New Developments in Magnetic Field and Rotation Induced Effects

Kenji Fukushima

The University of Tokyo

Review : 1812.08886 (PPNP in press)

— The 5th Workshop on
Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions —

⚠ DISCLAIMER ⚠

All new works are "new developments"

but…

simply impossible to cover all of them.

“Chosen” New Subjects

CME — Dynamical vs. Equilibrated

- ☐ Effect of the mass term — nontrivial cancellation
- ☐ Absence of “equilibrium” chiral magnetic effect

Chiral Barnett Effect

- ☐ Orbital angular mom. vs. Spin — electron vortex
- ☐ Longitudinal vs. Transverse

Axial Casimir Force

- ☐ No-go theorem and repulsive Casimir force
- ☐ Comment: Chiral vortical effect as a Casimir force

AXIAL WARD IDENTITY

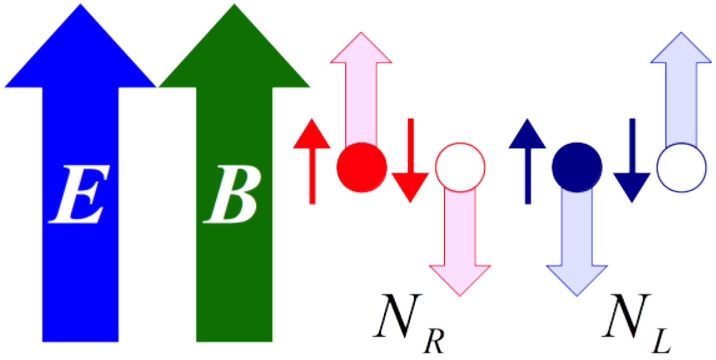
Axial Ward Identity (Abelian)

$$\partial_\mu j_5^\mu = -\frac{e^2}{16}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \cancel{2m\bar{\psi}i\gamma_5\psi}$$

frequently dropped

represents the chirality production rate

$$\frac{\partial n_5}{\partial t} = \frac{e^2 EB}{2\pi^2}$$ for parallel E and B and m=0



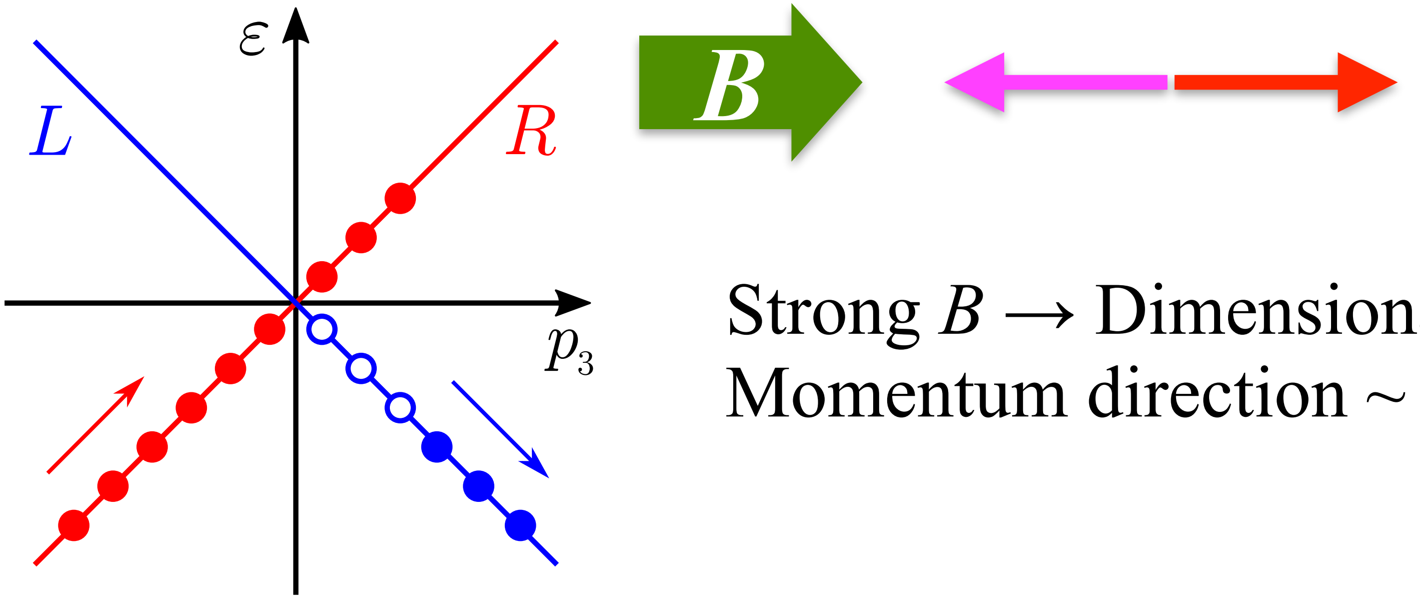
U(1)$_A$ symmetry is broken by the chiral anomaly

Mass Effect

Pair production induced by *E* and *B*

$$\omega = \frac{e^2 EB}{4\pi^2} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right)$$

not the chirality itself but when $B >> E$ (LLL)



$(\Delta N_5 = 2)$

Strong B → Dimensional Reduction
Momentum direction ~ Chirality

Mass Effect

Pair production induced by *E* (and *B*)

$$\omega \xrightarrow{B\gg E} \frac{e^2 EB}{4\pi^2}\exp\left(-\frac{\pi m^2}{eE}\right) = \frac{1}{2}\partial_t n_5$$

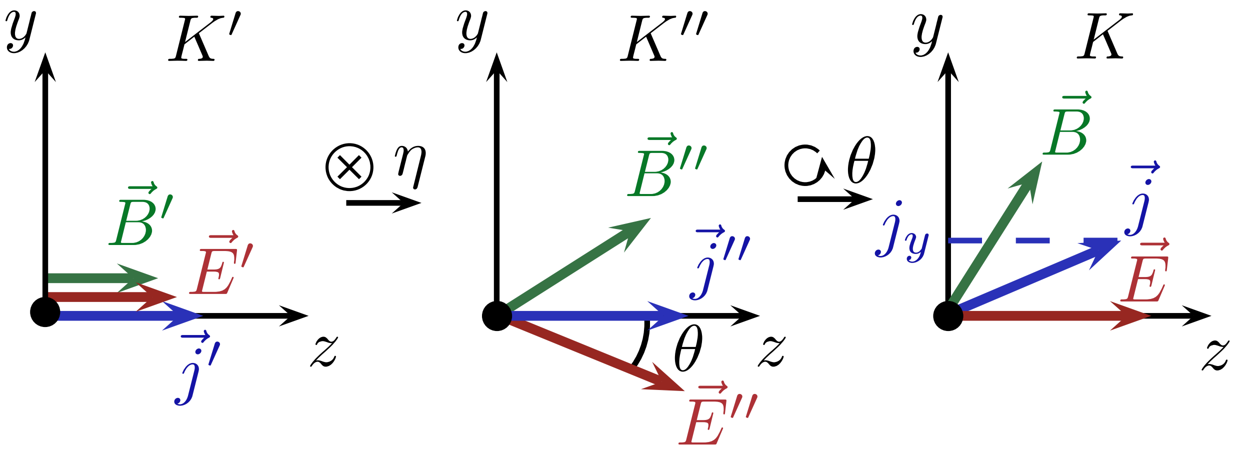
particle
antiparticle

left/right
handed

We already know the answer, but how can we get this from the AWI?

Extremely important question to understand the Chiral Magnetic Effect

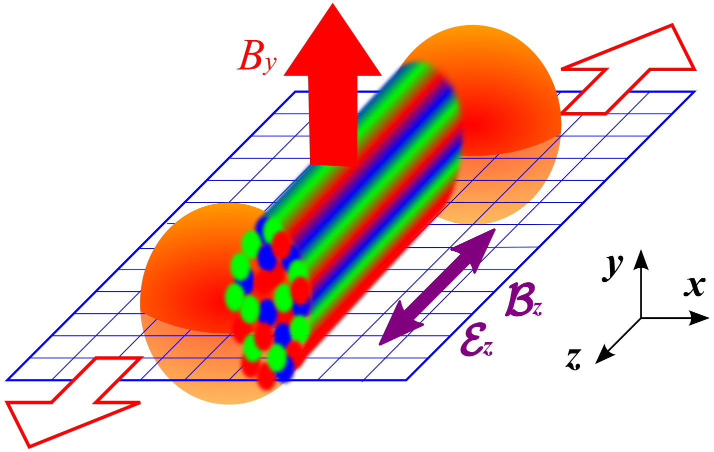
Chiral Magnetic Effect (no μ_5)



Fukushima-
-Kharzeev-
-Warringa, PRL (2010)

