Simulating chiral anomalies in a box system

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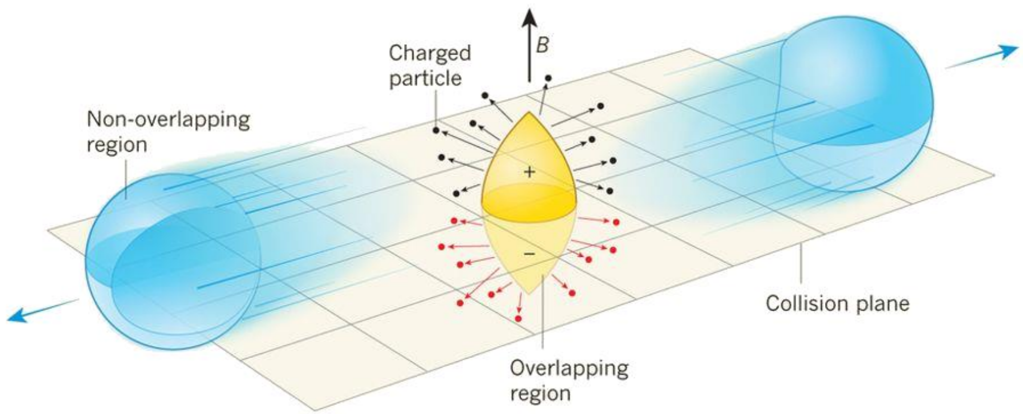
Based on: *PRC* 98, 044904 (2018); arXiv:1904.01834.

Introduction



A. Dobrin *Nature* 546, 7659 (2017)

$$\vec{J} = \frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B},$$

$$\vec{J}_5 = \frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}.$$

P-odd
CP-even

P-even
CP-odd

CME $\vec{J}^{\mu} = \vec{J}^{\mu}_{R} + \vec{J}^{\mu}_{L}$ $\vec{J}^{\mu} = \langle\bar{\psi}\gamma^{\mu}\psi\rangle$ $N_{C(+)} \neq N_{C(-)}$ ⇨ $\vec{J} \neq 0$

CSE $\vec{J}^{\mu 5} = \vec{J}^{\mu}_{R} - \vec{J}^{\mu}_{L}$ $\vec{J}^{\mu 5} = \langle\bar{\psi}\gamma^{\mu}\gamma^{5}\psi\rangle$ $N_{Q(+)} \neq N_{Q(-)}$ ⇨ $\vec{J}^{5} \neq 0$

Chiral kinetic Motion

Spin EOM (SEOM)

$$\dot{\vec{r}} = c\vec{\sigma}$$
$$\dot{\vec{k}} = \dot{\vec{r}} \times qe\vec{B}$$
$$\dot{\vec{\sigma}} = c\frac{2}{\hbar}\vec{k} \times \vec{\sigma}$$

Canonical equation

$$H = c\vec{\sigma} \cdot \vec{k}$$

$$\vec{k} = \vec{p} - qe\vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Adiabatic approximation

$$\vec{\sigma} \approx c\hat{k} - \frac{\hbar}{2k}\hat{k} \times \dot{\hat{k}}$$

E. van der Bijl and R. A. Duine, *PRL*. 107, 195302 (2011).
X.-G. Huang, *Sci. Rep*. 6, 20601 (2016).

Chiral EOM (CEOM)

$$\sqrt{G}\dot{\vec{r}} = \hat{k} + \hbar(c\vec{\Omega} \cdot \hat{k})qe\vec{B}$$
$$\sqrt{G}\dot{\vec{k}} = \hat{k} \times qe\vec{B}$$
$$\sqrt{G} = 1 + \hbar(qe\vec{B} \cdot c\vec{\Omega})$$

