Non-static chiral magnetic response

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5th Workshop on Chirality, Vorticity and Magnetic Field in HIC – 4. 9. 2019.

Outline

- **Motivation**

  about anomalous transport: features & relevance
  simple cartoon for CME & its phenomenology

- **CME in linear response**

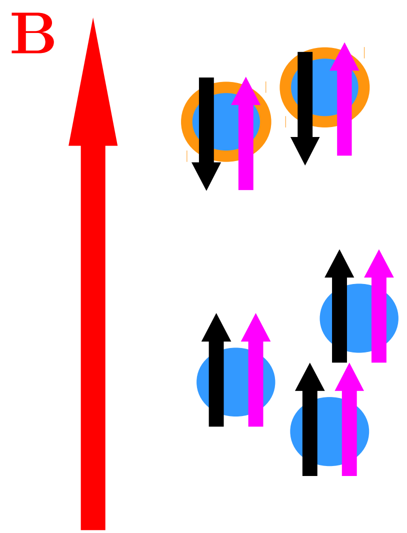
  charge conservation and axial anomaly
  nontrivial properties of the static limit

- **Non-static response functions**

  anomaly ruled current vs. absent response
  some details of the nonstatic calculation
  examples of static μ_5 (***B***) and arbitrary ***B*** (μ_5)

Overview

$$\partial_\mu j_A^\mu = C_A \mathbf{E}\cdot\mathbf{B}$$



CME

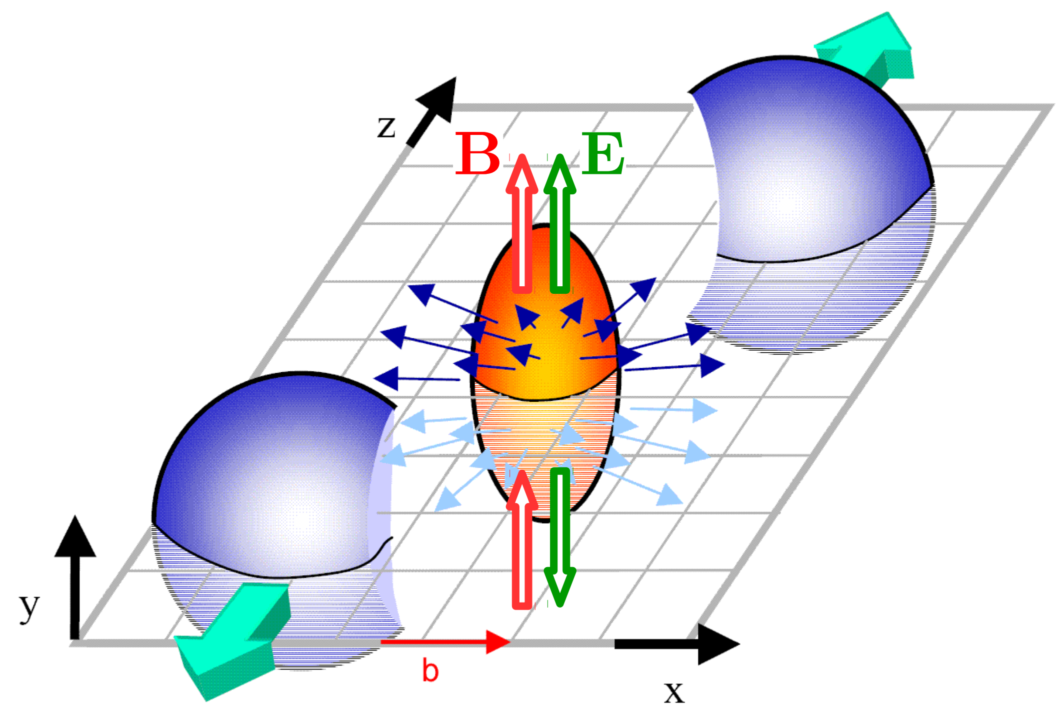
$\mu_5 \neq 0$

$n_R \neq n_L$

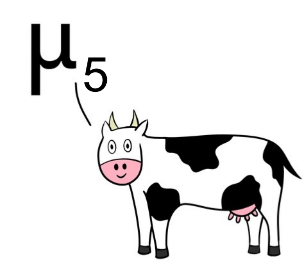
$\langle \mathbf{s} \rangle \sim \mathbf{B}$

$\langle \mathbf{p} \rangle \sim \mu_5 \langle \mathbf{s} \rangle$

(+ opposite charge)



	$\mathbf{J}$ =	$\sigma\mathbf{E}$ +	$\sigma_A\mathbf{B}$
$\mathcal{P}$	*odd*	*even* × *odd*	*odd* × *even*
$\mathcal{T}$	*odd*	*odd* × *even*	*even* × *odd*



Electric transport in chiral medium

$$\mathbf{J} = \sigma \mathbf{E} + C_A \mu_5 \mathbf{B}$$

$$\mathbf{J}_5 = \# \mu \mu_5 \mathbf{E} + C_A \mu \mathbf{B}$$

Consistent with Chern-Simons electrodynamics:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A^\mu J_\mu - \frac{C_A}{4} \theta \widetilde{F}^{\mu\nu} F_{\mu\nu}$$

