Mass correction to CVE&CSE and Frequency dependent Electric Conductivity from CKT

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Apr 09, 2019, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions
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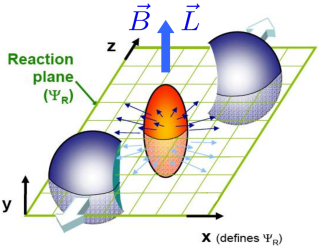
Outline

Macroscopic transport phenomena

- Vector current in CME & VCVE $\vec{j} = \sigma_B^V \vec{B} + \sigma_V^V \vec{\omega}$ chiral imbalance μ_5
- Axial current in CSE & ACVE $\vec{j}_5 = \sigma_B^A \vec{B} + \sigma_V^A \vec{\omega}$ dual effects: electric & chiral charge coupled by axial anomaly



CME & VCVE in heavy ion collisions

- Background contaminations & Efforts to excluding them

  Voloshin, PRL 2010
  F.Wang, PRC, 2010
  Bzdak, Koch, J.Liao, PRC 2010
  Schlichting, Pratt, PRC, 2011
  G.Wang [STAR Collaboration], NPA, 2013
  Bloczynski, X.G.Huang, X.Zhang, J.Liao, NPA, 2015
  W.T.Deng, X.G.Huang, G.L.Ma, G.Wang, PRC, 2016
  F.Wen, Bryon, L.Wen, G.Wang, CPC, 2018
  H.j.Xu, J.Zhao, X.Wang, H.Li, Z.W.Lin, C.Shen, F.Wang, CPC, 2018 $\cdots$

Transport coefficients in chiral (massless) limit

- Vector current in CME & VCVE $\vec{j} = C\mu_5 e\vec{B} + 2C\mu\mu_5\vec{\omega}$, $C \overset{QED}{=} \frac{1}{2\pi^2}$
- Axial current in CSE & ACVE $\vec{j}_5 = C\mu e\vec{B} + C\left[(\mu^2 + \mu_5^2) + \frac{\pi^2 T^2}{3}\right]\vec{\omega}$
  - Anomalous hydrodynamics Son, Surowka, PRL 2009; Neiman, Oz, JHEP 2011 ⋯
  - Chiral kinetic theory S.Pu, J.H.Gao, Q.Wang, PRD 2011; Son, Yamamoto, PRL 2012; Stephanov, Y.Yin, PRL 2012 ⋯
  - Field theory Kharzeev, Warringa, PRD 2009; Landsteiner, Megias, Pena-Benitez, PRL 2011; Hidaka, S.Pu, D.L.Yang, PRD 2017 ⋯

  Progress in Massive CKT X.L.Sheng, R.H.Fang, Q.Wang, Rischke, PRD 2019; J.H.Gao, Z.T.Liang, arXiv:1902.06510; Hattori, Hidaka, D.L.Yang, arXiv:1903.01653; Z.Y.Wang, X.Y.Guo, S.Z.Shi, P.F.Zhuang, arXiv:1903.03461 ⋯
- Mass correction exists (CSE)
  Metlitski, Zhitnitsky, PRD 2005; Gorbar, Miransky, Shovkovy, X.Wang, PRD 2013; E.d.Guo, S.Lin, JHEP 2017 ⋯
- Aim: mass correction to ACVE
  microscopic input $\rightarrow$ anomalous hydrodynamics ?
  clearer view $\rightarrow$ μ independent part $\sim$ gravitational anomaly ?

Read off CSE & ACVE coefficients

- Axial anomaly equation in massless QED

$$\partial_\mu j_5^\mu = -Ce^2 E\cdot B \leftarrow \nabla_\mu j_A^\mu = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2}F^B_{\mu\nu}F^C_{\rho\lambda} + \frac{b_A}{768\pi^2}\mathcal{R}^\alpha{}_{\beta\mu\nu}\mathcal{R}^\beta{}_{\alpha\rho\lambda}\right)$$

with fluid velocity u^μ, external fields

$$E^\mu = F^{\mu\nu}u_\nu, \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}, \quad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu \nabla_\rho u_\sigma$$