$$\vec{\Omega} = \frac{\vec{k}}{2k^3},$$ Berry curvature

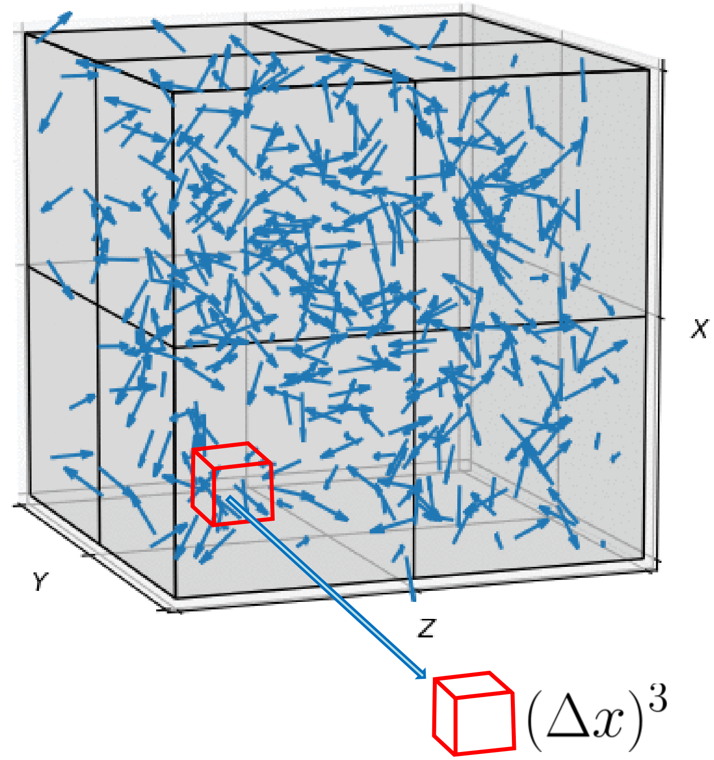
$$\hat{k} = \vec{k}/k$$

$q = \pm 1$: Charge number
$c = \pm 1$: Helicity

M. A. Stephanov, Y. Yin, J.-W. Chen, S. Pu, Q. Wang, X.-N. Wang, D. T. Son, N. Yamamoto…

Initialization

Cubic Periodic Box



- ***Coordinate***

Uniform Distribution

- ***Momentum***

Fermi-Dirac Distribution

$$f = \frac{1}{1 + e^{\frac{k - \mu_{qc}}{T}}}, \mu_{qc} = q\mu + c\mu_5$$

- ***Scattering process***

Pauli blocking

Stochastic method

Blocking probability: $1 - (1 - f_3)(1 - f_4)$

Collision probability: $P_{22} = v_{\mathrm{rel}} \sigma_{22} \frac{\Delta t}{(\Delta x)^3}$

$v_{\mathrm{rel}} = s/(2E_1 E_2)$

Z. Xu, C. Greiner, *PRC* 71, 064901 (2005).

CEOM

$$eB_y = 0.5\text{GeV/fm}$$

$$\eta/s = 0.08\hbar, T = 300\text{MeV}$$

Theoretical limit

Truncation: $\begin{cases} 0.3 < \sqrt{G} < 1.7 \\ k > 0.3 \text{ GeV/c} \end{cases}$

$$\int \frac{d^3k}{(2\pi\hbar)^3} \Rightarrow \int \sqrt{G}\frac{d^3k}{(2\pi\hbar)^3}$$