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J} + C_A \left(M\mathbf{B} - \mathbf{P} \times \mathbf{E} \right)$$

$$\nabla \cdot \mathbf{E} = \rho + C_A \mathbf{P} \cdot \mathbf{B}$$

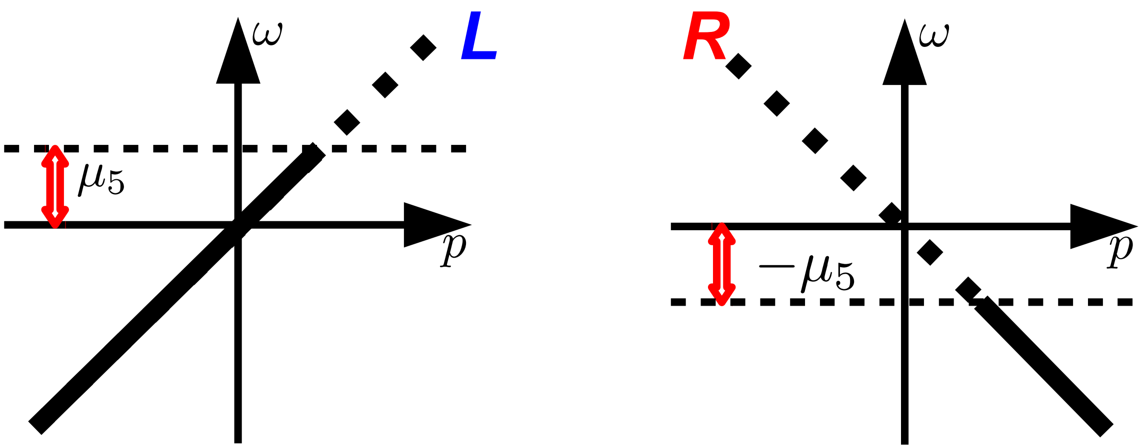
$$P^\mu = (M, \mathbf{P}) = \partial^\mu \theta$$

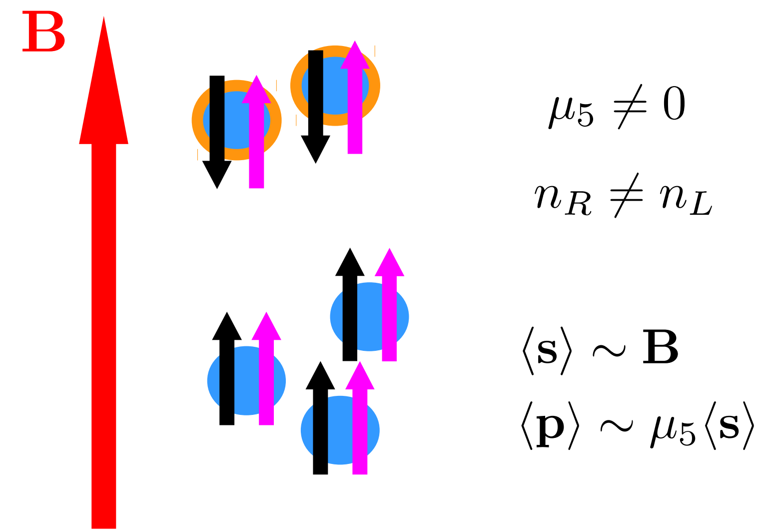
$$\mathbf{J} = \frac{e^2}{2\pi^2} (-\dot{\theta}) \mathbf{B}$$

see: Kharzeev aX:0911.3715

Simple picture of CME

chiral fermions in, affected by homog. E||B fields





(+ opposite charge)

QP contr.* + *fermionic kinematics:

$$\mathbf{J}_{\mathrm{CME}} = \frac{e^2}{2\pi^2}\left(\frac{1}{3} + \frac{2}{3}\right)\mu_5 \mathbf{B}$$

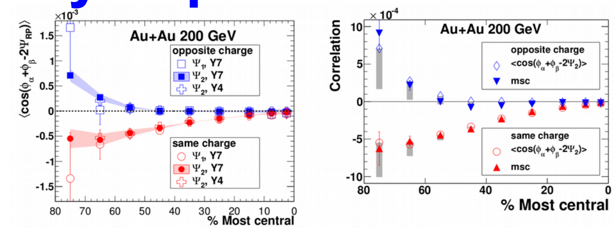
See: Kharzeev, Stephanov, Yee, PRD ***95**, 051901 (2016)*

CME = collective motion of vacuum particles with *arbitrarily large* momentum

How to measure CME in HIC?

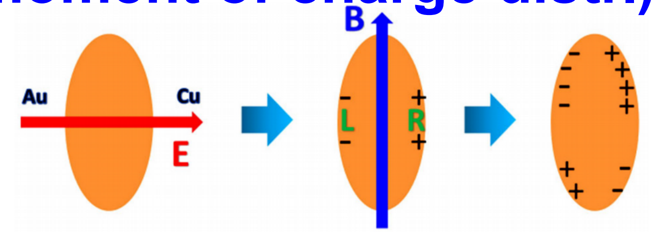
What signs to look for?

- **charge separation → dipole asymmetry in production**



- **CMW → Cu+Au coll. (quadrupole moment of charge distr.)**

*see: Burnier, Liao, Kharzeev, Yee PRL **107**, 052303 (2011), Huang & Liao, PRL **110**, 232302 (2013)*



- **other things:**

CSL ("chiral soliton lattice" nonzero quark masses → anoumalous Hall current & B—Omega coupling;
K. Nishimura, aX:1711.02190
transition radiation as a probe of chiral anomaly – circularly polarized photons at given angle to the jet direction
Tuchin PRL 121, 182301 (2018)

main theor. uncertainties: related to initial state & LT of sources
from experimental POV: background...