Schwinger process in *K'*

$$\Gamma = \frac{q^2 E'_z B'_z}{4\pi^2} \coth\left(\frac{B'_z}{E'_z}\pi\right) \exp\left(-\frac{m^2\pi}{|qE'_z|}\right)$$



Current generation rate

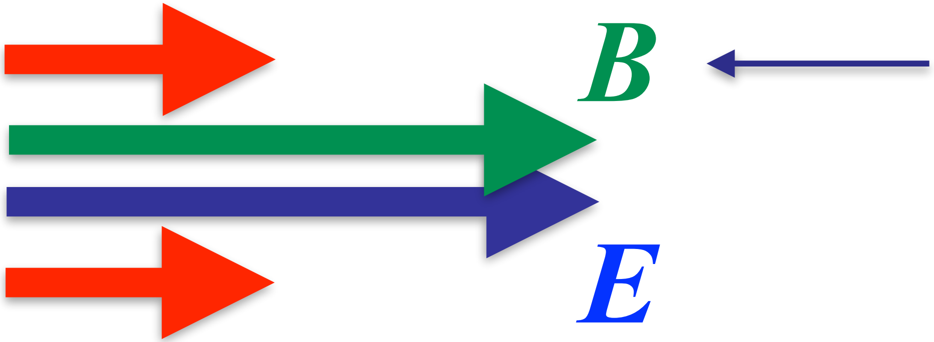
$$\partial_t j_y \simeq \frac{q^2 B_y}{2\pi^2} \frac{g\mathcal{E}_z\mathcal{B}_z^2}{\mathcal{B}_z^2 + \mathcal{E}_z^2} \coth\left(\frac{\mathcal{B}_z}{\mathcal{E}_z}\pi\right) \exp\left(-\frac{2m^2\pi}{|g\mathcal{E}_z|}\right)$$

Chiral Magnetic Effect (no μ_5)

Technology simplified for cond-mat exp.

from the Schwinger process

$$j_{\rm CME} = (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B} \quad \propto \quad B^2$$



Only this is external

$$j_{\rm Ohm} = \sigma\boldsymbol{E}$$

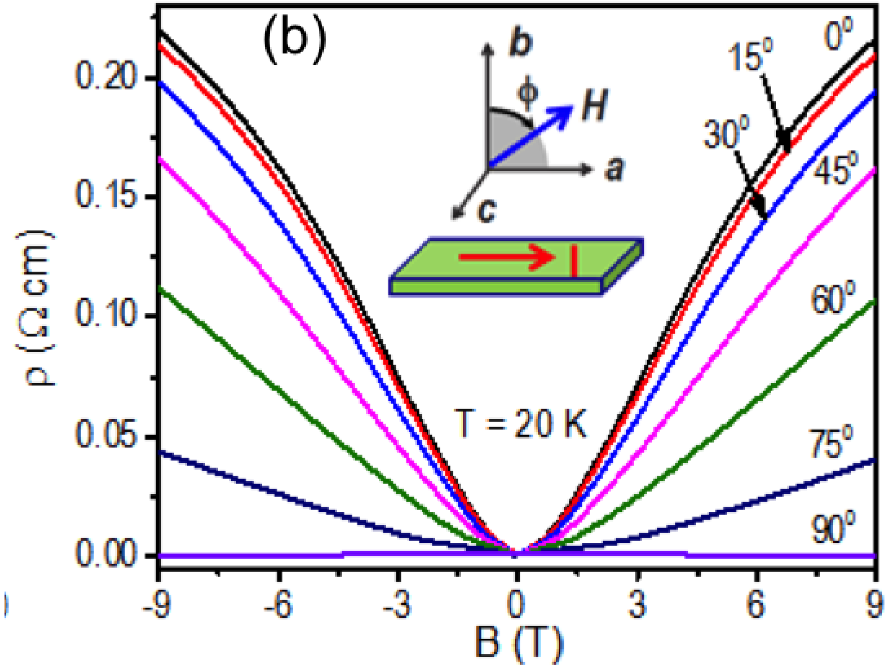
$$j = (\sigma_{\rm Ohm} + \sigma_{\rm CME})E \qquad \sigma_{\rm CME} \propto B^2$$

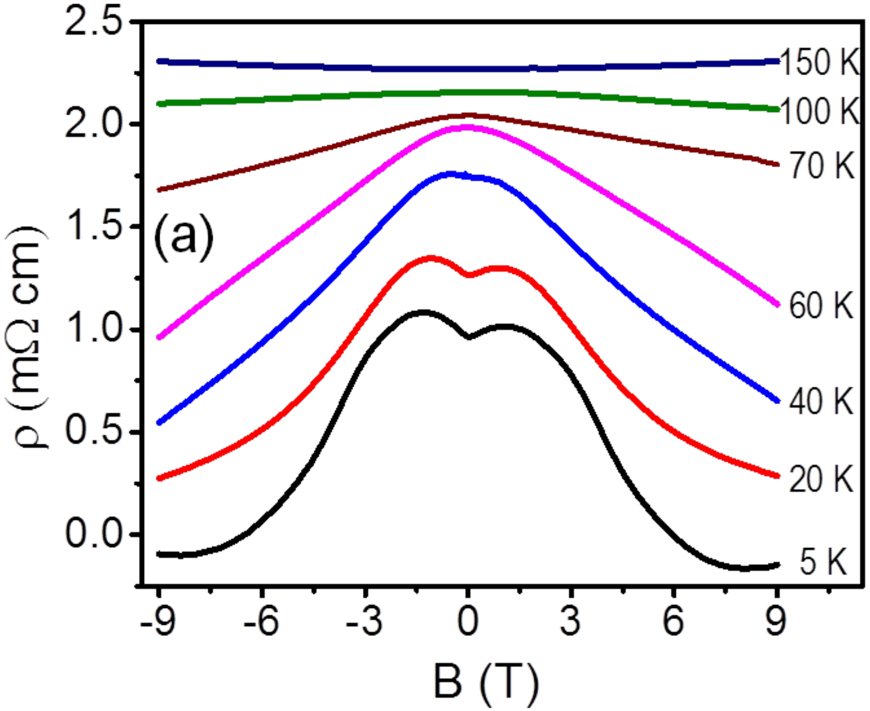
Son-Spivak, PRB (2012)

Chiral Magnetic Effect (no μ_5)

Lorentz force = “Classical” MR

Perpendicular *E* and *B* are Lorentz force free





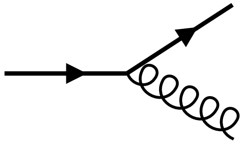
Li et al.
Nature Physics (2016)

Negative “magnetoresistance”

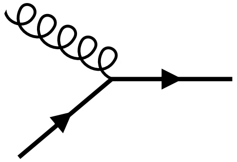
Chiral Magnetic Effect (no μ_5)

Fully field-theoretical calculation

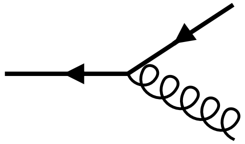
Fukushima-Hidaka, PRL (2018) **[full version in prep.]**



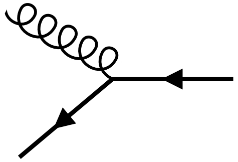
(a1) $q \to qg$



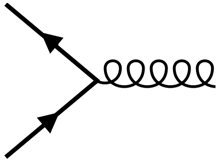
(a2) $qg \to q$



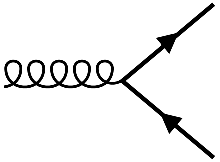
(b1) $\bar{q} \to \bar{q}g$



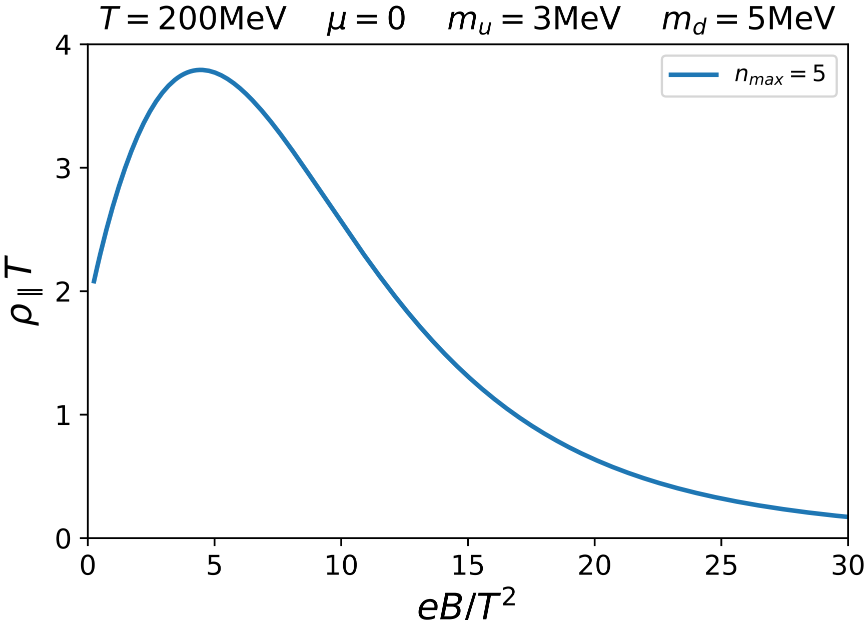
(b2) $\bar{q}g \to \bar{q}$



(c1) $q\bar{q} \to g$



(c2) $g \to q\bar{q}$



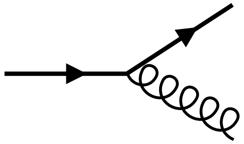
Good agreement with the cond-mat exp.