D. Xiao, J. Shi, and Q. Niu, *PRL* 95, 137204 (2005).

$$\langle A \rangle = \frac{\sum_i A_i\sqrt{G_i}}{\sum_i \sqrt{G_i}}$$

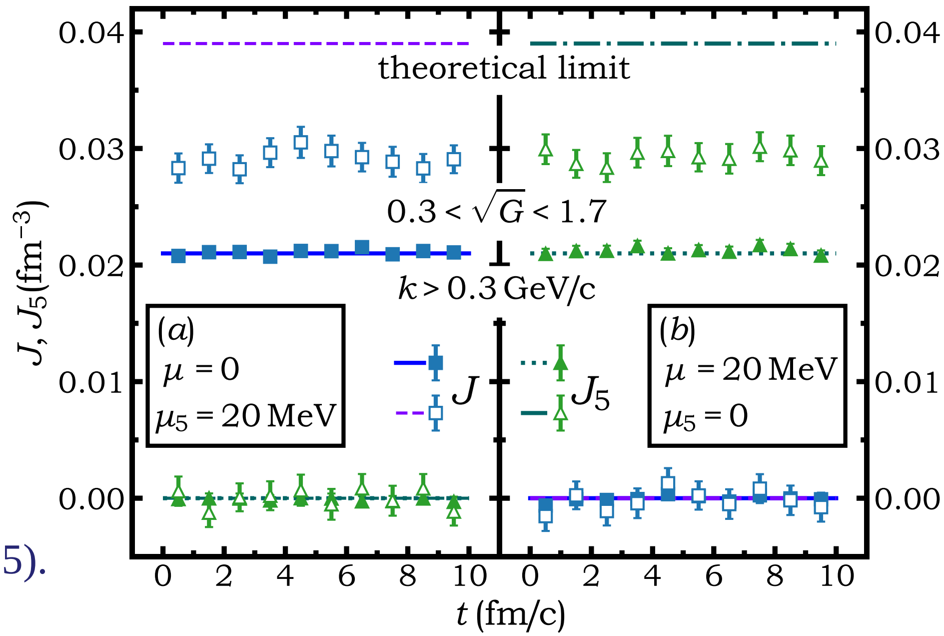
$\sqrt{G}$ as a weight factor

Current

Lines: Theoretical limits

Symbols: Simulating results

CME CSE



$N_{C(+)} \neq N_{C(-)}$

$N_{Q(+)} = N_{Q(-)}$

$N_{Q(+)} \neq N_{Q(-)}$

$N_{C(+)} = N_{C(-)}$

SEOM

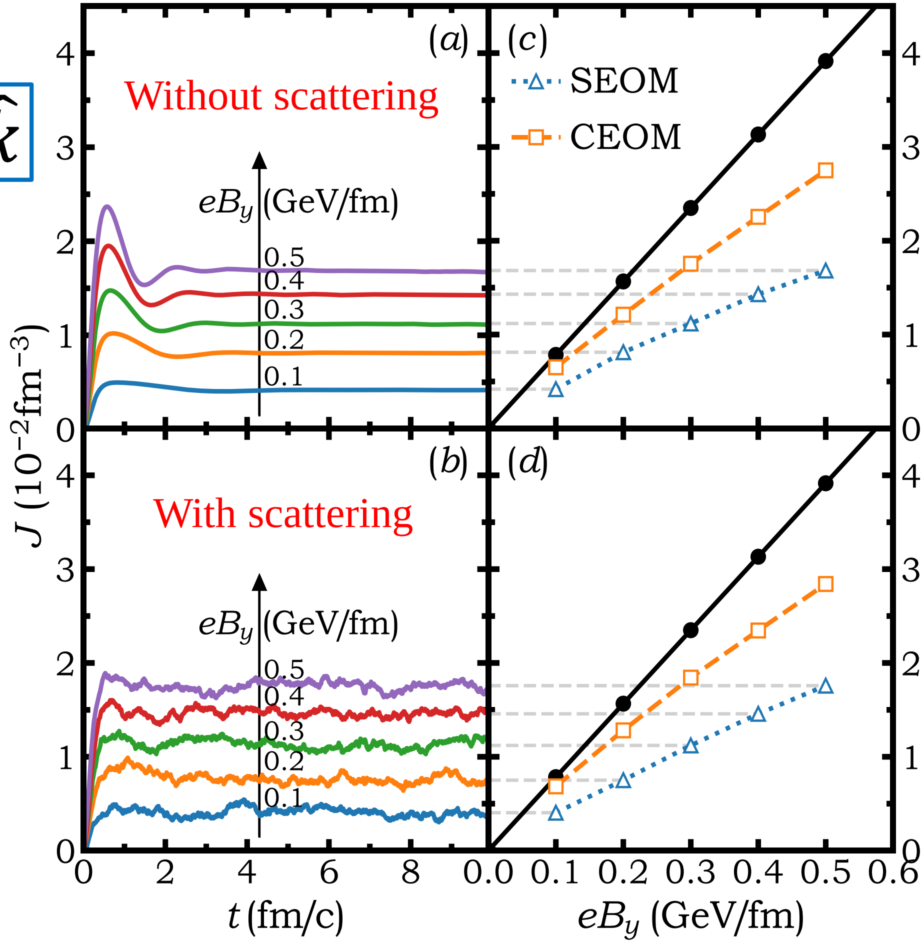
Current

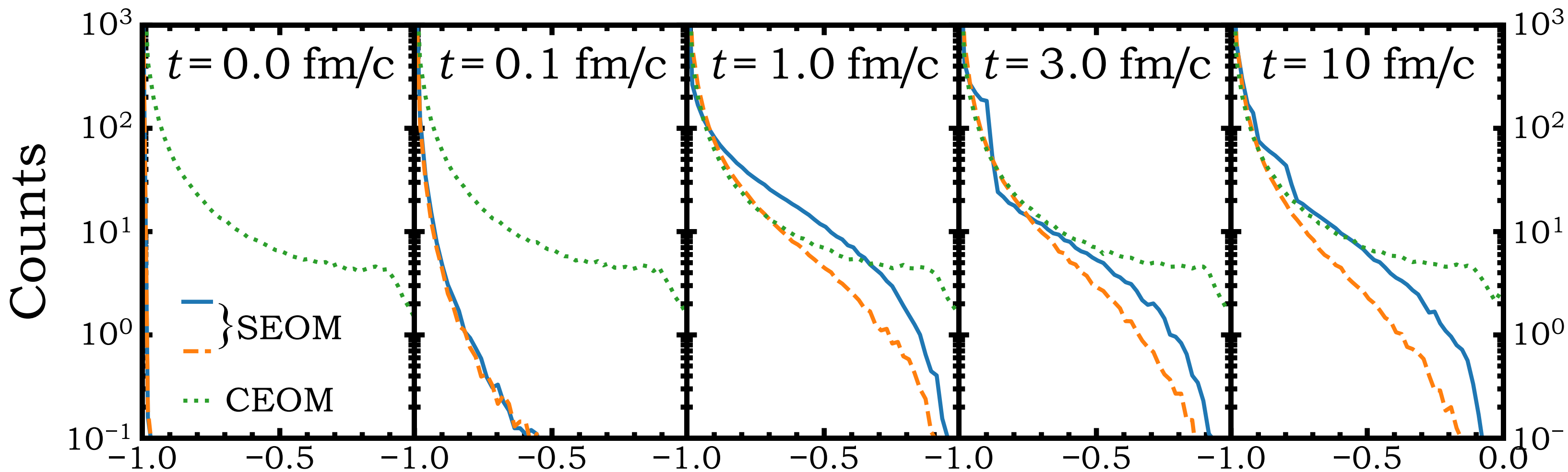
$T = 300\text{MeV}, \;\; \eta/s = 0.08\hbar$

