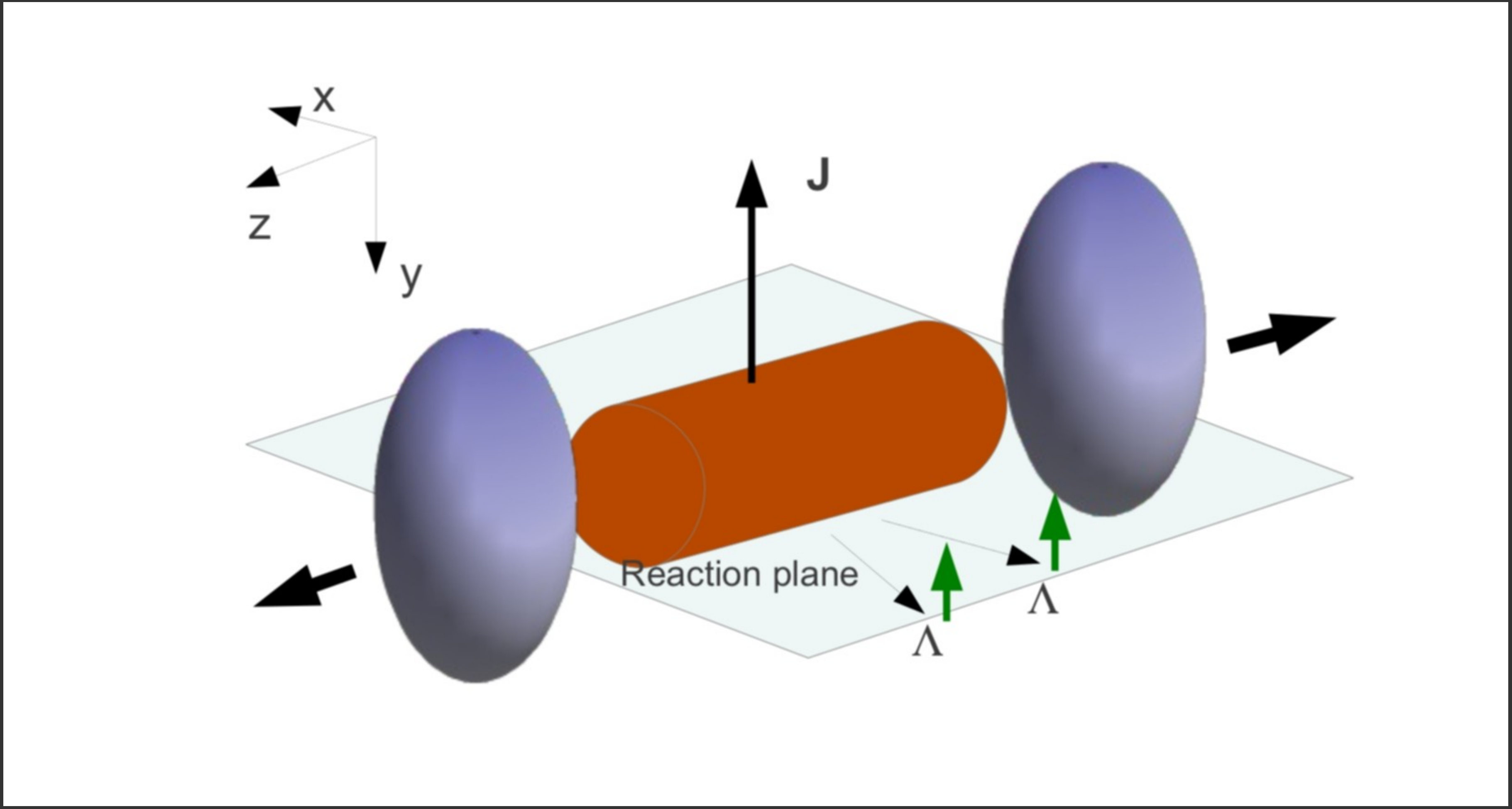
Francesco Becattini, University of Florence and INFN

Vorticity and polarization in QGP: where do we stand?

OUTLINE

- Introduction
- Spin in a relativistic fluid: a theory outline
- Puzzles in polarization measurements
- Ongoing theoretical developments

Heavy ion collisions and polarization



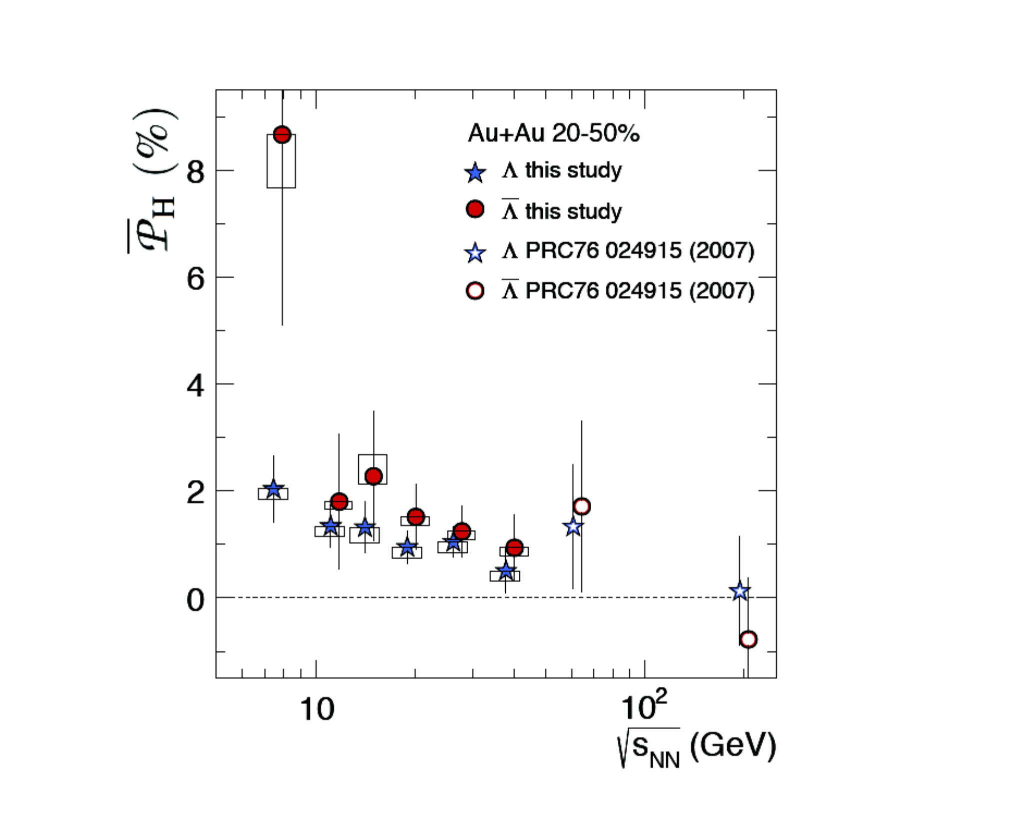
S. Voloshin, nucl-th: 0410089

Parton model: Z. T. Liang, X. N. Wang, Phys.Rev.Lett. 94 (2005) 102301

Statistical mechanics: F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Evidence of Λ polarization in relativistic heavy ion collisions

