Spin current hydrodynamics in holographic first order gravity

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Motivation

Recent interest in the role of the spin current operator in

Condensed matter Spintronics : Maekawa et al. (2012)
Heavy Ion Collisions Physics of Polarization: Talks by F. Becattini, R. Ryblewsky, E. Speranza, A. Kumar
Anomalous Transport phenomena Axial current is encoded in the spin current tensor

Introduction of spin current operator for strongly coupled plasmas in holography is immediate

Holographic Recipe: Banados, Miskovic and Theisen (2006)

Earlier Attempts: Hashimoto, Iizuka and Kimura (2015)

Our goal

Systematically study transport properties of spin current operator $S^{\lambda}{}_{\mu\nu}$ in strongly coupled theories using holographic techniques

This talk

Introduction of spin tensor in holography

Hydrodynamic treatment of the spin tensor

Order zero thermodynamics and hydrodynamics with non-trivial spin sources

First order transport in local equilibrium (computation of CVSE coefficient)

What is holography?

Ads/CFT tell us that operators in a d-dim strongly coupled QFT are dual to classical fields in a d+1-dim gravitational theory

$$Z_{QFT}^{J} = Z_{\text{on-shell}}^{\text{gravity}}[\phi^{(0)} = J_{\mathcal{O}}] = e^{\int J_{\mathcal{O}}\mathcal{O}}$$

Boundary Conditions in gravity = Sources in QFT

Its all about the sources!

Which field is dual to $S^{\lambda}{}_{\mu\nu}$?

External gauge fields couple to global currents

Operator	Bulk Field	Symmetry	Bulk Theory
J_{μ}	A_{μ}	U(1)	Einstein-Maxwell
$T_{\mu\nu}$	$g_{\mu\nu}$	Translations	Einstein Gravity
$S^{\lambda}{}_{\mu\nu}$	?	?	?

Claim: $S^{\lambda}{}_{\mu\nu}$ is dual to spin connection $\omega^{\lambda}{}_{\mu\nu}$ in a first order formulation of gravity

Symmetry argument: $S^{\lambda}{}_{\mu\nu}$ is related to Lorentz transformations

Supporting the claim

Spin Current for free Dirac fermions: $S^{\lambda}{}_{\mu\nu} = i\bar{\psi}\gamma^{\lambda}\gamma_{\mu\nu}\Psi$

Coupling the fermions to curved background

$$I[\psi] = \int d^4x|e|i\bar{\psi}\gamma^a e_a^\mu \left(\partial_\mu + \frac{i}{2}\omega^{ab}{}_\mu\gamma_{ab}\right)\psi + c.c.$$

We must use vielbein when working with fermions: $e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}$

Spin connection sources the spin current

In holography we don't work with free fermions but principle holds for more general theories

Sourcing both $T_{\mu\nu}$ and $S^{\lambda}{}_{\mu\nu}$ implies working in a first order background Bulk theory should admit propagating torsion

Sourcing $S^{\lambda}{}_{\mu\nu}$ and $T_{\mu\nu}$ in QFT

Consider a QFT coupled to a first order curved background

Invariance of the action under translation and Lorentz transformations implies conservation equations

$$\nabla_\mu \left(|e| T^\mu{}_\nu\right) = \frac{|e|}{2} S^\lambda{}_{\rho\sigma} R^{\rho\sigma}{}_{\lambda\nu} \qquad \nabla_\lambda \left(|e| S^\lambda{}_{\rho\sigma}\right) = \frac{|e|}{2} T_{[\rho\sigma]}$$

$T_{\mu\nu}$ and $S^{\lambda}{}_{\mu\nu}$ are related to τ_a and σ_{ab} : $\tau_a = \frac{\delta S}{\delta e^a} \qquad \sigma_{ab} = \frac{\delta S}{\delta \omega^{ab}}$

$T_{\mu\nu}$ is not necessarily symmetric.

