 0,\ G_V = 0$, uncorr	$\sigma_{con} = 0,\ G_V = 0$	$\sigma_{con} > 0,\ G_V = 1.1G_S$
$r(\%)$ for freeze-out $v_2(\bar{u}) - v_2(u)$	-3.244 ± 2.139	1.161 ± 2.073	0.385 ± 2.112	-0.828 ± 1.909
$r(\%)$ for initial $v_2(\pi^-) - v_2(\pi^+)$	2.488 ± 2.113	-0.475 ± 2.076	-2.699 ± 2.073	0.819 ± 1.999
$r(\%)$ for freeze-out $v_2(\pi^-) - v_2(\pi^+)$	0.301 ± 1.154	0.422 ± 1.139	0.861 ± 1.14	-0.596 ± 1.037

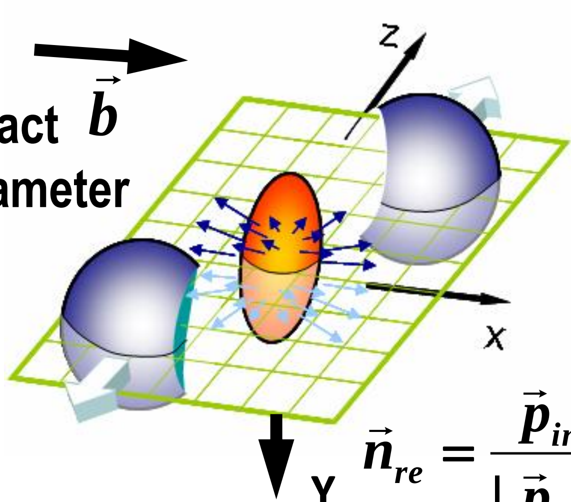
Slope modified during the hadronization and after the hadronic evolution

We can not obtain a positive slope as large as 3% observed experimentally.

Z.Z. Han and JX, arXiv: 1904.03544 [nucl-th]

Spin polarization in relativistic heavy-ion collisions

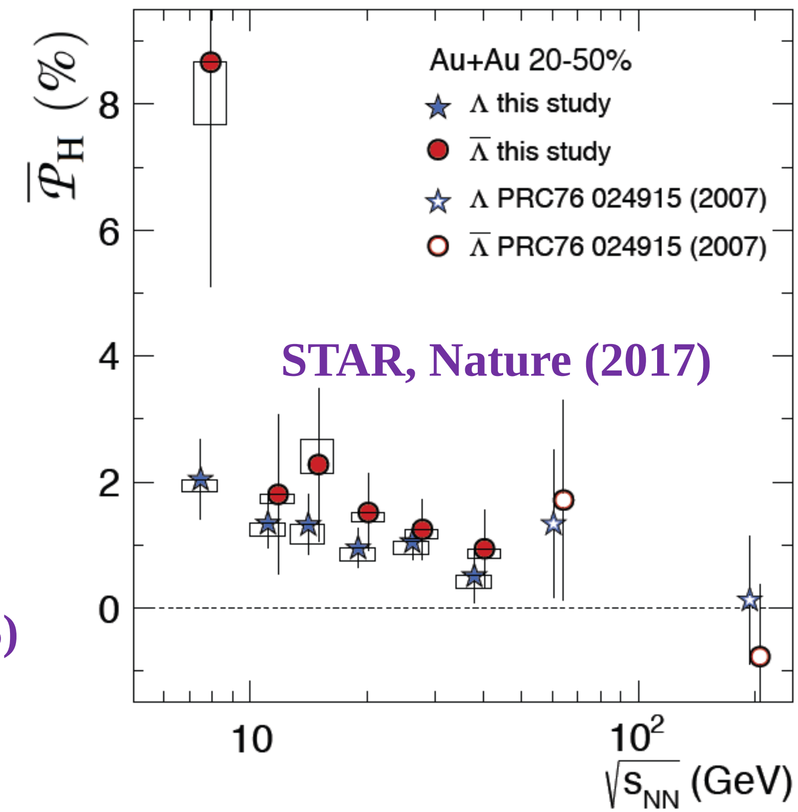
impact $\vec{b}$
parameter



$$\vec{n}_{re} = \frac{\vec{p}_{in} \times \vec{b}}{|\vec{p}_{in} \times \vec{b}|}$$

