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Local suppression and enhancement of pairing condensate under rotation

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Outline

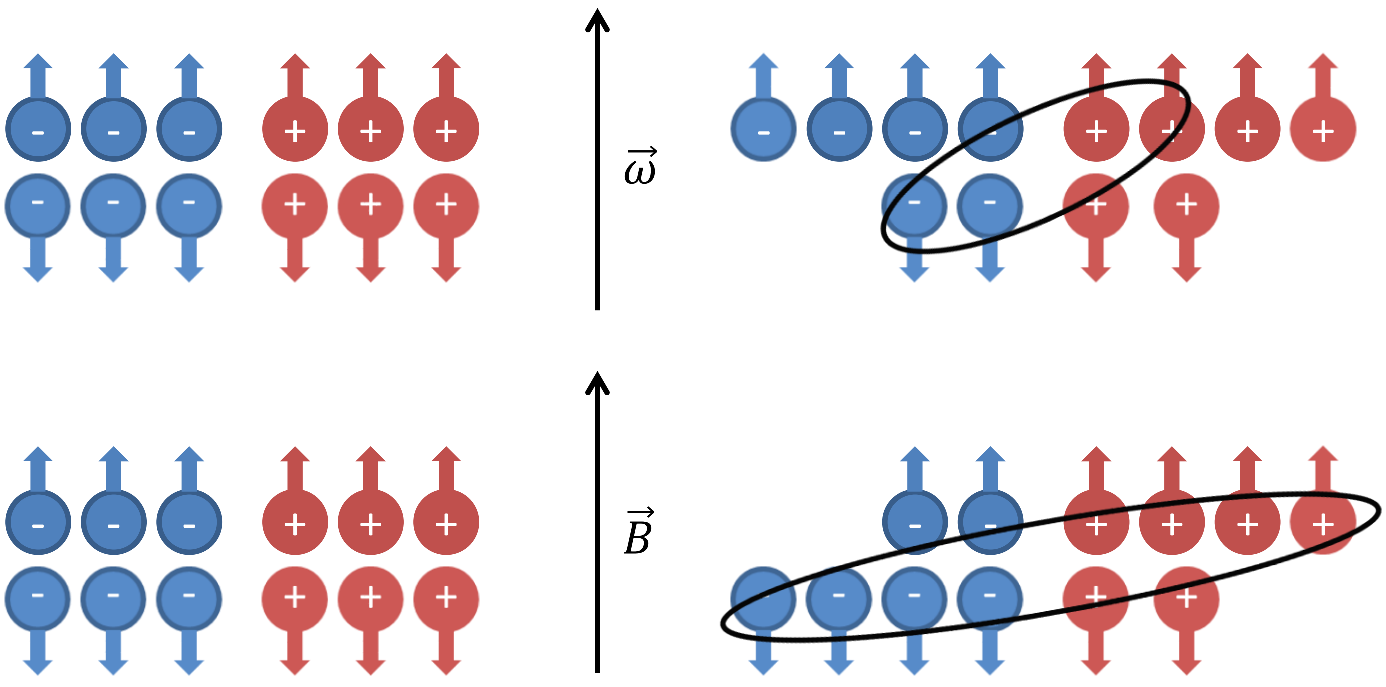
- Motivation
- NJL model with rotation
- Local density approx.
- Self-consistent results
- Conclusions and outlook

Motivation

- Rotation or vortex is a common phenomenon in cold atom, heavy ion collision and neutron star.
- Non-triviality by considering the limit of speed and centrifugal-like effects.
- Requiring a unified framework to deal with finite size and inhomogeneous effects.

Model choice

- Focus on scalar condensates



✓ Nambu—Jona-Lasinio model

NJL model with rotation

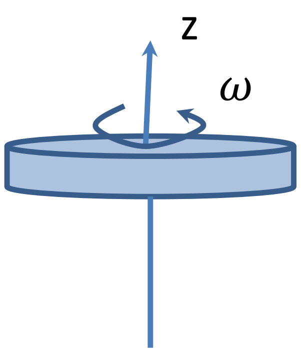
- Rotating system at angular velocity ω .
- In its rest frame, impacted by curved metrics

$$\gamma^\mu \to \bar{\gamma}^\mu = e_a^\mu \gamma^a$$
$$\partial_\mu \to \partial_\mu + \Gamma_\mu$$

$$H^{Dirac} = H_0^{Dirac} - \vec{\omega} \cdot \vec{J}$$

$$= \gamma^0 \vec{\gamma} \cdot \vec{p} + M\gamma^0 - \vec{\omega} \times \vec{r} \cdot \vec{p} - \vec{\omega} \cdot \frac{\vec{\sigma}}{2} \otimes$$

$$1_{2\times 2}$$



- Renormalizable in 2+1D case.
- Regularization needed in 3+1D case.
- Eigen states are unavailable for a general $\omega(\rho)$.