Consistent with lattice-QCD at *B*=0

Chiral Magnetic Effect (no μ_5)

Fully field-theoretical calculation

Fukushima-Hidaka, PRL (2018) **[full version in prep.]**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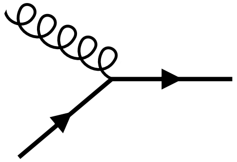
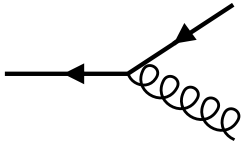
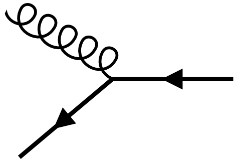

(a2) $qg \to q$


(b1) $\bar{q} \to \bar{q}g$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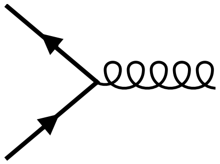
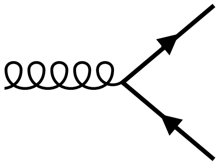
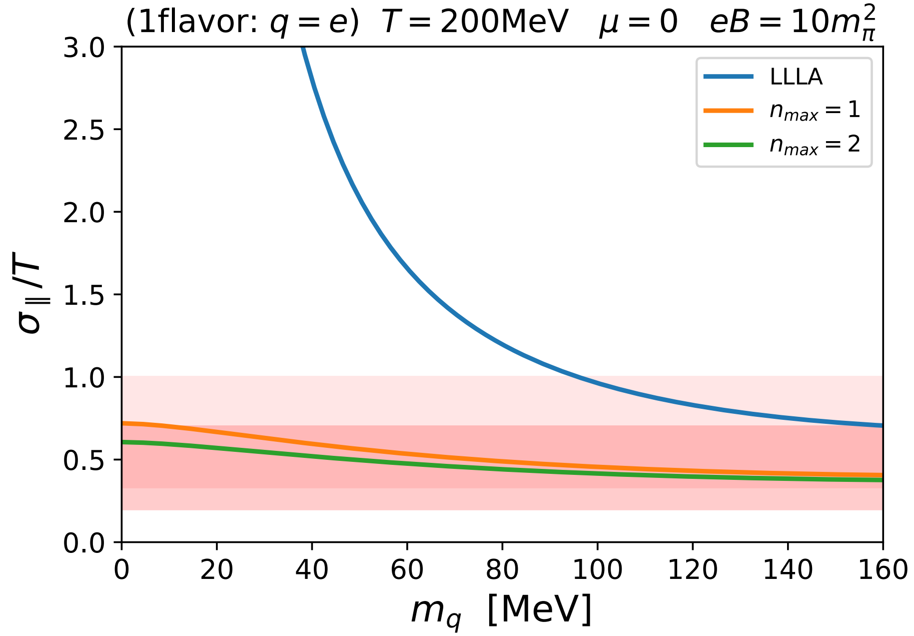

(c1) $q\bar{q} \to g$


(c2) $g \to q\bar{q}$



Good agreement with the cond-mat exp.

Consistent with lattice-QCD at *B*=0

Clarifying CME Controversies

Dynamical Chiral Magnetic Effect

$$j^3 = \langle \mathrm{in} | \bar{\psi} \gamma^3 \psi | \mathrm{in} \rangle = 2\omega \cdot t$$

This can be directly derived, and moreover, we find:

$$\bar{j}^3 = \langle \mathrm{out} | \bar{\psi} \gamma^3 \psi | \mathrm{in} \rangle = 0$$

In-Out amplitude can be interpreted as a *static* expectation value in Wick-rotated theory (*T*→0 theory)

No Chiral Magnetic Effect in equilibrium!

Yamamoto, PRB (2015) / Copinger-KF-Pu, PRL (2018)

Clarifying CME Controversies

μ_5 is a convenient but controversial bookkeeping device

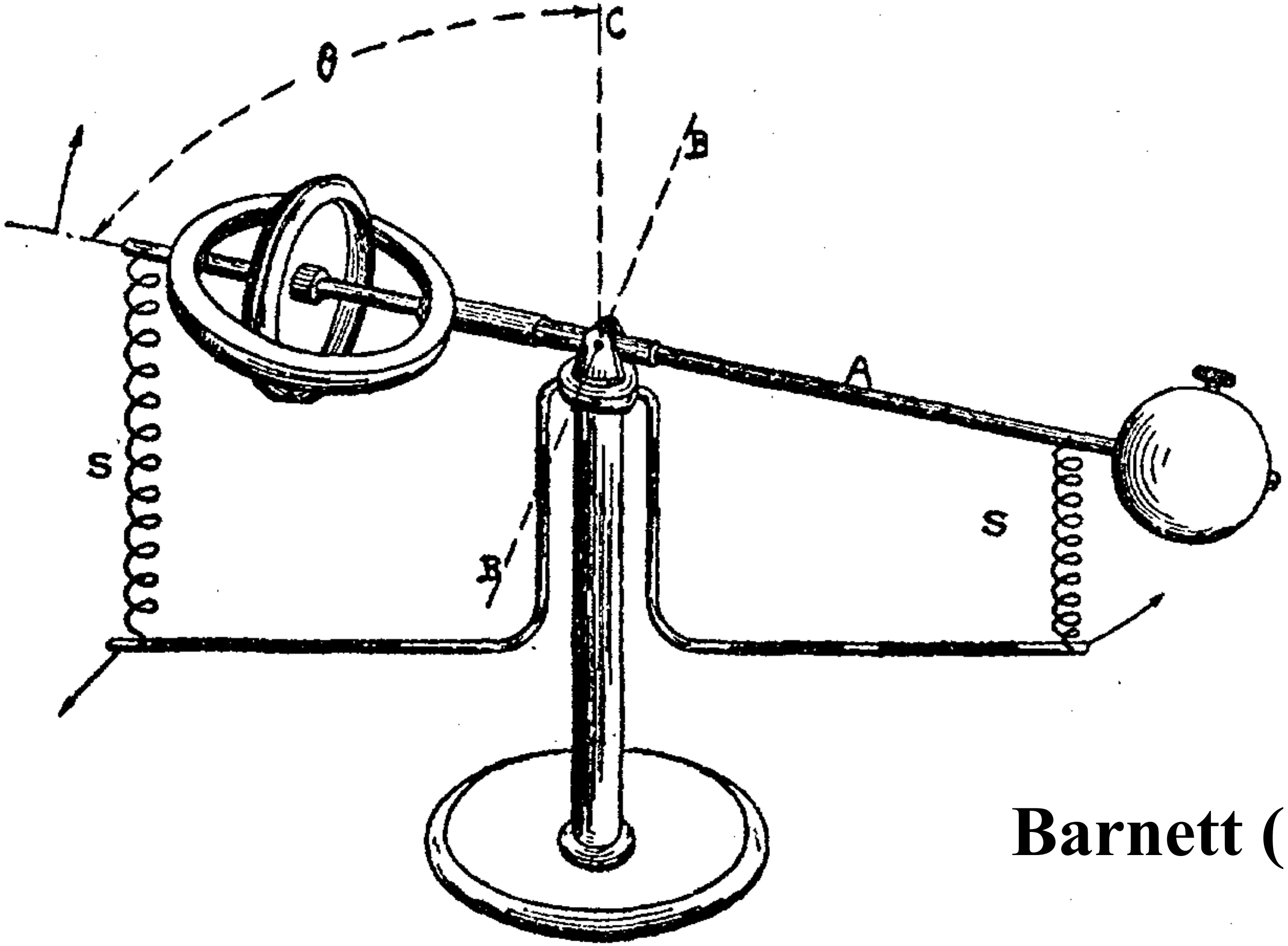
CME current is nonzero *wrongly* even in equilibrium lattice-QCD if μ_5 is coupled.

CME current must be *zero* in equilibrium lattice-QCD if Euclid electromagnetic fields are applied (testable prediction).

