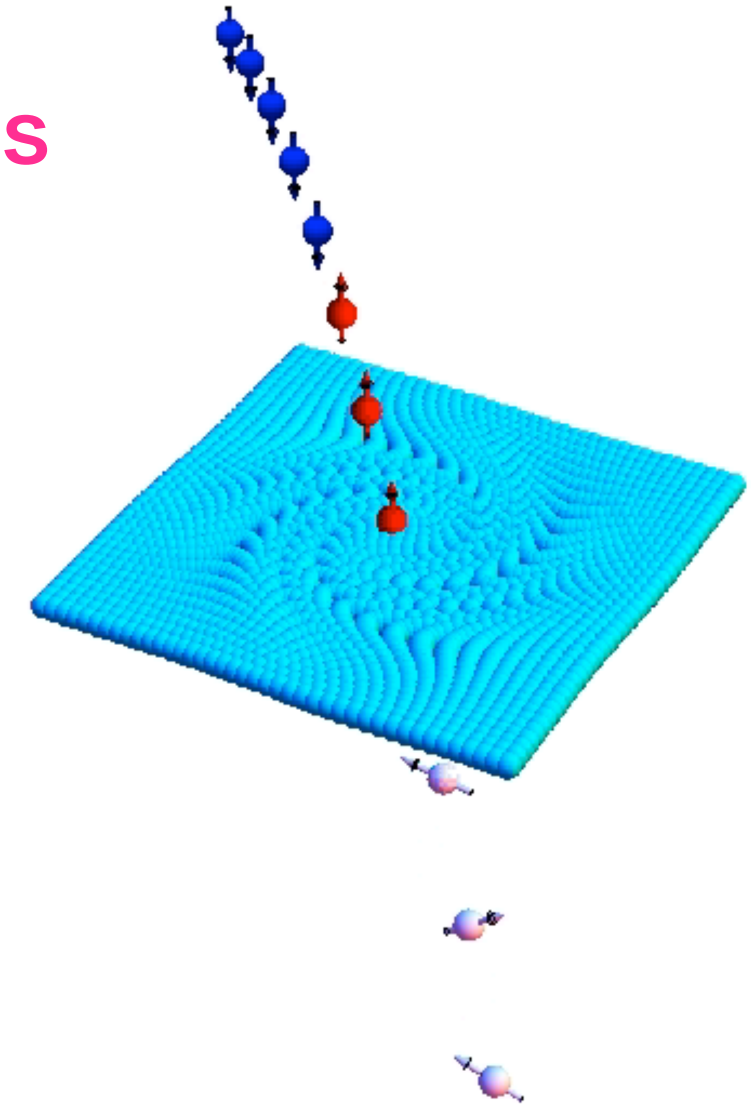
Spin hydrodynamics in condensed matter systems



Mamoru Matsuo
(Kavli Institute for Theoretical Sciences,
Univ. of Chinese Academy of Sciences)

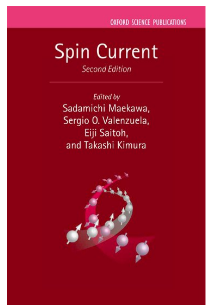
in collaboration with :

(Theory)
Y. Ohnuma, J. Ieda & S. Maekawa

(Experiment)
H. Chudo, R. Takahashi, M. Ono, K. Harii,
Y. Ogata, M. Imai, S. Okayasu, & E. Saitoh (JAEA)
R. Iguchi (NIMS)
D. Kobayashi, Y. Nozaki (Keio U.)

Ref.

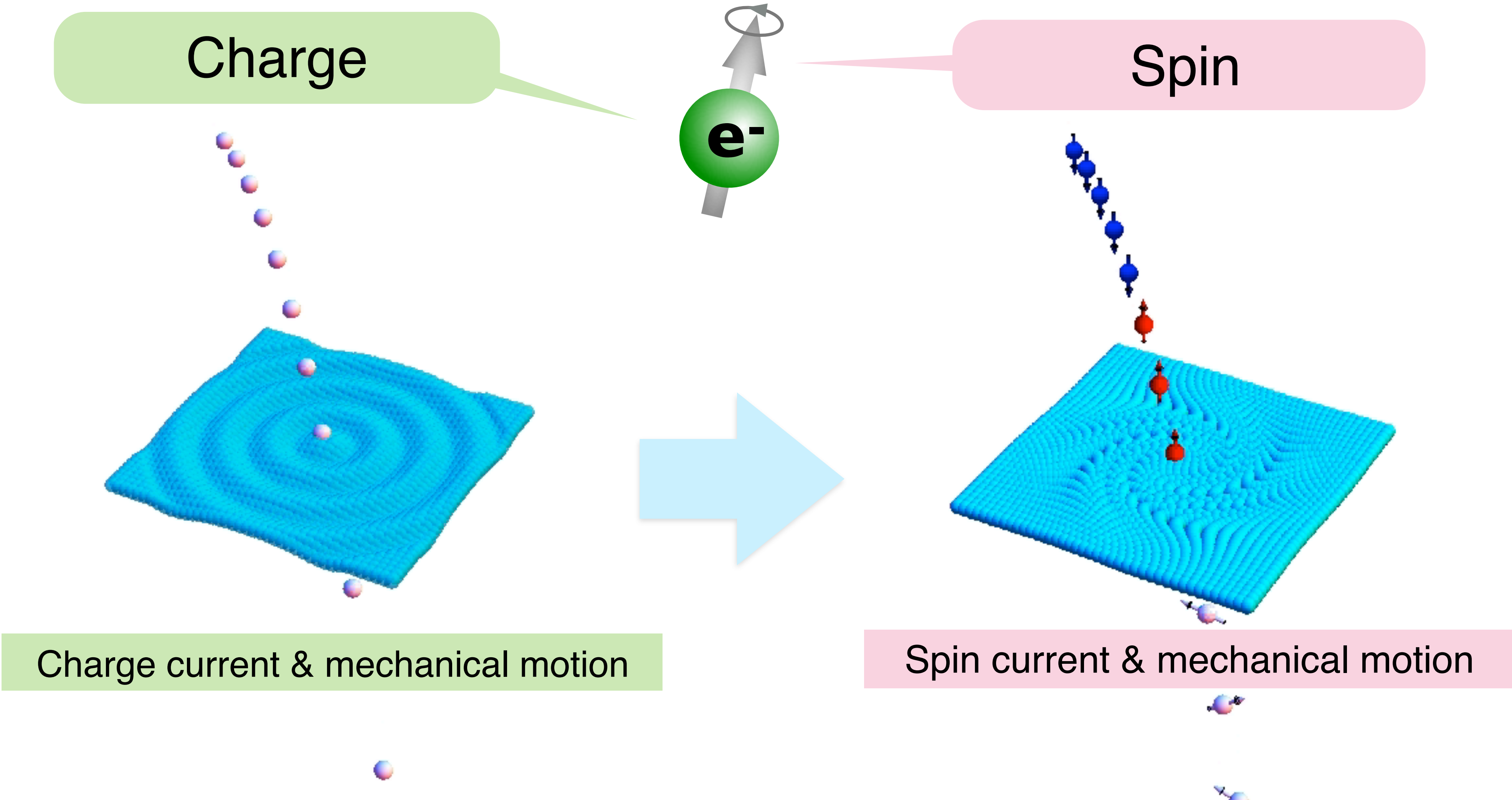
MM et al., “Spin-mechatronics” Chap. 25 in Spin current 2nd ed.(Oxford, 2017)
Takahashi et al., Nat. Phys.12, 52 (2016)
Kobayashi et al., Phys. Rev. Lett. 119, 077202 (2017)



See also.

K.Hattori, M.Hongo, X.-G.Huang, MM, H.Taya, arXiv:1901.06615

“Spin-mechatronics”



Observation of spin-current generation by

- Liquid metal motion in Hg (R.Takahashi et al., Nat. Phys. 2016)
- Surface acoustic wave in Cu (D.Kobayashi et al., PRL 2017)
- Rigid rotation in Pt (A.Hirohata et al., Sci.Rept.2018)

What is electron?

Electronics

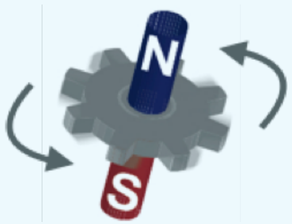
Charge
[electricity]

Spin
[magnetism]

Spintronics

Charge current = flow of charged spheres

robust ⇒ easy to control

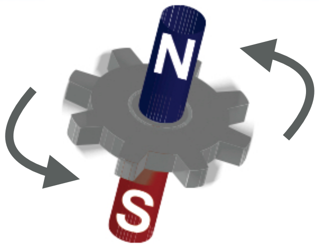


Spin current = flow of spinning gears

fragile ⇒ controlled by **nanotechnology**
to utilize **magnetism** and **rotation**

How to control spins?

