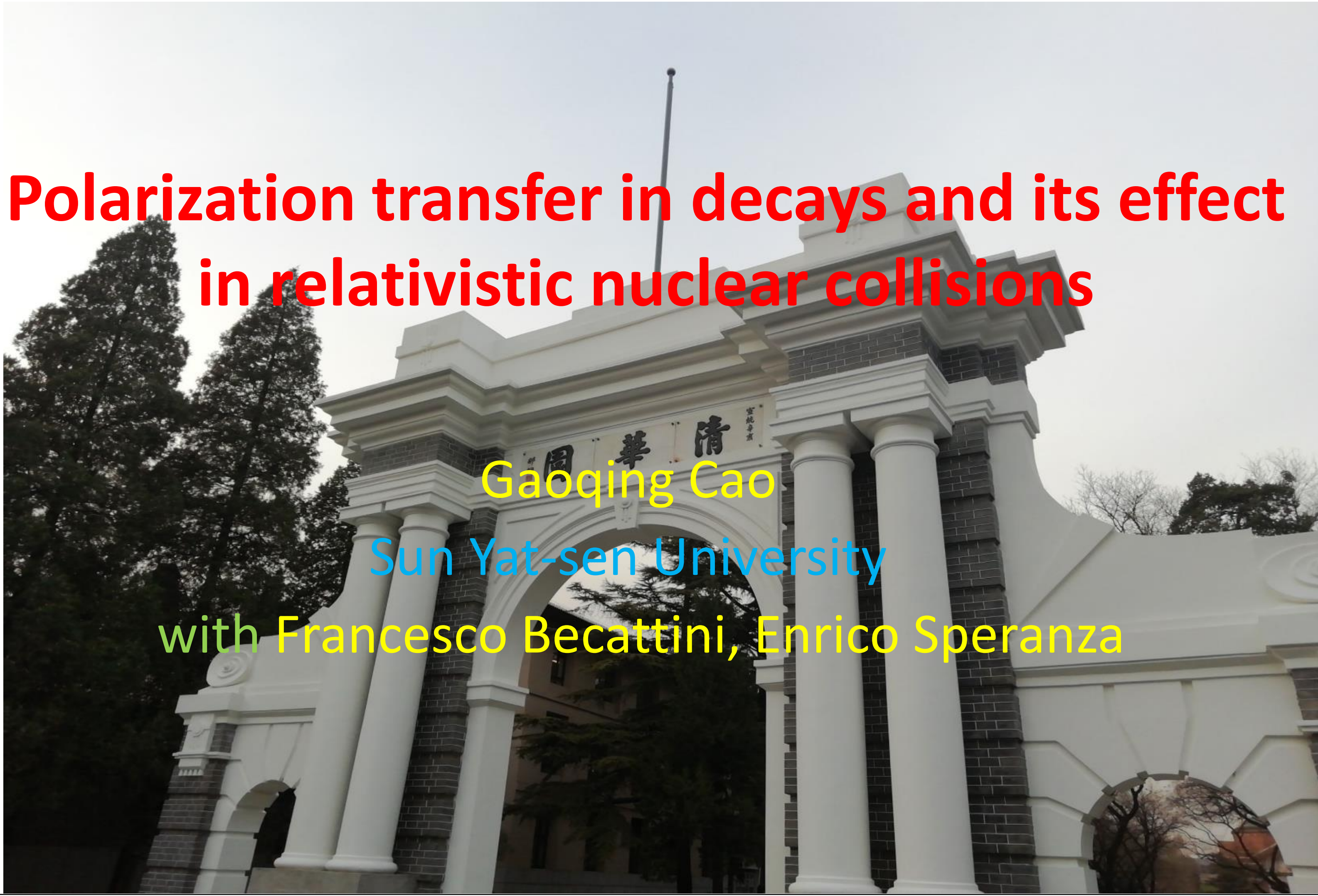
The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions



Polarization transfer in decays and its effect in relativistic nuclear collisions