Conservation equations will act as effective hydrodynamic equations

Hydrodynamic Ansatz

Look for a solution in terms of a gradient expansion

$$T^{\mu\nu} = a_1 u^\mu u^\nu + a_2 \eta^{\mu\nu} + a_3 u^\mu \omega^\nu + a_5 u^\mu a^\nu + \ldots$$

$$S^{\lambda\mu\nu} = b_1 \epsilon^{\lambda\mu\nu\sigma} u_\sigma + b_2 \eta^{\lambda[\mu} u^{\nu]} + b_3 u^\lambda \epsilon^{\mu\nu\rho\sigma} u_\rho \omega_\sigma + b_4 u^\lambda u^{[\mu} a^{\nu]} + \ldots$$

Considering u^μ as only source with a constant temperature T_0

Vorticity $\omega^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$ and acceleration $a_\mu = u^\rho \partial_\rho u_\mu$

This expansion can be enhanced by allowing for external sources of spin

More transport coefficients in the expansion

General sources of spin in 4D QFT

Most general group decomposition of $\omega^{ab}{}_{\mu}$

Lorentz 4D = (1/2,1/2) + (1/2,1/2) + (3/2,1/2) +(1/2,3/2)

Vector   Axial Vector   Chiral Tensor

Projection of the representations into the rest frame

$[(1/2,1/2) + (1/2,1/2)]_V$ = Scalar (μ_V) + Vector (V)

$[(1/2,1/2) + (1/2,1/2)]_A$ = PScalar(μ_A) + PVector (A)

$[(3/2,1/2) + (1/2,3/2)]_V$ = Vector (D) + ST-Matrix (H)

$[(3/2,1/2) + (1/2,3/2)]_A$ = PVector (W) + ST-PMatrix (C)

Even in the absence of sources these components will interact in the bulk

Propagating torsion is essential

What can holography tell us?

Holographic recipe give us < $T_{\mu\nu}$ > and < $S^{\lambda}{}_{\mu\nu}$ > from the bulk gravitational solution

Banados, Miskovic and Theisen (2006)
Cvetkovic, Miskovic and Simic (2017)
Gallegos and Gursoy (To appear 2019)

Thermodynamic energy is also given through the first law

Gallegos and Gursoy (To appear 2019)

$$U = \int d\Sigma_\mu \left[\langle T^{\mu\nu} \rangle u_\nu - \frac{1}{2} \langle S^{\mu\rho\sigma} \rangle \omega_{\rho\sigma\lambda} u^\lambda \right]$$

Using the hydrodynamic expansion in the gravitational side give us all transport coefficients and thermodynamic info

Hydro Conservation equations will appear as constraints on the gravitational equations of motion

First examples of transport allowing for a non-trivial $S^{\lambda}{}_{\mu\nu}$

Toy holographic model (5D Lovelock Chern-Simons gravity)

Consider two different set ups:

- Allow for non-trivial $O(\partial^0)$ spin sources and compute $O(\partial^0)$ constitutive relations (pressures and charges).
- Consider a rotating fluid in local equilibrium at finite (T_0, μ_A) and compute $O(\partial^1)$ constitutive relations (CVSE coefficient computation)

Holographic Setup

Toy model action (5D Lovelock Chern-Simons gravity)

$$S = \kappa \int \epsilon_{abcde} \left[R^{ab} R^{cd} e^{e} + \frac{2}{3} R^{ab} e^{c} e^{d} e^{e} + \frac{1}{5} e^{a} e^{b} e^{c} e^{d} e^{e} \right]$$

Disclaimer: Theory has a non-abelian anomaly and its relation to a chiral anomaly remains unclear

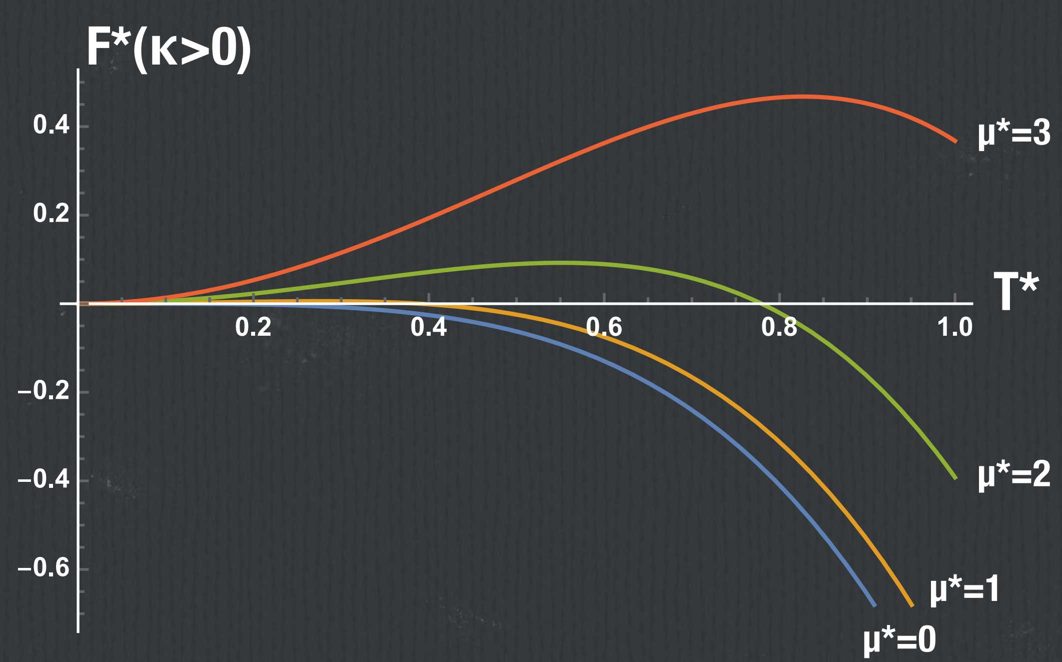
Personal Opinion:
Anomaly vanishes at zero boundary torsion → It can not take the form of a local gravitational anomaly