$eB_y = 0.5\text{GeV/fm}$ $\vec{\sigma}_{t=0} = c\hat{k}$

➢ *Scattering process*

1. Stochastic method;
2. Pauli blocking by considering spin state in discrete space;
3. Single particle Energy conservation $c\vec{\sigma}\cdot\vec{k} = c\vec{\sigma}'\cdot\vec{k}'$;
4. Spin tend to polarize to $q\vec{B}$, $q\vec{B}\cdot\vec{\sigma}'$ is the maximum value.





SEOM $\cos\langle\vec{\sigma},\vec{k}\rangle = \dfrac{\vec{\sigma}\cdot\vec{k}}{k}$

solid: without scattering

CEOM $\cos\langle\vec{\sigma},\vec{k}\rangle = \dfrac{c\dot{\vec{r}}\cdot\vec{k}}{|\dot{\vec{r}}|k}$

$\vec{\sigma} \approx c\dot{\vec{r}}$

For SEOM:

$\cos\langle\vec{\sigma},\vec{k}\rangle$ is ± 1 in the initial state;

It evolves gradually away from ± 1 and becomes equilibrated in the final state.

For CEOM:

$\cos\langle\vec{\sigma},\vec{k}\rangle$ is equilibrated at the beginning.

CMW

Current

$$\vec{J} = \frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B},$$

$$\vec{J}_5 = \frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}.$$

Density

$$\rho \approx \frac{N_c T^2}{3\hbar^3}\mu,$$

$$\rho_5 \approx \frac{N_c T^2}{3\hbar^3}\mu_5.$$

$\mu/T \ll 1, \mu_5/T \ll 1$

$$\vec{J}_{R/L} = \pm\frac{3\hbar e\vec{B}}{2\pi^2 T^2}\rho_{R/L}.$$

Continuity equation

$$\partial_t \rho_{R/L} + \vec{\nabla}\cdot\vec{J}_{R/L} = 0$$

Fick's law: $-D_L\vec{\nabla}\rho_{R/L}$

v_p is the velocity of the CMW,
D_L is the diffusion constant.

$$\left(\partial_t \pm v_p\partial_y - D_L\partial_y^2\right)\rho_{R/L} = 0$$

$$v_p = \frac{3\hbar e B}{2\pi^2 T^2}.$$

G. M. Newman, *JHEP* 01 (2006) 158,
D. E. Kharzeev, H.-U. Yee, *PRD* 83, 085007 (2011).

Initialization

$$\rho_{R/L}(y,0) = \pm\frac{1}{2}A_c n \sin(\beta y)$$

Periodic Bound condition $\rho_{R/L}|_{y=-l} = \rho_{R/L}|_{y=+l}$

Evolution:

$$\rho_{R/L}(y,t) = \pm\frac{1}{2}A_c n e^{-D_L\beta^2 t}\sin\left[\beta\left(y \mp v_p t\right)\right]$$

$$\rho = \rho_R + \rho_L = -A_c n e^{-D_L\beta^2 t}\sin\left(\beta v_p t\right)\cos(\beta y)$$
$$\rho_5 = \rho_R - \rho_L = +A_c n e^{-D_L\beta^2 t}\cos\left(\beta v_p t\right)\sin(\beta y)$$