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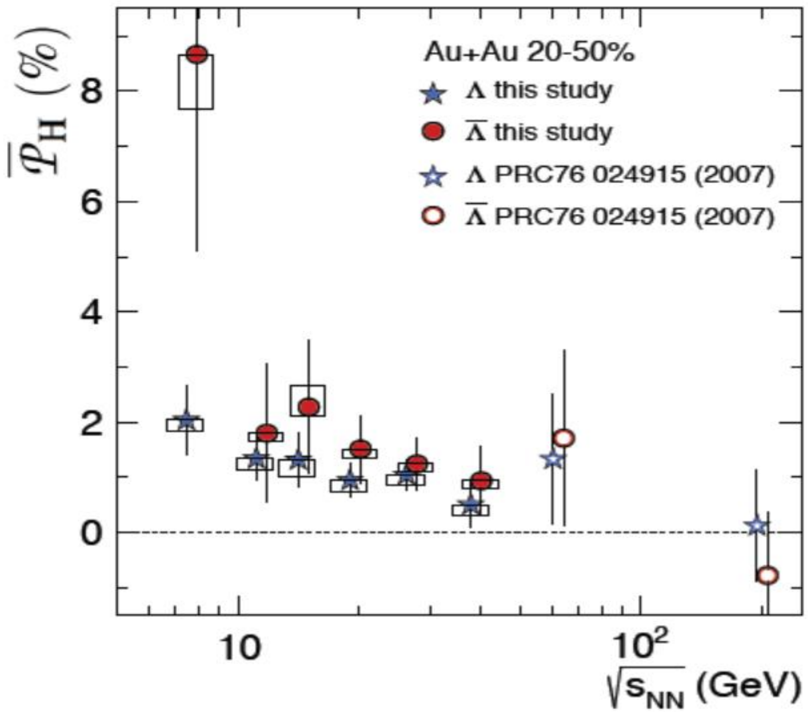
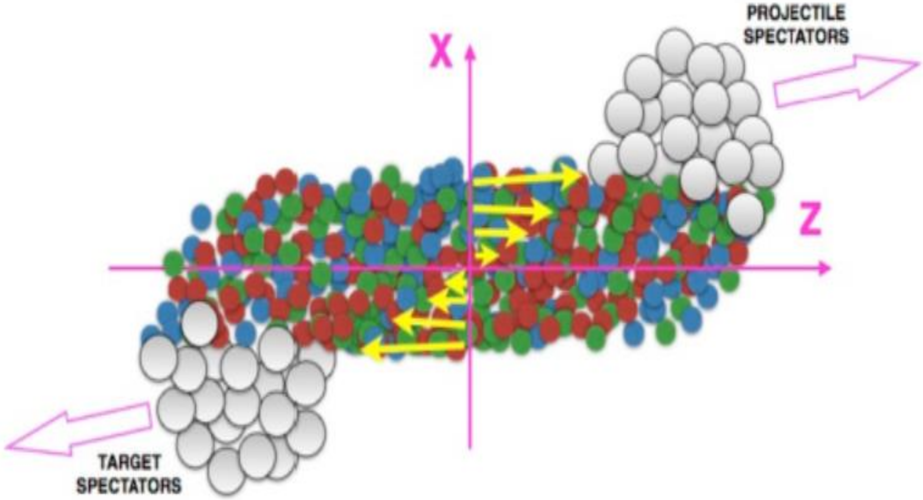
Outline

- Sign puzzle with longitudinal polarization
- Formalism at local thermodynamic equilibrium
  - A. Spin density matrix
  - B. Polarization transfer in two-body decays
  - C. Average longitudinal polarization
- Numerical results
- Summary