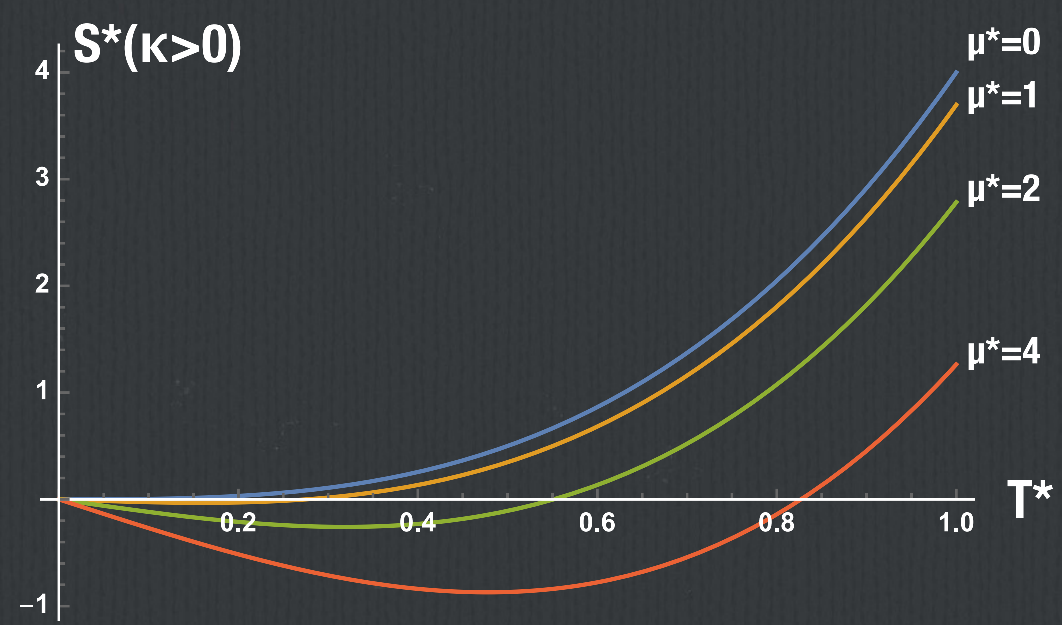
First Setup-Example 1: Conformal Charged Fluid

Consider only μ_A as source (pseudo scalar part of $\omega^{ab}{}_{\mu}$)

$$T^{\mu\nu} = 32\pi^4\kappa T_0^4\left(1+\frac{\mu^2}{2\pi^2 T_0^2}\right)(4u^\mu u^\nu + \eta^{\mu\nu})$$

$$S^{\lambda\mu\nu} = 32\pi^2\kappa T_0^2\mu_A\epsilon^{\lambda\mu\nu\rho}u_\rho$$