Local density approx.

- Mean field approximation gives

$$H = \left(i\gamma^0 \vec{\gamma} \cdot \overrightarrow{\partial} + M\,\gamma^0\right) - \vec{J}\cdot\vec{\omega}$$

where $M(\vec{r}) = m_0 - 2\,G\,\langle\bar{\psi}\psi\rangle$.

- Gap equation for chiral condensate ($\epsilon_m = E - \left(m + \frac{1}{2}\right)\omega$)

$$\langle\bar{\psi}\psi\rangle = \frac{M - m_0}{-2G} = \Sigma_{m\,k_z k_t^2} \frac{M}{E}\,\left[1 - 2\,n_f\left(\epsilon_m\right)\right]\left(J_m^2(k_t r) + J_{m+1}^2(k_t r)\right)$$

- With Eigen states quantizing $\psi(x)$ as

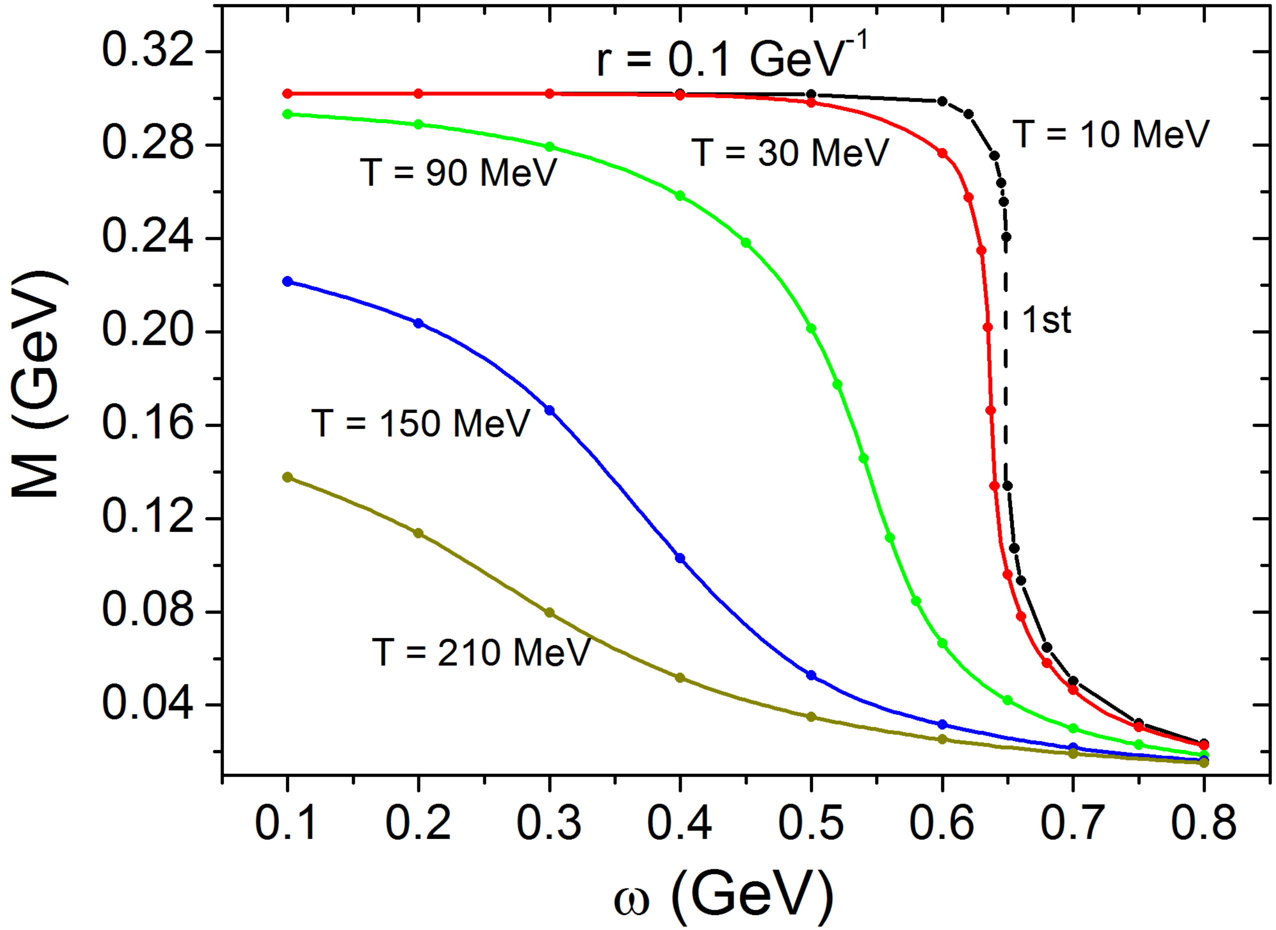
$$\psi(\vec{r},t) = \Sigma_{snk_z k_t^2}\left[a_{snk_z k_t^2}(t)\,u_{snk_z k_t^2} + b^\dagger_{snk_z k_t^2}(t)\,v_{snk_z k_t^2}\right].$$