Conventional spintronics:
spin as a tiny magnet



Spin mechatronics:
spin as a spinning gear

$$H_{\mathrm{Zeeman}} = -S \cdot \gamma B$$
$$H_{\mathrm{Spin-Orbit}} = -S \cdot (\lambda p \times E)$$

$$H_{\mathrm{Spin-vorticity}} = -S \cdot \frac{\omega}{2}$$
$$\mathrm{vorticity}: \ \omega = \nabla \times v$$

Electron in inertial frames
(Non-relativistic limit of
Special relativistic **Dirac equation)**

Electron in non-inertial frames
(Non-relativistic limit of
General relativistic **Dirac equation)**

w/ Magnets,
w/ strong spin-orbit
materials (Pt, W, ···)

w/o magnets,
w/o spin-orbit coupling!
(Cu, Al, ···)

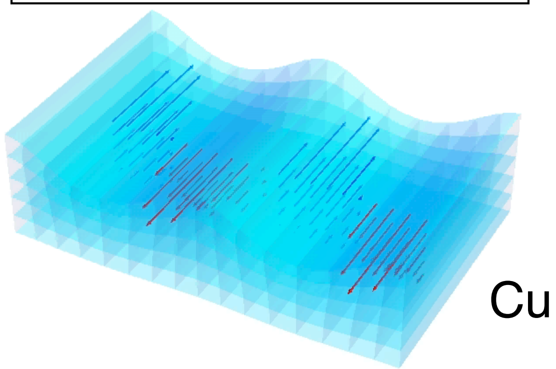
Mechanical generation of spin current

Dirac eq. **in non-inertial frame [spin connection]**
(Electron in moving materials)

$$H = \beta mc^2 + (c\alpha - v) \cdot (p + eA) + eA_0 - S \cdot \omega/2$$

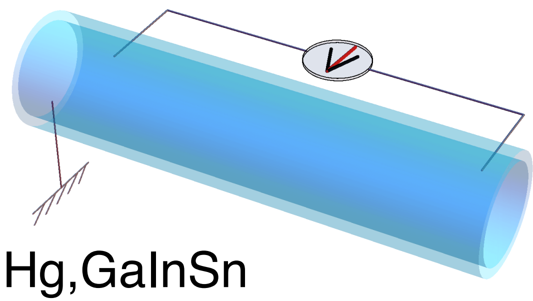
$$H = \frac{(p + eA)^2}{2m} + e\phi - S \cdot \gamma B - S \cdot \omega/2 - \frac{e\lambda}{\hbar} S \cdot p \times \left(E + (\omega \times r) \times B\right)$$

Surface acoustic wave



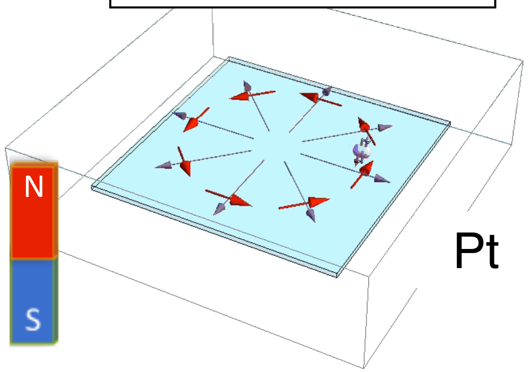
PRB(R)2013
PRL2017

Fluid vorticity



PRB(R)2017
Nat.Phys. 2016

Rigid rotation



PRL2011
Sci.Rept. 2018

Observation of spin-rotation coupling in quark-gluon plasma (Nature 2017)

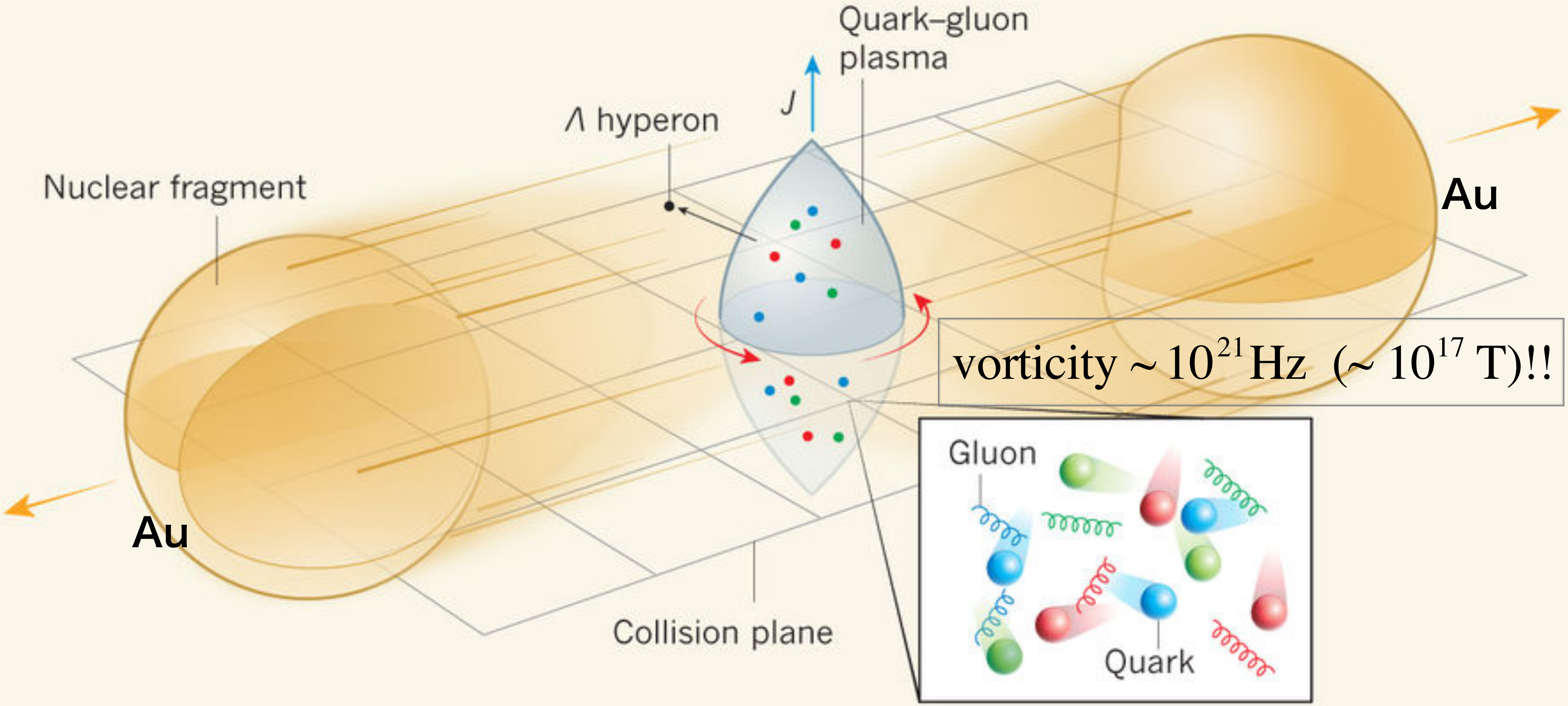
LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*

Au+Au, non-central collision in Relativistic Heavy Ion Collider (RHIC)



vorticity $\sim 10^{21}$ Hz ($\sim 10^{17}$ T)!!

Recently, Takahashi *et al.*[14] reported the first observation of a coupling between the vorticity of a fluid and the internal quantum spin of the electron, opening the door to a new field of fluid spintronics. In their study, the vorticity ω—a measure of the 'swirl' of the velocity flow field around any point (non-relativistically, $\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$)—is generated through shear viscous effects as liquid mercury flows next to a rigid wall.

Ref.14: R.Takahashi et al., Nature Physics 12, 52 (2016)

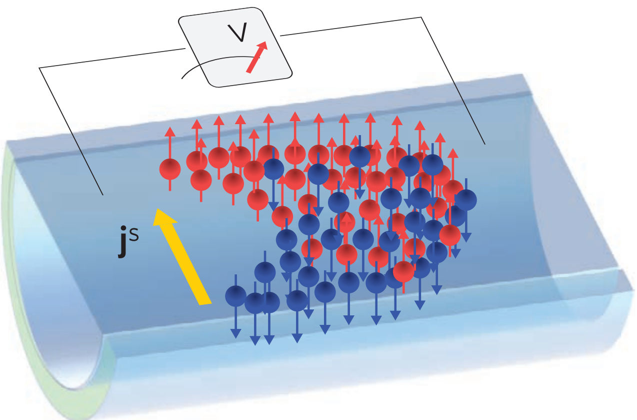
news & views

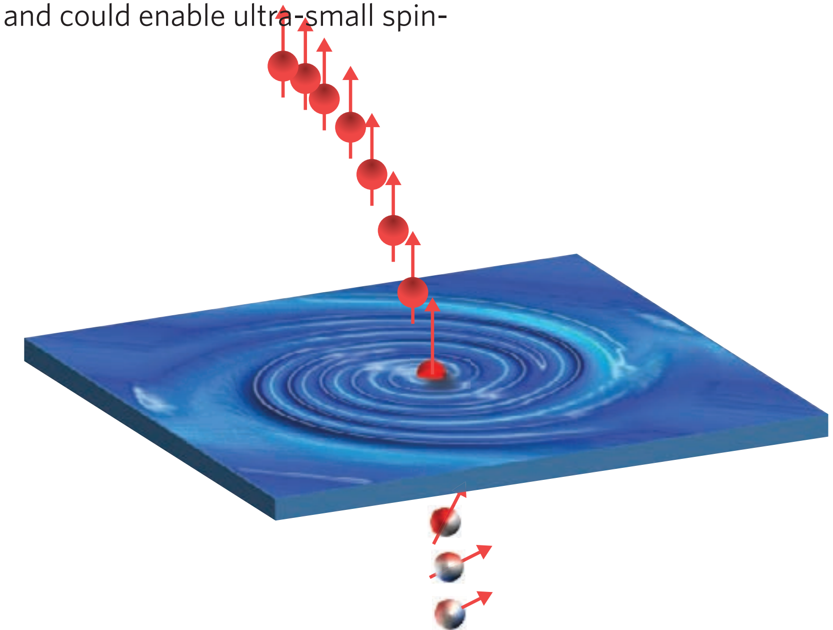
FLUID SPINTRONICS

Cause a stir

The rotational motion of liquids can induce a flow of electron spins, and could enable ultra-small spin-hydrodynamic generators that operate with liquid metals.