Anomaly in QED

U(1) vector current: $J^\mu = \overline{\Psi}\gamma^\mu\Psi$

U(1) axialvector current: $J_5^\mu = \overline{\Psi}\gamma^\mu\gamma^5\Psi$

$$\partial_\mu J^\mu = 0 \qquad\qquad \partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$$

fermions coupled to gauge fields:

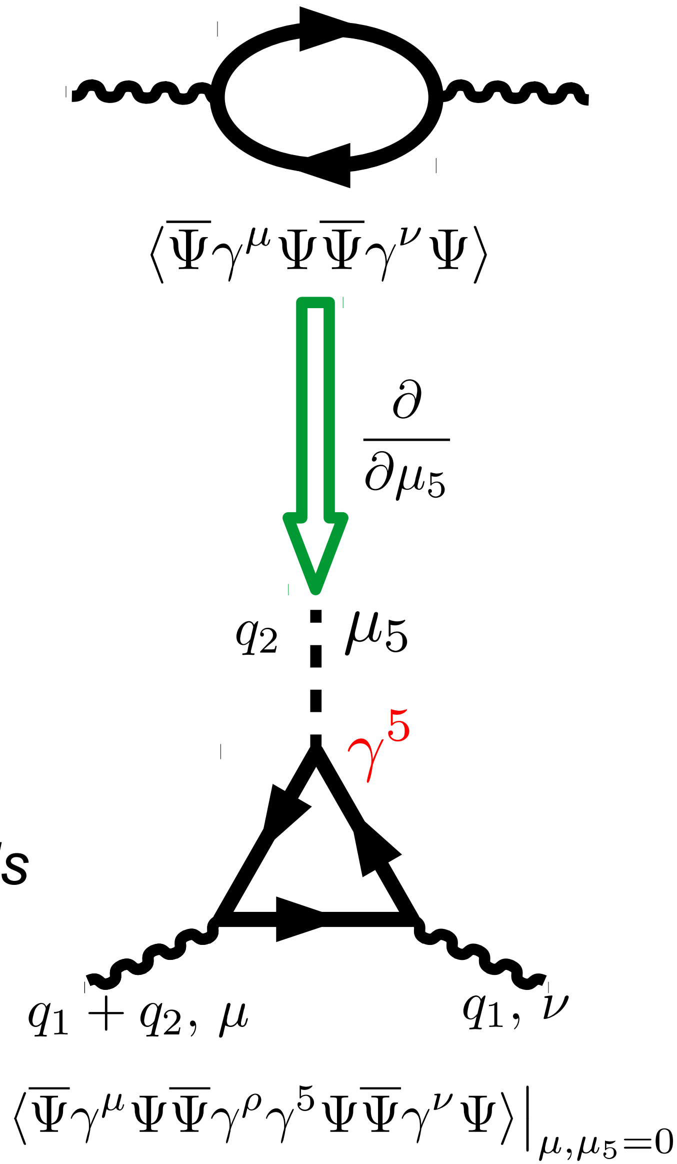
- maintaining gauge invariance
  - $\rightarrow$ *costs the anomalous divergence of the axial current*
- the anomaly comes from the UV behaviour of the fermionic propagator

Anomalous conductivities

- **static (↔ steady state) current: *universal***
  → *given by the anomaly (1-loop)*
  → *no further quantum corrections!*

- **BUT relaxation dynamics:**
  → ***depends on the underlying theory***

- **approximation: linear response**
  → *microscopic dynamics is not effected by the extarnal fields*
  → *gradient corrections to hydrodynamic fields*



Linear response

$$\delta\langle J^\mu\rangle \sim \langle J^\mu J^0\rangle\mu + \langle J^\mu J_5^0\rangle\mu_5 + \langle J^\mu J^\nu\rangle A_\nu^{\rm ext} + \langle J^\mu J_5^\nu\rangle A_{5,\,\nu}^{\rm ext} +$$

$$+\langle J^\mu J^0 J^0\rangle\mu\mu + \langle J^\mu J^0 J^\nu\rangle\mu A_\nu^{\rm ext} + \langle J^\mu J^\nu J^\rho\rangle A_\nu^{\rm ext} A_\rho^{\rm ext} + \langle J^\mu J^0 J_5^0\rangle\mu\mu_5 +$$

$$+\langle J^\mu J^\nu J_5^0\rangle A_\nu^{\rm ext}\mu_5 + \langle J^\mu J^0 J_5^\nu\rangle\mu A_{5,\,\nu}^{\rm ext} + \langle J^\mu J^\nu J_5^\rho\rangle A_\nu^{\rm ext} A_{5,\,\rho}^{\rm ext} + \langle J^\mu J_5^0 J_5^0\rangle\mu_5\mu_5 +$$

$$+\langle J^\mu J_5^0 J_5^\nu\rangle\mu_5 A_{5,\,\nu}^{\rm ext} + \langle J^\mu J_5^\nu J_5^\rho\rangle A_{5,\,\nu}^{\rm ext} A_{5,\,\rho}^{\rm ext}$$

$$\delta\langle J_5^\mu\rangle \sim \langle J_5^\mu J^0\rangle\mu + \langle J_5^\mu J_5^0\rangle\mu_5 + \langle J_5^\mu J^\nu\rangle A_\nu^{\rm ext} + \langle J_5^\mu J_5^\nu\rangle A_{5,\,\nu}^{\rm ext} +$$

$$+\langle J_5^\mu J^0 J^0\rangle\mu\mu + \langle J_5^\mu J^0 J^\nu\rangle\mu A_\nu^{\rm ext} + \langle J_5^\mu J^\nu J^\rho\rangle A_\nu^{\rm ext} A_\rho^{\rm ext} + \langle J_5^\mu J^0 J_5^0\rangle\mu\mu_5 +$$

$$+\langle J_5^\mu J^\nu J_5^0\rangle A_\nu^{\rm ext}\mu_5 + \langle J_5^\mu J^0 J_5^\nu\rangle\mu A_{5,\,\nu}^{\rm ext} + \langle J_5^\mu J^\nu J_5^\rho\rangle A_\nu^{\rm ext} A_{5,\,\rho}^{\rm ext} + \langle J_5^\mu J_5^0 J_5^0\rangle\mu_5\mu_5 +$$

$$+\langle J_5^\mu J_5^0 J_5^\nu\rangle\mu_5 A_{5,\,\nu}^{\rm ext} + \langle J_5^\mu J_5^\nu J_5^\rho\rangle A_{5,\,\nu}^{\rm ext} A_{5,\,\rho}^{\rm ext}$$