where

$$u_{n+}(k_t^2, k_z) = \frac{1}{2}\sqrt{\frac{E+M}{E}}\,e^{i\,k_z\,z} e^{i\,n\,\theta} \times \left(J_n(k_t r), e^{i\theta} J_{n+1}(k_t r), \frac{k_z - i\,k_t}{E+M} J_n(k_t r), -\frac{k_z - i\,k_t}{E+M} e^{i\theta} J_{n+1}(k_t r)\right)^\tau$$

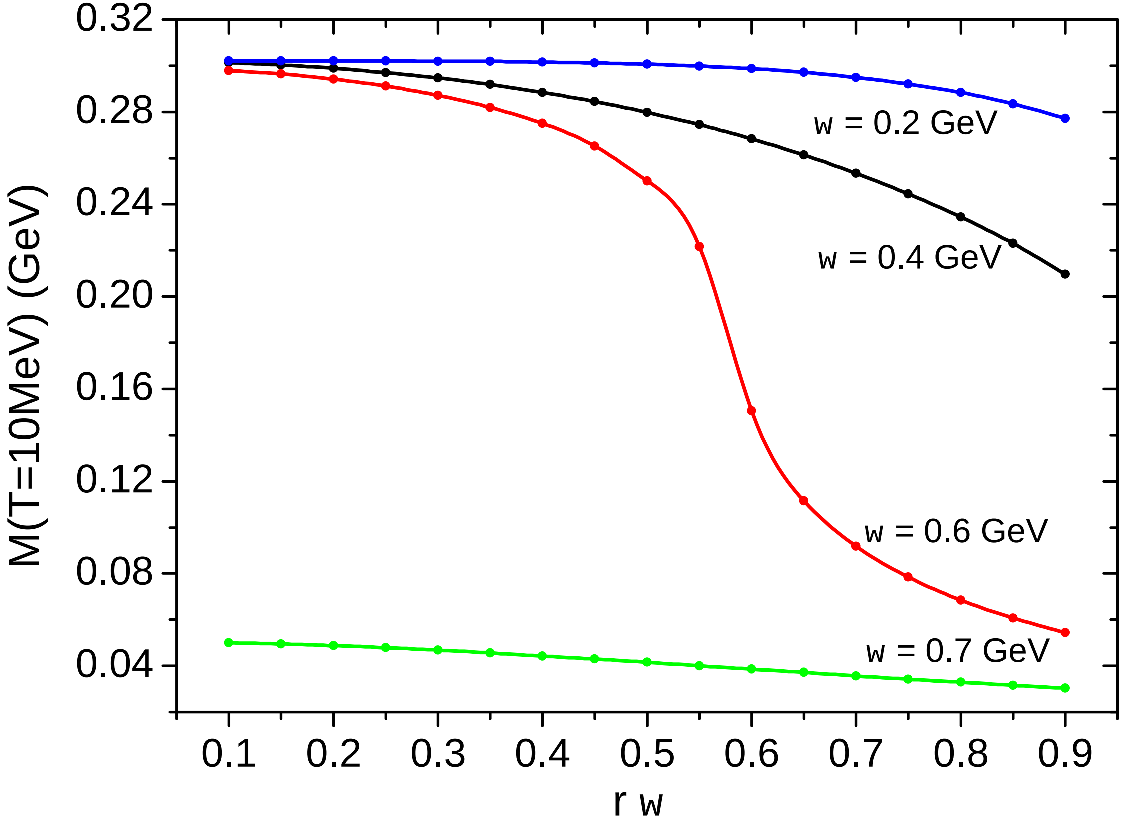
LDA: Rotation induced PT

- Small T : 1st order; large T: cross-over