Sign puzzle with longitudinal polarization

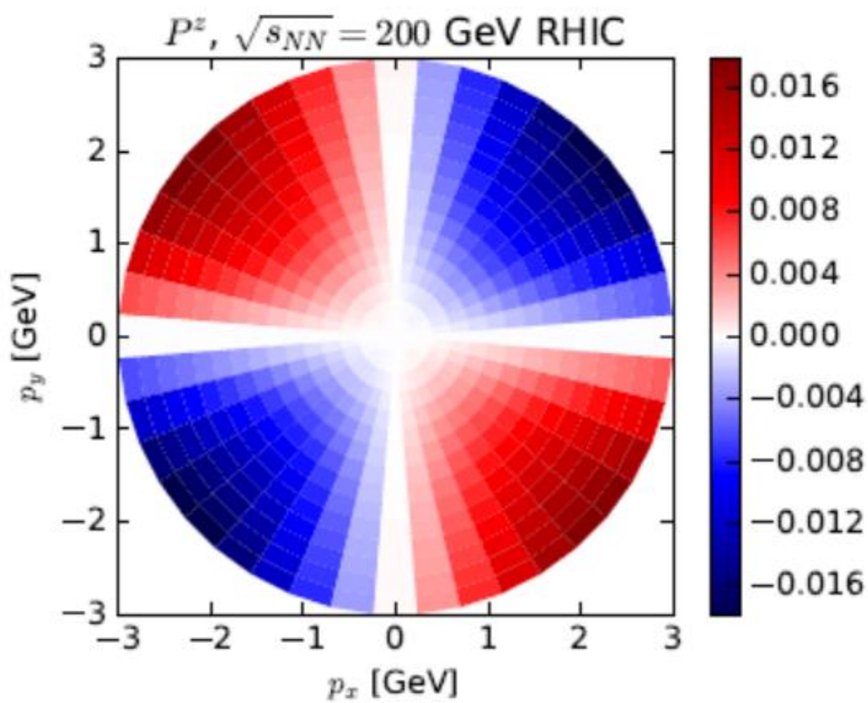


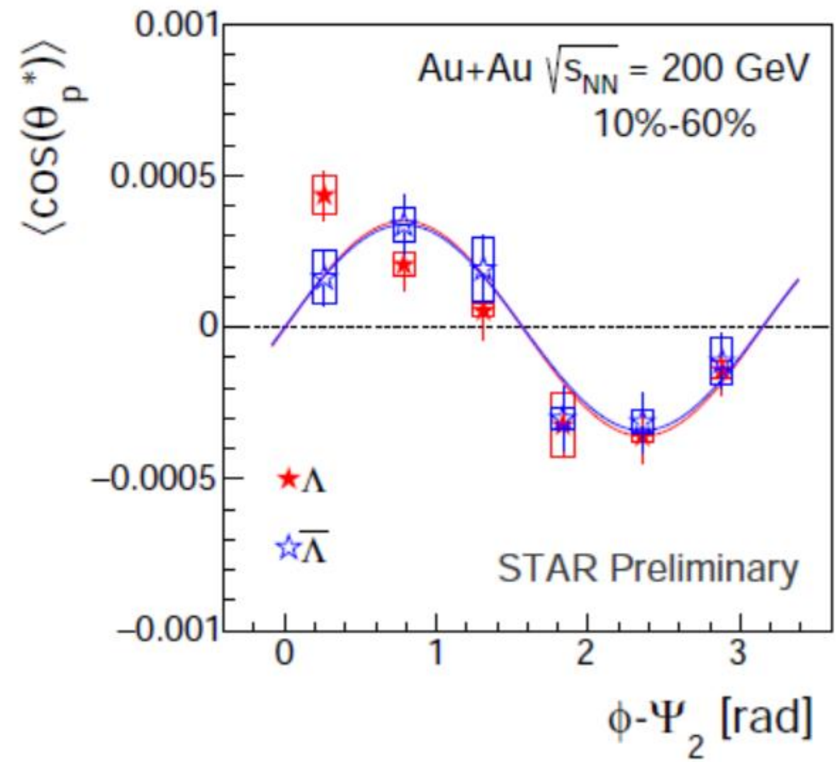


L. Adamczyk et al., Nature 548, 62 (2017)

Non-isotropic geometry induces local polarization

Sign puzzle with longitudinal polarization





F. Becattini and I. Karpenko, PRL, 2018

T. Niida, Nucl. Phys. A 982, 511 (2019)

Secondary decay contribution？

See also Hui Li's talk

A. Spin density matrix

Basic definition

$$\Theta(p)_{\sigma\sigma'} = \frac{\mathrm{tr}(\widehat{\rho} a^\dagger(p)_{\sigma'} a(p)_\sigma)}{\sum_\sigma \mathrm{tr}(\widehat{\rho} a^\dagger(p)_\sigma a(p)_\sigma)}$$

$$\widehat{\rho} = \frac{1}{Z}\exp[-b\cdot\widehat{P} + \frac{1}{2}\varpi : \widehat{J}]$$

In non-interacting approximation (single particle)

$$\Theta(p)_{\sigma\sigma'} = \frac{\langle p,\sigma|\widehat{\rho}_i|p,\sigma'\rangle}{\sum_\sigma \langle p,\sigma|\widehat{\rho}_i|p,\sigma\rangle}$$

Hermitian ⇩ Analytic continuation

$$\Theta(p) = \frac{D^S([p]^{-1}\exp[(1/2)\varpi : \Sigma][p]) + D^S([p]^\dagger \exp[(1/2)\varpi : \Sigma^\dagger][p]^{-1\dagger})}{\mathrm{tr}(\exp[(1/2)\varpi : \Sigma_S]) + \exp[(1/2)\varpi : \Sigma_S^\dagger])}$$

Small rotation ⇩ Thermal vorticity tensor $\varpi^{\mu\nu}[p]^{-1\alpha}_{\mu}[p]^{-1\beta}_{\nu} = \varpi_*^{\alpha\beta}(p)$

$$\Theta(p)^\sigma_{\sigma'} \simeq \frac{\delta^\sigma_{\sigma'}}{2S+1} + \frac{1}{2(2S+1)}\varpi_*(p)^{\alpha\beta}\epsilon_{\alpha\beta\rho\nu}D^S(J^\rho)^\sigma_{\sigma'}\hat{t}^\nu$$

A. Mean spin of single particle

Mean spin vector with momentum p

$$S^{\mu}(p) = [p]^{\mu}_{\nu}\mathrm{tr}(D^{S}(\mathsf{J}^{\nu})\Theta(p))$$

$$= [p]^{\mu}_{\kappa}\frac{1}{2(2S+1)}\varpi_{*}(p)^{\alpha\beta}\epsilon_{\alpha\beta\rho\nu}\mathrm{tr}\left(D^{S}(\mathsf{J}^{\rho})D^{S}(\mathsf{J}^{\kappa})\right)\hat{t}^{\nu}$$

$$= -\frac{1}{2(2S+1)}\frac{S(S+1)(2S+1)}{3}[p]^{\mu}_{\kappa}\varpi_{*}(p)^{\alpha\beta}\epsilon_{\alpha\beta\rho\nu}g^{\rho\kappa}\hat{t}^{\nu}$$

$$= -\frac{1}{2}\frac{S(S+1)}{3}[p]^{\mu}_{\rho}\varpi_{*}(p)_{\alpha\beta}\epsilon^{\alpha\beta\rho\nu}\hat{t}_{\nu} = -\frac{1}{2m}\frac{S(S+1)}{3}\varpi_{\alpha\beta}\epsilon^{\alpha\beta\mu\nu}p_{\nu}$$