Linear response

$\mu = 0,\ \mathbf{E} = 0,\ A_5 = 0$

$$\delta\langle J^\mu \rangle \sim \langle J^\mu J^0 \rangle \mu + \langle J^\mu J^\nu \rangle A^{\text{ext}}_\nu +$$

$$+ \langle J^\mu J^0 J^0_5 \rangle \mu\mu_5 +$$

$$+ \langle J^\mu J^\nu J^0_5 \rangle A^{\text{ext}}_\nu \mu_5 + \langle J^\mu J^0 J^\nu_5 \rangle \mu A^{\text{ext}}_{5,\,\nu} + \langle J^\mu J^\nu J^\rho_5 \rangle A^{\text{ext}}_\nu A^{\text{ext}}_{5,\,\rho}$$

only gradient corr. $\sim \nabla\mu_5$

$$\delta\langle J^\mu_5 \rangle \sim \langle J^\mu_5 J^0_5 \rangle \mu_5 + \langle J^\mu_5 J^\nu_5 \rangle A^{\text{ext}}_{5,\,\nu} +$$

$$+ \langle J^\mu_5 J^0 J^0 \rangle \mu\mu + \langle J^\mu_5 J^0 J^\nu \rangle \mu A^{\text{ext}}_\nu + \langle J^\mu_5 J^\nu J^\rho \rangle A^{\text{ext}}_\nu A^{\text{ext}}_\rho +$$

$$+ \langle J^\mu_5 J^0_5 J^0_5 \rangle \mu_5\mu_5 +$$

$$+ \langle J^\mu_5 J^0_5 J^\nu_5 \rangle \mu_5 A^{\text{ext}}_{5,\,\nu} + \langle J^\mu_5 J^\nu_5 J^\rho_5 \rangle A^{\text{ext}}_{5,\,\nu} A^{\text{ext}}_{5,\,\rho}$$

AVV triangle

$$i\delta G^{\rho\mu\nu}_{AVV}(q_1,q_2) = -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p+q_1)\gamma^\nu iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p+q_1)\gamma^\nu iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^R(p+q_1)\gamma^\nu iG^A(p)+$$

$$+\gamma^\mu iG^C(p+q_1+q_2)\gamma^\nu iG^A(p+q_2)\gamma^\rho\gamma^5 iG^C(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

$$G^{R/A}(p) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\frac{\rho(\omega,\mathbf{p})}{p_0-\omega\pm i0^+}$$

$$iG^{12/21}(p) = \rho(p)\cdot\begin{cases} -n_{FD}(p_0/T) \\ 1-n_{FD}(p_0/T)\end{cases}$$

$$G^{11/22} = \frac{G^{12}+G^{21}}{2}\pm\left(G^R+G^A\right)$$

$$G^C = (1-2n_{FD}(p_0/T))\,\rho(p)$$

– {same terms with *m=M>> all other scales*}

AVV triangle

$$q_{1\nu} \cdot \delta G^{\rho\mu\nu}_{AVV}(q_1, q_2) = 0$$

$$(q_1 + q_2)_\mu \cdot \delta G^{\rho\mu\nu}_{AVV}(q_1, q_2) = 0$$

$$\partial \cdot J = 0$$

$$q_{2\rho} \cdot \delta G^{\rho\mu\nu}_{AVV}(q_1, q_2) = -i\frac{e^2}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}$$

$$\Updownarrow$$

$$\partial \cdot J_5 = C_A \mathbf{E} \cdot \mathbf{B}$$

$$iG^{R/A}(p+q)q_\mu\Gamma^\mu_V iG^{R/A}(p) = G^{R/A}(p+q) - G^{R/A}(p)$$

$$iG^C(p+q)q_\mu\Gamma^\mu_V iG^{R/A}(p) = G^C(p+q)$$

$$iG^{R/A}(p+q)q_\mu\Gamma^\mu_V iG^C(p) = -G^C(p)$$

U(1) invariance (Ward-Takahasi)

Limiting cases

$$\delta\langle J^\mu_x\rangle = -\int_{q_1}\int_{q_2}\widetilde{A}^{\text{ext}}_\nu(q_1)\widetilde{\mu}_5(q_2)i\delta G^{0\mu\nu}_{AVV}(q_1,q_2)e^{ix\cdot(q_1+q_2)}$$

$$\delta\langle J^\mu_x\rangle = -\int \mathrm{d}^4y\int \mathrm{d}^4z A^{\text{ext}}_\nu(y)\mu_5(z)i\delta G^{0\mu\nu}_{AVV}(x-y,x-z)$$

q_{10} or $q_{20}\to 0$ setting external fields constant in time

$\mathbf{q}_1$ or $\mathbf{q}_2\to 0$ setting external fields homogeneous

Limiting cases – static point

$\mathbf{q}_2 \to 0$ percedes $q_{20} \to 0$ **ANOMALY** $\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B}$