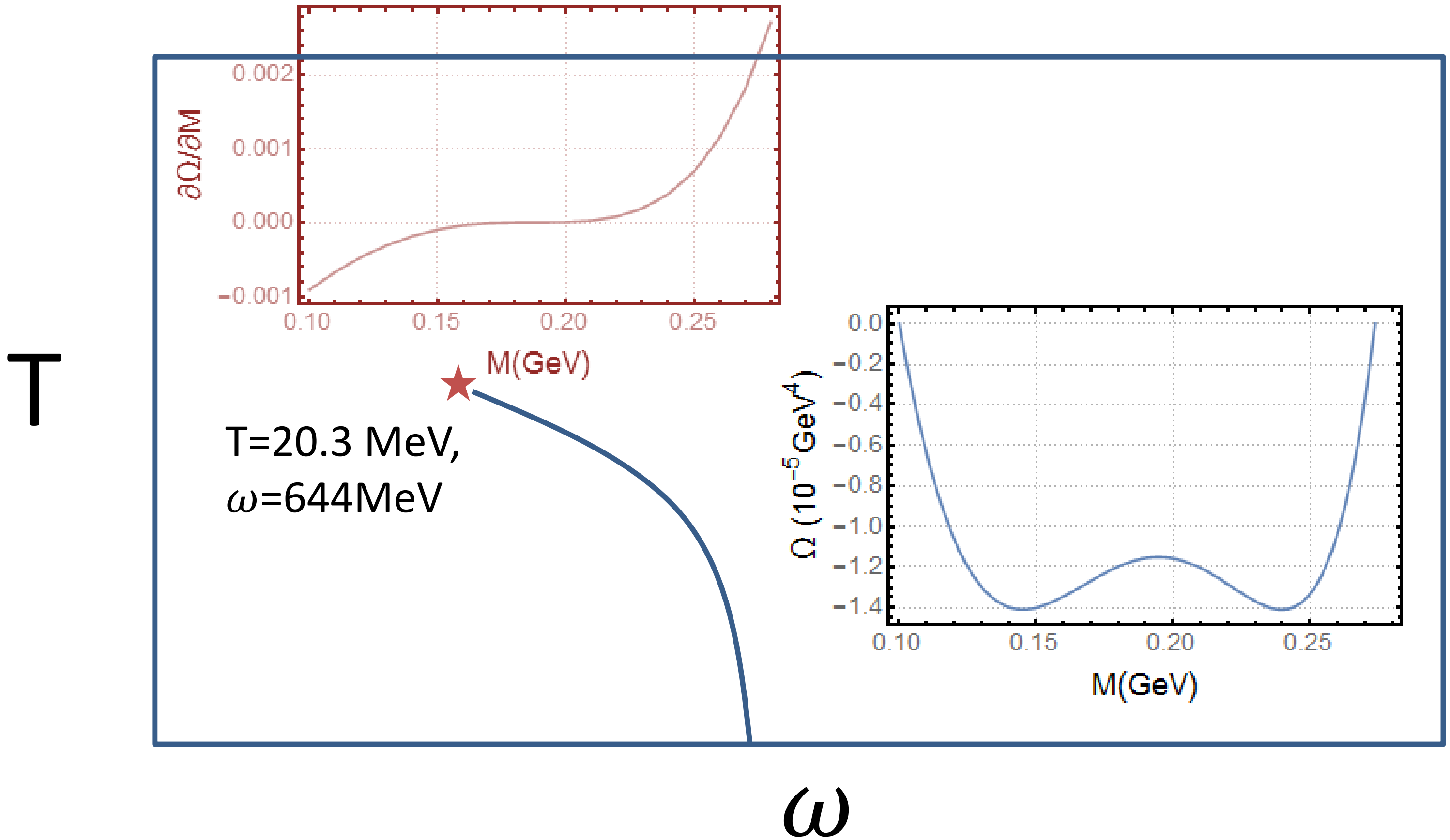
LDA: distance dependence

- Large distance suppression



Critical end point

- There should be a critical end point(2^{nd} order) in T-ω phase diagram.

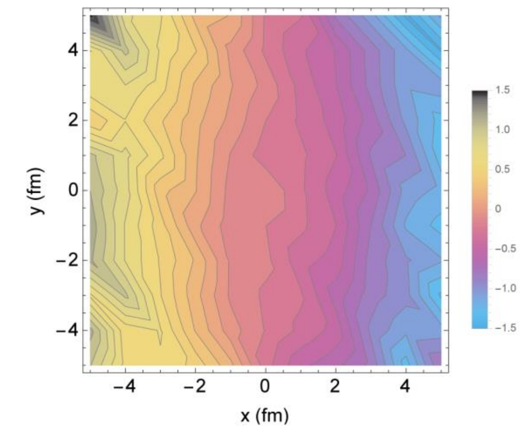


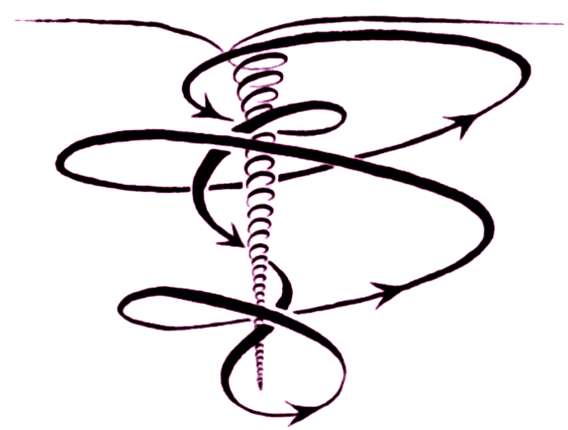
Go further

- Too ideal for a system rotating as a rigid body.
- Boundary conditions

  --static vacuum?