Lorentz transformation

$$[p] = \mathsf{R}(\varphi,\theta,0)\mathsf{L}_{z}(\xi) = \mathsf{R}_{z}(\varphi)\mathsf{R}_{y}(\theta)\mathsf{L}_{z}(\xi)$$

B. Polarization transfer in two-body decays

Two-body quantum superposition in helicity basis

$$|p_* j m \lambda_1 \lambda_2\rangle \propto T^j(\lambda_1, \lambda_2) \int \mathrm{d}\Omega_*\, D^j(\varphi_*, \theta_*, 0)_{\lambda}^{m\,*} |\mathbf{p}_* \lambda_1 \lambda_2\rangle$$

(Dynamic amplitude) (Wigner D matrix) $\lambda = \lambda_1 - \lambda_2$

Density matrix for Daughters

$$\sum_{m,n=-j}^{j} \Theta^m_n |p_* j m \lambda_1 \lambda_2\rangle\langle p_* j n \lambda_1 \lambda_2|$$

Differential density matrix

$$\widehat{\rho}(\mathbf{p}_*) \propto \sum_{m,n=-j}^{j} T^j(\lambda_1, \lambda_2) T^j(\lambda_1', \lambda_2')^* D^j(\varphi_*, \theta_*, 0)_{\lambda}^{m\,*} \Theta^m_n D^j(\varphi_*, \theta_*, 0)_{\lambda'}^{n} |\mathbf{p}_* \lambda_1 \lambda_2\rangle\langle \mathbf{p}_* \lambda_1' \lambda_2'|$$

Two-body

$$\Theta_{D\,\lambda_1'\lambda_2'}^{\;\lambda_1\lambda_2} = \frac{\sum_{m,n=-j}^{j} T^j(\lambda_1, \lambda_2) T^j(\lambda_1', \lambda_2')^* D^j(\varphi_*, \theta_*, 0)_{\lambda}^{m\,*} \Theta^m_n D^j(\varphi_*, \theta_*, 0)_{\lambda'}^{n}}{\sum_{\lambda_1,\lambda_2} \sum_{m,n=-j}^{j} |T^j(\lambda_1, \lambda_2)|^2 D^j(\varphi_*, \theta_*, 0)_{\lambda}^{m\,*} \Theta^m_n D^j(\varphi_*, \theta_*, 0)_{\lambda}^{n}}$$

B. Polarization transfer in two-body decays

Reduced spin density matrix

$$\Theta_{D\,\lambda_1'\lambda_2}^{\,\lambda_1\lambda_2} \simeq \frac{T^j(\lambda_1,\lambda_2)T^j(\lambda_1',\lambda_2)^*\left[\delta_{\lambda'}^{\lambda} + (1/2)\varpi_*(P)^{\alpha\beta}\epsilon_{\alpha\beta\rho\nu}D^j(\mathrm{J}^\tau)_{\lambda'}^{\lambda}\mathrm{R}(\varphi_*,\theta_*,0)_\tau^\rho\hat{t}^\nu\right]}{\sum_{\lambda_1,\lambda_2}|T^j(\lambda_1,\lambda_2)|^2}$$

Mean spin vector of the Daughter 1

$$\begin{aligned}
S_1^\mu(p_*) &= [p_*]_\nu^\mu \sum_{\lambda_1\lambda_1'} D^{S1}(\mathrm{J}^\nu)_{\lambda_1}^{\lambda_1'}\Theta_{D\,\lambda_1'\lambda_2}^{\,\lambda_1\lambda_2} \\
&= \frac{1}{2}\varpi_*(P)^{\alpha\beta}\epsilon_{\alpha\beta\rho\nu}\hat{t}^\nu \frac{\sum_{\lambda_1\lambda_2}T^j(\lambda_1,\lambda_2)T^j(\lambda_1',\lambda_2)^*[p_*]_\kappa^\mu D^{S1}(\mathrm{J}^\kappa)_{\lambda_1}^{\lambda_1'}D^j(\mathrm{J}^\tau)_{\lambda'}^{\lambda}\mathrm{R}(\varphi_*,\theta_*,0)_\tau^\rho}{\sum_{\lambda_1,\lambda_2}|T^j(\lambda_1,\lambda_2)|^2} \\
&= -\frac{3}{j(j+1)}S_{*M}(P)_\rho \frac{\sum_{\lambda_1\lambda_2}T^j(\lambda_1,\lambda_2)T^j(\lambda_1',\lambda_2)^*[p_*]_\kappa^\mu D^{S1}(\mathrm{J}^\kappa)_{\lambda_1}^{\lambda_1'}D^j(\mathrm{J}^\tau)_{\lambda'}^{\lambda}\mathrm{R}(\varphi_*,\theta_*,0)_\tau^\rho}{\sum_{\lambda_1,\lambda_2}|T^j(\lambda_1,\lambda_2)|^2}
\end{aligned}$$