- Divergence form in static case → CSE & ACVE coefficients

$$e\vec{E} = -\nabla\mu,\ \nabla\cdot\vec{B} = 2\vec{E}\cdot\vec{\omega},\ \nabla\cdot\vec{\omega} = 0,\ \nabla T = 0 \quad \text{in medium field}$$

$$\xrightarrow[\text{by parts}]{\text{integration}} \frac{1}{C}\nabla\cdot\vec{j}_5 = \nabla\cdot\left(e\mu\vec{B}\right) + \nabla\cdot\left(\mu^2\vec{\omega} + \# T^2\vec{\omega}\right) = \nabla\cdot\left(\sigma_B\vec{B} + \sigma_V\vec{\omega}\right)$$

Axial anomaly equation (AAE) in massive case

- AAE in QED $\partial_\mu j_5^\mu = -2mP - Ce^2 E \cdot B, \quad P = -i\bar{\psi}\gamma_5\psi$
- AAE in Chiral kinetic theory

$$\frac{1}{C}\partial_\mu j_5^\mu = -(e^2 E \cdot B)C_1(m, \beta, \mu) - m^2\beta(eE \cdot \omega)C_2(m, \beta, \mu)$$

$$P = \frac{1}{4\pi^2 m}(e^2 E \cdot B)[C_1(m, \beta, \mu) - 1] + \frac{\beta m}{4\pi^2}(eE \cdot \omega)C_2(m, \beta, \mu)$$

where $E_q = \sqrt{q^2 + m^2}$

$$C_1(m, \beta, \mu) = \beta \int_0^\infty dq \left[\frac{e^{\beta(E_q-\mu)}}{\left(e^{\beta(E_q-\mu)} + 1\right)^2} + \frac{e^{\beta(E_q+\mu)}}{\left(e^{\beta(E_q+\mu)} + 1\right)^2} \right]$$

$$C_2(m, \beta, \mu) = \int_0^\infty dq \frac{1}{E_q} \left[\frac{e^{\beta(E_q-\mu)}}{\left(e^{\beta(E_q-\mu)} + 1\right)^2} - \frac{e^{\beta(E_q+\mu)}}{\left(e^{\beta(E_q+\mu)} + 1\right)^2} \right]$$

Pseudoscalar condensate R.H.Fang, J.Y.Pang, Q.Wang, X.N.Wang, PRD.95.014032, 2017

Read off CSE & ACVE coefficients again

$$\frac{1}{C}\nabla\cdot\vec{j}_5 = \nabla\cdot\left[F_1(\beta,m,\mu)e\vec{B}\right] + \nabla\cdot\left[\left(m^2F_2(\beta,m,\mu)+2F_3(\beta,m,\mu)\right)\vec{\omega}\right]$$

$$= \nabla\cdot(\quad \sigma_B \quad \vec{B} \; + \quad \sigma_V \quad \vec{\omega})$$

where $\frac{\partial}{\partial\mu}F_1 = C_1,\ \frac{\partial}{\partial\mu}F_2 = \beta C_2,\ \frac{\partial}{\partial\mu}F_3 = F_1 \ \Rightarrow$

$$F_1(\beta,m,\mu) = \int_0^\infty dq\tilde{f}_-(E_q) + A_1(\beta,m)$$

F1 C-odd; β, m C-even

$$F_2(\beta,m,\mu) = \int_0^\infty dq\frac{1}{E_q}\tilde{f}_+(E_q) + A_2(\beta,m)$$

$$F_3(\beta,m,\mu) = \frac{1}{\beta}\int_0^\infty dqL(E_q) + A_3(\beta,m)$$

natural choice of A_2, A_3: $A \propto \int_0^\infty a\left(\tilde{n}(E_q\pm\mu)\right)$

$$L(E_q) = -\ln\left(\tilde{n}(-E_q+\mu)\right) - \ln\left(\tilde{n}(-E_q-\mu)\right)$$

$$\tilde{f}_\pm(E_q) = \tilde{n}(E_q-\mu)\pm\tilde{n}(E_q+\mu), \quad \tilde{n}(x) = \frac{1}{e^{\beta x}+1}$$