  --enhancement effect?



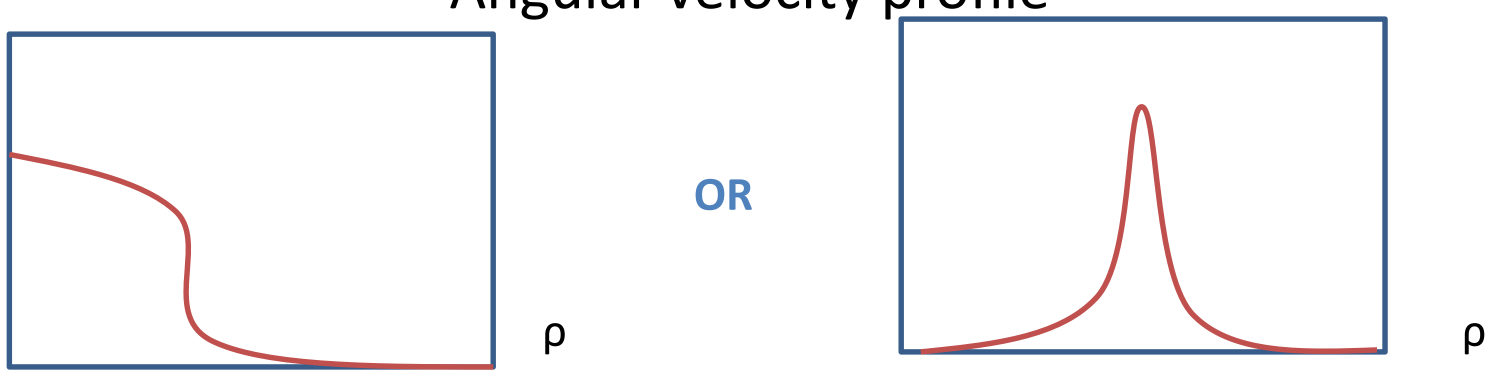


Full: Assumptions and scheme

- 2+1D case, renormalization $G(\Lambda) = \pi/(\Lambda - M_0)$
- Speed limit: $v = 2\, r^{-1} \int_0^r d\rho \rho \omega(\rho) < 1.$
- Space dependence or finite size.

~~Smooth condensate~~ ➡ Self-consistent computation

Angular velocity profile



Self-Consistent Gap equation

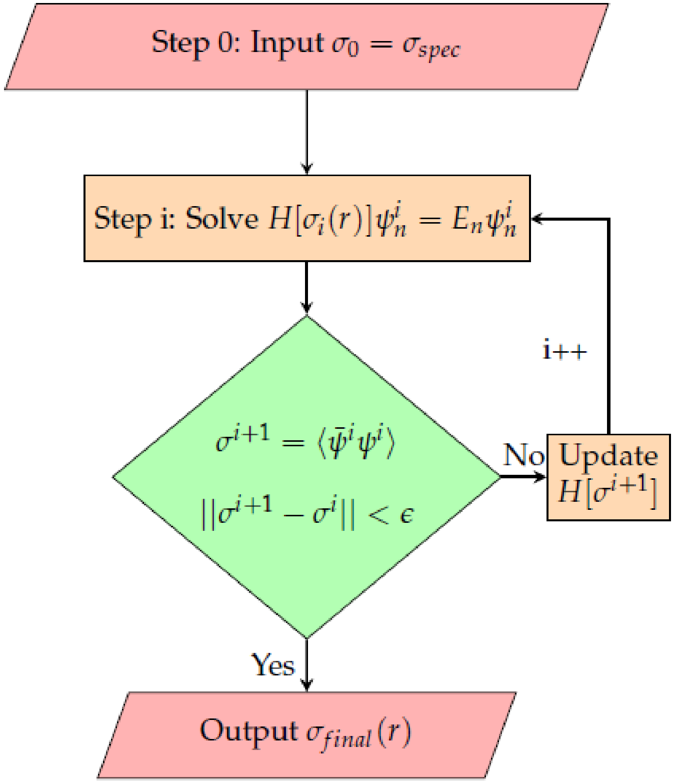
- Mean Field approximation

$$\sigma(\vec{r}) = -G\langle\bar{\psi}\psi\rangle$$

- Cylindrical symmetric basis

$$\psi_n^l(s) = \frac{1}{\sqrt{2\pi}}\sum_j c_{n,j}^{\uparrow}(s)\varphi_{j,l}^{s}(\rho,\theta)$$