CHIRAL BARNETT EFFECT

Rotation Effect

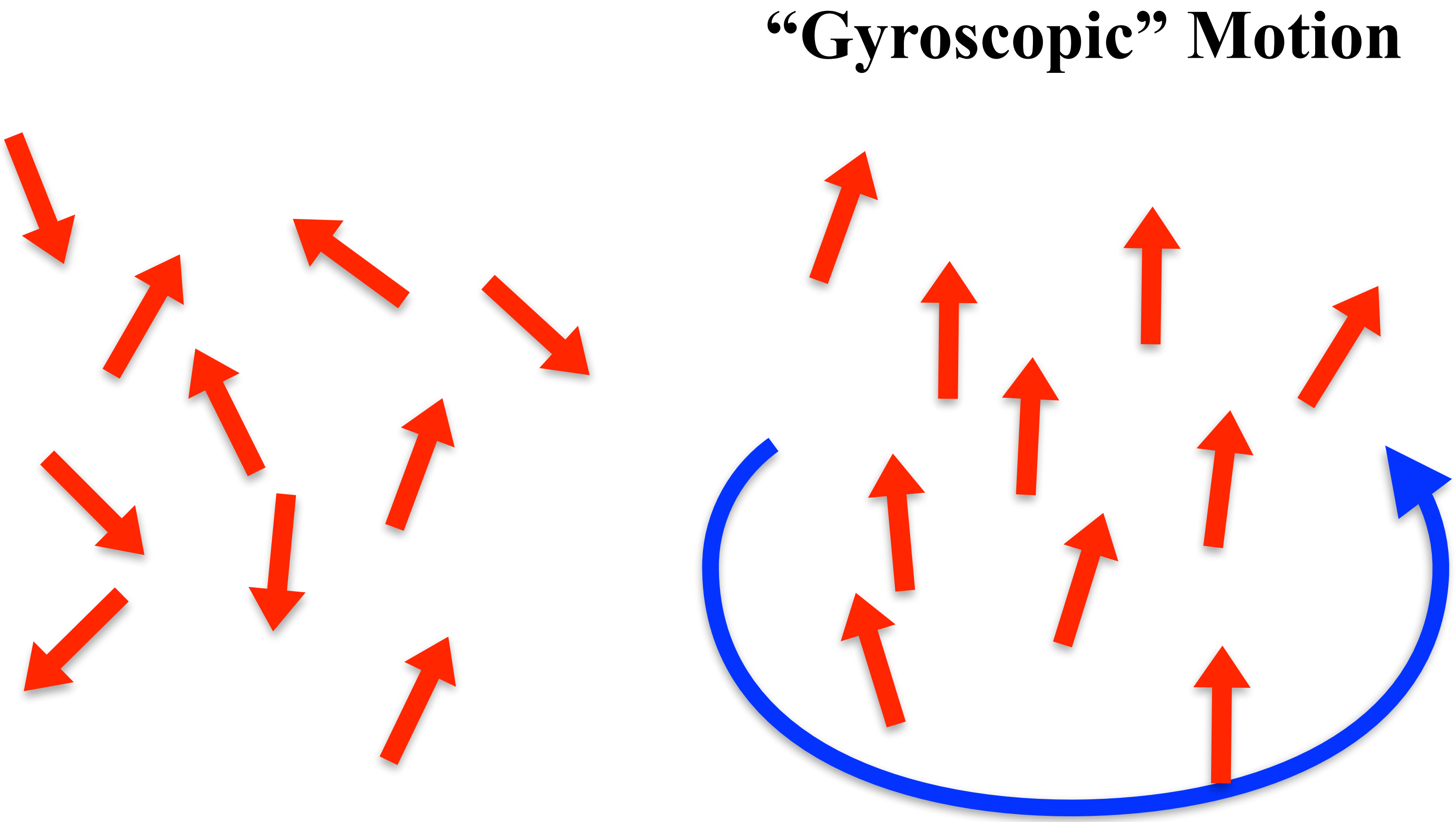
Gyromagnetic Effect



Barnett (1935)

Rotation Effect

Barnett Effect



Rotation Effect

Barnett Effect

$$\boldsymbol{\omega} \cdot \boldsymbol{J} = \boldsymbol{\mu} \cdot \boldsymbol{B}$$

Magnetization $\boldsymbol{M} = \chi_B \boldsymbol{B}$

magnetic susceptibility

Magnetic moment $\boldsymbol{\mu} = \gamma \boldsymbol{J}$

gyromagnetic ratio

$$\Longrightarrow \quad M = \frac{\chi_B}{\gamma}\omega$$

Standard formula for the Barnett effect

Rotation Effect

Angular Momentum
= Noether Current from Rotational Symmetry

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu}$$

$$\bar{\psi}\, i\hbar(\gamma^\lambda x^\mu \partial^\nu - \gamma^\lambda x^\nu \partial^\mu)\psi \qquad \frac{1}{4}\bar{\psi} i\hbar\gamma^\lambda[\gamma^\mu, \gamma^\nu]\psi$$

Neither *L* nor *S* conserved separately

$$\partial_\lambda L^{\lambda\mu\nu} = -\partial_\lambda S^{\lambda\mu\nu} = \bar{\psi}\, i\hbar(\gamma^\mu \partial^\nu - \gamma^\nu \partial^\mu)\psi$$

Rotation Effect

Different decomposition

$$\tilde{L}^{\lambda\mu\nu} = \frac{1}{2}L^{\lambda\mu\nu} + \frac{1}{2}\bar{\psi}\, i\hbar\big[(x^{\mu}\gamma^{\nu} - x^{\nu}\gamma^{\mu})\partial^{\lambda}\big]\psi$$

$$\tilde{S}^{\lambda\mu\nu} = J^{\lambda\mu\nu} - \tilde{L}^{\lambda\mu\nu}$$

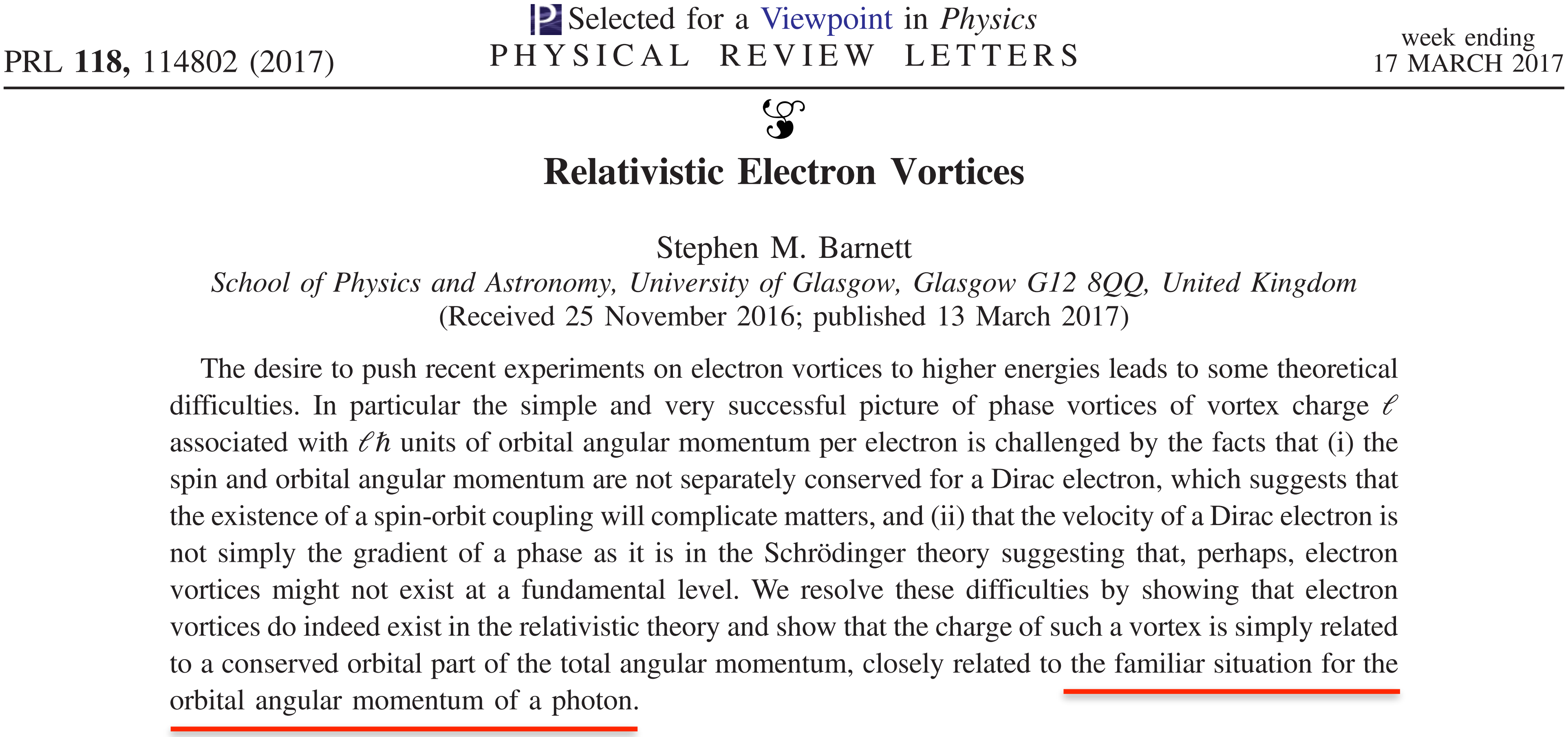
$$\Longrightarrow \quad \partial_{\lambda}\tilde{L}^{\lambda\mu\nu} = \partial_{\lambda}\tilde{S}^{\lambda\mu\nu} = 0$$

Separately conserved?