μ_5 first set to homogeneous

$q_{20} \to 0$ percedes $\mathbf{q}_2 \to 0$

μ_5 first set to time independent

- $q_{10} \to 0$ lastly: $\frac{1}{3} \times$ **ANOMALY**
- not $q_{10} \to 0$ lastly: **ZERO**

See: Hou, Hui, Ren, *JHEP* **5**, 46 (2011); Wu, Hou, Ren, *Phys. Rev. D* **96**, 096015 (2017)

Limiting cases – absent current

$$\frac{\partial}{\partial q_{1k}} i\delta G^{0ij}_{AVV} =$$

$$= -\frac{e^2}{2}\int_p \mathrm{tr}\left\{\gamma^i\gamma^5\left(-\frac{\partial}{\partial p_0}G^A(p_0,\mathbf{p})\right)\gamma^j\frac{\partial}{\partial p_k}G^A(p_0,\mathbf{p}) + \right.$$
$$-\gamma^i\gamma^5\left(-\frac{\partial}{\partial p_0}G^R(p_0,\mathbf{p})\right)\gamma^j\frac{\partial}{\partial p_k}G^R(p_0,\mathbf{p}) +$$
$$-\gamma^i\gamma^5\frac{\partial}{\partial p_k}G^A(p_0,\mathbf{p})\gamma^j\left(-\frac{\partial}{\partial p_0}G^A(p_0,\mathbf{p})\right) +$$
$$\left. +\gamma^i\gamma^5\frac{\partial}{\partial p_k}G^R(p_0,\mathbf{p})\gamma^j\left(-\frac{\partial}{\partial p_0}G^R(p_0,\mathbf{p})\right)\right\}(1-2n(p_0))$$
$$-\{\text{same with } m = M \gg \text{ all other scales}\} +$$

→ only m=0 contributes (or small m)

→ fermionic interactions can change it!

$$e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j\frac{\partial G^A(p)}{\partial p_k}\right\}(-2n'(p_0))$$

$$+\frac{e^2}{2}\int_p \mathrm{tr}\left\{\gamma^i\gamma^5 g^A_M(p_0,\mathbf{p})\gamma^0 G^A_M(p_0,\mathbf{p})\gamma^j\frac{\partial}{\partial p_k}G^A_M(p_0,\mathbf{p}) + \right.$$
$$-\gamma^i\gamma^5 g^R_M(p_0,\mathbf{p})\gamma^0 G^R_M(p_0,\mathbf{p})\gamma^j\frac{\partial}{\partial p_k}G^R_M(p_0,\mathbf{p}) +$$
$$+\gamma^i\gamma^5 g^A_M(p_0,\mathbf{p})\frac{\partial}{\partial p_k}G^A_M(p_0,\mathbf{p})\gamma^j G^A_M(p_0,\mathbf{p})\gamma^0 +$$
$$\left. -\gamma^i\gamma^5 g^R_M(p_0,\mathbf{p})\frac{\partial}{\partial p_k}G^R_M(p_0,\mathbf{p})\gamma^j G^R_M(p_0,\mathbf{p})\gamma^0\right\}(1-2n(p_0)) =$$

→ contribution from the regulator term only

→ fermionic interactions could not change it!

$$e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0 G^A_M(p)\gamma^j\frac{\partial G^A_M(p)}{\partial p_k}\right\} g^A_M(p)(1-2n(p_0))$$

Limiting cases – constant μ_5

μ_5 is set first constant then homogeneous

B can be set to homogeneous with still non-trivial time-dependence

Limiting cases – constant μ_5

μ_5 is set first constant then homogeneous

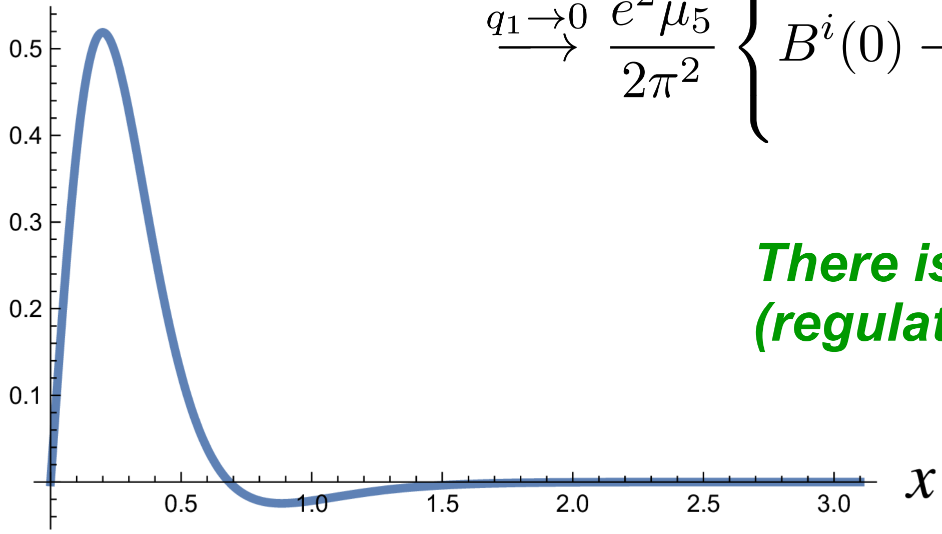
B can be set to homogeneous with still non-trivial time-dependence

$$\widetilde{J}^i = \mu_5 \int_{-\infty}^{\infty} \mathrm{d}\tau \widetilde{B}^i(t+\tau, \mathbf{q}_1) \frac{e^2}{2\pi^2} \{\delta(\tau) +$$

$$-4\theta(-\tau)\left[\frac{\partial}{\partial\tau}\left(\frac{\partial}{\partial\tau}\left(\frac{\sin(q_1\tau)}{q_1\tau}\right)\frac{TF_1(\tau T)}{q_1^2}\right) - \frac{\sin(q_1\tau)}{q_1\tau} TF_2(\tau T)\right]\Bigg\}$$

$$\xrightarrow{q_1\to 0} \frac{e^2\mu_5}{2\pi^2}\left\{B^i(0) - 4\int_0^{\infty} \mathrm{d}\tau B^i(t+\tau) TF_3(\tau T)\right\}$$