perpendicular to the reaction plane

Z. T. Liang and X. N. Wang, PRL (2005); PLB (2005)



Λ polarization

$$\frac{dN}{d\cos\theta^*} \propto 1 + \alpha_H p_H \cos\theta^*$$

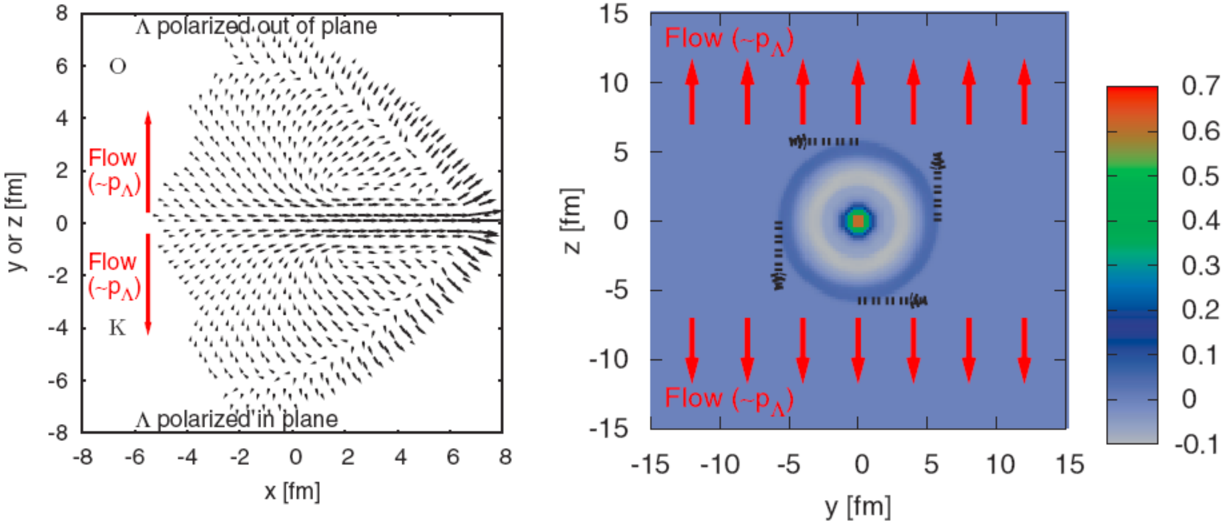
$$P_H = -\frac{8}{\pi\alpha_H}\langle\sin(\phi_P^* - \Psi_{RP})\rangle = \frac{\Lambda^\uparrow - \Lambda^\downarrow}{\Lambda^\uparrow + \Lambda^\downarrow} \neq 0$$

$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

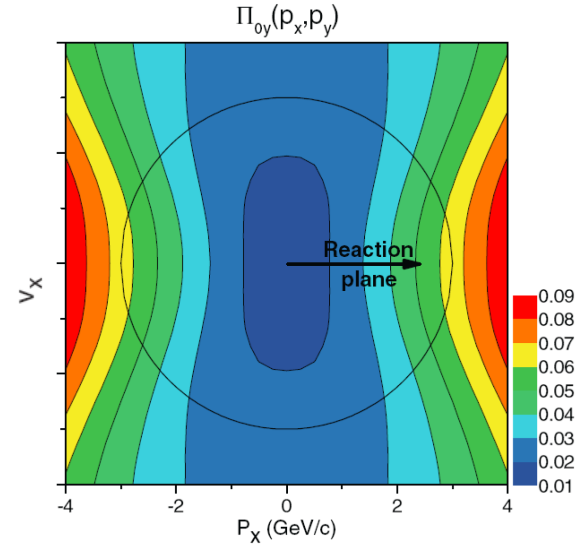
$$P_{\overline{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$$