B I. $\Sigma^* \to \Lambda + \pi$

Specific conditions $\lambda_2 = 0,\ j = 3/2 \text{ and } S_1 = 1/2$

$$T(1/2) = T(-1/2)$$

$$S^{\mu}_{\Lambda}(p_*) = \frac{3}{2j(j+1)}\left[L_{\hat{\mathbf{p}}_*}(\xi)^{\mu}_{\rho} S_{*M}(P)^{\rho} - \frac{1}{2} S_{*M}(P)_{\rho} \mathsf{R}(\varphi_*, \theta_*, 0)^{\mu}_{\nu} \mathsf{L}_z(\xi)^{\nu}_{3} \mathsf{R}(\varphi_*, \theta_*, 0)^{\rho 3}\right]$$
$$= \frac{3}{2j(j+1)}\left[\mathsf{L}_{\hat{\mathbf{p}}_*}(\xi)^{\mu}_{\rho} S_{*M}(P)^{\rho} - \frac{\varepsilon_*}{2m_{\Lambda}} \mathbf{S}_{*M} \cdot \hat{\mathbf{p}}_* \hat{\mathbf{p}}^{\mu}_* - \frac{\mathbf{p}_*}{2m_{\Lambda}} \mathbf{S}_{*M} \cdot \hat{\mathbf{p}}_* \delta^{\mu}_{0}\right]$$

Λ rest frame

$$\mathbf{S}_{0\Lambda}(p_*) = \mathbf{S}_{\Lambda}(p_*) - S^{0}_{\Lambda}(p_*) \frac{\mathbf{p}_*}{\varepsilon_* + m_{\Lambda}} = \frac{2}{5}\left[\mathbf{S}_{*M} - \frac{1}{2} \mathbf{S}_{*M} \cdot \hat{\mathbf{p}}_* \hat{\mathbf{p}}_*\right]$$

B II. $\Sigma^0 \to \Lambda + \gamma$

Specific conditions $j = 1/2,\ S_1 = 1/2 \text{ and } |\lambda_2| = 1$

⇩ Helicity conservation

$$\lambda_1' = \lambda_1,\ \lambda = \lambda_1 - \lambda_2 = -\lambda_1,\ \lambda_2 = -2\lambda_1$$

$$S_\Lambda^\mu(p_*) = -\frac{3}{2j(j+1)} S_{*M}(P)_\rho \sum_{\lambda_1} [p_*]^\mu_\kappa D^{1/2}(\mathrm{J}^\kappa)^{-\lambda_1}_{-\lambda_1} D^{1/2}(\mathrm{J}^\tau)^{\lambda_1}_{\lambda_1} \mathrm{R}(\varphi_*, \theta_*, 0)^\rho_\tau$$

$$= -\frac{3}{4j(j+1)} \mathbf{S}_{*M} \cdot \hat{\mathrm{p}}_* \left(\frac{\varepsilon_*}{m_\Lambda} \hat{\mathrm{p}}^\mu_* + \frac{\mathrm{p}_*}{m_\Lambda} \delta^\mu_0 \right)$$

⇩ Λ rest frame

$$\mathbf{S}_{0\Lambda}(p_*) = \mathbf{S}_\Lambda(p_*) - S^0_\Lambda(p_*) \frac{\mathbf{p}_*}{\varepsilon_* + m_\Lambda} = -\mathbf{S}_{*M} \cdot \hat{\mathbf{p}}_* \hat{\mathbf{p}}_*$$

C. Average longitudinal polarization

The magnitude of momentum for Daughter fixed in Mother frame

$$p_{*D} = \frac{1}{2m_M} \prod_{s,t=\pm} (m_M + s\, m_{D1} + t\, m_{D2})^{1/2}$$

Mean spin vector average over Mother distribution

$$\langle \mathrm{S}_{0\Lambda}(\mathrm{p}) \rangle = \frac{\int d^3\mathrm{p}_*\, n(\mathrm{P}) \left| \frac{\partial \mathrm{P}}{\partial \mathrm{p}_*} \right| \mathrm{S}_{0\Lambda}(p_*) \delta(\mathrm{p}_* - p_{*D})}{\int d^3\mathrm{p}_*\, n(\mathrm{P}) \left| \frac{\partial \mathrm{P}}{\partial \mathrm{p}_*} \right| \delta(\mathrm{p}_* - p_{*D})} = \frac{\int d\Omega_*\, n(\mathrm{P}) \left| \frac{\partial \mathrm{P}}{\partial \mathrm{p}_*} \right| \mathrm{S}_{0\Lambda}(p_*)}{\int d\Omega_* n(\mathrm{P}) \left| \frac{\partial \mathrm{P}}{\partial \mathrm{p}_*} \right|}$$

Momentum of Mother

$$\mathrm{P} = 2m_M \frac{(\varepsilon_* + \varepsilon)(\mathrm{p} - \mathrm{p}_*)}{(\varepsilon_* + \varepsilon)^2 - (\mathrm{p} - \mathrm{p}_*)^2}$$

$$\left| \frac{\partial \mathrm{P}}{\partial \mathrm{p}_*} \right| = \frac{\varepsilon_* (\varepsilon_* + \varepsilon)^4 M^3}{(\varepsilon_* \varepsilon + \mathrm{p}_* \cdot \mathrm{p} + m_{D1}^2)^4}$$