We will see a similar situation in the optical sector
Separately conserving optical *L* and *S* definable!?

Rotation Effect

Different decomposition



Selected for a Viewpoint in *Physics*
PRL **118,** 114802 (2017)
PHYSICAL REVIEW LETTERS
week ending
17 MARCH 2017

Relativistic Electron Vortices

Stephen M. Barnett
School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom

(Received 25 November 2016; published 13 March 2017)

The desire to push recent experiments on electron vortices to higher energies leads to some theoretical difficulties. In particular the simple and very successful picture of phase vortices of vortex charge ℓ associated with $\ell\hbar$ units of orbital angular momentum per electron is challenged by the facts that (i) the spin and orbital angular momentum are not separately conserved for a Dirac electron, which suggests that the existence of a spin-orbit coupling will complicate matters, and (ii) that the velocity of a Dirac electron is not simply the gradient of a phase as it is in the Schrödinger theory suggesting that, perhaps, electron vortices might not exist at a fundamental level. We resolve these difficulties by showing that electron vortices do indeed exist in the relativistic theory and show that the charge of such a vortex is simply related to a conserved orbital part of the total angular momentum, closely related to the familiar situation for the orbital angular momentum of a photon.

Rotation Effect

Different decomposition

Foldy-Wouthuysen transformation

$$H' = e^{iS} H e^{-iS} = \beta(p^2 + m^2)^{1/2}$$

$$e^{iS} = e^{\beta \boldsymbol{\alpha} \cdot \mathbf{p} \theta / p}, \qquad \tan(2\theta) = \frac{p}{m}$$

Conserved *L* and *S* defined for non-rela. theory

Rotation Effect

Different decomposition

$$\tilde{\mathbf{L}} = e^{-iS}\mathbf{x}\times\mathbf{p}e^{iS} = \mathbf{x}\times\mathbf{p} + i\frac{\beta\boldsymbol{\alpha}\times\mathbf{p}}{\sqrt{m^2+p^2}} + \left(1-\frac{m}{\sqrt{m^2+p^2}}\right)\left(\mathbf{S}-\frac{(\mathbf{p}\cdot\mathbf{S})\mathbf{p}}{p^2}\right)$$

$$\tilde{\mathbf{S}} = e^{-iS}\mathbf{S}e^{iS} = \mathbf{S} - i\frac{\beta\boldsymbol{\alpha}\times\mathbf{p}}{\sqrt{m^2+p^2}} - \left(1-\frac{m}{\sqrt{m^2+p^2}}\right)\left(\mathbf{S}-\frac{(\mathbf{p}\cdot\mathbf{S})\mathbf{p}}{p^2}\right)$$

Commute with a free Dirac Hamiltonian

Rotation Effect

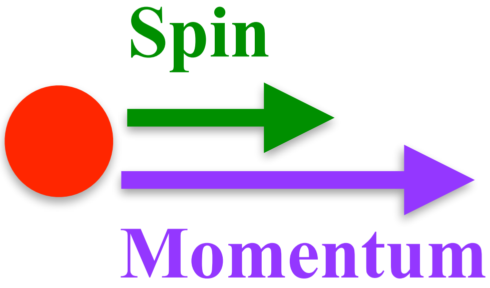
We prefer the former decomposition because:

1) **Reduced to ordinary *L* and *S* in non-rela. limit**

2) ***S* is related to the axial current**

$$S^{0ij} = \epsilon^{ijk}\,\frac{\hbar}{2}\,\bar{\psi}\gamma^{k}\gamma_5\psi = \epsilon^{ijk}\,\frac{j_5^k}{2}$$

Corresponding Spin Operator:



$$\boldsymbol{S} \;\to\; \hbar\lambda\left(\hat{\boldsymbol{p}} - \underline{\hbar\lambda\frac{\hat{\boldsymbol{p}}}{p}\times\boldsymbol{\nabla}}\right)$$

Torque from gyromagnetic effect

Rotation Effect

Spin Expectation Value

Energy in a rotating fluid $\varepsilon_{\rm rot} = p - \underline{\boldsymbol{\omega}\cdot\left(\boldsymbol{x}\times\boldsymbol{p}+\hbar\lambda\hat{\boldsymbol{p}}\right)}$

$= \ \boldsymbol{\omega}\cdot\boldsymbol{J}$

$$\langle\boldsymbol{S}\rangle = \int_{\boldsymbol{p}} \lambda\hbar\left(\hat{\boldsymbol{p}} - \lambda\hbar\frac{\hat{\boldsymbol{p}}}{p}\times\boldsymbol{\nabla}\right) f(\varepsilon_{\rm rot})$$

Vilenkin (1978)

$$\approx \underline{-\hbar\lambda(\boldsymbol{\omega}\times\boldsymbol{x})\int_{\boldsymbol{p}}\frac{p}{3}f'(p)} - \underline{\hbar^2\lambda^2\boldsymbol{\omega}\int_{\boldsymbol{p}}f'(p)}$$

$\langle\boldsymbol{S}\rangle_\perp$ **"Transverse" Barnett Effect**

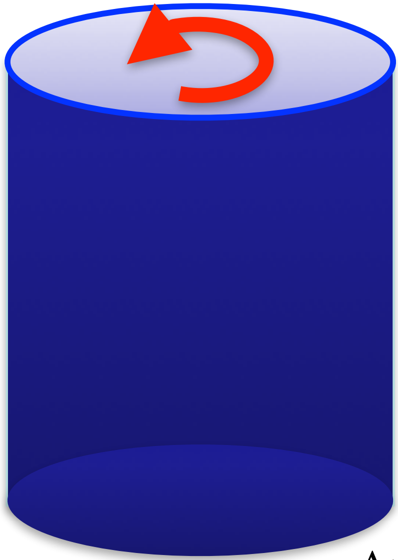
Chiral Vortical Effect
~ Barnett Effect

Rotation Effect

Spin Expectation Value

$$\langle \boldsymbol{S} \rangle_\perp = -\hbar \sum_{R,L} \lambda(\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} \frac{p}{3} f'_\lambda(p)$$

$$= \frac{\hbar}{2}(\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} [f_R(p) - f_L(p)] = \frac{\hbar}{2}(\boldsymbol{\omega} \times \boldsymbol{x})\, n_5$$