Kubo formulas for massive field theory

Transport Coefficients → Retarded Correlators

$$\sigma_B = \lim_{p_k \to 0} \frac{-1}{2ip_k} \epsilon^{ijk} G^R_{ij}(p)|_{p_0=0}$$

$$\sigma_V = \lim_{p_k \to 0} \frac{-1}{ip_k} \epsilon^{ijk} G^R_{i,0j}(p)|_{p_0=0}$$

where

$$G^R_{ij}(p) = i \int d^4x e^{ip\cdot x} \langle [j^i_5(x), j^j(0)] \rangle \theta(x^0)$$

$$G^R_{i,0j}(p) = i \int d^4x e^{ip\cdot x} \langle [j^i_5(x), T^{0j}(0)] \rangle \theta(x^0)$$

$$j^i = \bar{\psi}\gamma^i\psi, \quad j^i_5 = \bar{\psi}\gamma^i\gamma_5\psi, \quad T^{0i} = \frac{i}{2}\bar{\psi}(\gamma^0\partial^i + \gamma^i\partial^0)\psi$$

Mass in propagator only

Retarded correlators in free limit

- Massive fermion propagator (Thermal field theory)

$$S(Q) = \frac{1}{\gamma^0(i\tilde{\omega}_m + \mu) - \vec{\gamma}\cdot\vec{q} - m} = \frac{(i\tilde{\omega}_m + \mu + m\gamma^0)\gamma^0 - \vec{\gamma}\cdot\vec{q}}{(i\tilde{\omega}_m + \mu)^2 - q^2 - m^2}$$

- Euclidean correlator in imaginary time formalism

$$\epsilon^{ijk} G^E_{i,0j}(P) = \frac{-1}{2\beta} \sum_{\tilde{\omega}_m} \int \frac{d^3q}{(2\pi)^3} \epsilon_{ijk}\, Tr\left[S(Q)\gamma^i\gamma_5 S(P+Q)(\gamma^0 q^j + \gamma^j i\tilde{\omega}_m)\right]$$

$$\frac{1}{2}\epsilon^{ijk} G^E_{ij}(p) = \frac{-e}{2\beta} \sum_{\tilde{\omega}_m} \int \frac{d^3q}{(2\pi)^3} \epsilon_{ijk}\, Tr\left[S(Q)\gamma^i\gamma_5 S(P+Q)\gamma^j\right]$$

↑ Matsubara frequencies ↑ Dirac trace

Equality

Equality	σ_B/Ce	σ_V/C
axial anomaly Eq.	$F_1(\beta, m, \mu)$	$2F_3(\beta, m, \mu) + m^2 F_2(\beta, m, \mu)$
Kubo formulas	$\int_0^\infty dq \tilde{f}_-(E_q)$	$\int_0^\infty \tilde{f}_+(E_q) \frac{2q^2 + m^2}{E_q} dq$

Integration by part

$$\int_0^\infty dq \frac{2q^2}{E_q} \tilde{f}_+(E_q) = -\int_0^\infty dq \frac{2q}{\beta} \frac{\partial}{\partial q} L(E_q) = \int_0^\infty dq \frac{2}{\beta} L(E_q) = 2F_3(\beta, m, \mu)$$

with

$$F_1(\beta, m, \mu) = \int_0^\infty dq \tilde{f}_-(E_q), \quad F_2(\beta, m, \mu) = \int_0^\infty dq \frac{1}{E_q} \tilde{f}_+(E_q)$$

$$F_3(\beta, m, \mu) = \frac{1}{\beta} \int_0^\infty dq L(E_q)$$

Zero m, T, μ Limits

Limits	$m=0$	$T=0$	$\mu=0$
σ_B/Ce	μ	$\sqrt{\mu^2-m^2}$	0
σ_V/C	$\mu^2+\frac{\pi^2T^2}{3}$	$\mu\sqrt{\mu^2-m^2}$	$2\int_0^\infty \tilde{n}(E_q)\frac{2q^2+m^2}{E_q}dq$

For ACVE coefficient

$$\sigma_V = \int_0^\infty \tilde{f}_+(E_q)\frac{2q^2+m^2}{E_q}dq$$

$$\sigma_V(\mu=0) = 2\int_0^\infty \tilde{n}(E_q)\frac{2q^2+m^2}{E_q}dq \neq 0$$

Origin? $\sigma_V(\mu=0)\sim\frac{\pi^2T^2}{3}\rightarrow$ gravitational anomaly

$\sigma_V(\mu)-\sigma_V(\mu=0)\rightarrow$ axial anomaly

Small m expansion at finite T & μ

Mass correction $\propto m^2$ only, $m^2 \ln m$ is vanishing

$$\sigma_V/C = \frac{1}{2\pi^2}\left(\mu^2 + \frac{\pi^2 T^2}{3} - \frac{m^2}{2}\right) + O(m^4)$$

agree Flachi, Fukushima, PRD 2018

$$\sigma_B/Ce = \mu + \frac{m^2}{2T}\frac{\partial}{\partial s}\left(\mathrm{Li}_s(-e^{\mu/T}) - \mathrm{Li}_s(-e^{-\mu/T})\right)\Big|_{s=-1} + O(m^4)$$