Vorticity
+
magnetic field
（？）

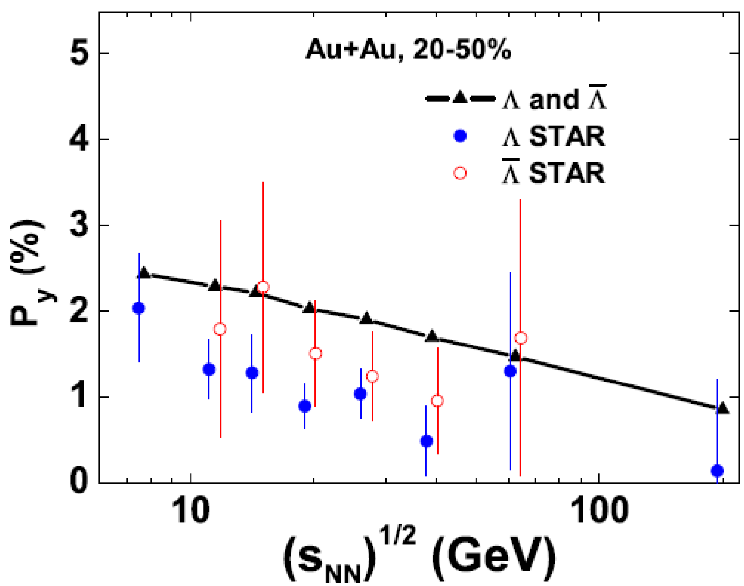
vorticity lead to same $\Lambda(\bar{\Lambda})$ polarization



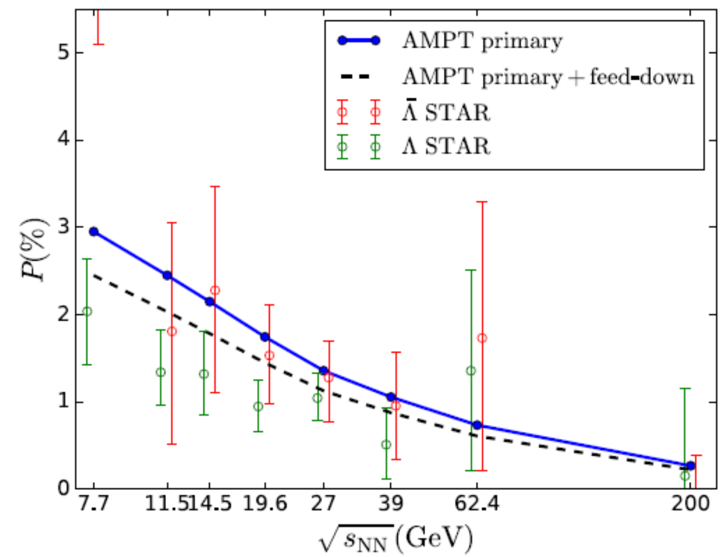
B. Betz et al., Phys. Rev. C, 2007



F. Becattini et al., Phys. Rev. C, 2013



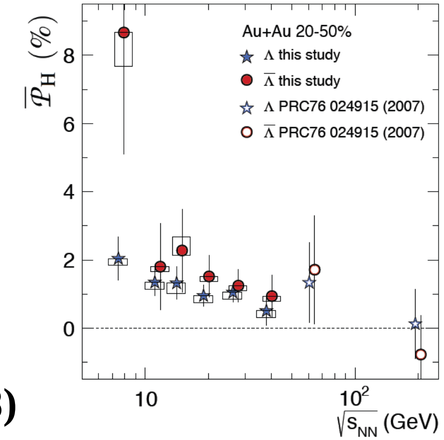
Y.F. Sun and C.M. Ko, Phys. Rev. C, 2017