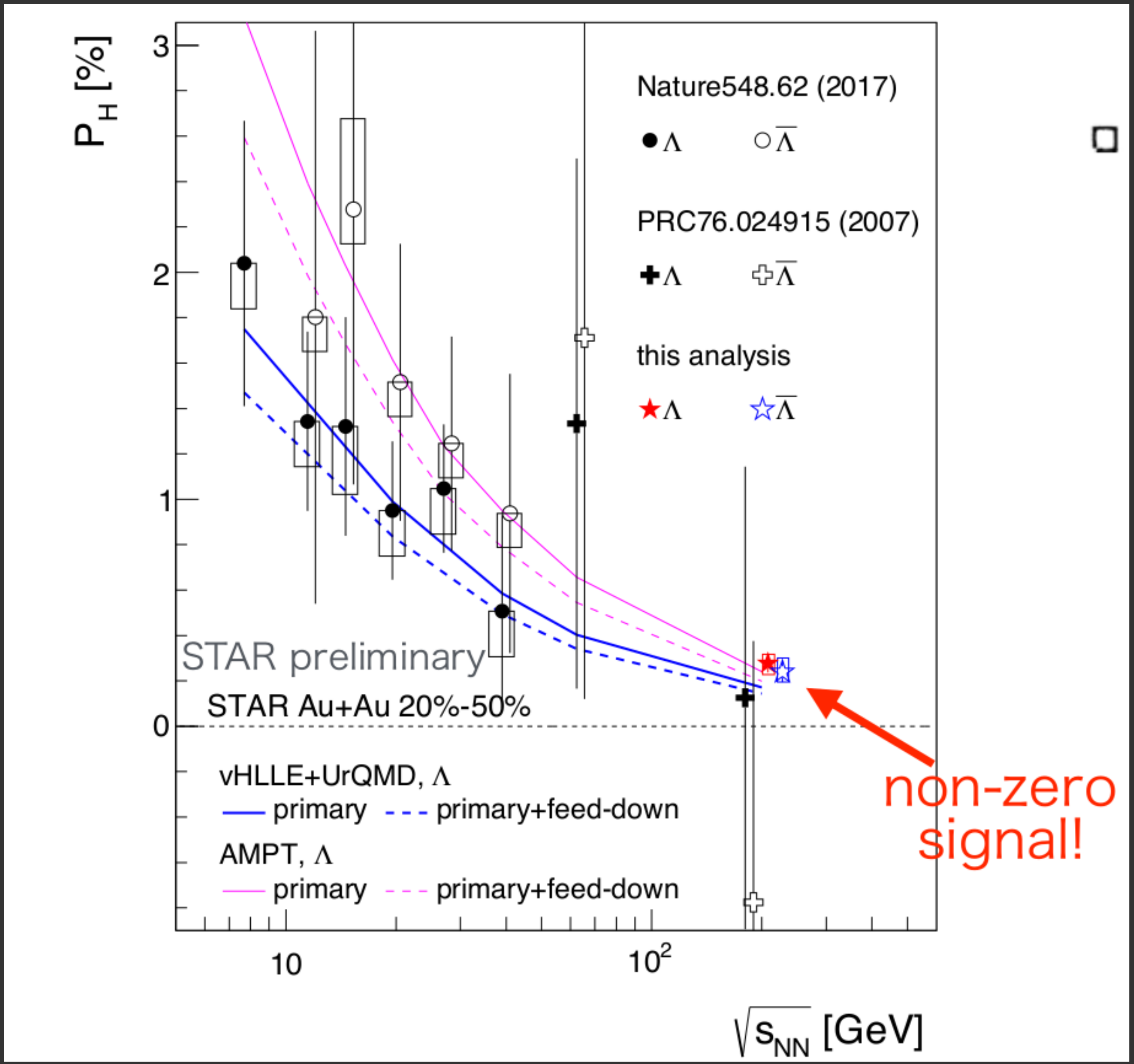
STAR collaboration, Nature 548 (2017) 62



M. Lisa, talk given at Chirality 2016 UCLA

STAR collaboration, Phys.Rev. C98 (2018) 014910

Lambda global polarization from STAR



In agreement with hydro-based quantitative calculation:

L. Csernai, G. Inghirami, L. G. Pang, X. N. Wang, X. G. Wang, Q. Wang, X. L. Xia, J. Liao, I. Karpenko, F.B. …...

See talks by Y.L. Xie and X.L. Xia

Spin in a relativistic fluid

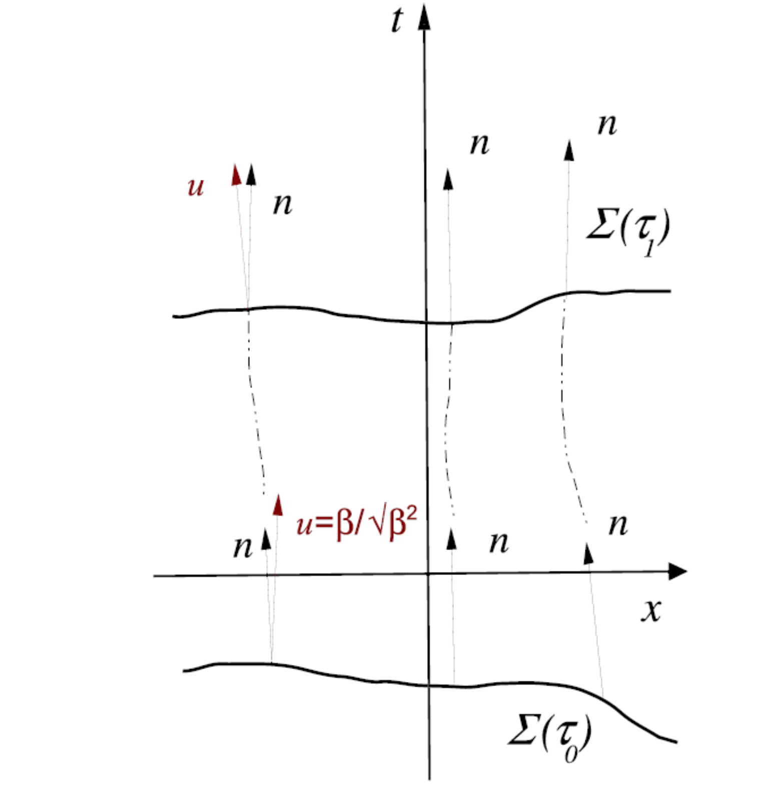
How to attack the problem? It should be quantum from the outset

$$\beta = \frac{1}{T} u$$

$$\zeta = \frac{\mu}{T}$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$



The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho} \log \widehat{\rho})$$

with the constraints of fixed energy-momentum density

Zubarev, 1979, Ch, Van Weert 1982

F. B., L. Bucciantini, E. Grossi, L. Tinti,
Eur. Phys. J. C 75 (2015) 191 (β frame)

T. Hayata, Y. Hidaka, T. Noumi, M. Hongo,
Phys. Rev. D 92 (2015) 065008

Global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_\Sigma \mathrm{d}\Sigma_\mu \; \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Σ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad\qquad \partial_\mu \zeta = 0$$

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp \left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

True statistical operator
(Zubarev theory)

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right].$$

With the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_{\Theta} \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right],$$

Local equilibrium, non-dissipative terms

Dissipative terms

$$T^{\mu\nu}(x) = \mathrm{tr}(\widehat{\rho} \widehat{T}^{\mu\nu}(x))$$

Calculation of the local equilibrium mean values

For instance:

$$T^{\mu\nu}(x)_{\rm LE} = {\rm tr}(\widehat{\rho}_{\rm LE}\widehat{T}^{\mu\nu}(x))$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_B \beta_\nu - \zeta\,\widehat{j}^\mu\right)\right],$$

Hydrodynamic limit: Taylor expand the field β and ζ starting from *x*

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp[-\beta(x)_\mu \widehat{P}^\mu + \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity

Calculation of the mean spin at local equilibrium

Polarization (2S+1)x(2S+1) density matrix

$$\Theta(p)^{\sigma}_{\sigma'} = \frac{\mathrm{tr}(\widehat{\rho} a^{\dagger}_{\sigma'}(p) a_{\sigma}(p))}{\sum_{\sigma} \mathrm{tr}(\widehat{\rho} a^{\dagger}_{\sigma}(p) a_{\sigma}(p))}$$

In practice the derivation required an intermediate educated *ansatz* of the Wigner function of the Dirac field at global equilibrium (F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013), see also R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904)

One gets to:

$$S^{\mu}(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_{\sigma} \frac{\int_{\Sigma} \mathrm{d}\Sigma_{\tau} p^{\tau} n_F (1 - n_F) \partial_{\nu} \beta_{\rho}}{\int_{\Sigma} \mathrm{d}\Sigma_{\tau} p^{\tau}\, n_F}$$

$$n_F = (\mathrm{e}^{\beta \cdot p - \xi} + 1)^{-1}$$

Simplest formula meeting the requirements:

- 1st order in thermal vorticity
- Freeze-out integral
- Vanishing for a fully degenerate Fermi gas (i.e. $n_F = 1$)
- $S \cdot p = 0$

The exact solution at general global equilibrium is still missing

A simpler derivation

F. B., G. Cao, E. Speranza, in progress

Definition of the mean spin vector

$$S^\mu(p) = \sum_{i=1}^{3} \sum_{\sigma\sigma'} D^S(\mathsf{J}^i)_{\sigma\sigma'} \Theta(p)_{\sigma'\sigma} n_i(p)^\mu$$

$$= \sum_i \mathrm{tr}(D^S(\mathsf{J}^i)\Theta(p))[p](\hat{e}_i)^\mu = \sum_{i=1}^{3} [p]^\mu_i \mathrm{tr}(D^S(\mathsf{J}^i)\Theta(p))$$

Standard Lorentz transformation for a massive particle (in the helicity basis)

$$[p] = \mathsf{R}(\varphi, \theta, 0)\mathsf{L}_z(\xi) = \mathsf{R}_z(\varphi)\mathsf{R}_y(\theta)\mathsf{L}_z(\xi)$$

$D^S(\ \ )$ (0,S) representation of the Lorentz group SL(2,C)

Density operator for single particles

At global equilibrium ($\zeta = 0$)

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu}\right]$$

Approximation: QFT ⟶ Set of N distinguishable relativistic particles (Boltzmann approximation)

If they are non-interacting:

$$\widehat{P}^\mu = \sum_i \widehat{P}^\mu_i \qquad \widehat{J}^{\mu\nu} = \sum_i \widehat{J}^{\mu\nu}_i = \sum_i x^\mu_i \widehat{P}^\nu_i - x^\nu_i \widehat{P}^\mu_i$$

$$\widehat{\rho} = \otimes_i \widehat{\rho}_i = \otimes_i \frac{1}{Z_i} = \prod_i \frac{1}{Z_i} \exp[-b \cdot \widehat{P}_i + \frac{1}{2}\varpi : \widehat{J}_i]$$

Factorization and polarization density matrix

$$\exp[-b \cdot \widehat{P}_i + \frac{1}{2}\varpi : \widehat{J}_i] = \exp[-\tilde{b}(\varpi) \cdot \widehat{P}_i] \exp[-\frac{1}{2}\varpi : \widehat{J}_i]$$

by using Baker-Cambbpell-Hausdorff relations

Polarization density matrix

$$\Theta_i(p)_{\sigma\sigma'} = \frac{\langle p, \sigma | \widehat{\rho}_i | p, \sigma' \rangle}{\sum_\sigma \langle p, \sigma | \widehat{\rho}_i | p, \sigma \rangle}$$

Analtyic continuation technique: for imaginary ϖ we deal with an actual Lorentz transformation

$$\Theta(p)_{\sigma\sigma'} = \frac{\langle p, \sigma | \widehat{\Lambda} | p, \sigma' \rangle}{\sum_\sigma \langle p, \sigma | \widehat{\Lambda} | p, \sigma \rangle} = \frac{2\varepsilon\delta^3(\mathbf{p} - \Lambda(p)) W(p)_{\sigma\sigma'}}{\sum_\sigma 2\varepsilon\delta^3(\mathbf{p} - \Lambda(p)) W(p)_{\sigma\sigma}}$$

$W(p) = D^S([\Lambda p]^{-1}\Lambda[p])$ is the Wigner rotation

Result after analytic continuation

$$\Theta(p) = \frac{D^S([p]^{-1}\exp[(1/2)\varpi : \Sigma][p]) + D^S([p]^\dagger \exp[(1/2)\varpi : \Sigma^\dagger][p]^{-1\dagger})}{\mathrm{tr}(\exp[(1/2)\varpi : \Sigma_S]) + \exp[(1/2)\varpi : \Sigma_S^\dagger])}$$
$$= \frac{D^S(\exp[(1/2)\varpi_*(p) : \Sigma]) + D^S(\exp[(1/2)\varpi_*(p) : \Sigma^\dagger])}{\mathrm{tr}(\exp[(1/2)\varpi : \Sigma_S]) + \exp[(1/2)\varpi : \Sigma_S^\dagger])}$$

$\varpi_*(p)$ is the thermal vorticity in the particle rest frame $\Sigma_S = D^S(\mathsf{J})$

At the leading order in thermal vorticity:

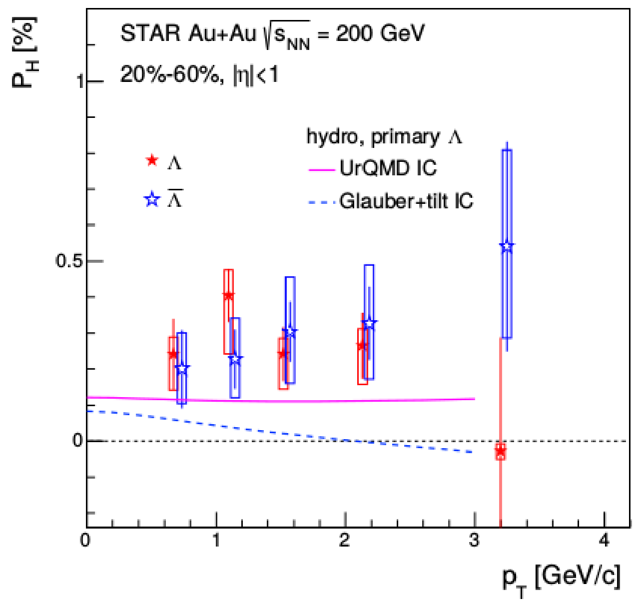
$$S^\mu(p) = [p]^\mu_\kappa \frac{1}{2(2S+1)} \varpi_*(p)^{\alpha\beta} \epsilon_{\alpha\beta\rho\nu} \mathrm{tr}\left(D^S(\mathsf{J}^\rho) D^S(\mathsf{J}^\kappa)\right) \hat{t}^\nu$$
$$= -\frac{1}{2(2S+1)} \frac{S(S+1)(2S+1)}{3} [p]^\mu_\kappa \varpi_*(p)^{\alpha\beta} \epsilon_{\alpha\beta\rho\nu} g^{\rho\kappa} \hat{t}^\nu$$
$$= -\frac{1}{2} \frac{S(S+1)}{3} [p]^\mu_\rho \varpi_*(p)_{\alpha\beta} \epsilon^{\alpha\beta\rho\nu} \hat{t}_\nu = -\frac{1}{2m} \frac{S(S+1)}{3} \varpi_{\alpha\beta} \epsilon^{\alpha\beta\mu\nu} p_\nu$$

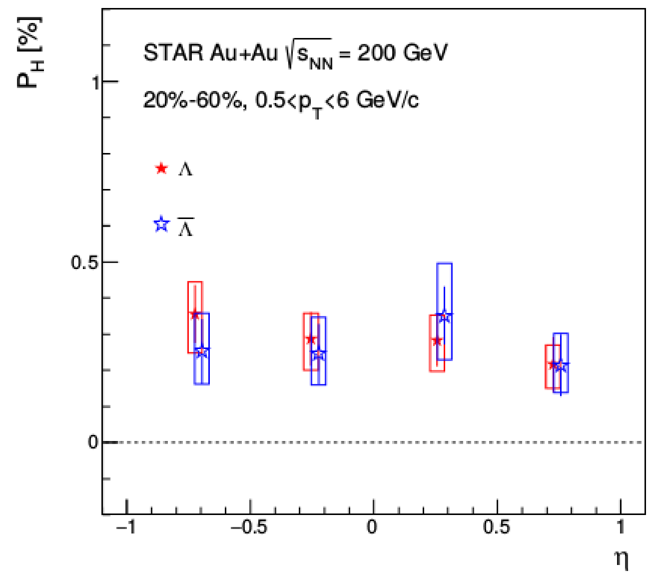
In agreement with:

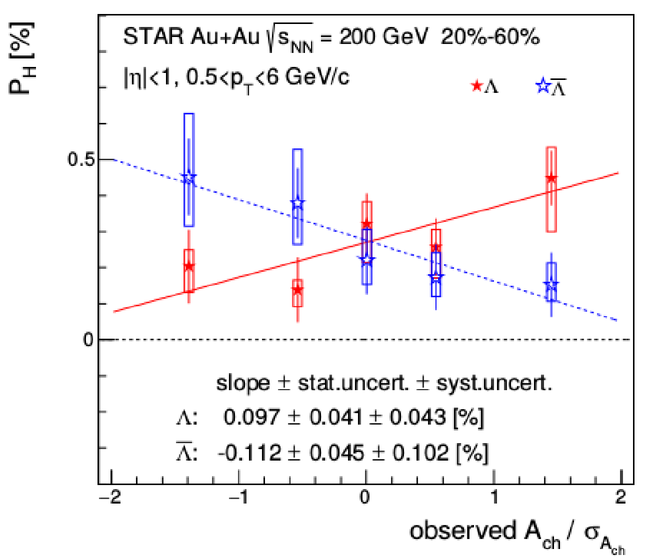
F. B., I. Karpenko, M. Lisa, S. Voloshin, Phys. Rev. C 95 (2017) 054902

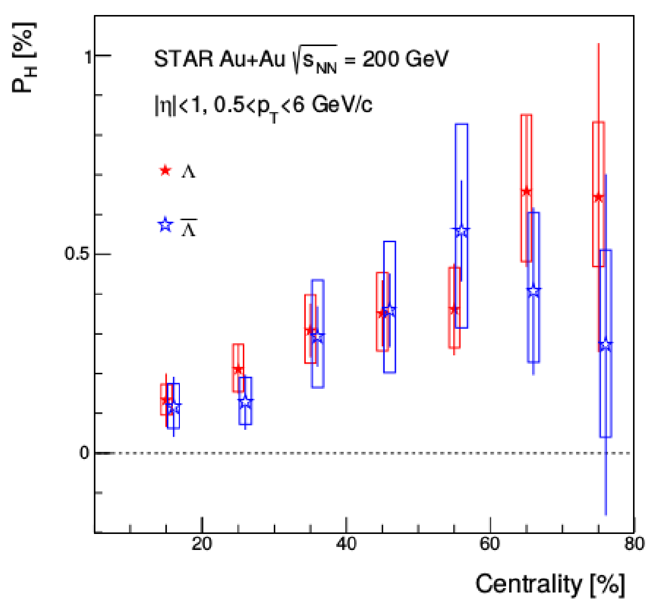
Differential measurements (1)

STAR collaboration, Phys.Rev. C98 (2018) 014910



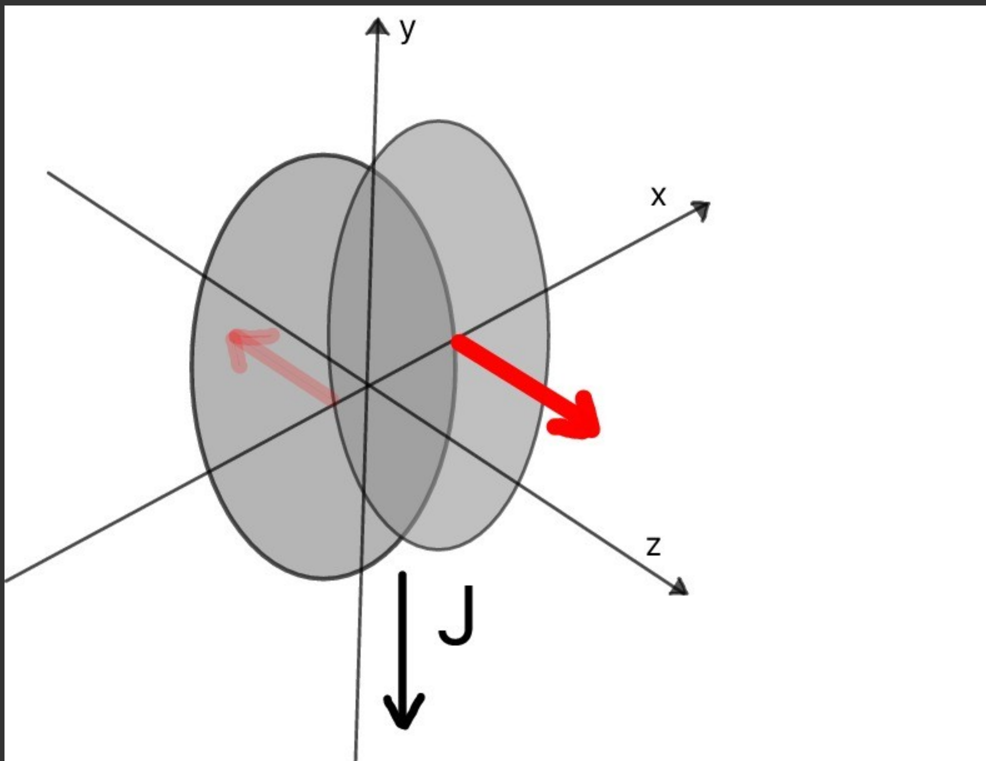






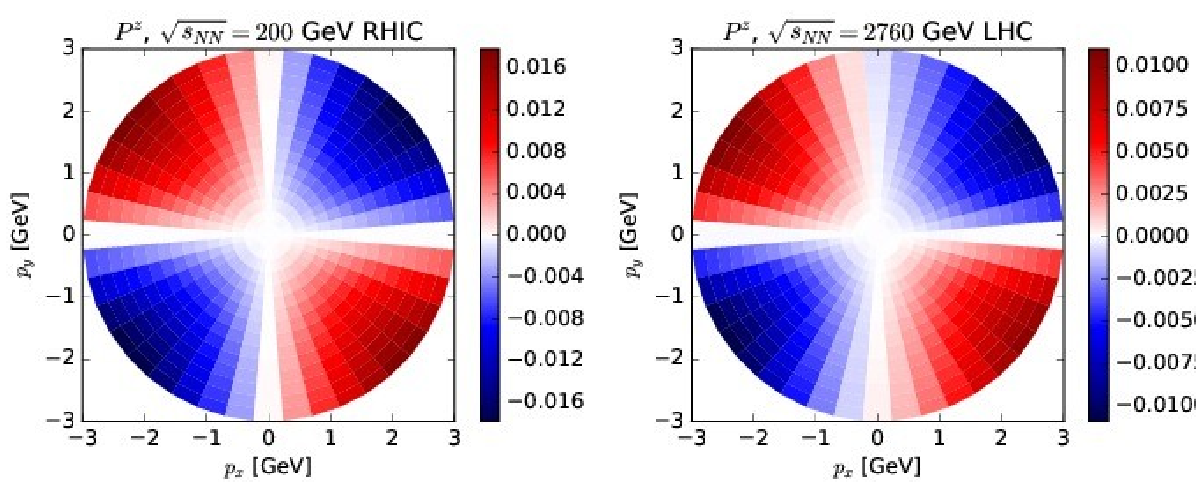
Collective *longitudinal* polarization: quadrupole structure

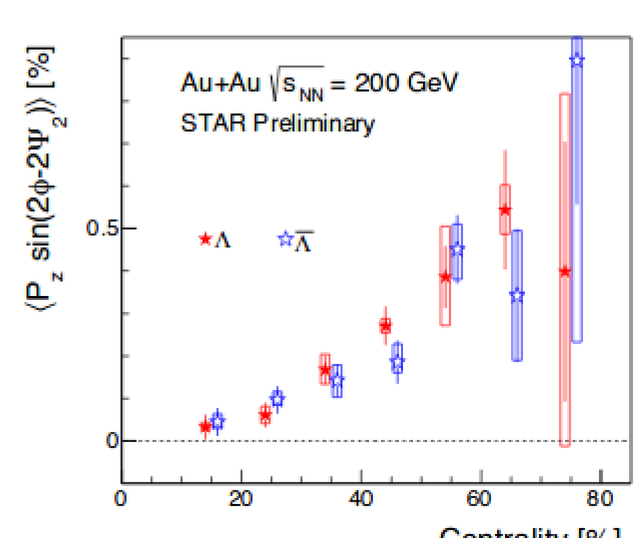
F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302; S. Voloshin, EPJ Conf. Ser. 17 (2018) 10700