- Brute-force diagonalization



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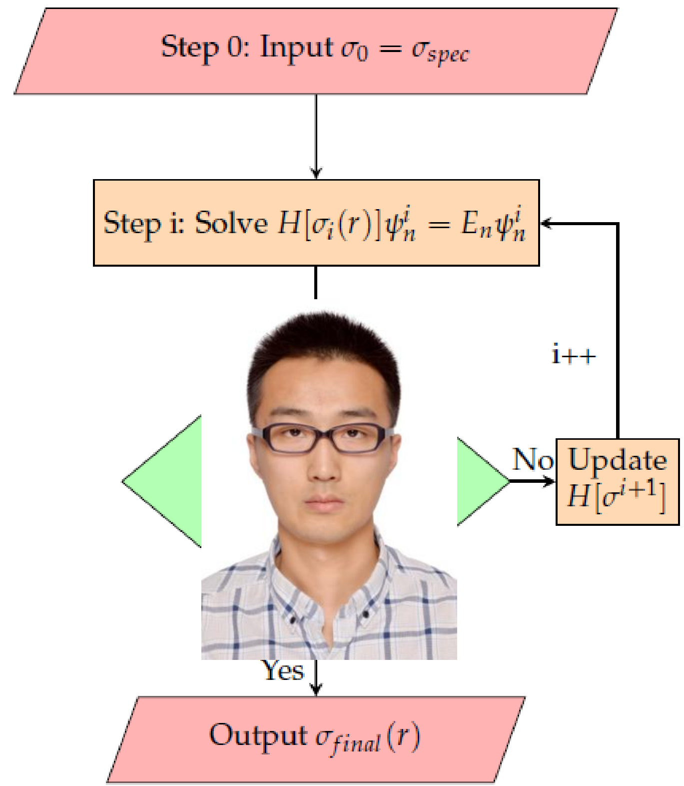
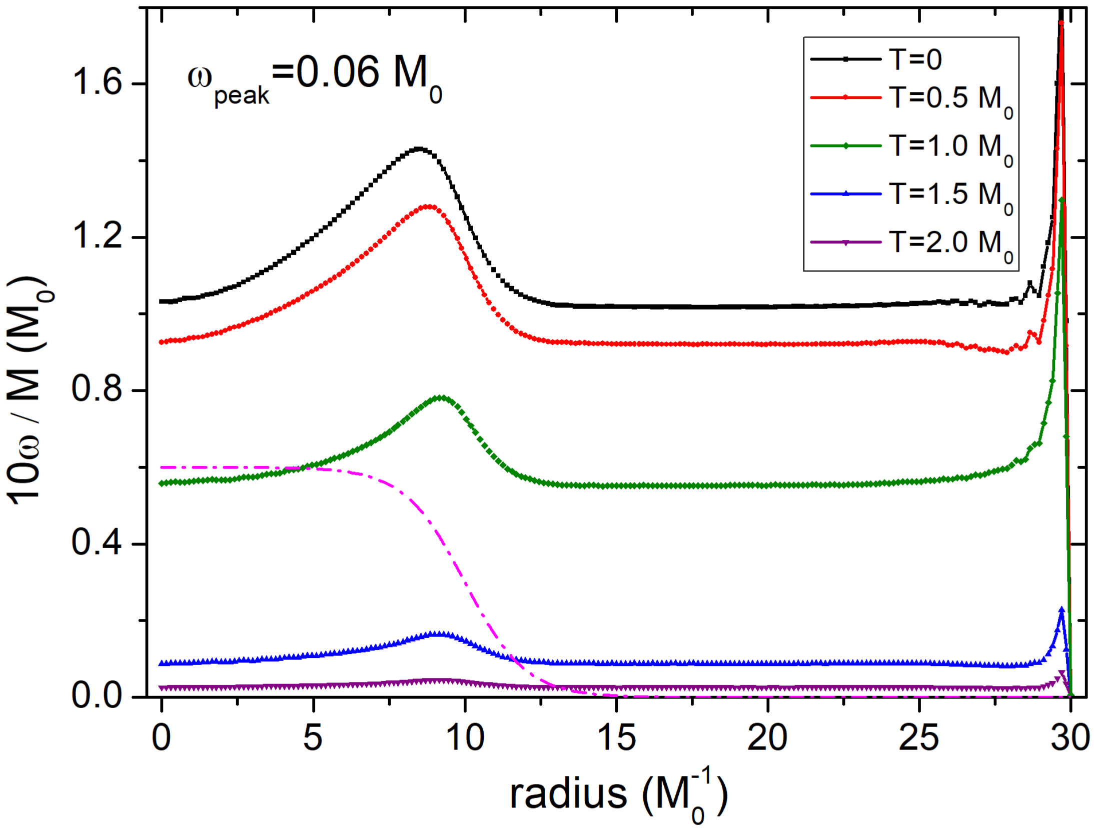