Igor Žutić and Alex Matos-Abiague





Contents

Gyromagnetic effect

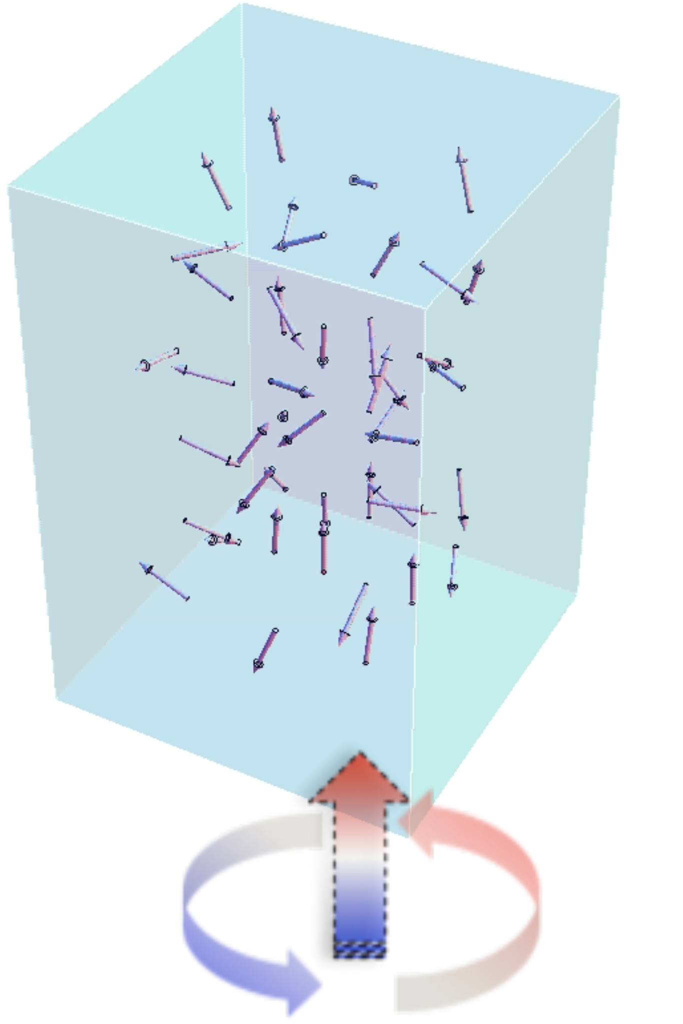
Einstein—de Haas/Barnett effect

Gyrospintronic effect

Spin current generation via. spin-vorticity coupling

Magnetization by rotation: Barnett effect (1915)

$$H_{\text{Spin-rotation}} = -S \cdot \Omega$$



$$H_{\text{Cor}} = -L \cdot \Omega$$

$$H_{\text{Zeeman}} = -S \cdot \gamma B$$

$$B_{\Omega} = \frac{\Omega}{\gamma} \left[\gamma = \frac{e}{m} \text{:gyromagnetic ratio} \right]$$

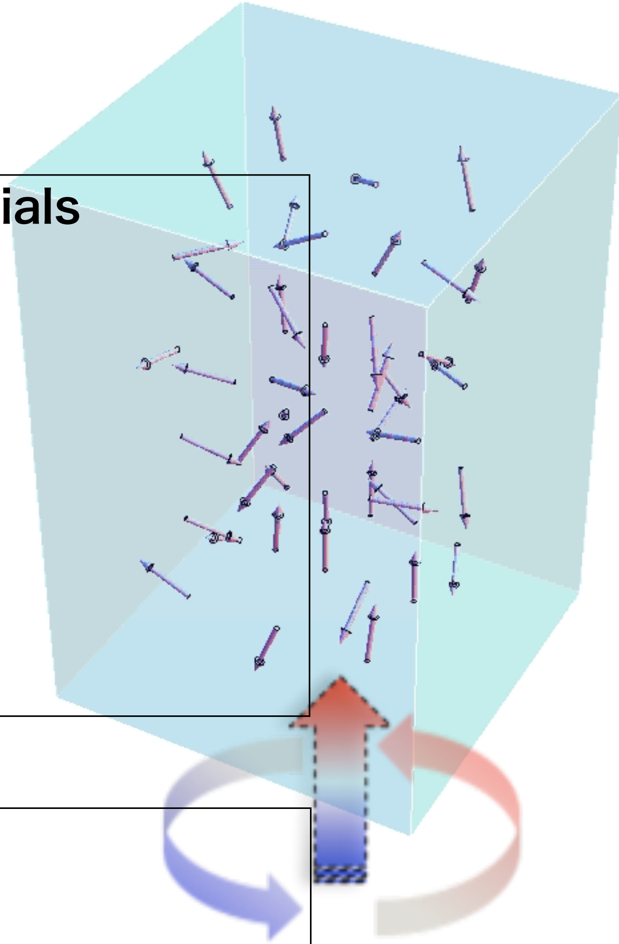
$$H_{\text{Spin-rotation}} = -S \cdot \Omega$$

Observation of spin-vorticity coupling

- Ferromagnets: Barnett's original exp. (1915)

- Theoretical predictions: MM et al., PRL(2011)

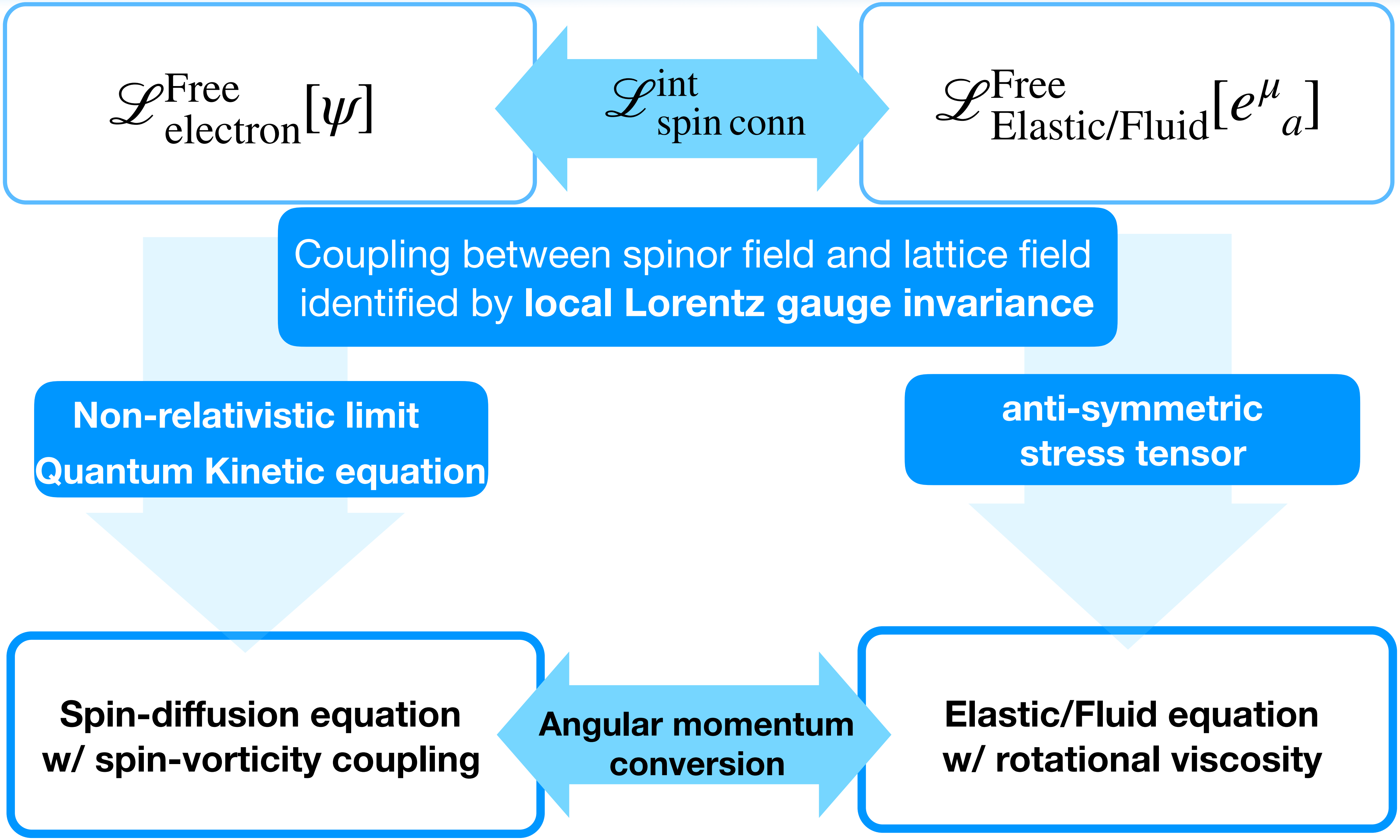
$$H_{\mathrm{SV}} = -S \cdot \omega / 2$$



Spin-vorticity coupling arise universally in rotating materials
- Paramagnetic states (Gd, Tb, Dy):
  Ono et el., PRB(2015),
  Ogata et al., APL(2017); JMMM(2017)
- Ferrimagnetic states
  Imai et al., APL(2018, 2019)
- Nuclear spin:
  Chudo et al., APEX(2014), JPSJ(2015)

Spin-current generation by rotation
- Liquid metal flow: Takahashi et al, Nat.Phys.(2016)
- Surface acoustic wave: Kobayashi et al., PRL(2017)
- Rigid rotation under magnetic field: Hirohata et al., Sci.Rept (2018)

Theoretical framework



Eringen (1964)