Peripheral heavy ion collisions feature two discrete symmetries: reflection w.r.t. reaction plane and rotation by 180 around its perpendicular direction. This reflects into the quadrupole pattern of the longitudinal component of Λ polarization at midrapidity

$$S^z(\mathbf{p}_T, Y=0) = \frac{1}{2}\sum_{k=1}^{\infty} f_{2k}(p_T)\sin 2k\varphi$$





T. Niida, talk given at Quark Matter 2018

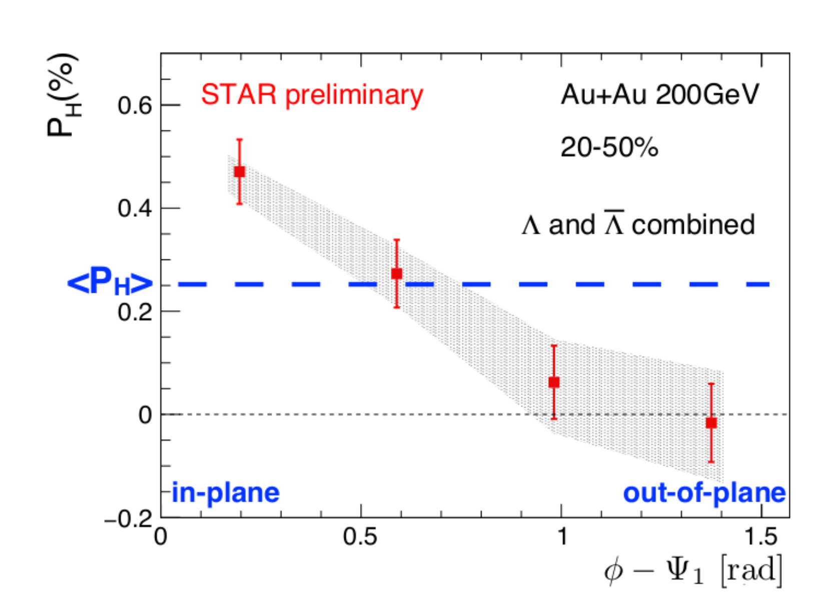
Nucl.Phys. A982 (2019) 511-514

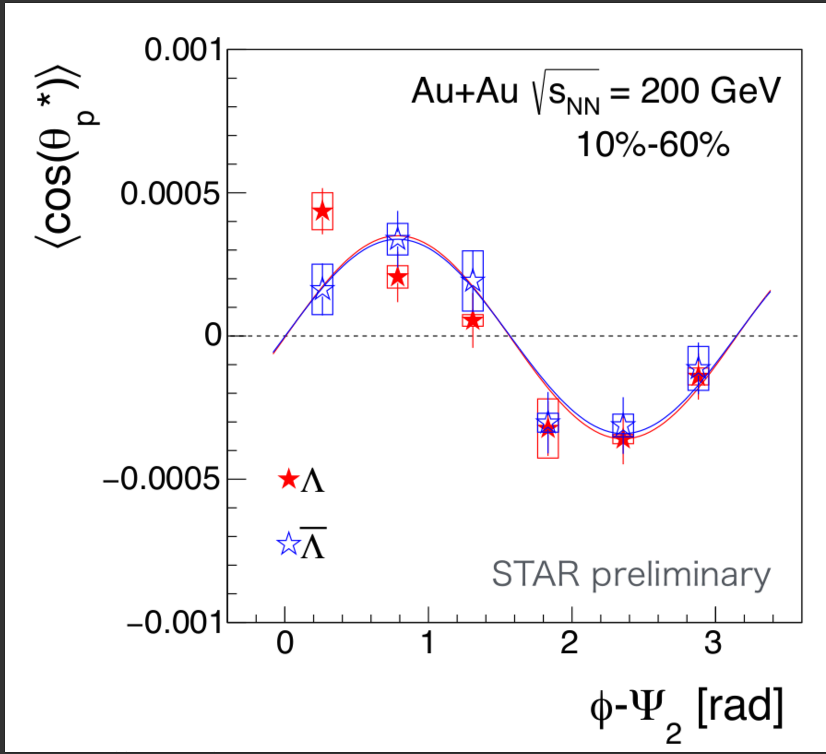
Differential measurements (2)

Disagreement between hydro and data

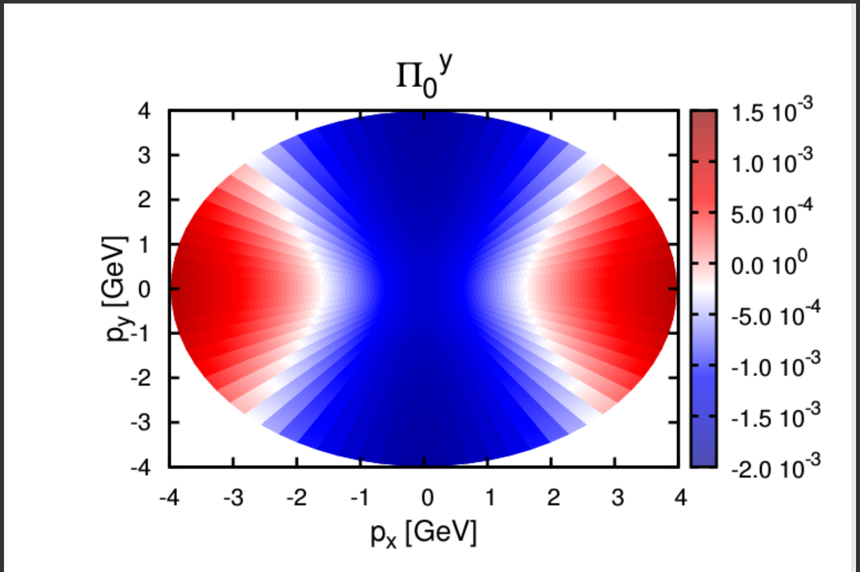
See talk by T. Niida

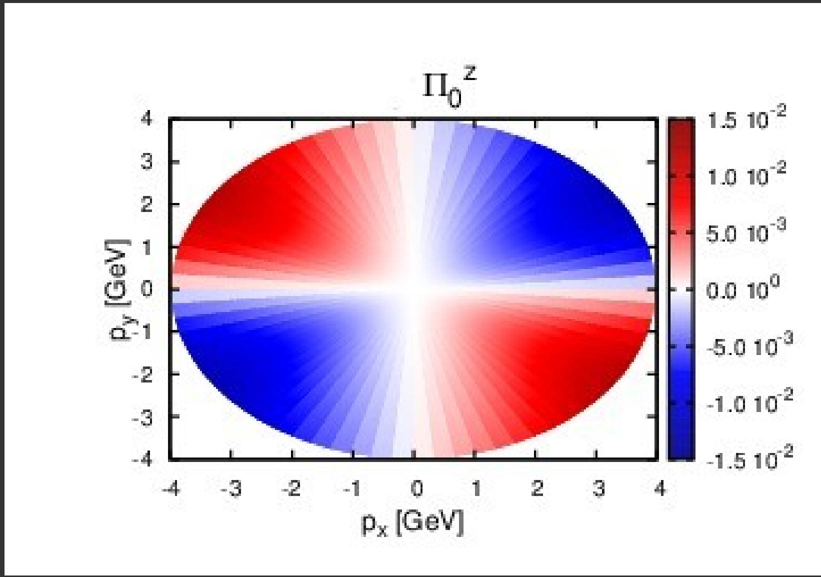
T. Niida, talk given at Quark Matter 2018
Nucl.Phys. A982 (2019) 511-514





F. B., G. Inghirami et al., Eur. Phys. J C 75 (2015) 406





What is the source of this discrepancy?