C. Average longitudinal polarization

High energy collision, longitudinal polarization dominates

$$\langle \mathbf{S}_{0\Lambda}(\mathbf{p})\rangle \cdot \hat{\mathbf{k}} = \frac{\int d\Omega_* \, n(\mathbf{P}) \left| \frac{\partial \mathbf{P}}{\partial \mathbf{p}_*} \right| [A S_{*Mz} + B(\mathbf{S}_{*M} \cdot \hat{\mathbf{p}}_*) \cos\theta_*]}{\int d\Omega_* n(\mathbf{P}) \left| \frac{\partial \mathbf{P}}{\partial \mathbf{p}_*} \right|}$$

$$\simeq \frac{\int d\Omega_* \, n(\mathbf{P}) \left| \frac{\partial \mathbf{P}}{\partial \mathbf{p}_*} \right| S_{*Mz} \, (A + B\cos^2\theta_*)}{\int d\Omega_* n(\mathbf{P}) \left| \frac{\partial \mathbf{P}}{\partial \mathbf{p}_*} \right|}$$

Primary mean Spin $S_{*Mz} \simeq S_M f_2(\mathrm{P}_T, b) \sin 2\varphi_M$

Variable redefinition $\psi = \varphi_* - \varphi$

$$\sin 2\varphi_M = \mathcal{A}(\theta, \psi) \sin 2\varphi + \mathcal{B}(\theta, \psi) \cos 2\varphi$$

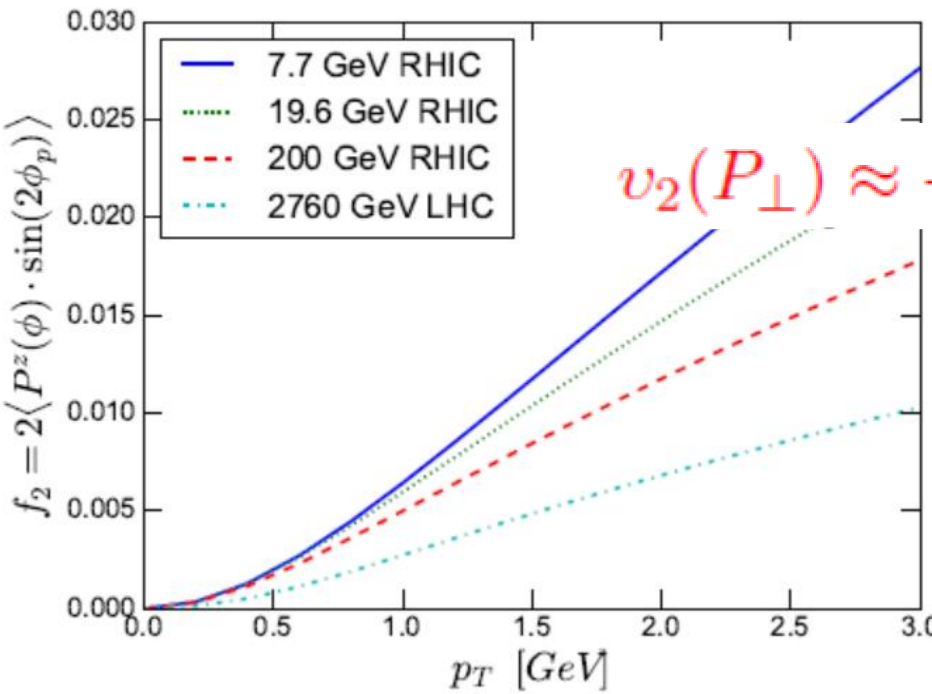
$$\cos 2\varphi_M = \mathcal{A}(\theta, \psi) \cos 2\varphi - \mathcal{B}(\theta, \psi) \sin 2\varphi$$