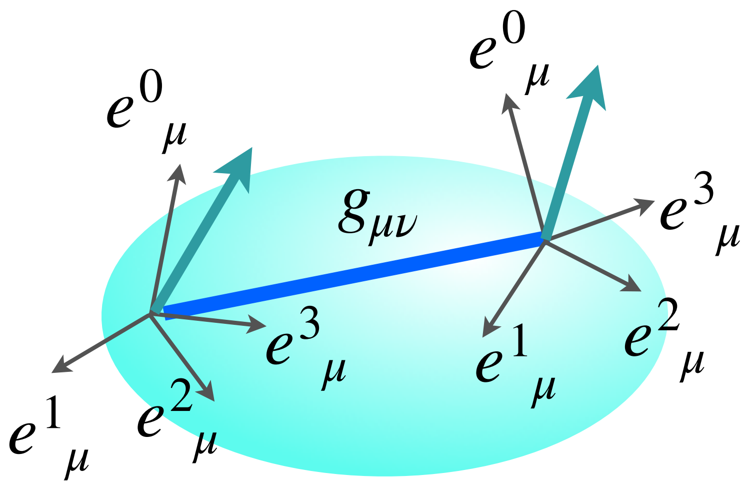
MM et al., PRB(R) 2017

Takahashi, MM et al., NatPhys2016

Tetrad (vierbein) formalism and local Lorentz invariance

Cartan (1922)

Gravity w/ spin & torsion



$$g_{\mu\nu}(x) = \eta_{ab} e^a{}_\mu(x) e^b{}_\nu(x)$$

Hehl et al.,(1976)

Dirac algebra in curved spacetime

$$\tilde{\gamma}_\mu(x) := \gamma_a e^a{}_\mu(x) \quad \{\tilde{\gamma}_\mu(x), \tilde{\gamma}_\nu(x)\} = 2 g_{\mu\nu}(x)$$

$$\begin{aligned} \{\tilde{\gamma}_\mu(x), \tilde{\gamma}_\nu(x)\} &= \{\gamma_a, \gamma_b\} e^a{}_\mu e^b{}_\nu \\ &= 2\eta_{ab} e^a{}_\mu(x) e^b{}_\nu(x) = 2 g_{\mu\nu}(x) \end{aligned}$$

Local Lorentz inv. Lagrangian

$$\mathscr{L}_{\mathrm{tot}} = -\bar{\psi}\left[i e^\mu{}_a \gamma^a \left(p_\mu - \frac{i}{2}\omega_\mu{}^{ab}\Sigma_{ab}\right) + m \right]\psi + \mathscr{L}_{\mathrm{gravity}}[e^\mu{}_a]$$

Spin connection

$$\psi \to \psi' = \exp[i\theta^{ab}(x)\Sigma_{ab}]\psi \quad \left(\Sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]\right)$$

Spin connection absorbs $\partial_\mu \theta^{ab}(x)\Sigma_{ab}$

Spin connection assures **local angular momentum conservation law.**

Spin connection

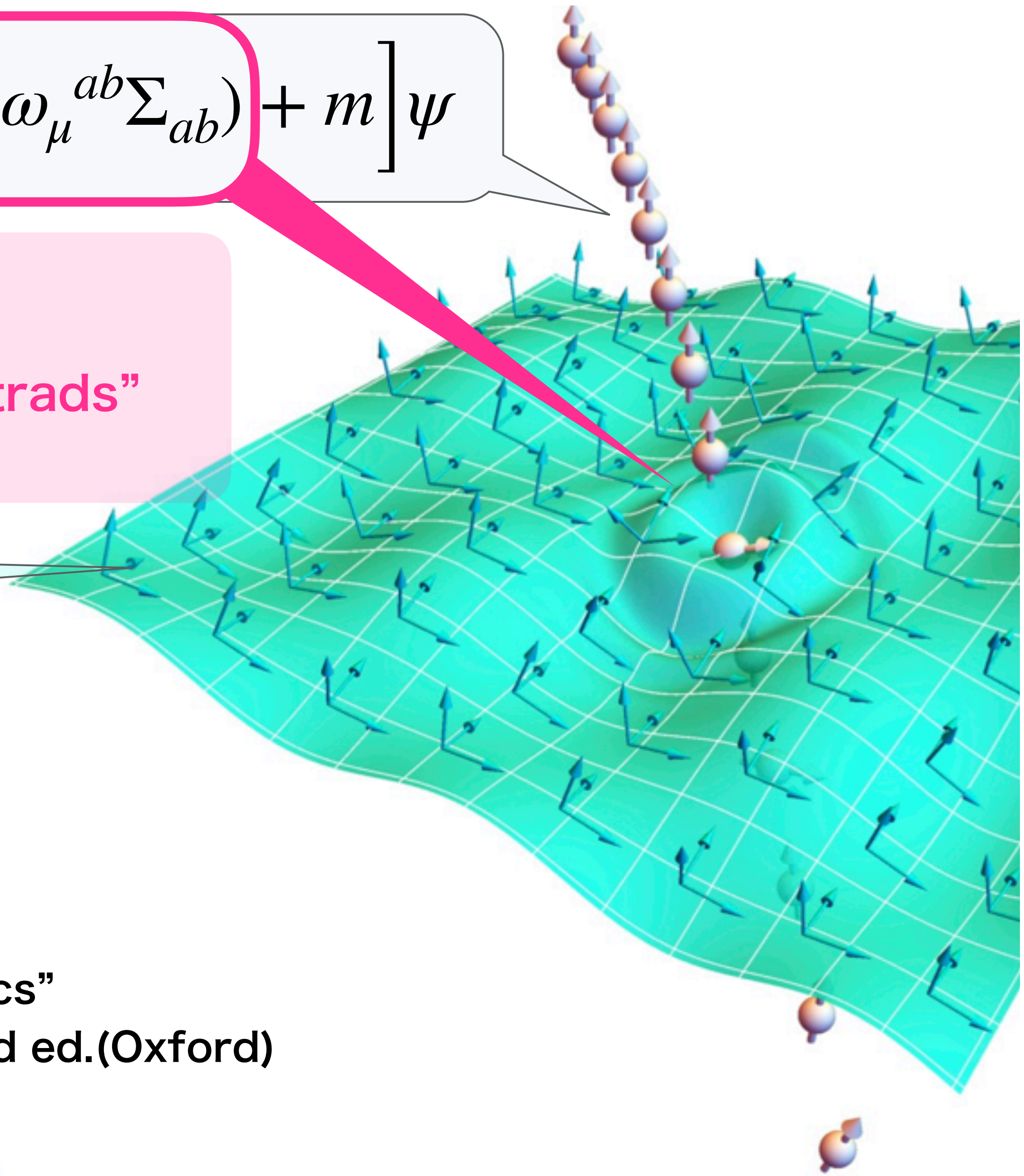
$$\mathscr{L}_{\text{electron}} = -\bar{\psi}\left[ie^{\mu}{}_{a}\gamma^{a}\left(p_{\mu} - \frac{i}{2}\omega_{\mu}{}^{ab}\Sigma_{ab}\right) + m\right]\psi$$

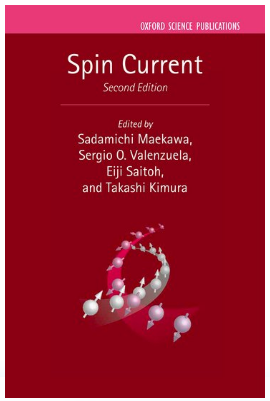
$$\omega^{ab}{}_{\mu}\,dx^{\mu} := e^{a}\cdot de^{b}$$

Spin connection = “Twist of tetrads”

→ spin-vorticity coupling

$$\mathscr{L}_{\text{Elastic/Fluid}} = \mathscr{L}[e^{\mu}{}_{a}]$$





MM et al., “Spin-mechatronics”

Chap. 25 in Spin current 2nd ed.(Oxford)

Hydrodynamics w/ angular momentum

Momentum conservation

$$\rho \frac{Dv}{Dt} = \nabla \cdot T^S + \nabla \times T^A$$

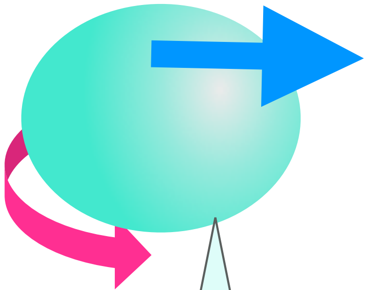
Angular momentum conservation

$$I \frac{D\dot{\theta}}{Dt} = T^A$$

$$T_{ij} = T_{ij}^{\ S} + T_{ij}^{\ A}$$

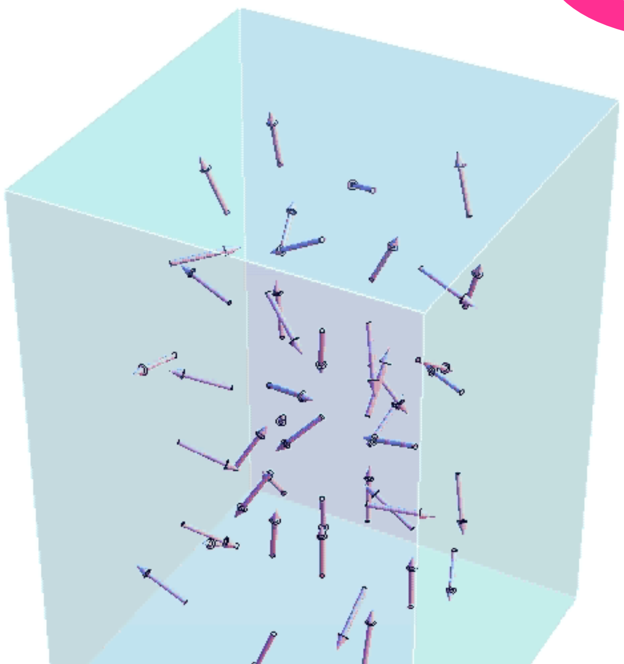
Eringen (1964)