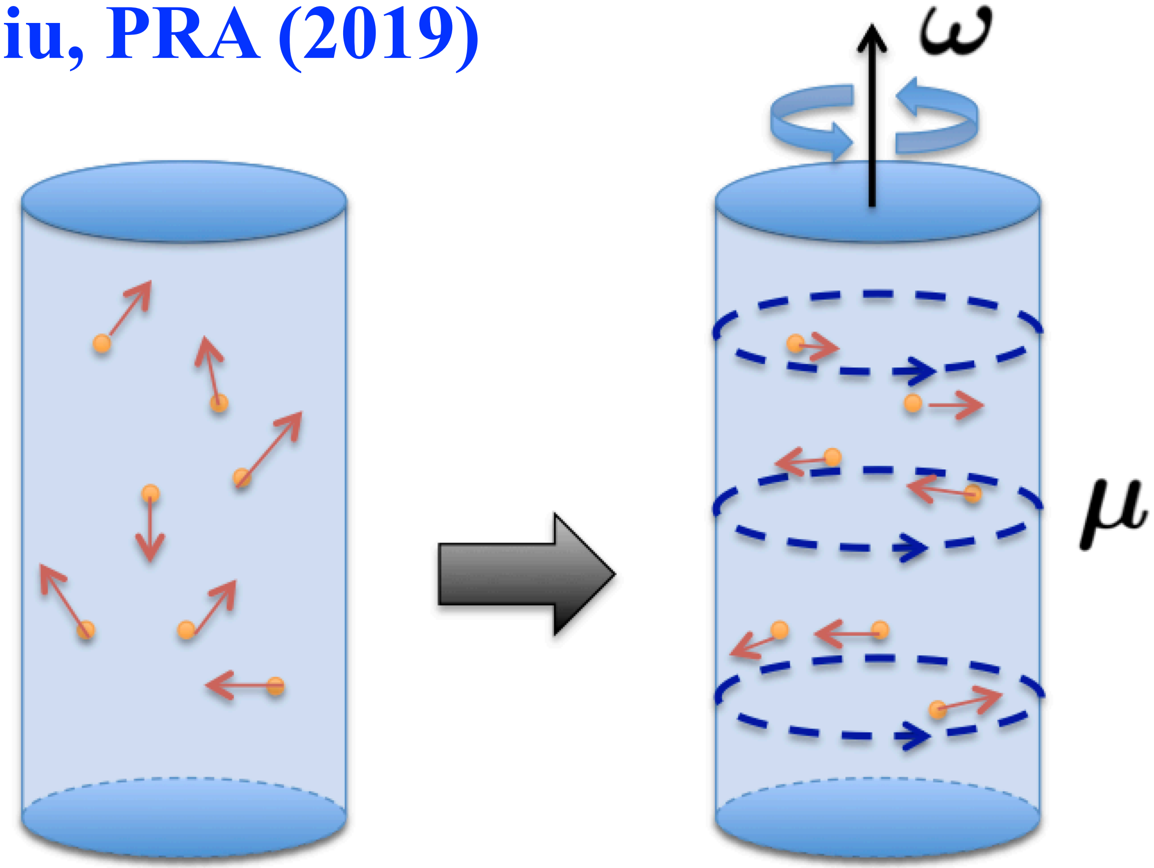


$$\boldsymbol{j}_5 = n_5 \boldsymbol{v}$$

Transverse Barnett appears for massless and chirally imbalanced fermions

Rotation Effect

KF-Pu-Qiu, PRA (2019)

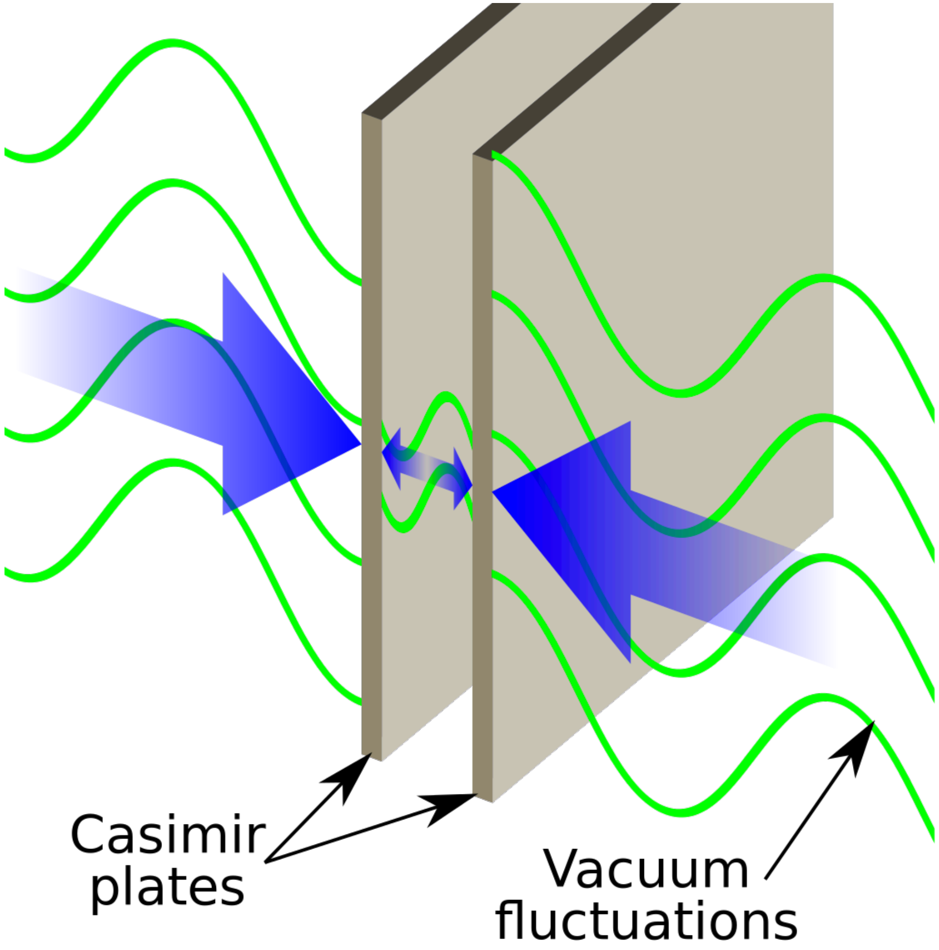


"Eddy magnetization" ← rotation + chiral imbalance

AXIAL CASIMIR FORCE

Casimir Effect

Wikipedia



No-Go Theorem

Opposites Attract: A Theorem about the Casimir force
Kenneth-Klich, PRL97, 160401 (2006)

Casimir force between two bodies related by reflection is always attractive.

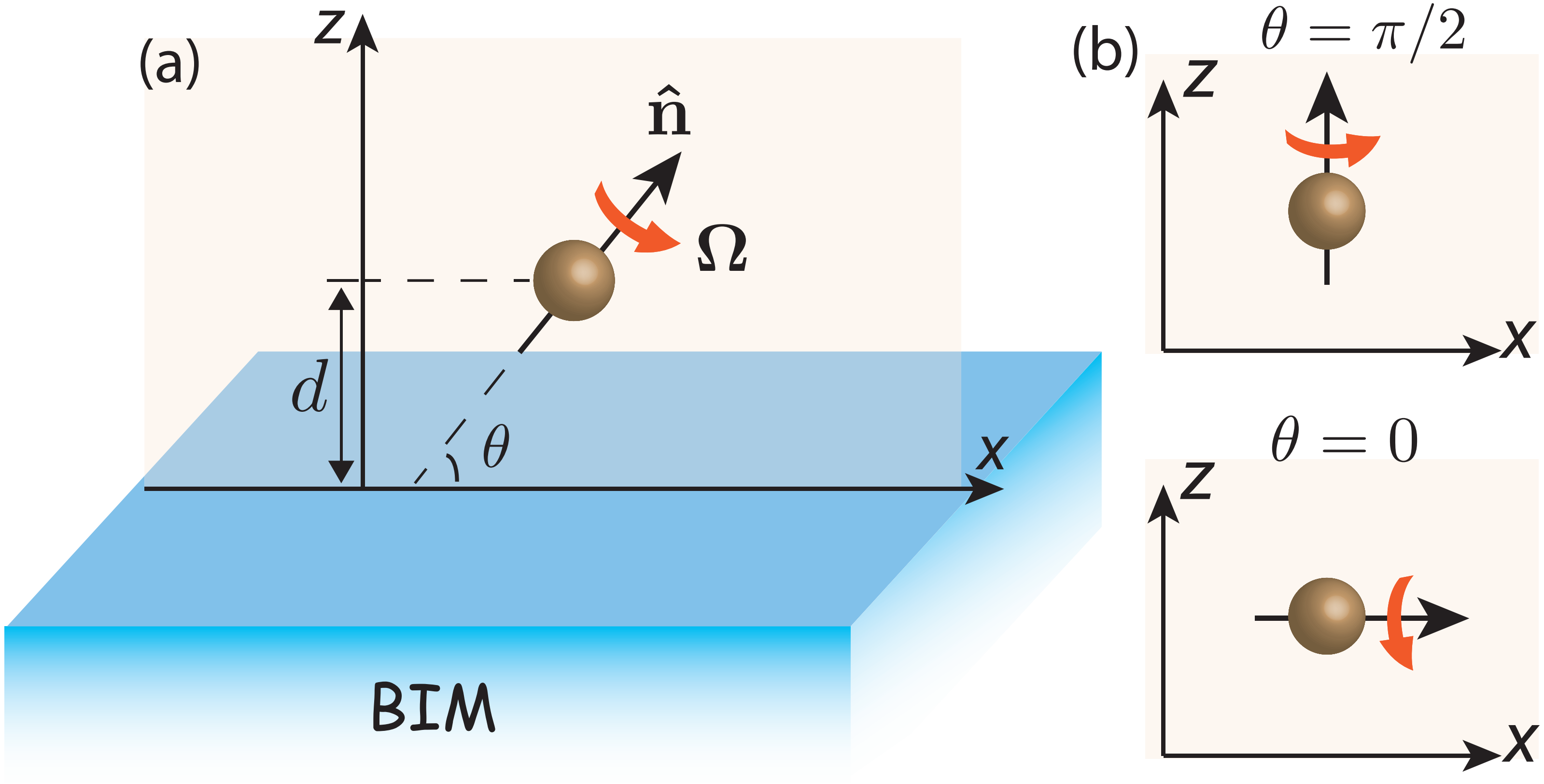
Looking for “repulsive” Casimir force

Flachi-Nitta-Takada-Yushii, PRL (2017)