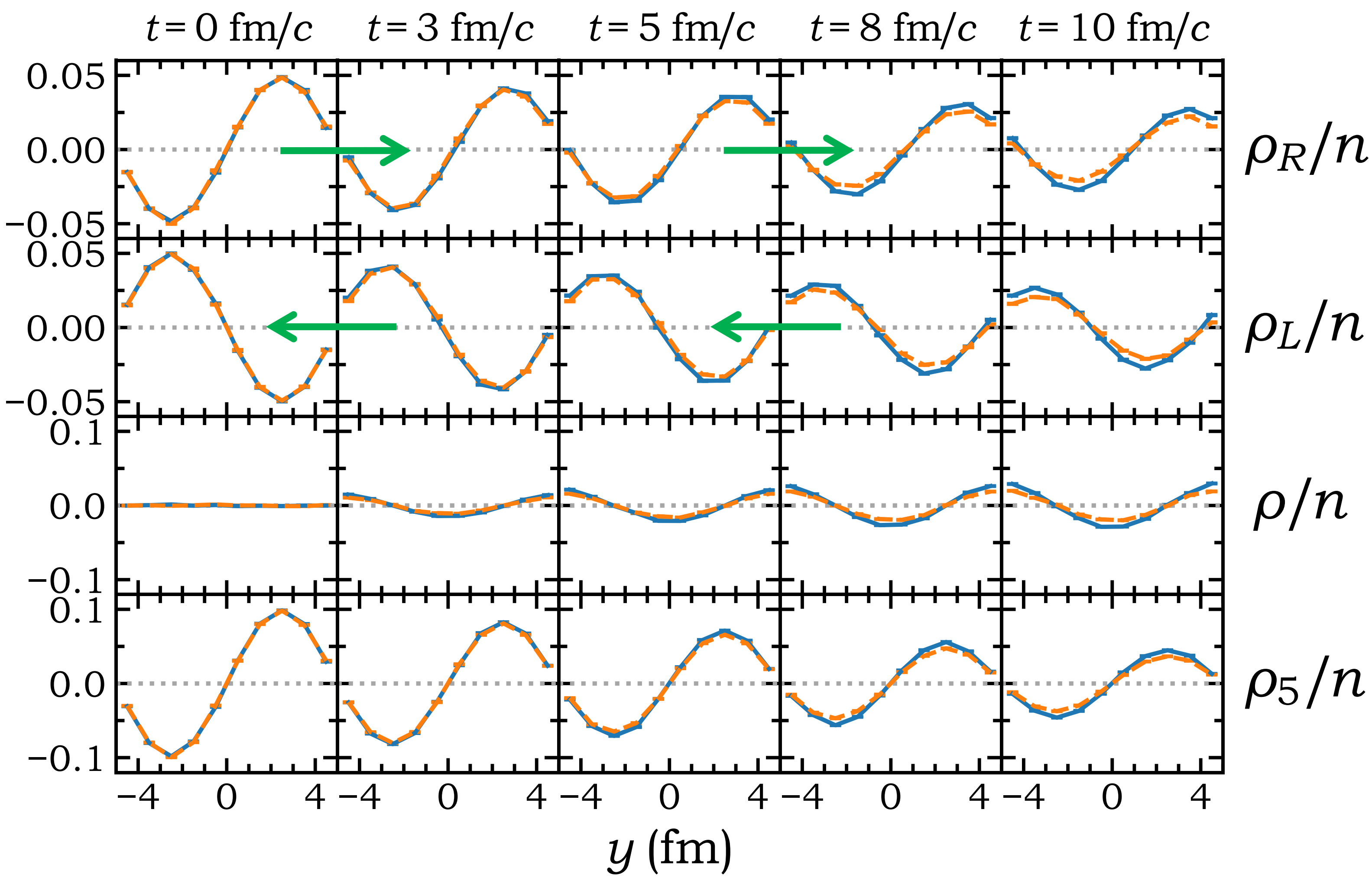
$$A_c = \frac{N_+ - N_-}{N_+ + N_-}$$

$$\beta = \frac{\pi}{l},\ n = \frac{N}{V}$$

Modification:

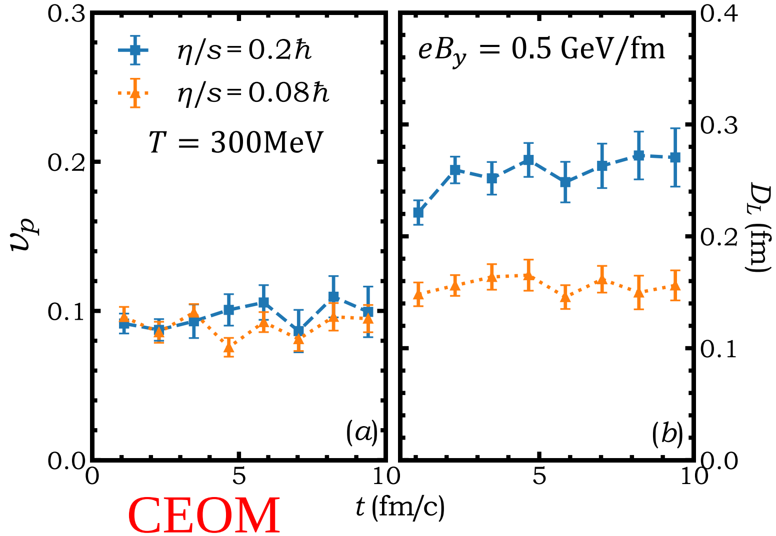
$$\vec{J} \Rightarrow \alpha\vec{J}$$
$$\vec{J}_5 \Rightarrow \alpha\vec{J}_5$$
$$\vec{v}_p \Rightarrow \alpha\vec{v}_p$$