Disagreement with the theory means:

- *Disagreement with the local equilibrium ansatz*

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma\frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

- *Disagreement with hydrodynamic description*

$$\partial\beta_{\mathrm{EXP}} \neq \partial\beta_{\mathrm{THEO}}$$

The formula provides an excellent quantitative prediction for the GLOBAL polarization, and this must be taken into account

FIRST CLASS (spin formula not adequate)

- Decay of resonances ? *(We now have an answer: NO* – See talk by G. Q. Cao and Hui Li)
- Need of dissipative corrections (viscosity-like) ?
- Need of second-order corrections ?
- Local equilibrium of spin not reached ? (Kinetic spin theory)
- Evidence for the need of a spin tensor ? (see talks by A. Kumar, R. Ryblewski, E. Speranza)

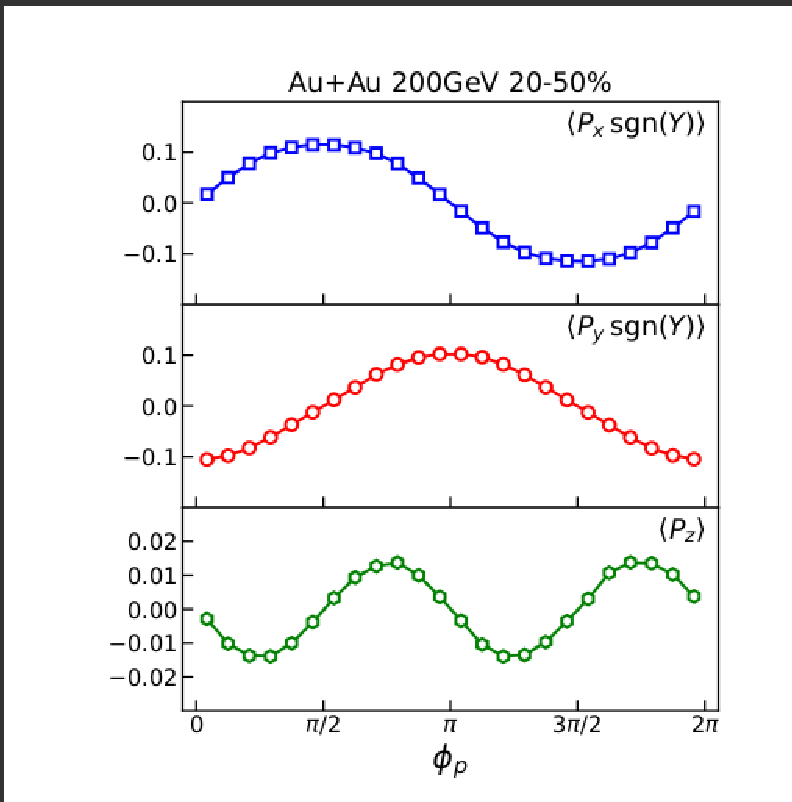
SECOND CLASS (disagreement of thermal vorticity)

- Are hydro initial conditions correct ?
- Do we have a good hydro model to predict thermal vorticity in the late stage?

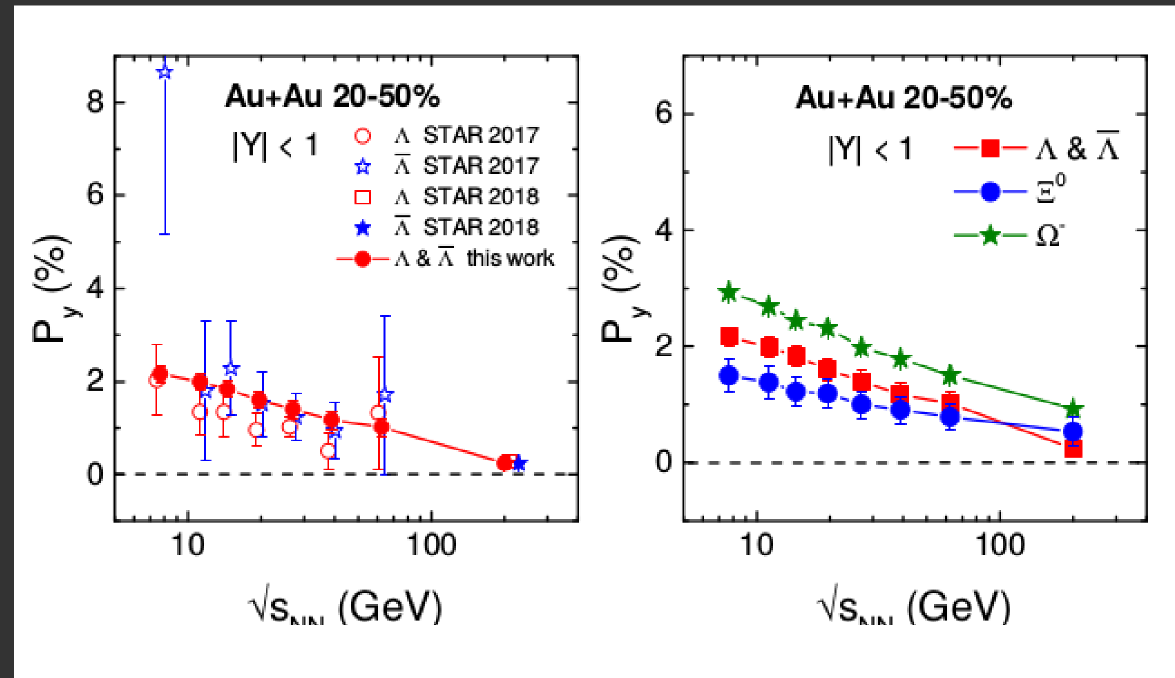
AMPT+thermal vorticity calculation

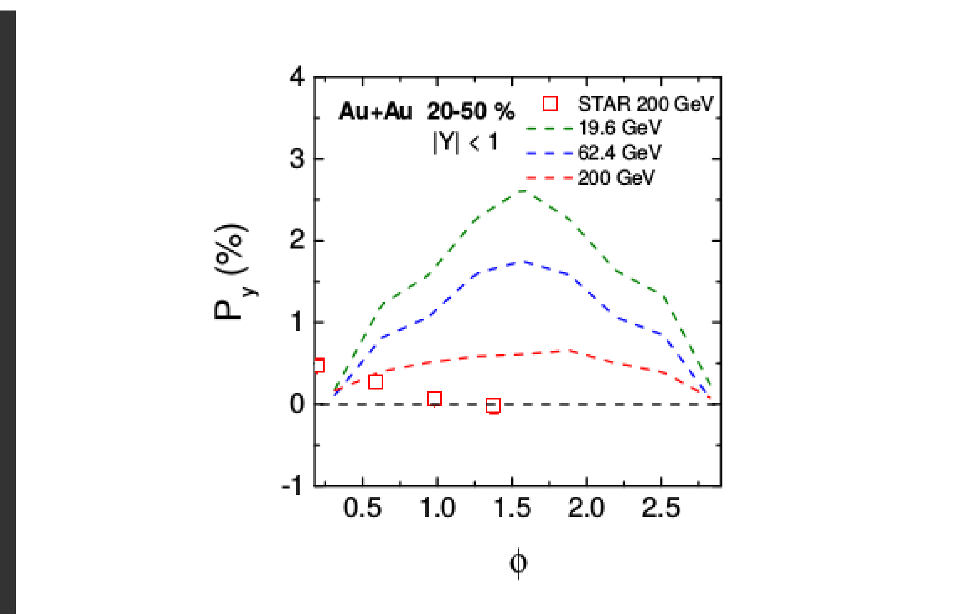
(in principle, Second class test)

D.X. Wei, W.-T. Deng, X.-G. Huang, arXiv:1810.00151



X. L. Xia, H. Li, Z. B. Tang and Q. Wang, Phys.Rev. C98 (2018) 024905





See also talk by C. M. Ko

Kinetic theory (Chiral and Massive)