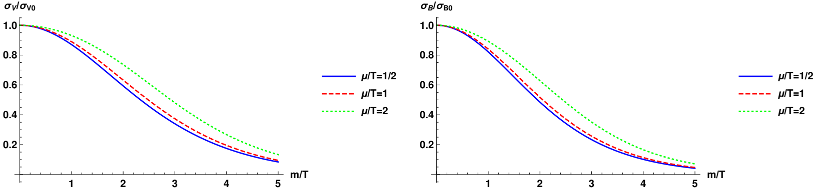
Radiative corrections to CSE Gorbar, Miransky, Shovkovy, X.Wang, PRD 2013

$$\langle j_5^3\rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

In weakly interacting massive theory at $T = 0$

Retarded Correlator with medium dependent only $\rightarrow$
Possible logarithmic corrections from vacuum contributions ?

Plot general m-dependence & Generalization to QCD



$$\sigma_V = \sum_f CN_c \int_0^\infty dq \tilde{f}_+(\sqrt{q^2+m_f^2}) \frac{2q^2+m_f^2}{E_q},$$

$$\sigma_B = \sum_f CeN_c \int_0^\infty dq \tilde{f}_-(\sqrt{q^2+m_f^2})$$

$m_u = m_d \simeq 0,\ m_s = 100\mathrm{MeV} \rightarrow$ Mass correction to σ_B & $\sigma_V < 1\%$
with $50\mathrm{MeV} < \mu < 200\mathrm{MeV}$ & $200\mathrm{MeV} < T < 400\mathrm{MeV}$

Summary for mass correction to CVE&CSE

- Mass correction to ACVE and CSE

  Linear response(Kubo formulas) / Massive QED/QCD } perfect agreement { Anomaly equation / Natural choice

- ACVE coefficient $\sigma_V(\mu = 0) \neq 0 \xrightarrow{?}$ gravitational anomaly
- Mass dependence of ACVE and CSE coefficients
  - Mass suppresses transport coefficients with less suppression at larger μ.
  - Small mass correction is proportional to m^2, with $m^2 \ln m$ vanishing in medium dependent part.

Motivation for Frequency dependent Conductivity

- From CME in HIC
  Strong&rapid decaying B $\xrightarrow{-\partial_t \vec{B} = \nabla \times \vec{E}}$ Rapid changing E
  $\rightarrow$ Frequency dependent conductivity
- From chiral kinetic theory(CKT)
  - In $O(\hbar EB)$ $\rightarrow$ Electric conductivity Gorbar, Shovkovy et al. PRD 2016

$$\vec{j} = \frac{\tau}{9}\left(T^2 + \frac{3\mu^2}{\pi^2}\right)\left(\vec{E} - \tau\frac{\partial \vec{E}}{\partial t}\right) + \frac{\tau^2\mu}{3\pi^2}\vec{E} \times \vec{B} + \cdots$$

  - Higher orders $O(B^n)$ $\rightarrow$ Landau quantization
  - Longitudinal conductivity $(E \parallel B)$ under lowest Landau level(LLL)
    Hattori, Satow, PRD 2016; Hattori, S.Y.Li, Satow, H.U.Yee, PRD 2017
    beyond LLL Fukushima, Hidaka, PRL 2018

Frequency dependent transverse conductivity($E \perp B$)

- $\vec{E} \cdot \vec{B} = 0 \quad \rightarrow \quad$ No axial anomoly
- Relaxation time approximation $\rightarrow$ simple collision term

$$C_n = -\frac{f_n - f_{n0}}{\tau}$$

- Kinetic equation ?
  - No longitudinal motion $\quad \rightarrow \quad \langle n, p_z' | S(E) | n, p_z \rangle = 0$
  - Transverse motion $\quad \rightarrow \quad \langle n', p_z | S(E) | n, p_z \rangle \neq 0$
  - Beyond LLL $\quad \rightarrow \quad$ transition $\mathcal{M}$ for P-odd Landau levels
  - $\cdots$

Thank you!