$F_3(x)$

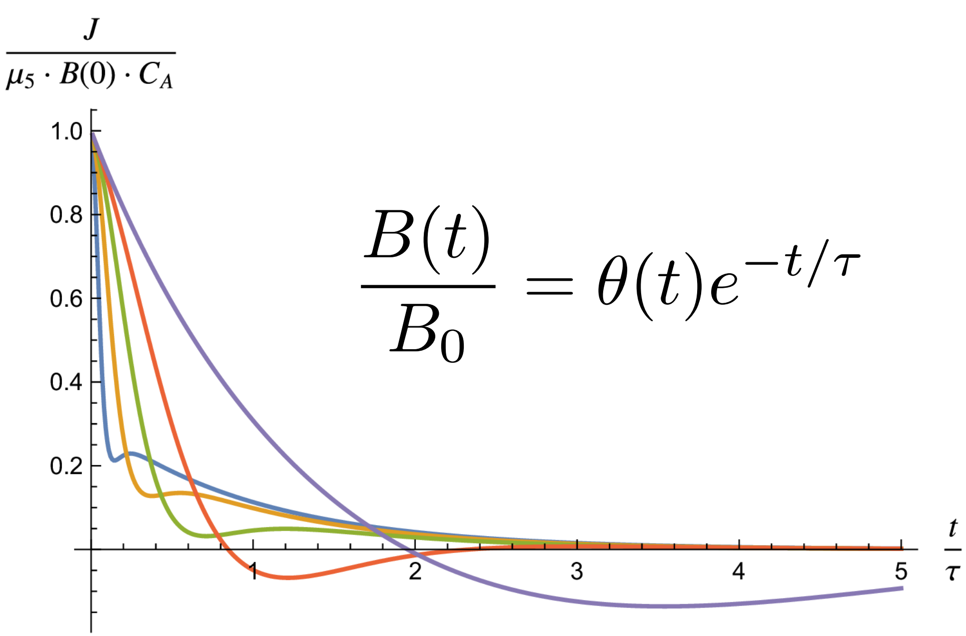


There is an instantaneous response (regulator terms!)

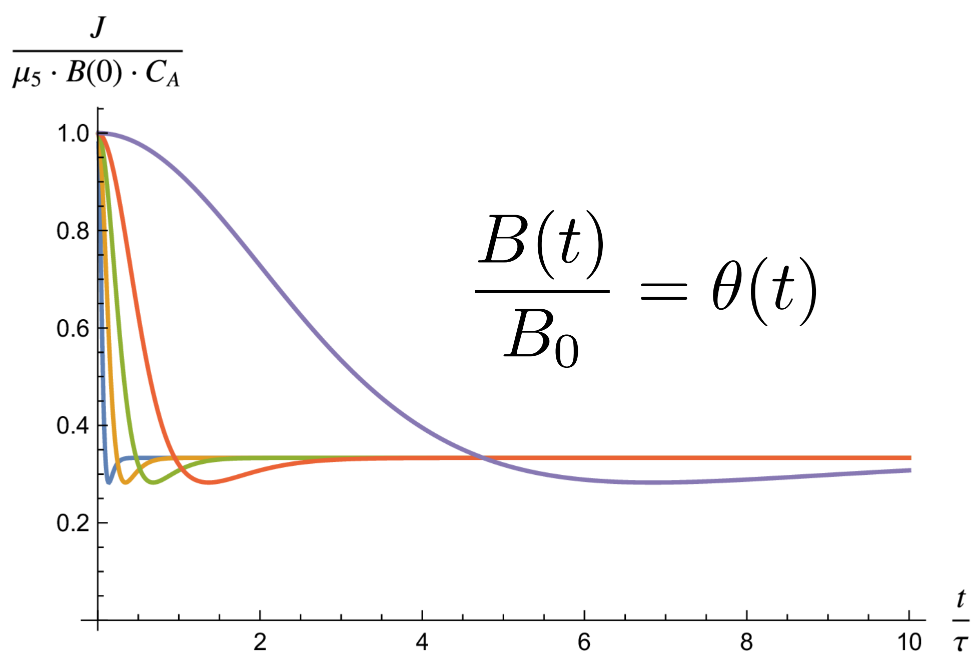
Limiting cases – constant μ_5

Magnetic field is homogeneous but time-dependent

Retardation is more pronounced for smaller temperatures



$$\frac{B(t)}{B_0} = \theta(t)e^{-t/\tau}$$



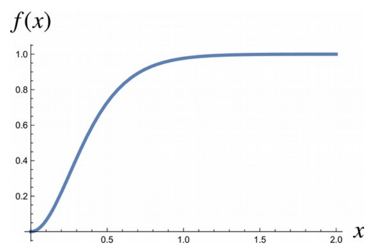
$$\frac{B(t)}{B_0} = \theta(t)$$

$$\tau T = 5.0,\ 2.0,\ 1.0,\ 0.5,\ 0.1$$

Limiting cases – constant *B*

Limiting cases – constant ***B***

$$\widetilde{J}^i(t,\mathbf{q}_2) = \int\limits_{-\infty}^{\infty} \mathrm{d}\tau \widetilde{\mu_5}(t+\tau,\mathbf{q}_2)\frac{e^2}{2\pi^2}\left\{B^i\left(\delta(\tau)+\frac{\theta(-\tau)}{2}\left[q_2\sin(q_2\tau)-\frac{\partial}{\partial\tau}\left(\frac{\sin(q_2\tau)}{q_2\tau}\right)f(\tau T)\right]+\right.\right.$$