dashed: SEOM solid: CEOM

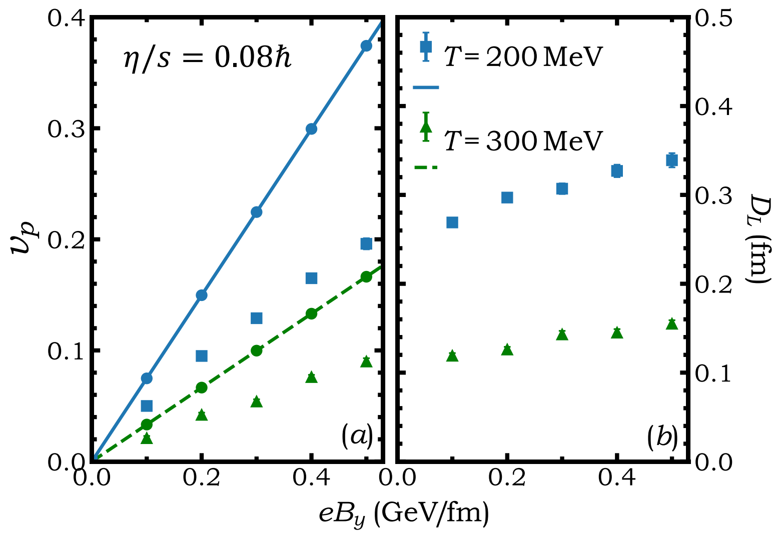


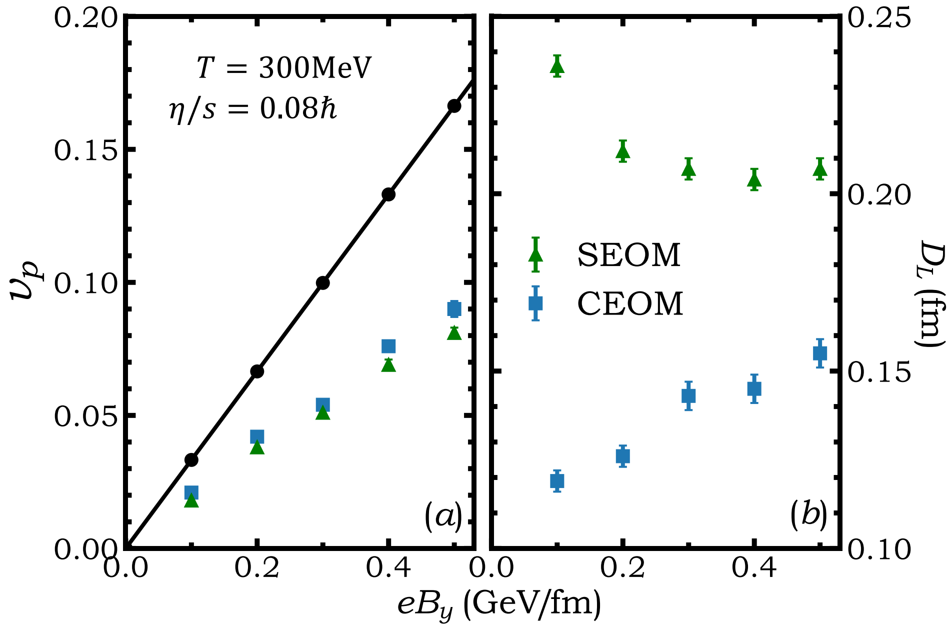
phase velocity v_p and damping D_L of CMW

$$\rho_{R/L}(y,t) = \pm\frac{1}{2}A_c n e^{-D_L\beta^2 t}\sin\left[\beta\left(y \mp v_p t\right)\right]$$