Backup - Frequency sum & Dirac trace

- Dirac trace

$$\epsilon_{ijk} Tr[\gamma_\mu \gamma^i \gamma_5 \gamma_\nu \gamma^0] a^\mu b^\nu = 4i(a_j b_k - a_k b_j)$$

$$\epsilon_{ijk} Tr[\gamma_\mu \gamma^i \gamma_5 \gamma_\nu \gamma^j] a^\mu b^\nu = 8i(a^0 b^k - a^k b^0) = 8i(a_k b_0 - a_0 b_k)$$

- Sum over fermionic Matsubara frequencies

$$\frac{1}{\beta} \sum_{\tilde{\omega}_m} \Delta_u(Q) \Delta_v(P+Q) = \frac{u\,\tilde{n}(E_q - u\mu) - v\,\tilde{n}(E_{p+q} - v\mu) + \frac{1}{2}(v-u)}{i\omega_n + uE_q - vE_{p+q}}$$

$$\frac{1}{\beta} \sum_{\tilde{\omega}_m} \Delta_u(Q) \Delta_v(P+Q) i\tilde{\omega}_m = \frac{(u\,E_q - \mu)(\frac{1+u}{2} - u\tilde{n}(E_q - u\mu))}{i\omega_n + uE_q - vE_{p+q}}$$

$$- \frac{(v\,E_{p+q} - \mu - i\omega_n)(\frac{1+v}{2} - v\tilde{n}(E_{p+q} - vi\omega_n - v\mu))}{i\omega_n + uE_q - vE_{p+q}}$$

Backup - Linear response in a restricted setting

- Static inhomogeneous sources (gauge fields & metric perturbation)

$$u^\mu = (1,0,0,0) \Rightarrow B^i = -\epsilon^{ijk}\partial_j A_k,\ \omega^i = -\frac{1}{2}\epsilon^{ijk}\nabla_j u_k = -\frac{1}{2}\epsilon^{ijk}\partial_j h_{0k}$$

- Absence of pseudoscalar condensate

Vanishing mass term $\xleftarrow[\nabla T=0]{\nabla\mu=0}$ $\begin{cases} \text{symmetries: P-odd, T-odd, C-even} \\ \text{possibile forms: } \nabla\mu\cdot\vec{B},\ \mu\nabla\mu\cdot\vec{\omega},\ T\nabla T\cdot\vec{\omega} \end{cases}$

- Constitutive equation for axial current

$$j_5^i(x) = -\sigma_B\epsilon^{ijk}\partial_j A_k(x) - \frac{1}{2}\sigma_V\epsilon^{ijk}\partial_j h_{0k}(x)$$

$$\xrightarrow[\text{transform}]{\text{Fourier}} j_5^i(p) = -\sigma_B\epsilon^{ijk}ip_jA_k(p) - \frac{1}{2}\sigma_V\epsilon^{ijk}ip_jh_{0k}(p)$$

Equality & Limits & Small m expansion

Equality	σ_B/Ce	σ_V/C
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Limits	$m=0$	$T=0$	$\mu=0$
σ_B/Ce	μ	$\sqrt{\mu^2-m^2}$	0
σ_V/C	$\mu^2+\frac{\pi^2T^2}{3}$	$\mu\sqrt{\mu^2-m^2}$	$2\int_0^\infty \tilde{n}(E_q)\frac{2q^2+m^2}{E_q}dq$

"Gradient expansion breaks down" K. Jensen et al., JHEP 1302, 088 (2013)

$$\sigma_B/Ce = \mu + \frac{m^2}{2T}\frac{\partial}{\partial s}\left(\mathrm{Li}_s(-e^{\mu/T}) - \mathrm{Li}_s(-e^{-\mu/T})\right)\Big|_{s=-1} + O(m^4)$$

E. V. Gorbar et al., PRD 88, 025025

$$\sigma_V/C = \frac{1}{2\pi^2}\left(\mu^2 + \frac{\pi^2T^2}{3} - \frac{m^2}{2}\right) + O(m^4)$$