Centrifugal-like enhancement

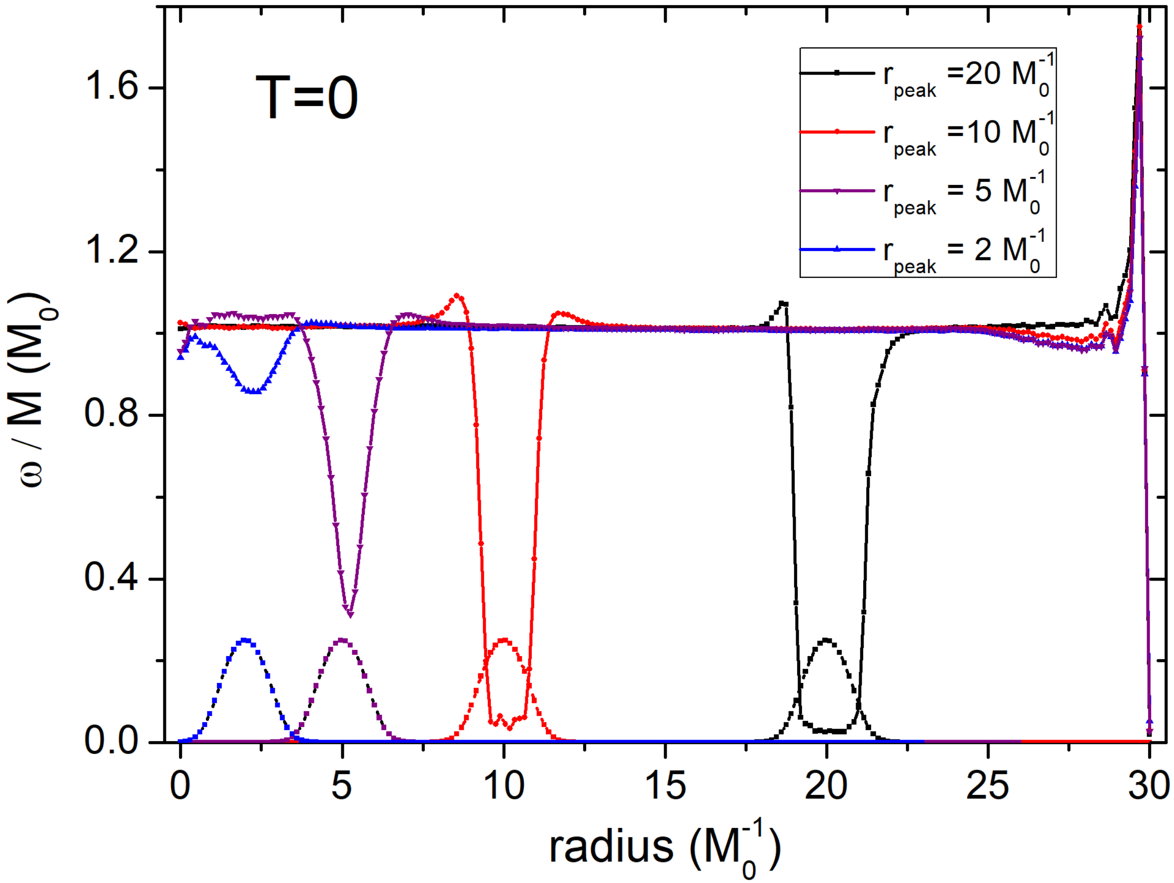
Condensate bump @ peak of $\partial_\rho \omega$