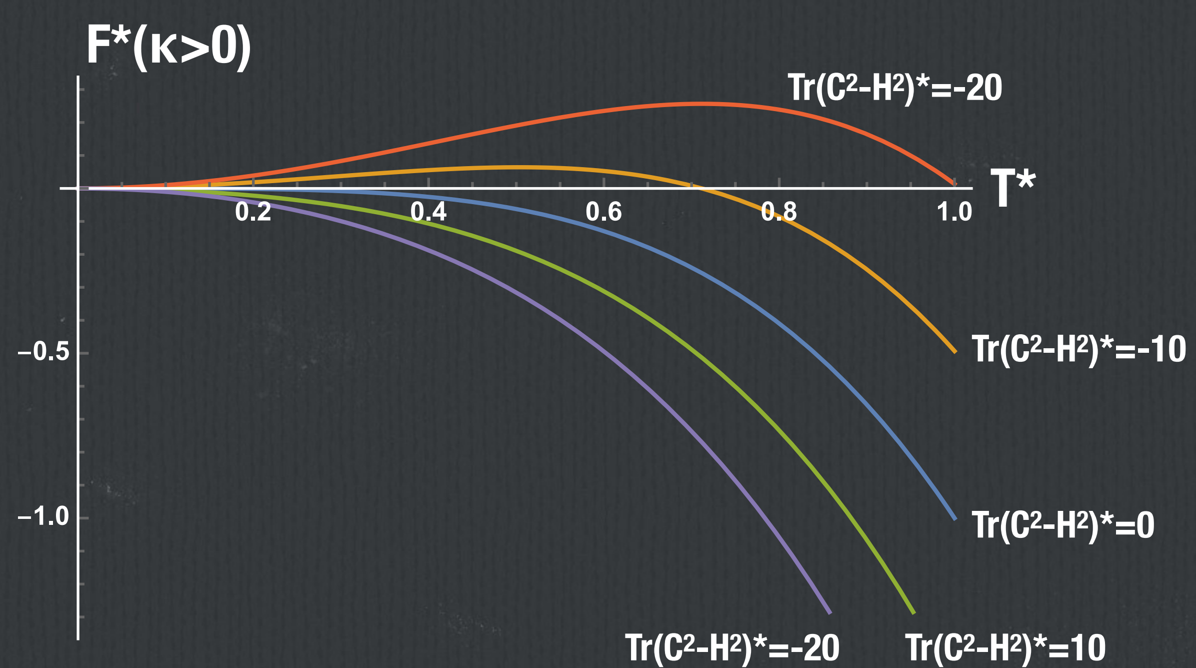


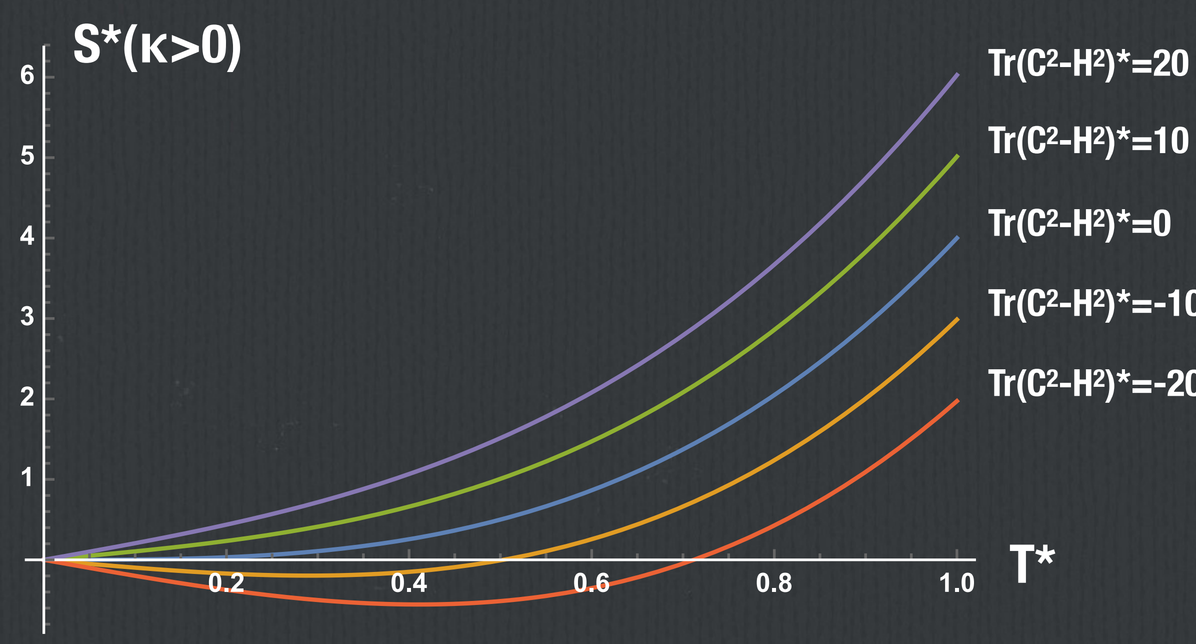
First Setup-Example 2: Fluid with Tensor Charges