H. Li, L.G. Pang, X.N. Wang, and X.L. Xia,
Phys. Rev. C, 2018

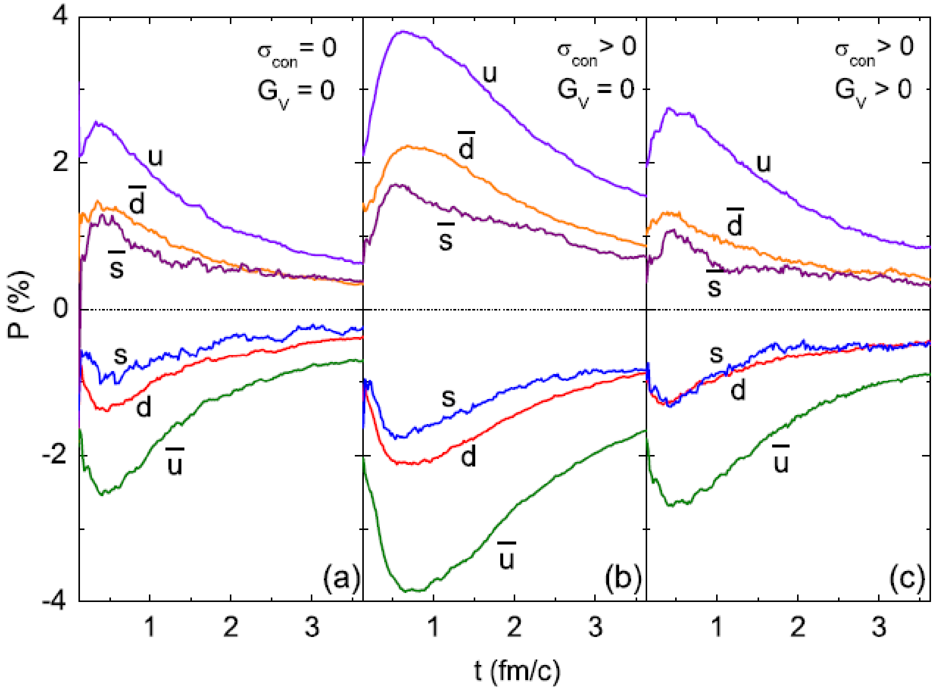
Splitting of quark-antiquark spin polarizations from vector potential

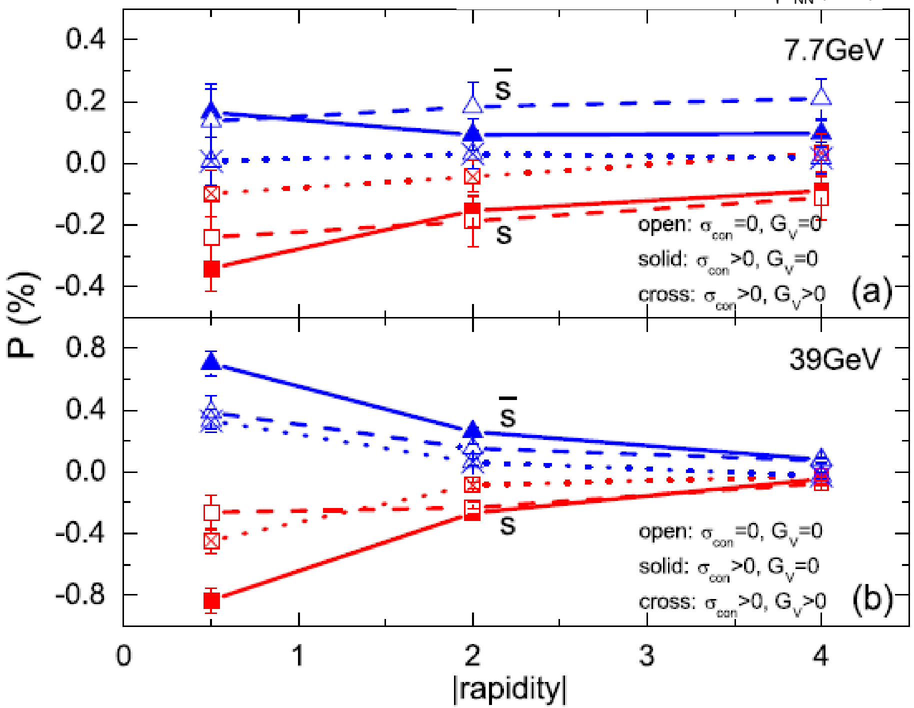
Thermal limit: $$\langle \vec{P} \rangle = \frac{\int \frac{d^3\vec{k}}{(2\pi)^3} c\dot{\vec{r}} \sqrt{G} \exp(-k/T)}{\int \frac{d^3\vec{k}}{(2\pi)^3} \sqrt{G} \exp(-k/T)} = \frac{\hbar \vec{B}}{4T^2}$$