Fluid element



$\rho v(r,t)$ **Momentum**

$I\dot{\theta}(r,t)$ **Angular momentum**



Angular momentum of fluid
is converted into
spin angular momentum of electrons
via. spin-vorticity coupling

Takahashi,MM et al., Nat. Phys (2016)

Anti-symmetric stress tensor

Eringen, Int. J. Engng. Sci. 2, 205 (1964)

Momentum conservation

$$\rho \frac{Dv}{Dt} = \nabla \cdot T^S + \nabla \times T^A$$

Angular momentum conservation

$$I \frac{D\dot{\theta}}{Dt} = T^A$$

$$T_{ij} = T_{ij}^{\ S} + T_{ij}^{\ A}$$

$$T_{ij}^{\ S} = -p\delta_{ij} + \mu\left(\partial_i v_j + \partial_j v_i\right)$$

Viscosity
> momentum relaxation

$$T_{ij}^{\ A} = \mu_{rot}\left[\left(\partial_i v_j - \partial_j v_i\right) - 2\varepsilon_{ijk}\dot{\theta}_k\right]$$

vorticity

rotation of fluid element

Rotational viscocity
> angular momentum relaxation

Relative angular velocity between fluid element and vorticity

Non equilibrium Green function

Lessor function (noneq. number density)

$$G^<_{12} := (-i)\,\mathrm{Tr}\rho\,\psi^\dagger_{r_2t_2}\psi_{r_1t_1}$$

Density matrix

$$G^R_{12} := (-i)\theta_{12}\left\langle\left[\psi_{r_1t_1},\psi^\dagger_{r_2t_2}\right]\right\rangle$$

$$G^A_{12} := (+i)\theta_{21}\left\langle\left[\psi_{r_1t_1},\psi^\dagger_{r_2t_2}\right]\right\rangle$$

Wigner tr.

$$\begin{pmatrix} k \\ \omega \end{pmatrix} \Leftarrow \begin{pmatrix} r_1 - r_2 \\ t_1 - t_2 \end{pmatrix}, \begin{pmatrix} r \\ t \end{pmatrix} = \begin{pmatrix} (r_1 + r_2)/2 \\ (t_1 + t_2)/2 \end{pmatrix}$$

$$G^<_{k\omega rt} = 2i\,\mathrm{Im}\,G^R_{k\omega} \times f^{(2)}_{k\omega rt}$$

Number Density | **Spectral Function** | **2 point Dist. Fn**

Spin current

$$J^\sigma_{i,s} = \frac{\hbar}{2}\mathrm{Tr}\left[\int_{\omega,k}\{\sigma, v_{k,i}\}G^<_{k\omega,\sigma}\right]$$

Quantum kinetic equation (Kadanoff-Baym equation) w/ spin-vorticity coupling

MM et al., PRB(R)2017

$$H_{\mathrm{svc}} = -\frac{1}{2}S\cdot\omega$$

Mechanical analogue of Stern-Gerlach effect

$$H_{\text{Zeeman}} = -S \cdot \gamma B$$

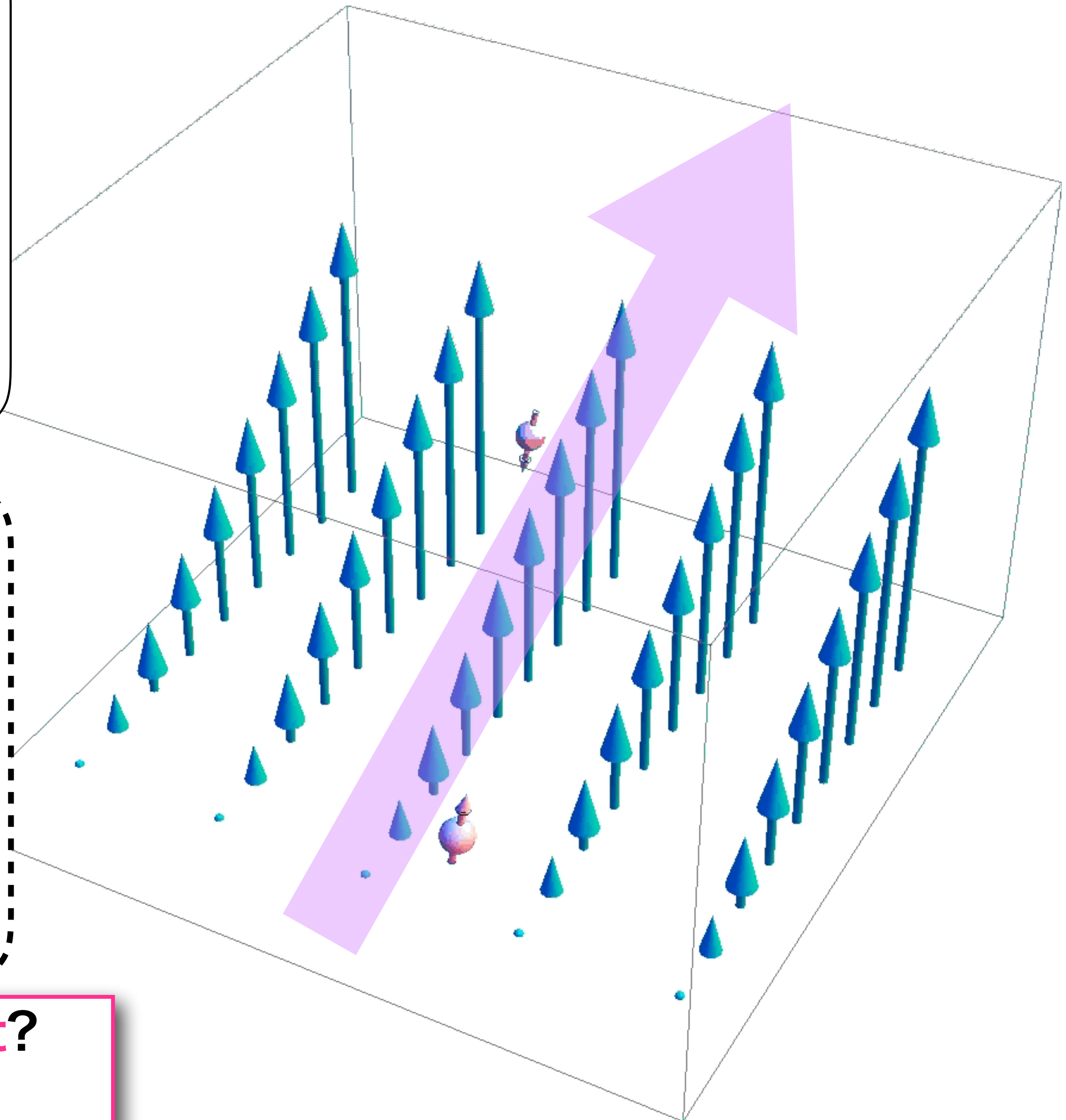
$$\Rightarrow F = -\nabla H_{Zeeman} = S \cdot \nabla(\gamma B)$$

Spin current is generated **along gradient of mag. field**.

$$H_{\text{sv}} = -S \cdot \frac{\omega}{2}$$

$$F = -\nabla H_{\text{sv}} = \frac{1}{2} S \cdot \nabla \omega$$

Spin current is generated **along rotation gradient.**



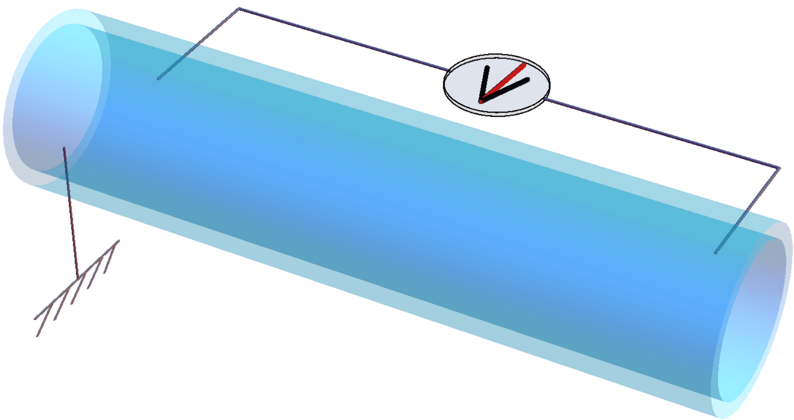
How to create **rotation gradient**?