Towards a First class test

Extend kinetic theory to particles with spin and helicity and obtain the single spin density matrix from the solution of a kinetic equation without assuming local equilibrium

Wigner function (no external field)

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \rangle$$

Wigner equation (collisionless, no external field)

$$(m - \not{k} - \frac{i}{2}\not{\partial})W(x,k) = 0$$

For free particles we have

$$\Theta(p)_{\sigma}^{\sigma'} = \frac{\bar{u}_\sigma(k)\int \mathrm{d}^4x W(x,k)^+ u_{\sigma'}(k)}{\sum_\sigma \bar{u}_\sigma(k)\int \mathrm{d}^4x W(x,k)^+ u_\sigma(k)}$$

Kinetic spin theory: Wigner equation and its solutions

Ansatz proposed for the Wigner function at global equilibrium with non-vanishing thermal vorticity based on an educated extension of Boltzmann statistics and imposing on-mass shell in *F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)*
But it was based on an on-shell approximation and it might have missed some quantum corrections

The exact solution at general global equilibrium is yet to be found

Systematic theoretical studies of the general Wigner equation for massive fermions as an expansion in ħ, including the EM field

W. Florkowksi, A. Kumar, R. Ryblewski, Phys.Rev. C98 (2018) 044906 arXiv:1806.02616
J.-H. Gao, and Z.-T. Liang, arXiv:1902.06510
N. Weickgenannt, X.-l. Sheng, E. Speranza, Q. Wang, D. Rischke, arXiv:1902.06513
Z. Wang, X. Guo, S. Shi and P. Zhuang, arXiv:1903.03461
K. Hattori, Y. Hidaka, Di-Lun Yang, arXiv:1903.01653

An intriguing possibility: hydrodynamics with spin tensor

W. Florkowski et al., Phys. Rev. C 97 (2018) 041901, F. B., W. Florkwoski, E. Speranza, Phys.Lett. B789 (2019) 419

See talks by A. Kumar, R. Ryblewski, E. Speranza

In quantum field theory there are conserved currents arising from Noether theorem (canonical currents):

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

Spin tensor

Pseudo-gauge transformation (F. W. Hehl, Rep. Mat. Phys. 9 (1976) 55)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

Leave conservation equations and P, J unchanged

Special case: Belinfante symmetrized stress-energy tensor, spin tensor vanishing. Tacitly understood in relativistic hydrodynamics

$$\widehat{T}^{\mu\nu}_{\mathrm{B}} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\mathcal{S}}^{\alpha,\mu\nu} - \widehat{\mathcal{S}}^{\mu,\alpha\nu} - \widehat{\mathcal{S}}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Question: does it make any difference in our theoretical calculations ?

$$T^{\mu\nu} = \langle \widehat{T}^{\mu\nu}_{\rm can} \rangle \qquad\qquad T^{\mu\nu} = \langle \widehat{T}^{\mu\nu}_{\rm B} \rangle$$

ANSWER: it all depends on what we measure. In fact we measure spectra, not energy density.
If their theoretical expression is not affected by the pseudo-gauge transformation, any tensor is good.
In other words: spatial densities in the QGP are “objective” up to quantum corrections.

Polarization ultimately depends on

$$\mathrm{tr}(\widehat{\rho} a^{\dagger}_{\sigma}(p) a_{\sigma'}(p))$$

Does the polarization density operator depend on pseudo-gauge transformations?

Global equilibrium: No
Local equilibrium : Maybe

Local equilibrium

Belinfante pseudo-gauge →

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \widehat{T}^{\mu\nu}_{\rm B}\beta_\nu(x)\right]$$

But if there is a spin tensor exists, the constraint of angular momentum density involves a new Lagrange multiplier Ω

$$n_\mu {\rm tr}\left(\widehat{\rho}\,\widehat{\mathcal{S}}^{\mu,\lambda\nu}\right) = n_\mu \mathcal{S}^{\mu,\lambda\nu}.$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta\widehat{j}^\mu\right)\right].$$

Transforming the tensors back to Belinfante set:

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_B\beta_\nu - \frac{1}{2}(\Omega_{\lambda\nu} - \varpi_{\lambda\nu})\widehat{\mathcal{S}}^{\mu,\lambda\nu} + \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{\mathcal{S}}^{\nu,\mu\lambda}\right) - \zeta\widehat{j}^\mu\right)\right],$$

In general, it is not the same as it would be obtained from Belinfante constraints, except Global Equilibrium!

$$\xi_{\lambda\nu} = \frac{1}{2}(\nabla_\nu\beta_\lambda + \nabla_\lambda\beta_\nu)$$

Hydrodynamics with spin tensor

The local thermodynamic equilibrium operator

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}_{\mathrm{C}}^{\mu\nu} \beta_\nu(x) - \frac{1}{2} \Omega_{\lambda\nu}(x) \widehat{\mathcal{S}}_{\mathrm{C}}^{\mu,\lambda\nu} \right]$$

$$\Omega_{\mu\nu} \neq \frac{1}{2} \partial_\mu \beta_\nu - \partial_\nu \beta_\mu$$

6 additional thermodynamic fields to be evolved

$$\zeta = \mu/T \qquad \beta_\mu = \frac{1}{T} u_\mu$$

→

$$+ \qquad \Omega_{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$
$$\partial_\mu \mathcal{S}^{\mu,\lambda\nu} = T^{\nu\lambda} - T^{\lambda\nu}$$

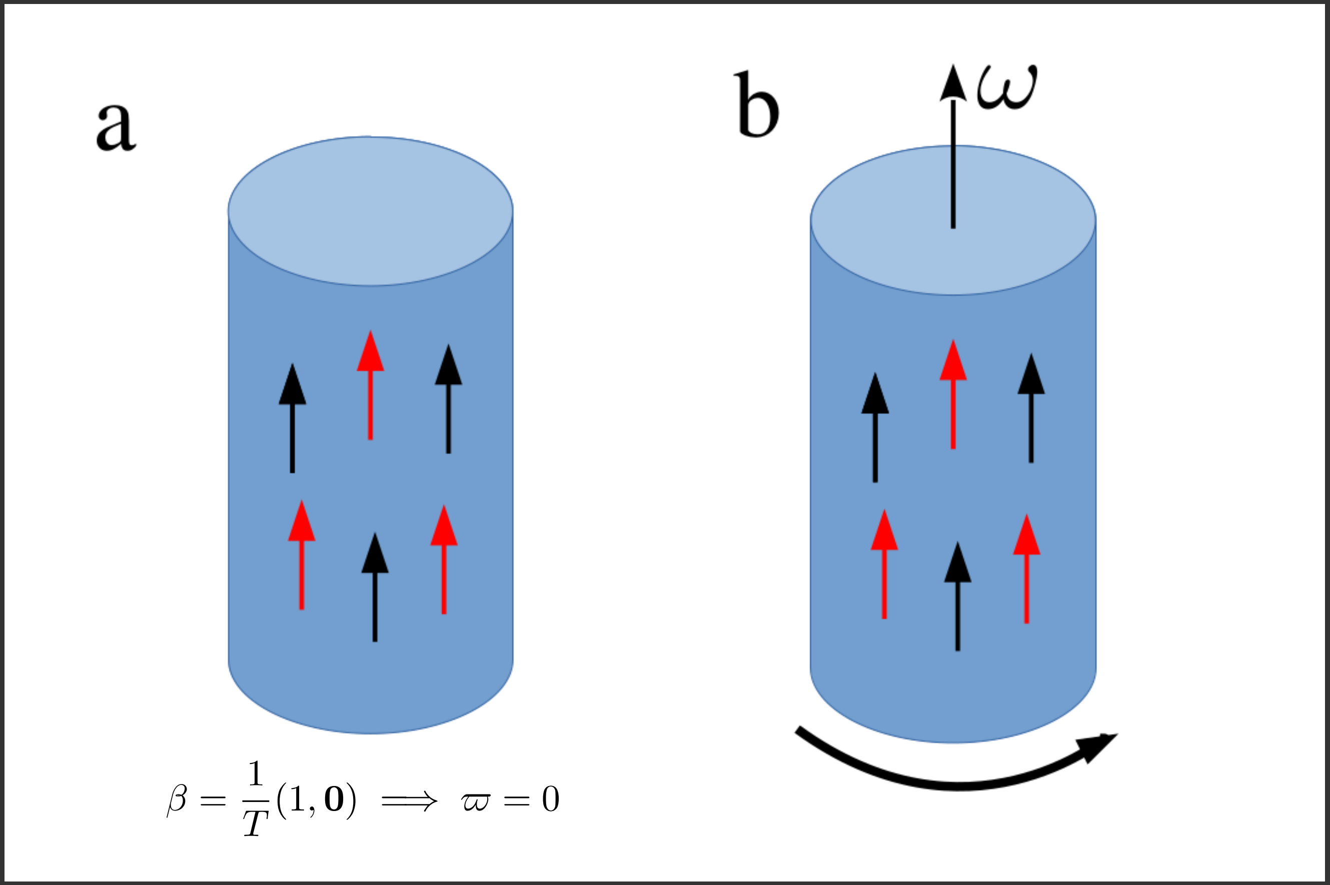
11 spin-hydrodynamical equations for 11 unknowns and 11 initial conditions