CEOM

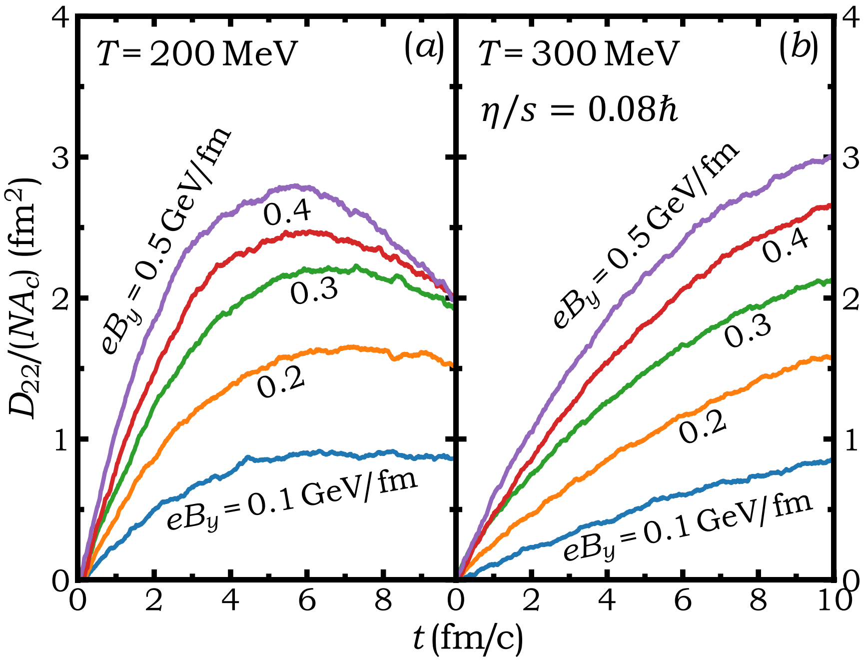


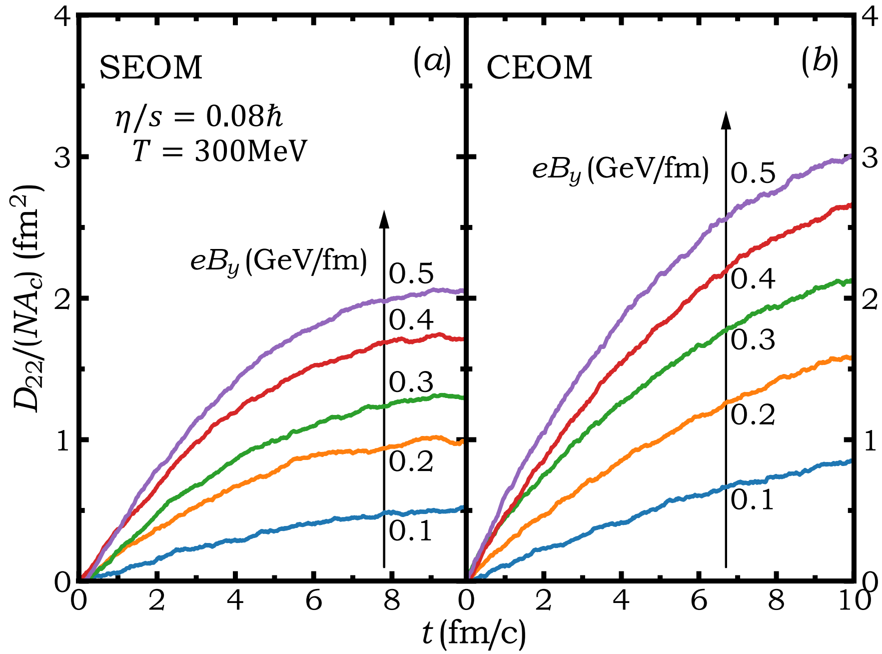


Electric Quadrupole Momentum

$$\mathcal{D}_{22} = \int \rho(\vec{r}) \left(3y^2 - r^2\right) \mathrm{d}^3 r \;\Longrightarrow\; \mathcal{D}_{22} = \frac{4A_c N}{\beta^2} e^{-D_L \beta^2 t} \sin\left(\beta v_p t\right)$$