Shows up in $n(p)$ Non-vanishing

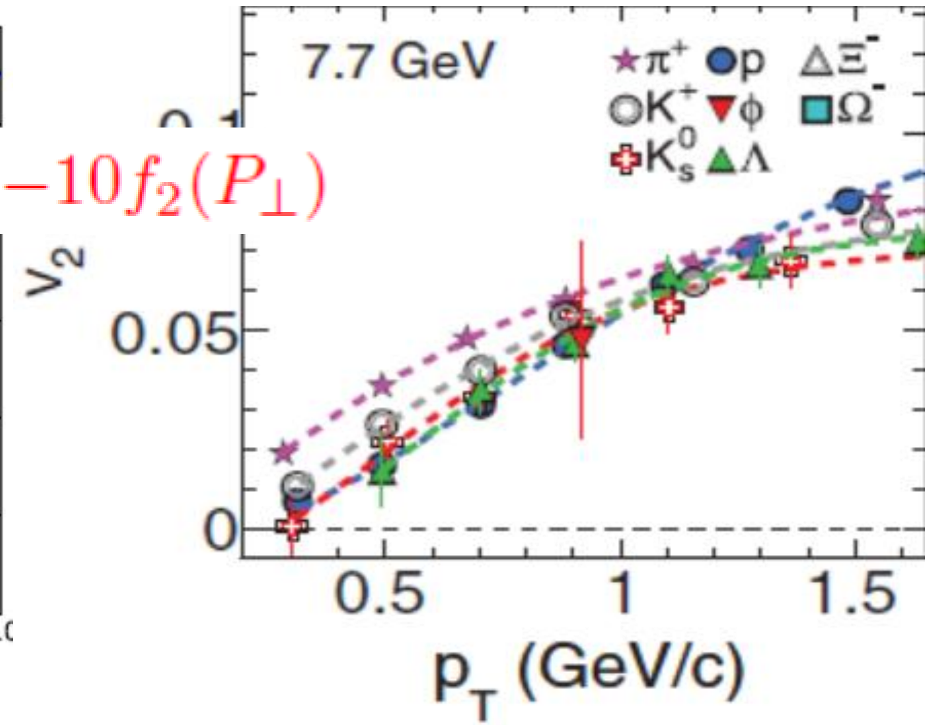
Numerical results

Number density $n(P) = a\, e^{2v_2(P_\perp)\cos(2\varphi_M)}$

$$f_2(p_T) = 2\frac{\mathrm{d}T}{\mathrm{d}\tau}\frac{1}{mT}v_2(p_T)$$



$$v_2(P_\perp) \approx -10 f_2(P_\perp)$$



F. Becattini and I. Karpenko, PRL, 2018

L. Adamczyk *et al.* ,PRC, 2013

Numerical results

Fitting to the case $\sqrt{S} = 200\ GeV$

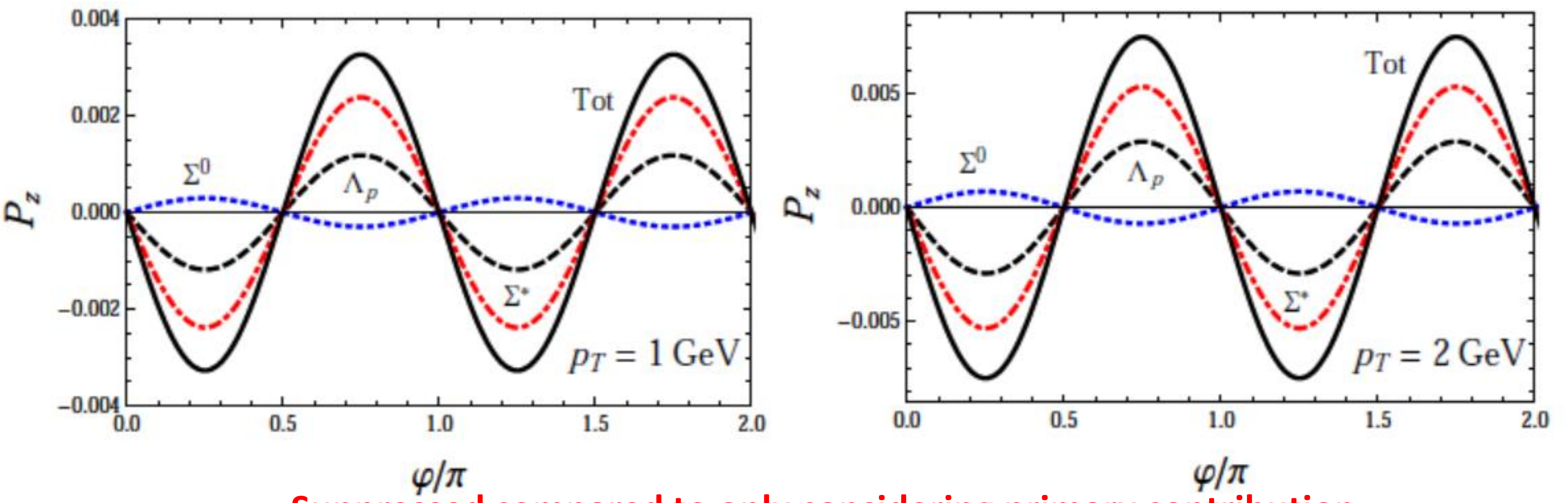
$$f_2(P_\perp) = -7.71*10^{-3}P_\perp^2 + 3.32*10^{-3}P_\perp^3 - 4.71*10^{-4}P_\perp^4$$

R

Hydro estimation: Primary Λ : 24.3%,

Σ^*: 35.9%, (almost primary)