Need to determine the constitutive equations:

$$j^\mu = j^\mu(\beta, \zeta, \Omega), \qquad T^{\mu\nu} = T^{\mu\nu}(\beta, \zeta, \Omega), \qquad \mathcal{S}^{\lambda,\mu\nu} = \mathcal{S}^{\lambda,\mu\nu}(\beta, \zeta, \Omega).$$

This approach makes it possible to describe as local thermodynamic equilibrium

polarized C-even matter at rest



If the relaxation time of spin degrees of freedom is slow enough, configurations like a) should be described as hydrodynamic

Conclusions

- Polarization has opened a new window in heavy ion physics.

- We have entered a new stage. After the initial success, more detailed measurements have shown discrepancies.

- From a phenomenological viewpoint, this urges us to reexamine and revise the usual assumptions and perhaps even some firm belief about the hydrodynamic model of the collision.

- From a theory viewpoint, a new outlook and to reconsider the foundations of relativistic hydrodynamics and kinetic theory in a fully quantum framework.

- The study of the quantum features of Quark Gluon Plasma is not just important for the field, but it may have important connections with fundamental physics problems

SPARE SLIDES

Summary

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta\widehat{j}^\mu\right)\right].$$

$$\widehat{\rho}'_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \widehat{T}_{\rm B}^{\mu\nu}\beta_\nu(x)\right]$$

These two are different and they also differ for $\Sigma(\tau_0)$ which defines the full density operator

For instance, polarization will be different because it depends on ρ

$$\mathrm{tr}(\widehat{\rho}a^\dagger_\sigma(p)a_{\sigma'}(p))$$

The operators are the same at global thermodynamic equilibrium, when $\beta_\mu = b_\mu + \varpi_{\mu\nu}x^\nu$

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-b_\mu\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right],$$

Hydrodynamic formula

Primary particles

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F}$$

$$n_F = (\mathrm{e}^{\beta\cdot p - \xi} + 1)^{-1}$$

$$\beta = \frac{1}{T} u$$

F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

Secondary particles: global polarization transfer in two-body STRONG and EM decays

$$\mathbf{S}^*_{\text{daughter}} = C \mathbf{S}^*_{\text{parent}}$$

$$C = \sum_{\lambda_A,\lambda_B,\lambda'_A} T^J(\lambda_A,\lambda_B) T^J(\lambda'_A,\lambda_B)^* \sum_{n=-1}^{1} \langle \lambda'_A | \widehat{S}_{A,-n} | \lambda_A \rangle \times \frac{c_n}{\sqrt{J(J+1)}} \langle J\lambda | J1 | \lambda' n \rangle \left(\sum_{\lambda_A,\lambda_B} |T^J(\lambda_A,\lambda_B)|^2 \right)^{-1}$$

F. B., I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95 054902 (2017)

ALL calculations based on different hydro (or even non-hydro) models use the above formula

L. Csernai, L. G. Pang, X. N. Wang, C. Ko, X. G. Wang, Q. Wang, X. L. Xia, J. Liao, A. Sorin, O. Teryaev report good agreement with the data

Intermezzo: Acceleration-vorticity-grad T decomposition

$$\partial_\mu \beta_\nu = \partial_\mu \left(\frac{1}{T}\right) u_\nu + \frac{1}{T}\partial_\mu u_\nu$$

$$\begin{aligned} A^\mu &= u \cdot \partial u^\mu \\ \omega^\mu &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu u_\rho u_\sigma \end{aligned}$$

$$\begin{aligned} S^\mu(p) \int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F &= \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1-n_F) \nabla_\nu (1/T) u_\rho \\ &+ \frac{1}{8m}\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1-n_F) 2 \frac{\omega^\mu u \cdot p - u^\mu \omega \cdot p}{T} \\ &- \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1-n_F) \frac{1}{T} A_\nu u_\rho \end{aligned}$$

Grad T

Vorticity

Acceleration

In the rest frame of the particle:

$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u} \times \nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A} \times \mathbf{u}/c^2$$

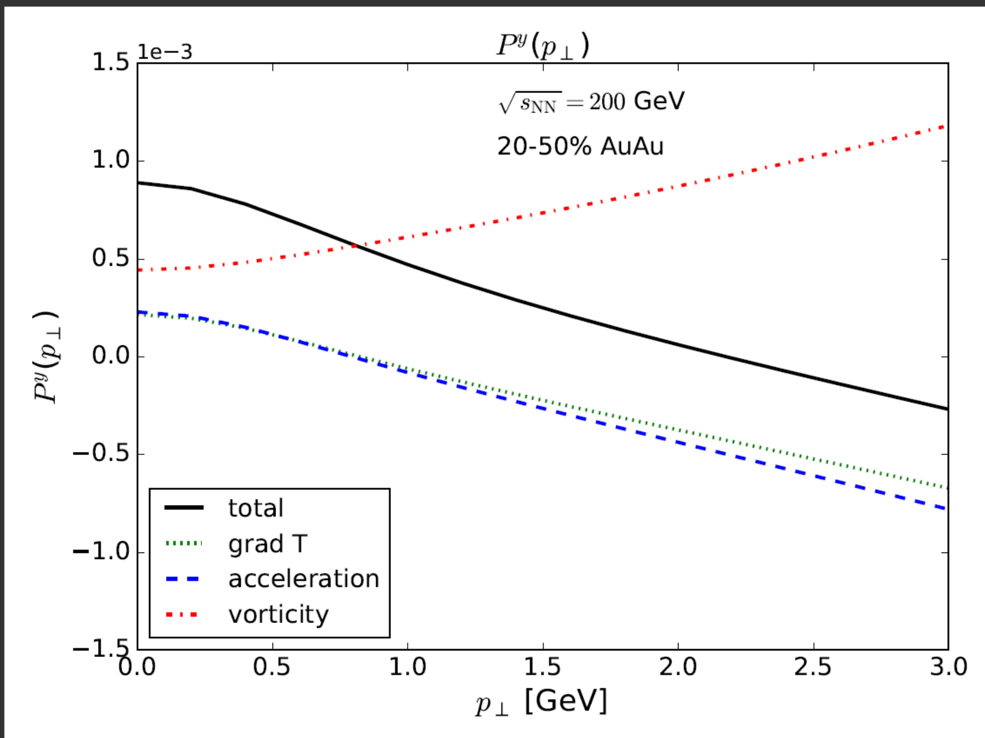
Thermal term (new effect)

Vorticous term (known)

Acceleration term (purely relativistic)

Are all these components needed?

I. Karpenko, QM 2018



GLOBAL J- COMPONENT

LONGITUDINAL COMPONENT