$$\Lambda^{\uparrow} \sim s^{\uparrow}, \Lambda^{\downarrow} \sim s^{\downarrow}, \overline{\Lambda}^{\uparrow} \sim \bar{s}^{\uparrow}, \overline{\Lambda}^{\downarrow} \sim \bar{s}^{\downarrow}$$

Z.Z. Han and JX,
Phys. Lett. B 786, 255 (2018)

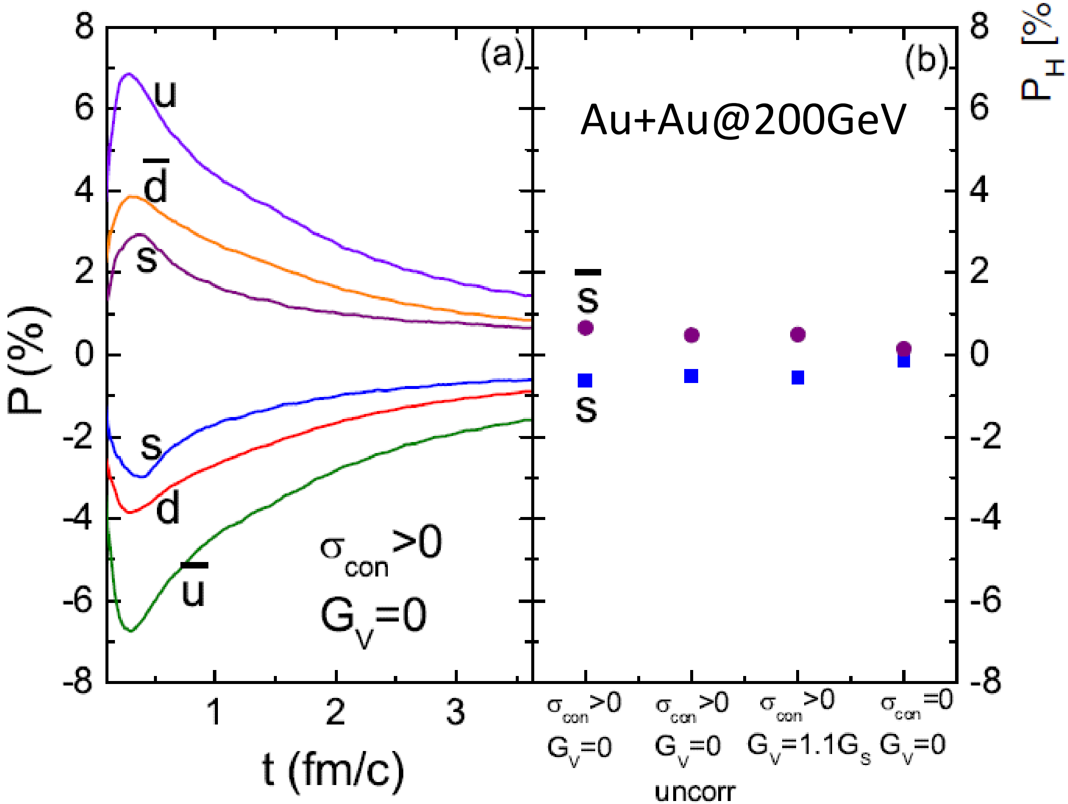


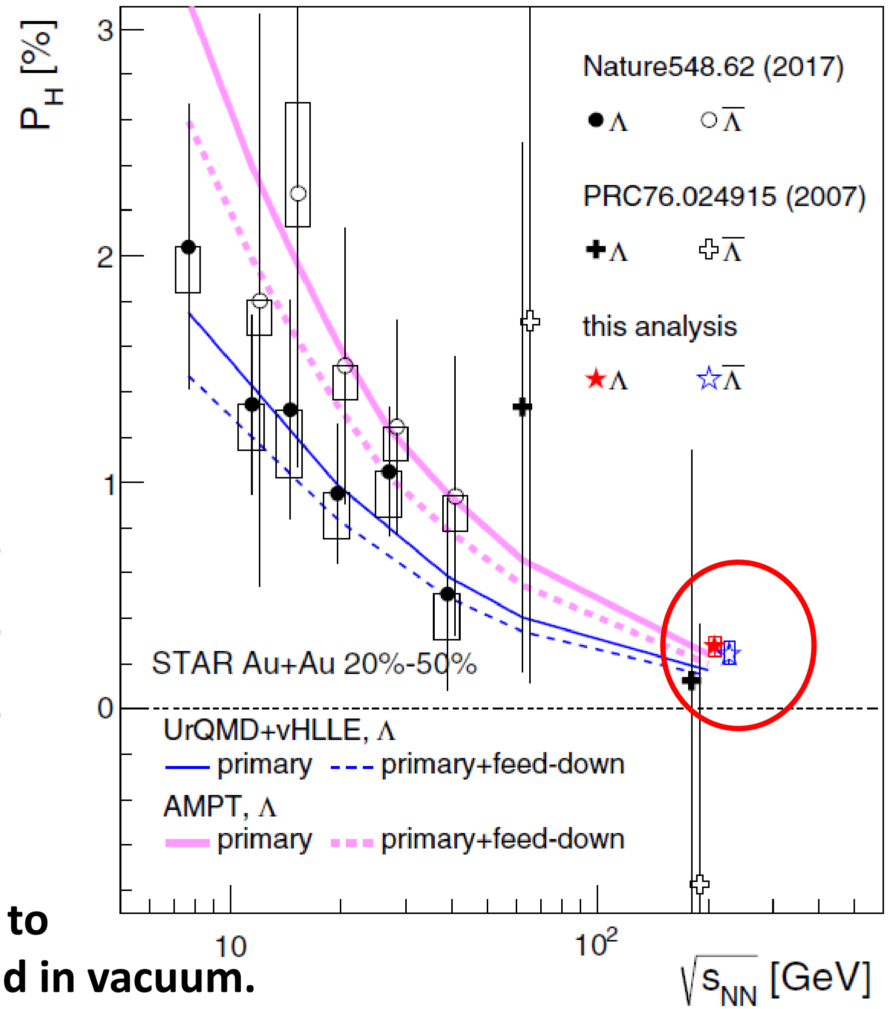


Smaller $\Lambda(s)$ spin polarization than $\overline{\Lambda}(\bar{s})$, consistent with exp data. Their splitting is sensitive not only to eB_y but also to G_V.

The large splitting at 7.7 GeV can not be obtained in the thermal limit under maximum/initial eB_y.

Splitting of quark-antiquark spin polarizations at 200 GeV





The experimentally observed small splitting seems to favor the space-time evolution of the magnetic field in vacuum.

Z.Z. Han and JX, arXiv: 1904.03544 [nucl-th]

STAR, PRC (2018)

Concluding remarks

- **SEOM leads to qualitatively similar but quantitatively weaker chiral effects compared with CEOM.**
- **Splitting of spin polarizations between Lambda and antiLambda is generated by both the magnetic field and the strong vector interaction.**
- **The positive slope of $v_2(\pi^-)$-$v_2(\pi^+)$ ~ A_{ch} observed experimentally is not likely due to CMW.**

Thank you!

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