CEOM

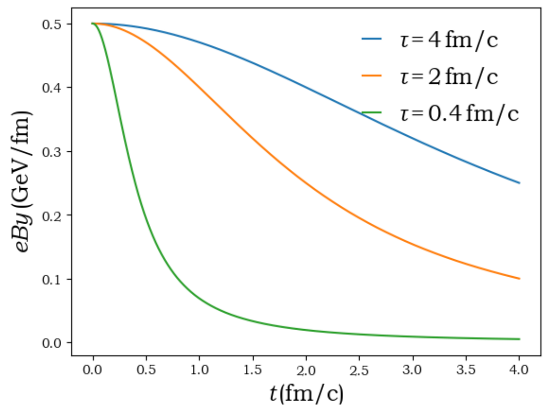




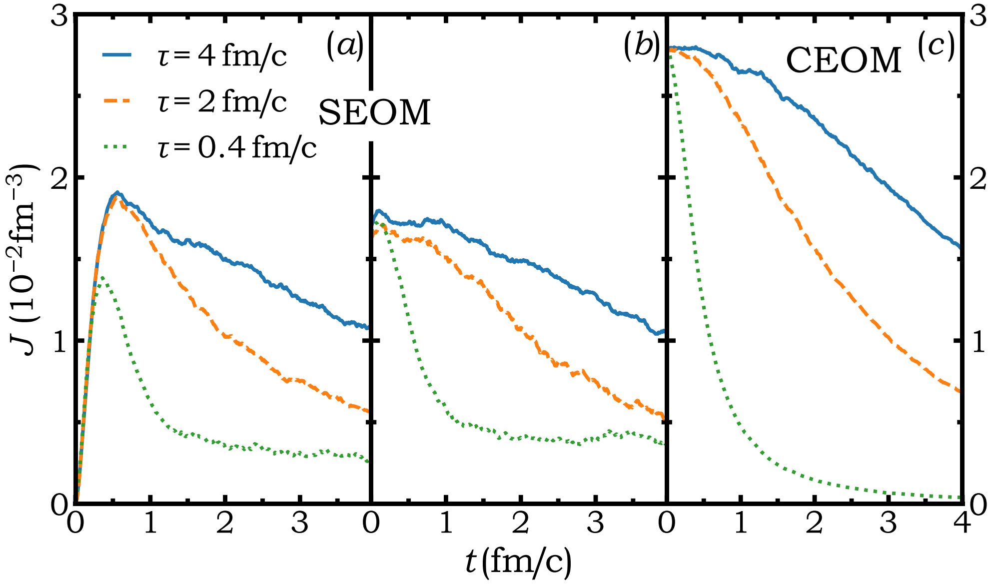
With a damping magnetic field

$$eB_y(t) = \frac{eB_y^0}{1+(t/\tau)^2} \qquad T = 300\text{MeV}$$

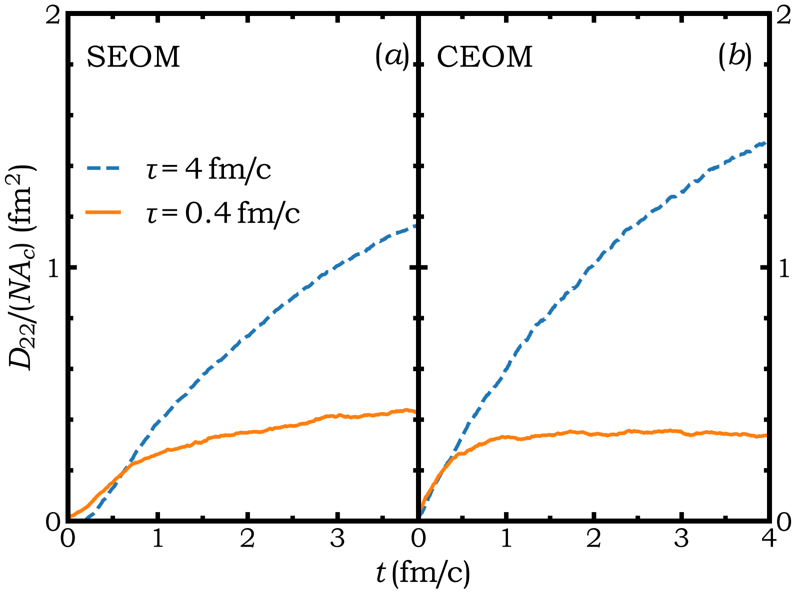
$$eB_y^0 = 0.5\text{GeV/fm} \qquad \eta/s = 0.08\hbar$$



Current



Electric Quadrupole Momentum



Summary

- The artificial truncation is needed for CEOM and underestimates the chiral effects compared to the theoretical limit.
- SEOM can be away from the artificial truncation but also leads to weaker chiral effects.
- The chiral magnetic wave in a box system can be described reasonably well with both CEOM and SEOM.
- Chiral effects are less sensitive to the fast decay of the magnetic field in SEOM compared to that in CEOM.

Thank you for your attention!

Backup

In a thermalized medium, the shear viscosity:

$$\eta = \frac{4\langle k\rangle}{15\sigma_{\mathrm{tr}}}$$

The transport cross section:

$$\sigma_{\mathrm{tr}} = \int d\Omega \frac{d\sigma_{22}}{d\Omega}\left(1-\cos^2\theta\right)$$

The isotropic cross section:

$$\sigma_{\mathrm{tr}} = \frac{2}{3}\sigma_{22}$$

$$f = 1/(e^{k/T}+1)$$

The entropy density:

$$s = -4N_c \int \frac{d^3k}{(2\pi\hbar)^3}[f \ln f - (1-f)\ln(1-f)]$$

$$\eta/s \in (0.08\hbar, 0.20\hbar)$$

H.-C. Song, S. A. Bass, U. Heinz, T. Hirano, and C. Shen, *PRL*. 106, 192301 (2011).

$$\sigma_{22} \in (1.74, 4.36)\frac{\hbar^2}{T^2}$$

Where σ_{22} in fm^2, T in GeV.

Backup

Phase space

Particle density: $n_{qc} = N_c \int \frac{d^3k}{(2\pi\hbar)^3} \sqrt{G} f\left(\frac{k-\mu_{qc}}{T}\right)$

$$\vec{J}_{qc} = N_c \int \frac{d^3k}{(2\pi\hbar)^3} \sqrt{G} \dot{\vec{r}} f\left(\frac{k-\mu_{qc}}{T}\right) \quad \sim \langle \vec{v} \rangle$$

$$\rho_R = n_{q(+)c(+)} - n_{q(-)c(-)}$$
$$\rho_L = n_{q(+)c(-)} - n_{q(-)c(+)}$$
$$\vec{J}_R = \vec{J}_{q(+)c(+)} - \vec{J}_{q(-)c(-)}$$
$$\vec{J}_L = \vec{J}_{q(+)c(-)} - \vec{J}_{q(-)c(+)}$$

$$\rho = \rho_R + \rho_L, \quad \rho_5 = \rho_R - \rho_L$$
$$\vec{J} = \vec{J}_R + \vec{J}_L, \quad \vec{J}_5 = \vec{J}_R - \vec{J}_L$$