Consider only tensor sources H and C

$$T^{\mu\nu} = 32\pi^4 \kappa T_0^4 \left[.4u^\mu u^\nu \left(1 + \frac{\mathrm{Tr}\left(C^2 - H^2\right)}{8\pi^2 T_0^2} \right) + \eta^{\mu\nu} \left(1 + \frac{\mathrm{Tr}\left(C^2 - H^2\right)}{4\pi^2 T_0^2} \right) - 2(C^2 - H^2)^{\mu\nu} \right]$$

$$S^{\lambda\mu\nu} = 32\pi^2 \kappa T_0^2 \left[H^{\lambda[\mu} u^{\nu]} + \epsilon^{\lambda\rho\sigma[\mu} C_\rho^{\nu]} u_\sigma \right]$$





First Example: Conclusions

Sensible answers for energy density, charge density, and thermodynamic potentials

Plethora of $O(\partial^0)$ constitutive relations arise from holography (instances of non-symmetric $T_{\mu\nu}$)

Gallegos and Gursoy (To appear 2019)

Procedure of general spin sourcing can be a starting point for studying transport in system with broken rotational invariance Obvious extension beyond hydrodynamic limit

Second Setup

(Preliminary)

Study conformal charged fluid with non vanishing vorticity $\omega^\mu=\epsilon^{\mu\nu\rho\sigma} u_\nu\partial_\rho u_\sigma$.

Choosing appropriate boundary conditions and taking the $\mu_A\to 0$ the following can be found

$$T^{\mu\nu} = 32\pi^4\kappa T_0^4\left[4u^\mu u^\nu + \eta^{\mu\nu}\right]$$

$$S^{\lambda\mu\nu} = -16\pi^2\kappa T_0^2 u^\lambda \epsilon^{\mu\nu\rho\sigma} u_\rho\omega_\sigma + 8\pi^2 T_0^2 \epsilon^{\lambda\mu\nu\rho}\omega_\rho$$

CVSE coefficient from totally antisymmetric part

Second Example: Observations

CVSE coefficient shows up due to the coupling of vorticity and the spin connection

CVSE without chiral gravitational anomaly?

Many arguments relating the CVSE coefficient with local and global gravitational anomalies

Landsteiner, Megias and Pena-Benitez (2013)
Liu, Glorioso and Rajagopal (2017)
Golkar and Sethi (2016)

Personal Opinion: The theory should have a global gravitational anomaly when space is a torus

Model admits more general form of sources

Generalization of anomalous transport related to vorticity to a more general class of theories?

CVSE coefficient: Comparing with standard approach

Local gravitational anomaly is introduced through CS term

Landsteiner, Megias and Pena-Benitez (2013)

$$S_{CS} = \lambda \int A \mathrm{Tr}\left(RR\right)$$

A is a U(1) axial Gauge Field sourcing axial current

λ is fixed by gravitational anomaly

Evaluating this term on-shell up to first order in derivatives reproduces

$$S_{\text{on-shell}} = 8\pi^2 T_0^2 \lambda \int A u du$$

u is the velocity of the fluid

Functional differentiating with respect to A gives CVSE coefficient

CVSE coefficient: Standard approach with new eyes

For higher derivative theories first and second order formulation of gravity are inequivalent

Only in first order is the variational problem well defined for a term of the form of S_{CS}

Include vector chemical potential μ_V on the spin connection and evaluate CS term on-shell

$$S_{\text{on-shell}} = 2\lambda \int \left(4\pi^2 T_0^2 + \mu_V^2\right) A u du$$

Fixes two anomalous transport coefficients via the gravitational anomaly

Take home message

With a non-trivial spin tensor we have to be careful when including local gravitational anomalies

Naive implementation give us more than what we asked for

Role of gravitational anomaly in CVSE coefficient must be carefully analyzed

Summary

We have been able to introduce the spin current operator in an holographic framework

We have done some simple computations testing the power of the holographic description

External sources of spin and anomalous transport

Personal comment:

- Belinfante choice is dual to Levi-Civita choice.
- First order formulation and second order formulation (Levi-Civita choice) are physically inequivalent.
- Inequivalence is dual to pseudo-gauge inequivalence

Outlook

Incorporation of dissipation effects is straightforward in the holographic prescription.

Hydrodynamic equations and transport coefficients for strongly coupled plasmas with spin can be derived systematically from holography.

Duality between spin tensor and connection goes beyond hydrodynamics and can find applications in other settings.