$$\sigma = G \sum \bar{\psi}\psi\left(\frac{1}{2} + f_{FD}(E_n)\right)$$

Inhomogeneous rotating vacuum

Condensate suppression @ large speed



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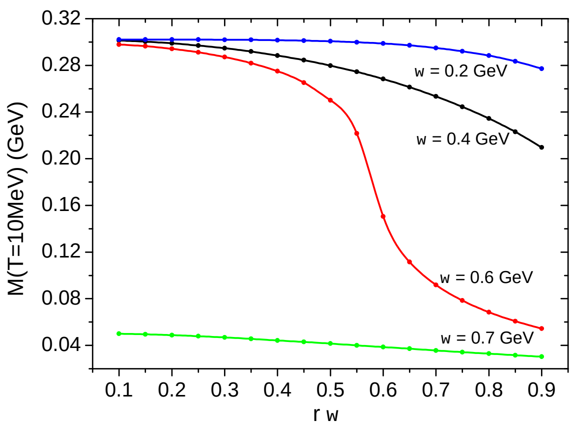
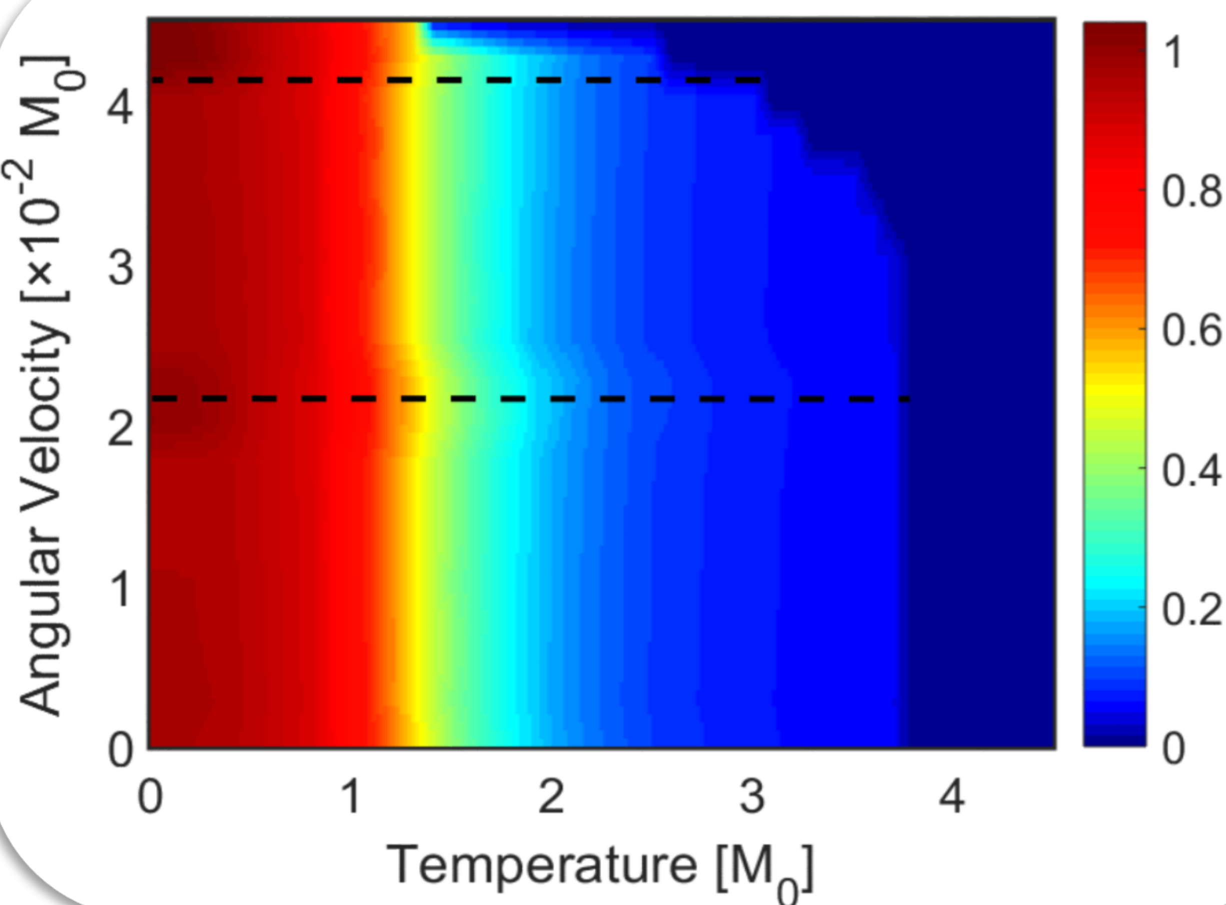


Phase diagram in Finite systems



Conclusions & Outlook

- Fermion pairing studied with 2+1D NJL model in Self-consistent BdG framework.
- Centrifugal effects and Chiral condensate suppression are observed in rotating systems.
- T-ω phase diagram is roughly similar, but NOT equivalent to T-μ diagram in finite systems.
- More novel states(vortex states) are worth to exploring.

Thank you

for your attention!

Cookbook of PT study

- Choose the model
- Write down the interaction (Lagrangian density)
- Order parameter(σ) & Approximation(Mean field)
- Calculate the free energy $Z = \int D\phi \, e^{-S[\phi,\sigma]} = e^{-\Omega[\sigma]V}$
- Minimize the free energy
- Gap equation for order parameter $f(\sigma, T, \mu, \ldots) = 0$

Model choice

- Aiming at the chiral condensate.
- A relativistic model is necessary for quark matter.
- A fermionic model is better than a bosonic one.

✓ Nambu—Jona-Lasinio model is a suitable one.

- Mean field approximation as the 1st step

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + 2G\langle\bar{\psi}\psi\rangle_{\vec{r}}\right)\psi + \mu\,\bar{\psi}\gamma^{0}\psi - G\langle\bar{\psi}\psi\rangle^{2} + \mathcal{L}_{rotation}$$