Jiang-Wilczek, PRB (2019) × 2

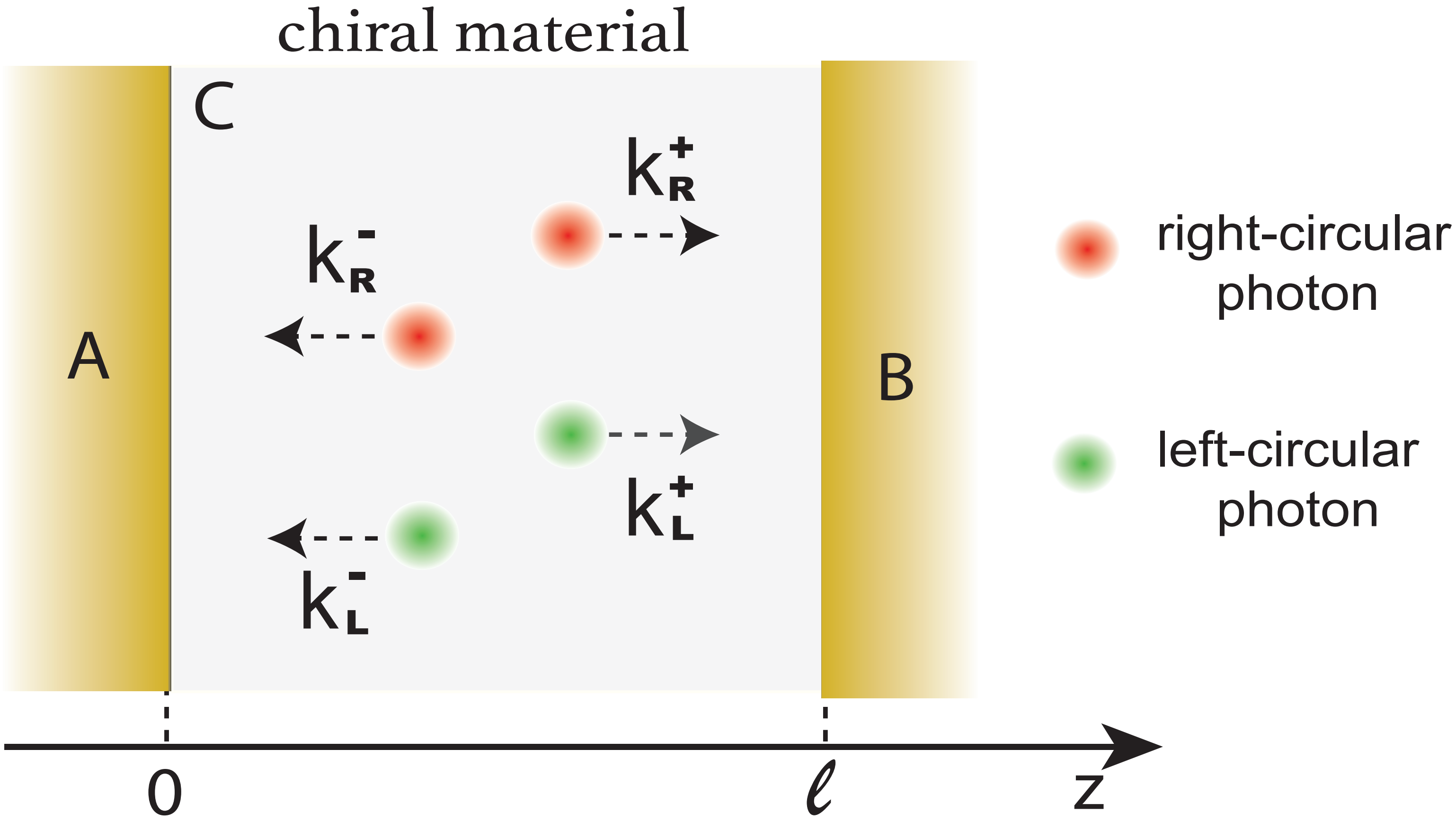
Breaking No-Go Theorem

Jiang-Wilczek, PRB (2019)



Breaking No-Go Theorem

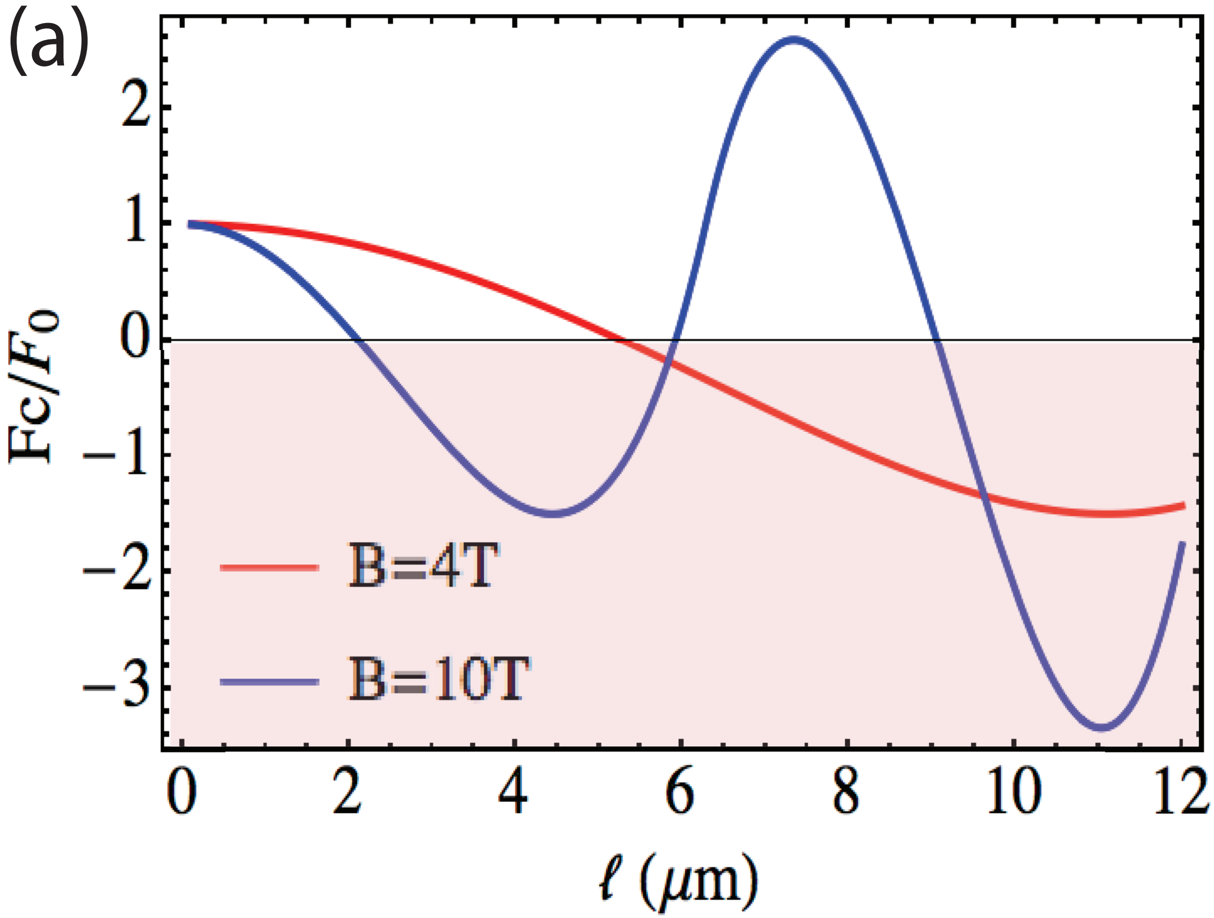
Jiang-Wilczek, PRB (2019)



Breaking No-Go Theorem

Jiang-Wilczek, PRB (2019)