→ 1. Surface acoustic wave,

2. Fluid motion of liquid metal !!

Spin current by vorticity gradient

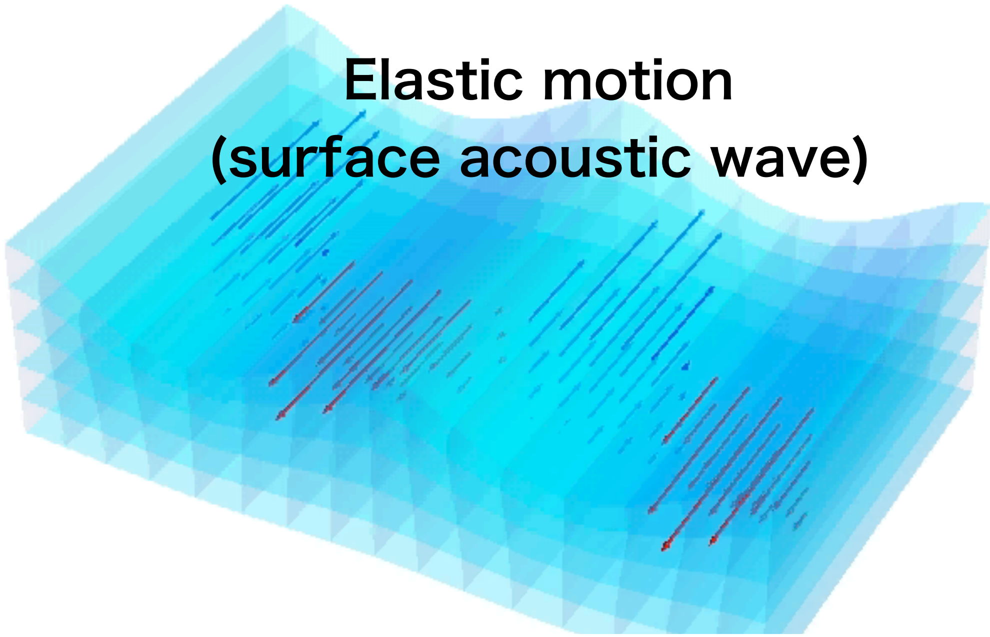
Fluid motion

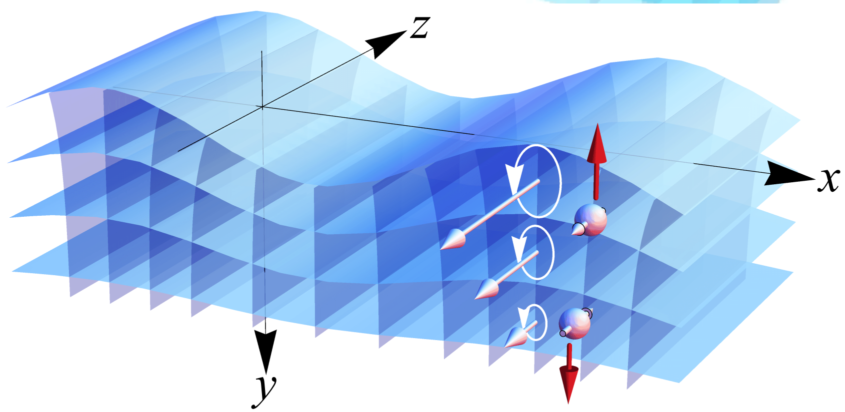


R. Takahashi, MM. et al.,
Nature Physics 2016
MM et al., PRB(R)2017

Science, Editor's choice
Nature Physics, N&V
Nature Materials, N&V

Elastic motion
(surface acoustic wave)

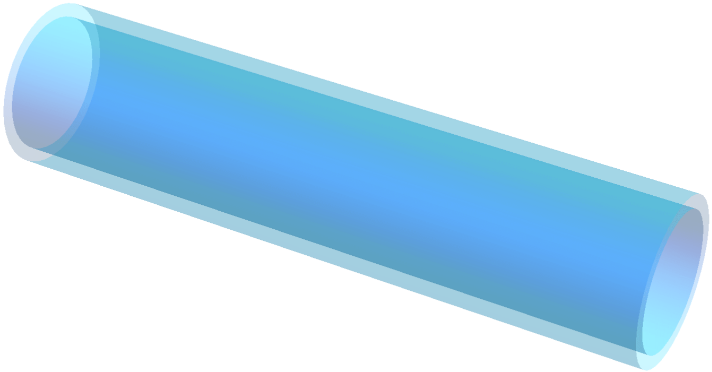




MM et al., PRB(R)2013

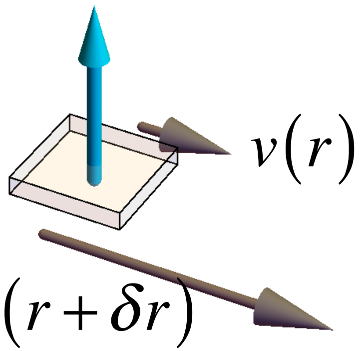
Kobayashi, Nozaki, MM et al.,
PRL2017 (Editors' Suggestion)

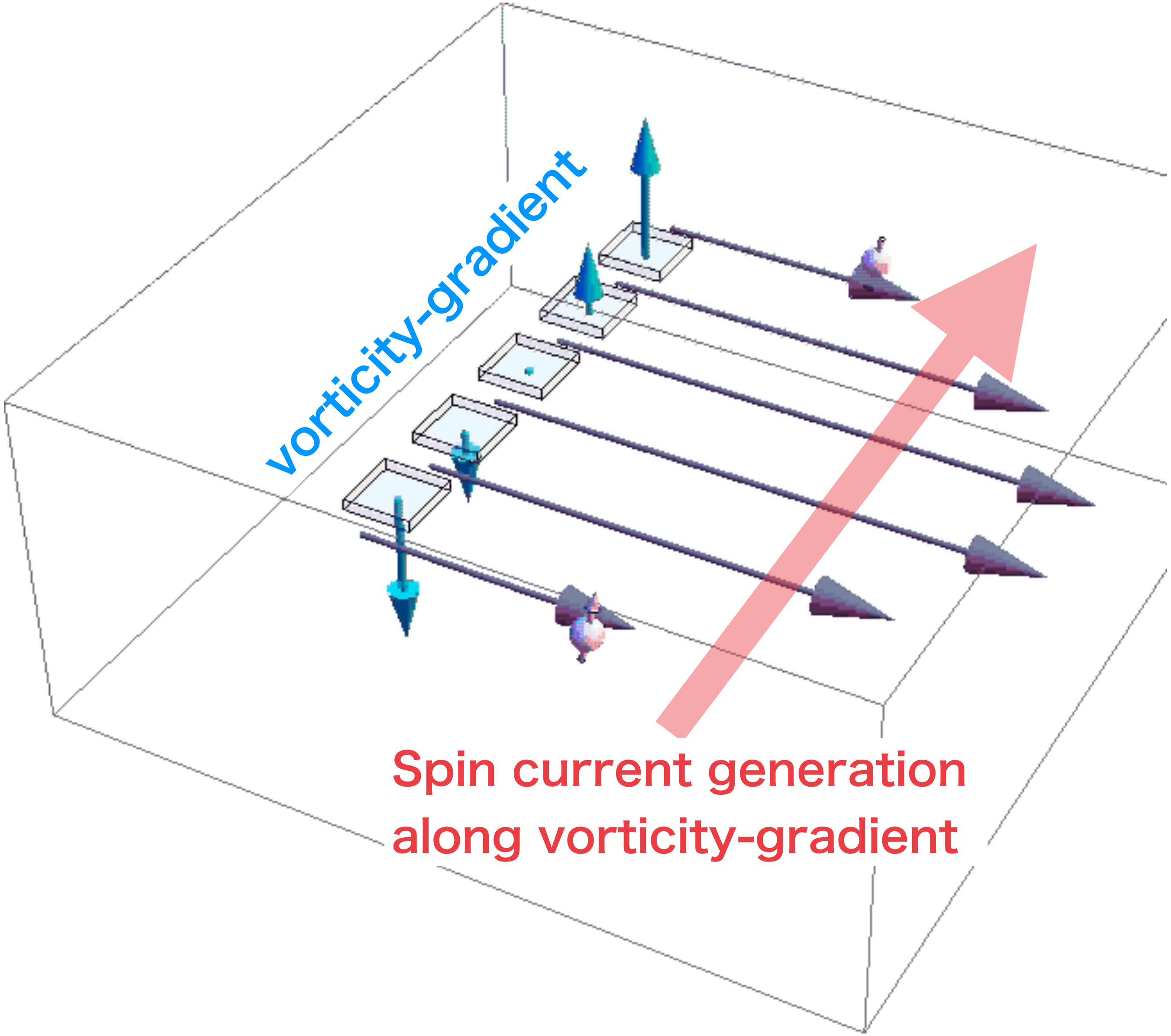
Rotation (vorticity) -gradient in a pipe flow of liquid metal