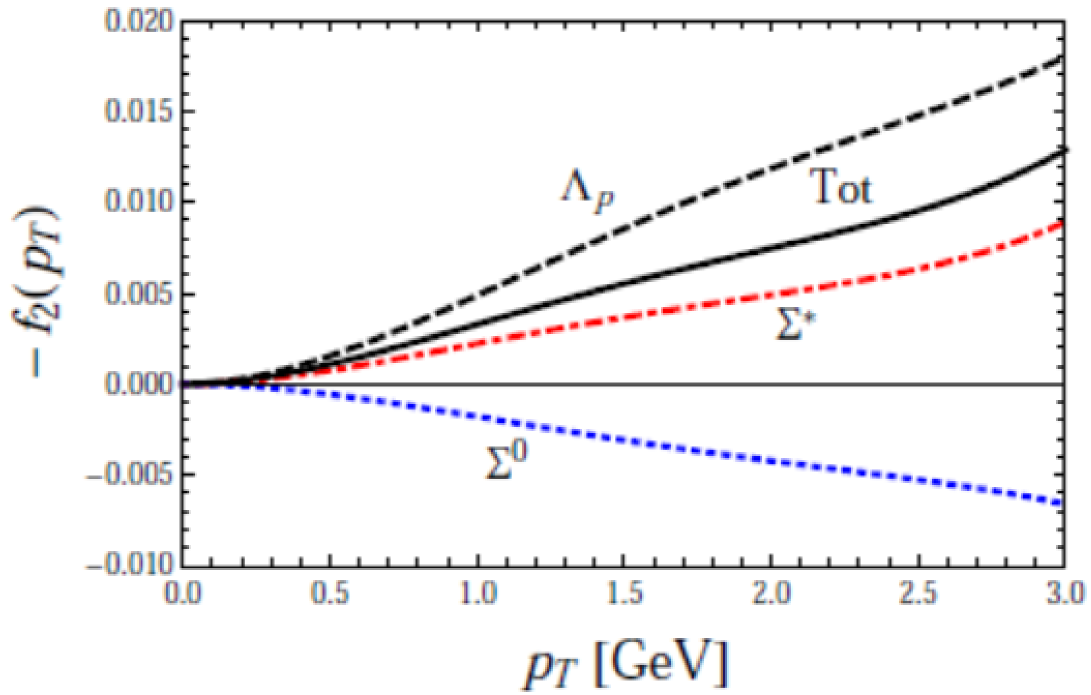
Σ^0: 27.5% (60% primary)



Suppressed compared to only considering primary contribution

Numerical results

Good $\sin(2\varphi)$ feature with the polarization transfer factor



$$P_z \approx f_2^{Tot} \sin 2\varphi$$

$$f_2^{Tot} = f_2^{\Lambda_p} R_{\Lambda_p} + 3 f_2^{\Sigma^*} R_{\Sigma^*} + f_2^{\Sigma^0} R_{\Sigma^0}$$

Summary

- Longitudinal polarization is studied within relativistic quantum mechanics by taking into account secondary contributions from decay
- Σ^* contributes to the same polarization heritage, Σ^0 contributes to the opposite one
- The sign puzzle can't be explained by the polarization transfer

Thanks!

backup

$$\mathcal{A}(\theta,\psi) = \frac{\mathrm{p}_*^2 \sin^2\theta_* \cos 2\psi - 2\mathrm{p}\mathrm{p}_* \sin\theta_* \cos\psi + \mathrm{p}^2}{\mathrm{p}_*^2 + \mathrm{p}^2 - 2\mathrm{p}\mathrm{p}_* \sin\theta_* \cos\psi},$$

$$\mathcal{B}(\theta,\psi) = \frac{\mathrm{p}_*^2 \sin^2\theta_* \sin 2\psi - 2\mathrm{p}_*\mathrm{p} \sin\theta_* \sin\psi}{\mathrm{p}_*^2 + \mathrm{p}^2 - 2\mathrm{p}\mathrm{p}_* \sin\theta_* \cos\psi}.$$

$$D^{3/2}_{\mathrm{RED}}(\mathsf{J}^1)) = \sigma_1 \qquad D^{3/2}_{\mathrm{RED}}(\mathsf{J}^2)) = \sigma_2 \qquad D^{3/2}_{\mathrm{RED}}(\mathsf{J}^3)) = \frac{\sigma_3}{2}$$

$$\begin{aligned} & \mathsf{R}(\varphi_*,\theta_*,0)^\mu_\nu \mathsf{L}_z(\xi)^\nu_3 \mathsf{R}(\varphi_*,\theta_*,0)^{\rho 3} \\ =& \mathsf{R}(\varphi_*,\theta_*,0)^\mu_3 \mathsf{L}_z(\xi)^3_3 \mathsf{R}(\varphi_*,\theta_*,0)^{\rho 3} + \mathsf{R}(\varphi_*,\theta_*,0)^\mu_0 \mathsf{L}_z(\xi)^0_3 \mathsf{R}(\varphi_*,\theta_*,0)^{\rho 3} \\ =& -\cosh\xi\, \hat{\mathrm{p}}^\mu_* \hat{\mathrm{p}}^\rho_* - \sinh\xi\, \hat{\mathrm{p}}^\rho_* \delta^\mu_0 = -\frac{\varepsilon_*}{m_\Lambda} \hat{\mathrm{p}}^\mu_* \hat{\mathrm{p}}^\rho_* - \frac{\mathrm{p}_*}{m_\Lambda} \hat{\mathrm{p}}^\rho_* \delta^\mu_0 \end{aligned}$$