$$\left.+(\mathbf{B}\cdot\widehat{\mathbf{q}_2})\widehat{q_2}^i\frac{\theta(-\tau)}{2}\left[q_2\sin(q_2\tau)+\frac{\partial}{\partial\tau}\left(\frac{\sin(q_2\tau)}{q_2\tau}\right)f(\tau T)\right]\right\}$$

T=0

$$J^i(t,\mathbf{r}) = \frac{e^2}{2\pi^2}\left\{B^i\mu_5(t,\mathbf{r})-\frac{2}{3}B^i\mu_5(t,\mathbf{r})+\right.$$

$$+\frac{1}{8\pi}\int \mathrm{d}^2\widehat{\mathbf{r}'}\int\limits_0^{\infty}\mathrm{d}r'\left[\left(r'\partial_1^2\mu_5(t-r',\mathbf{r}+\mathbf{r}')\right)\left(B^i+B^i_{\|,\mathbf{r}'}\right)+\right.$$

$$-\left(\partial_1\mu_5(t-r',\mathbf{r}+\mathbf{r}')+\frac{\mu_5(t-r',\mathbf{r}+\mathbf{r}')-\mu_5(t,\mathbf{r}+\mathbf{r}')}{r'}\right)\left(B^i-3B^i_{\|,\mathbf{r}'}\right)+$$

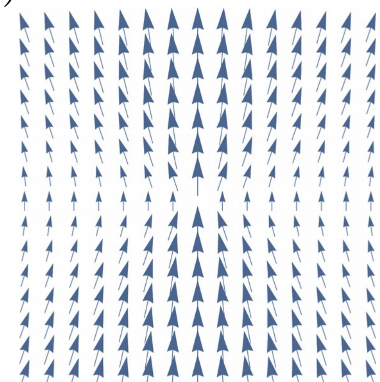
$$\left.\left.+\left(\partial_1\mu_5(t-r',\mathbf{r}+\mathbf{r}')f(r'T)-\mu_5(t-r',\mathbf{r}+\mathbf{r}')Tf'(r'T)\right)\left(B^i-B^i_{\|,\mathbf{r}'}\right)\right]\right\}$$

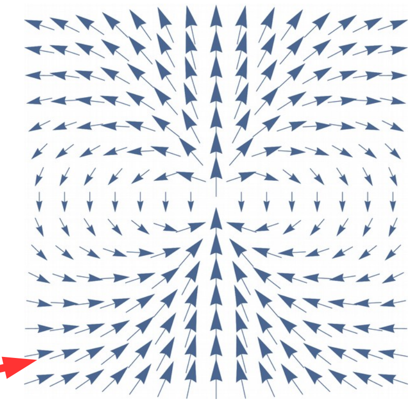
T>0

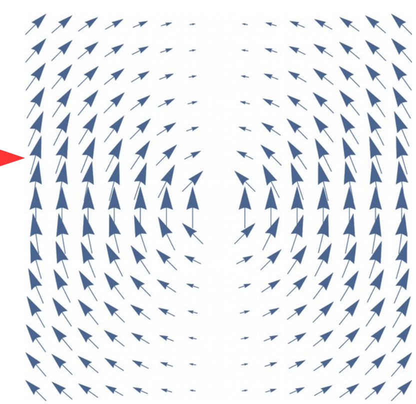
Limiting cases – constant *B*

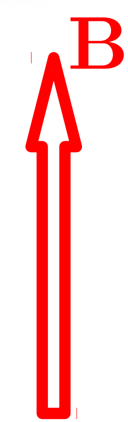
For a centered source: $\mu_5(\mathbf{r},t) = \mu_5(t)\delta^{(3)}(\mathbf{r})$

$$\mathbf{J}(t,\mathbf{r}) = \frac{e^2}{2\pi^2}\left\{\frac{1}{3}\mathbf{B}\mu_5(t)\delta^{(3)}(\mathbf{r}) + \right.$$
$$+\frac{1}{2}\left[\frac{\mu_5''(t-r)}{r}\boxed{(\mathbf{B}+\mathbf{B}_{\parallel,\mathbf{r}})} + \right.$$
$$-\left(\frac{\mu_5'(t-r)}{r^2} + \frac{\mu_5(t-r)-\mu_5(t)}{r^3}\right)\boxed{(\mathbf{B}-3\mathbf{B}_{\parallel,\mathbf{r}})}$$
$$\left.\left.+\frac{\mu_5'(t-r)f(rT)+\mu_5(t-r)Tf'(rT)}{r^2}\boxed{(\mathbf{B}-\mathbf{B}_{\parallel,\mathbf{r}})}\right]\right\}$$









Integrating out radially around the origin

$$\int_0^R \mathrm{d}r r^2 \mathbf{J}(\mathbf{r},t) \overset{R\ll t}{\approx} \begin{cases} T\to\infty & \frac{e^2}{2\pi^2}\frac{1}{6}\mu_5(t)(-\mathbf{B}+3\mathbf{B}_{\parallel,\mathbf{r}}), \\ T=0 & \frac{e^2}{2\pi^2}\frac{1}{3}\mu_5(t)\mathbf{B} \end{cases}$$

$$\mathbf{B}_{\parallel,\mathbf{r}} = (\mathbf{B}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}$$