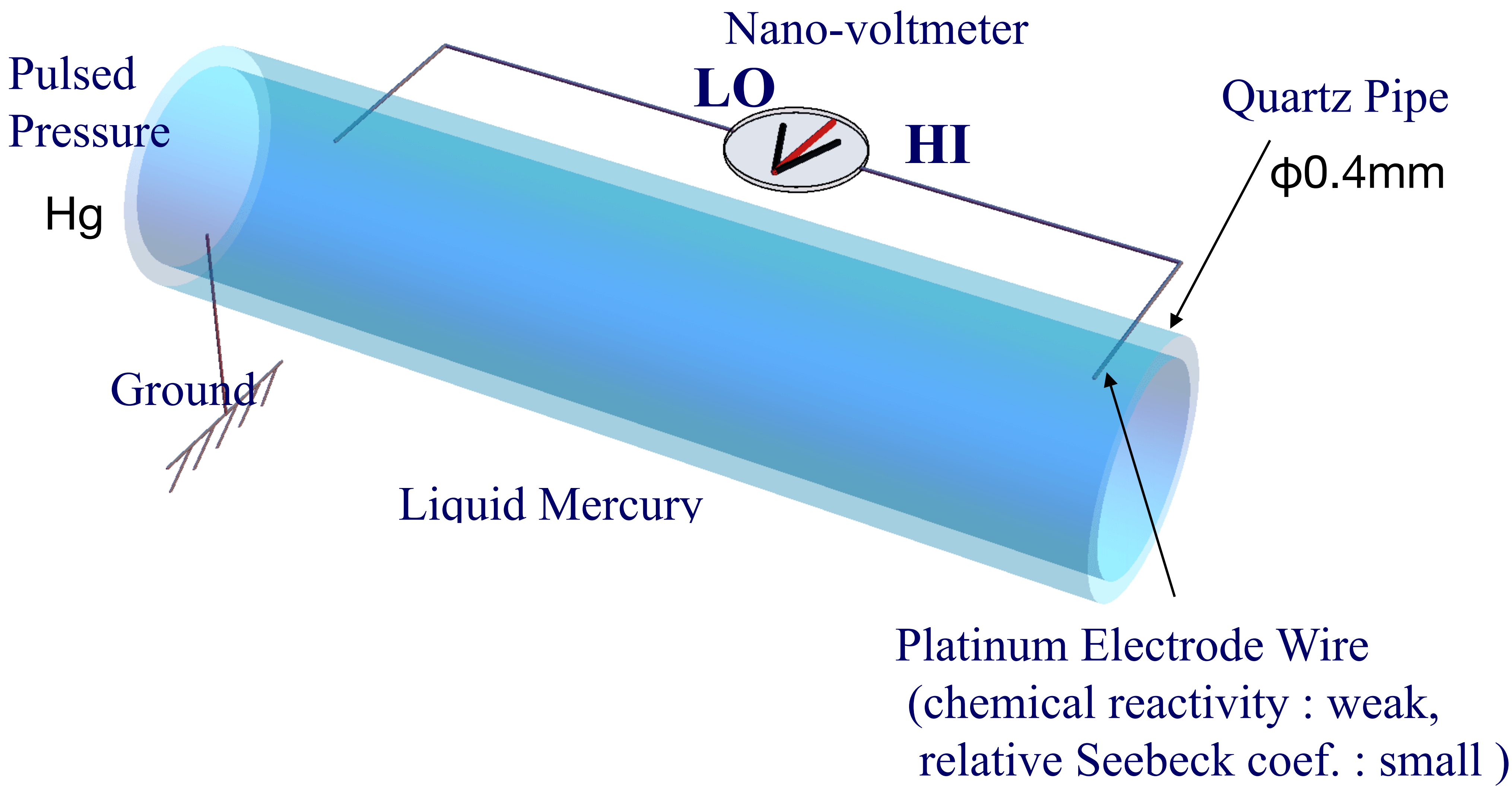
Vorticity:
local rotation of fluid

$$\nabla \times v$$



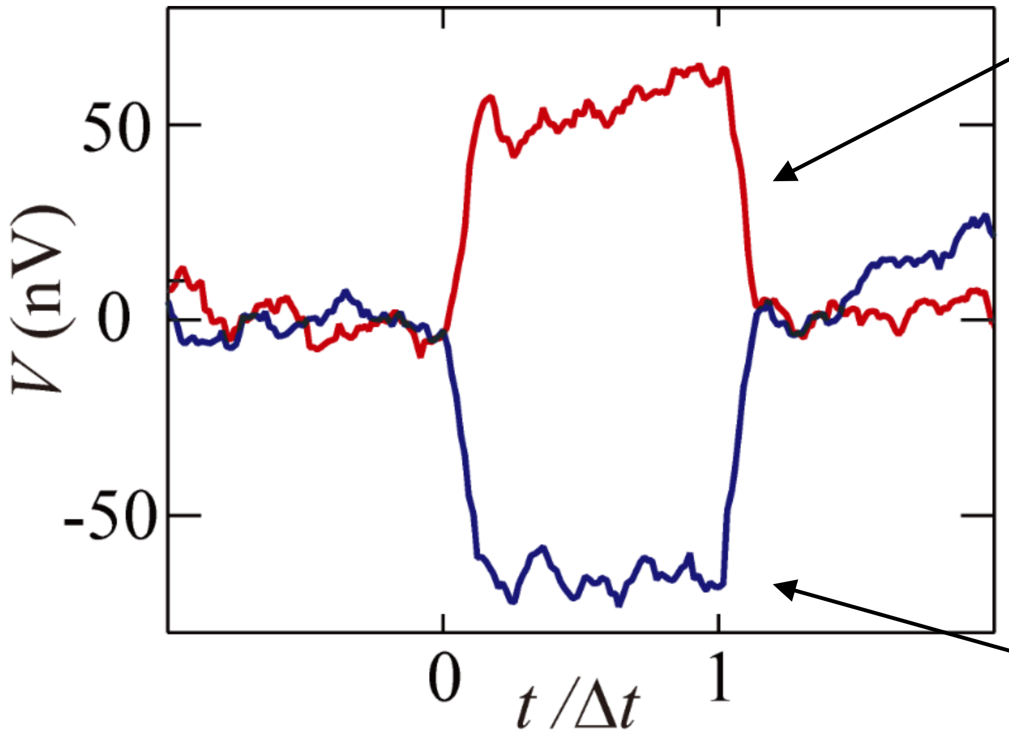


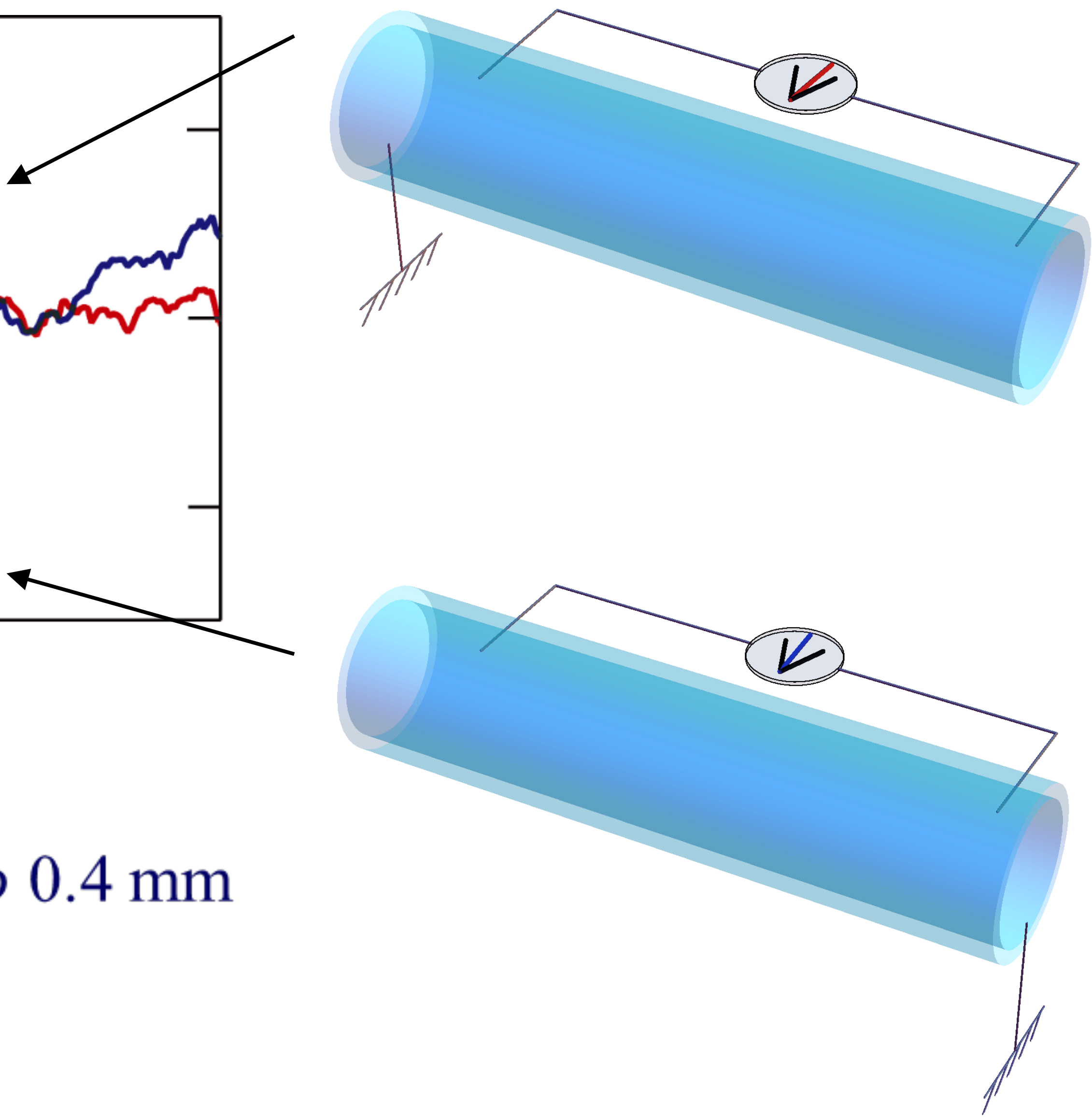
Experimental setup for spin hydrodynamic generation