Faraday materials

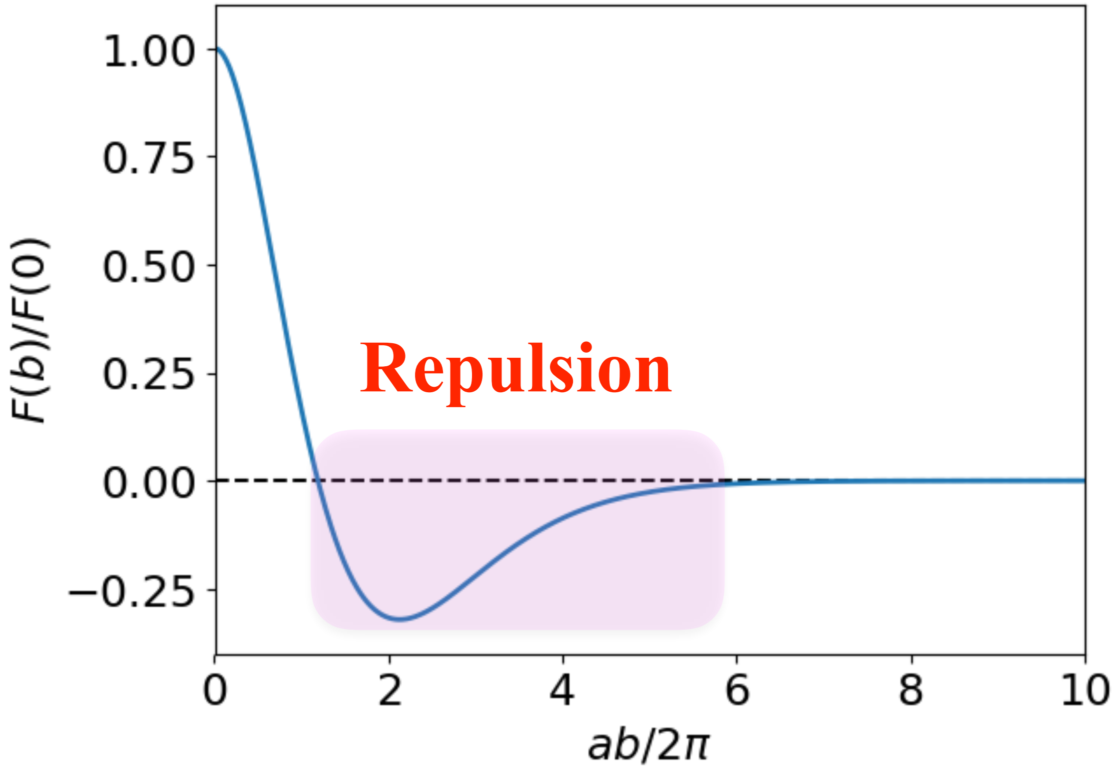


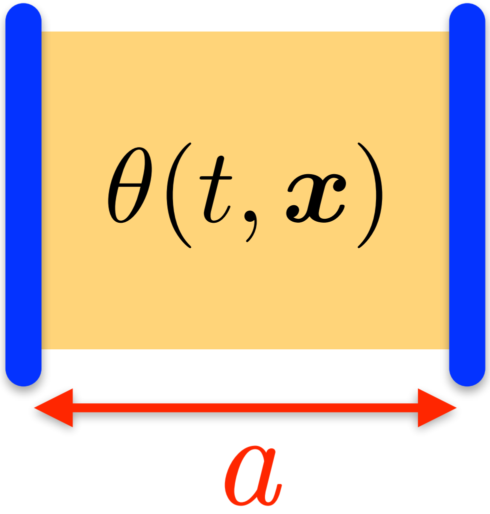
Breaking No-Go Theorem

KF-Imaki-Zebin, in prep.

CME Term

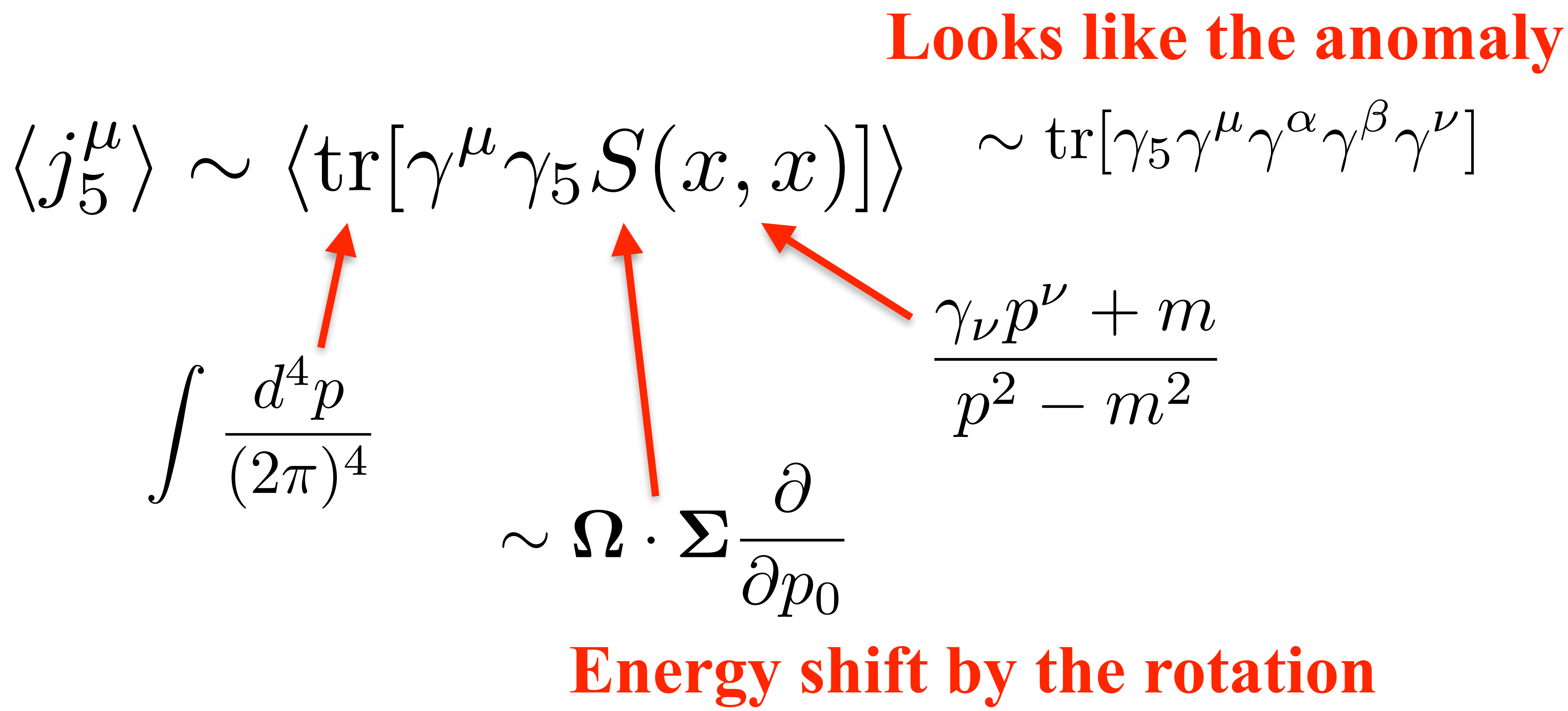
Repulsion for $\theta(t,\boldsymbol{x}) = \cancel{b_0}t - \boldsymbol{b}\cdot\boldsymbol{x}$ **?**





Comment: Casimir vs. CVE

CVE more like the Casimir force



Looks like the anomaly

$$\langle j_5^\mu \rangle \sim \langle \mathrm{tr}[\gamma^\mu \gamma_5 S(x,x)] \rangle$$

$$\sim \mathrm{tr}[\gamma_5 \gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\nu]$$

$$\frac{\gamma_\nu p^\nu + m}{p^2 - m^2}$$

$$\int \frac{d^4 p}{(2\pi)^4}$$

$$\sim \boldsymbol{\Omega} \cdot \boldsymbol{\Sigma} \frac{\partial}{\partial p_0}$$

Energy shift by the rotation

Comment: Casimir vs. CVE

CVE more like the Casimir force

$$j_5 \quad \propto \int \frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial p_0}\frac{p_0}{p^2 - m^2}$$

$$= \left(T\sum_n - \int \frac{dp_0}{2\pi} \right) \frac{\partial}{\partial p_0}\frac{p_0}{p^2 - m^2}$$

$$= \cdots = \frac{i}{12}T^2$$

Comment: Casimir vs. CVE

Duality: Thermodynamics ~ Casimir Effect

KF-Ohta, Physica A299, 248 (2001)

[arXiv: quant-ph/0108145]

"Explicit conversion from the Casimir force to Planck's law of radiation"

$$2l \Longleftrightarrow \beta = \frac{\hbar c}{k_{\rm B}T}, \qquad p \Longleftrightarrow -u$$

I am suspecting some dual Casimir system which has the physics of the chiral vortical effect

Conclusions

- **Chiral Magnetic Effect is a dynamical phenomenon, which is unclear with μ_5, but very clear with parallel electromagnetic fields**

- **Rotating chiral matter exhibits the Barnett effect not only the longitudinal but also the transverse (eddy) directions, but the decomposition to L and S still has ambiguity**

- **Axial (chiral) Casimir effect is a new detectable phenomenon which may give us a hint to CVE**