R. Takahashi, MM et al., Nat. Phys. **12**, 52-56 (2016)

Result - Spin-hydrodynamic signal measurement





Δt 5.9 sec, 2.7 m/s

Internal Diameter ϕ 0.4 mm

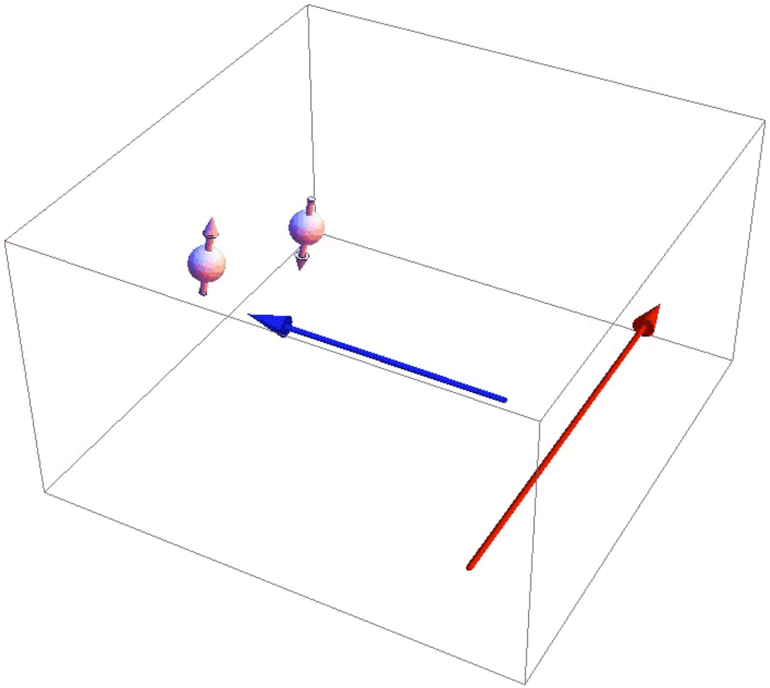
Length L 80 mm

R. Takahashi, MM et al., Nat. Phys. **12**, 52-56 (2016)

Charge to spin/spin to charge conversion

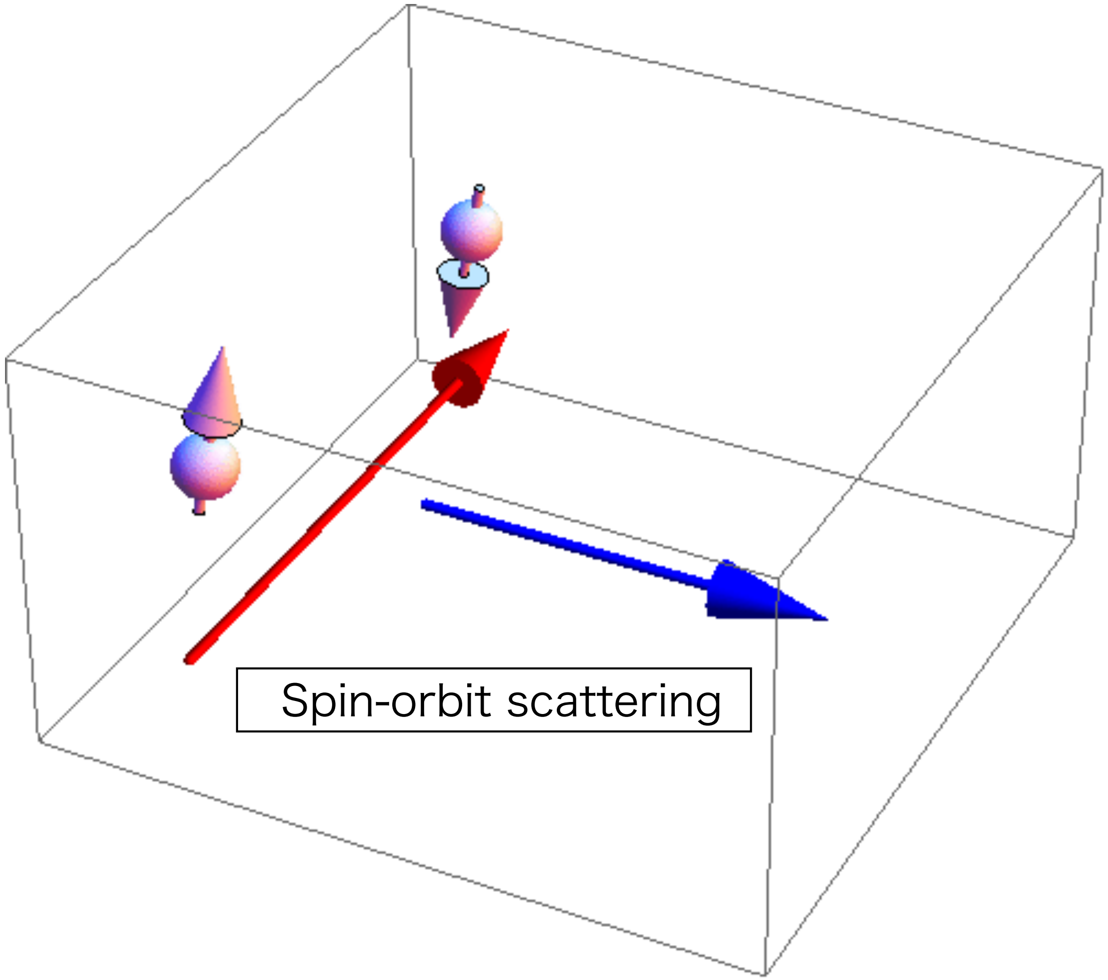
Spin Hall Effect

charge current
→spin current



InverseSpin Hall Effect

spin current
→charge current



Spin-Orbit Coupling

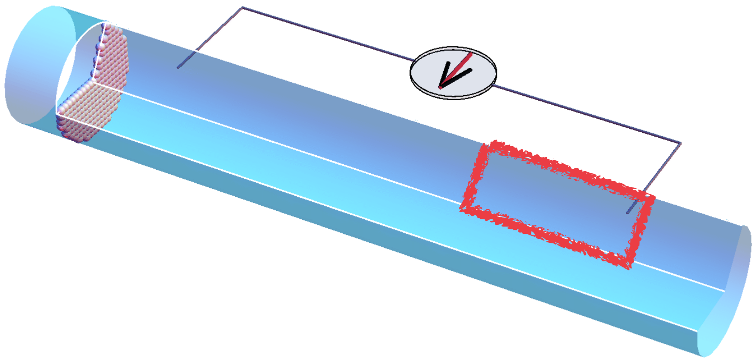
$$H_{SOI} = \frac{\bar{\lambda}}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p}+e\mathbf{A})\times(-e)\mathbf{E}\right]$$

$$v_{\sigma} = \frac{1}{i\hbar}\left[r, H_{SOI}\right] = \frac{(-e)\bar{\lambda}}{\hbar}\boldsymbol{\sigma}\times\mathbf{E}$$

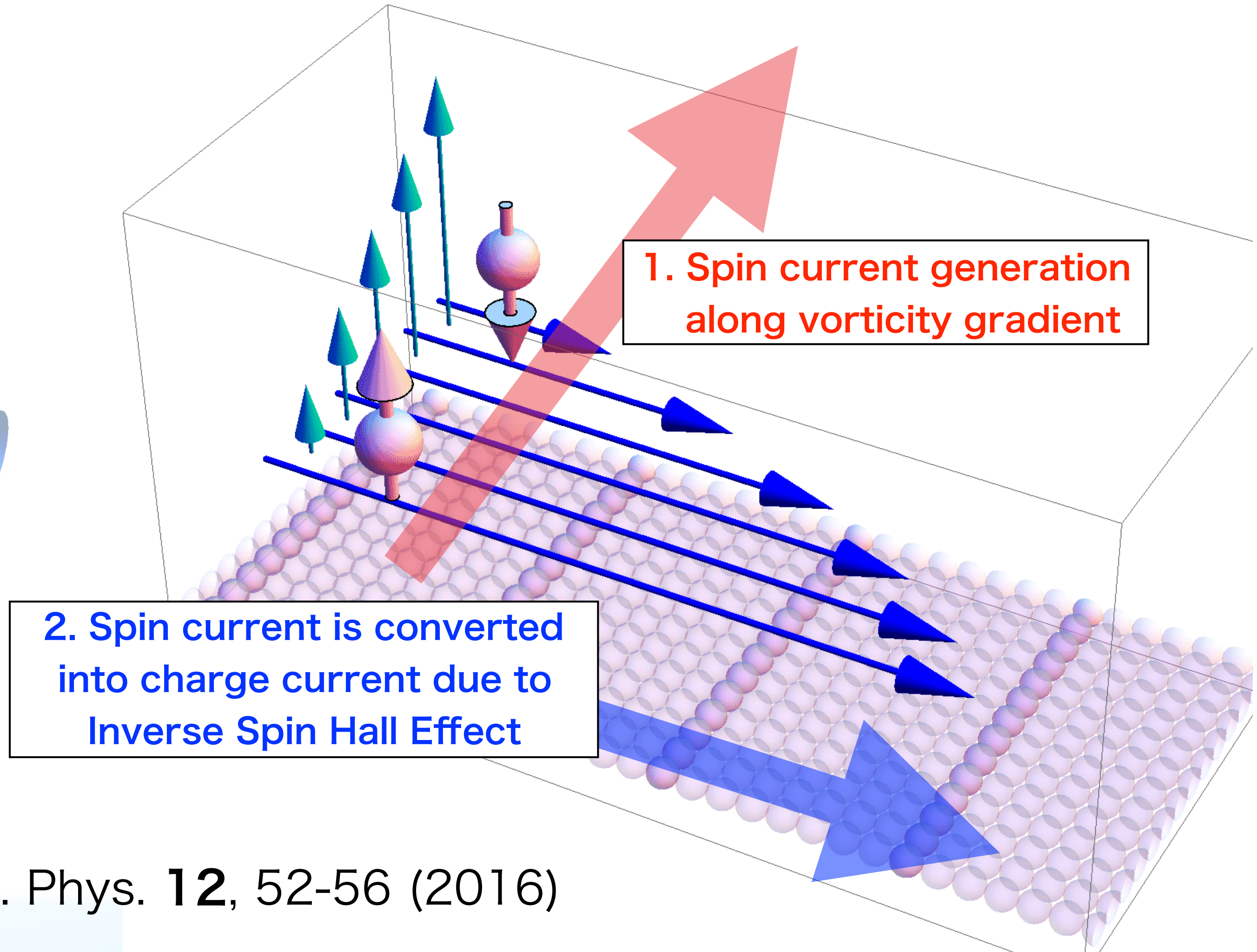
Mechanism of Spin-hydrodynamic voltage generation

"Spin-hydrodynamic generation"

1. Spin current generation along vorticity gradient + **2. Spin current is converted into charge current by ISHE**



flow of liquid metal: Hg



R. Takahashi, MM et al., Nat. Phys. **12**, 52-56 (2016)