Perspectives

- **Work in progress:**
  practical expression, suitable for implementation
  (maybe high T expansion?)
  realistic sources of n_5 from QCD:
  → stochastic generation of axial charge

- **Other plans to improve**
  finite lifetime effects, effects of interaction on the nonstatic response (cf. energy loss of partons)
  transport coefficients by grad. expansion → MHD use

- **higher loop non-static contributions?**
  – *see: Feng, Hou, Ren, Phys. Rev. D 99, 036010 (2019)*
  – *also talk by Hui Liu*

Take home message

- **Order of limits matters!**

  → different order of limits correspond to regimes dominated by different scales

  → constant μ_5 corresponds to the absence of the current means: CME is a nonequilibrium phenomenon

- **Retardation matters!**

  → corrections to the static anomaly current

  → sizable delay in response on low-T

Thank you for listening!

Questions? Comments?

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Backup

AVV triangle

$$q_{1\nu}\delta G^{\rho\mu\nu}_{AVV}(q_1,q_2) =$$

$$iG(p+q)q\cdot\Gamma_V iG(p) = G(p+q) - G(p)$$

$$= -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p+q_1) - \gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p)$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p+q_1)+$$

$$-\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p)+$$

$$+\gamma^\mu G^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$-\gamma^\mu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p) - \gamma^\mu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

– {same terms with *m=M>>all other scales*}

AVV triangle

$$q_{1\nu}\delta G^{\rho\mu\nu}_{AVV}(q_1,q_2) =$$

$$iG(p+q)q\cdot\Gamma_V iG(p) = G(p+q) - G(p)$$

$$= -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p+q_1) - \gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p)$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p+q_1)+$$

$$-\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p)+$$

$$+\gamma^\mu G^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$-\gamma^\mu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p) - \gamma^\mu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

– {same terms with *m=M>>all other scales*}

AVV triangle

$$G(p)\gamma^5 + \gamma^5 G(p) = \gamma^5 g(p)$$

$$q_{2\rho} \cdot i\delta G^{\rho\mu\nu}_{AVV}(q_1, q_2) = -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu\gamma^5 \overbrace{iG^C(p+q_1+q_2)\not{q}_2 iG^A(p+q_1)}^{G^C(p+q_1+q_2)}\gamma^\nu iG^A(p) +$$

$$+\gamma^\mu\gamma^5 \overbrace{iG^R(p+q_1+q_2)\not{q}_2 iG^C(p+q_1)}^{-G^C(p+q_1)}\gamma^\nu iG^A(p) +$$

$$+\gamma^\mu\gamma^5 \overbrace{iG^R(p+q_1+q_2)\not{q}_2 iG^R(p+q_1)}^{G^R(p+q_1+q_2)-G^R(p+q_1)}\gamma^\nu iG^A(p) +$$

$$+\gamma^\mu\gamma^5 iG^C(p+q_1+q_2)\gamma^\nu \overbrace{iG^A(p+q_2)\not{q}_2 iG^C(p)}^{G^A(p+q_2)-G^A(p)} +$$

$$+\gamma^\mu\gamma^5 iG^R(p+q_1+q_2)\gamma^\nu \overbrace{iG^C(p+q_2)\not{q}_2 iG^A(p)}^{G^C(p+q_2)} +$$

$$+\gamma^\mu\gamma^5 iG^R(p+q_1+q_2)\gamma^\nu \overbrace{iG^R(p+q_2)\not{q}_2 iG^C(p)}^{-G^C(p)} \Big\}$$

$$-\underbrace{\{m = M \gg q_1,\ q_2;\ g(p) \neq 0\}}_{@ = -i\frac{e^2}{2\pi^2}\epsilon^{\mu\nu\rho\sigma} q_{1\sigma} q_{2\rho}}$$

AVV triangle

$$G(p)\gamma^5 + \gamma^5 G(p) = \gamma^5 g(p)$$

$$-@ = \frac{ie^2}{2} \int_p \mathrm{tr}\Big\{ \gamma^\mu \gamma^5 (1 - 2n_{FD}(p_0/T)) \Big[$$
$$ig_M^R(p+q_1+q_2)\not{q}_2 iG_M^R(p+q_1)\gamma^\nu iG_M^R(p) + iG_M^R(p+q_1+q_2)\gamma^\nu iG_M^R(p+q_2)\not{q}_2 ig_M^R(p) +$$
$$ig_M^A(p+q_1+q_2)\not{q}_2 iG_M^A(p+q_1)\gamma^\nu iG_M^A(p) + iG_M^A(p+q_1+q_2)\gamma^\nu iG_M^A(p+q_2)\not{q}_2 ig_M^A(p) \Big]\Big\} =$$

$$\approx ie^2(-8iM^2)\epsilon^{\mu\nu\rho\sigma} q_{1\sigma} q_{2\rho} \int_p (1 - 2n_{FD}(p_0/T)) \left[\frac{1}{\left[(p_0+i0^+)^2 - p^2 - M^2\right]^3} - \frac{1}{\left[(p_0-i0^+)^2 - p^2 - M^2\right]^3} \right] =$$
$$= -\frac{4e^2}{\pi^3}\epsilon^{\mu\nu\rho\sigma} q_{1\sigma} q_{2\rho} 2\pi i \left(\frac{3}{8} \int_0^\infty \mathrm{d}y \frac{y^2}{(y^2+1)^{5/2}} - \lim_{N\to\infty} \sum_{n=0}^{N} \int_0^\infty \mathrm{d}y \frac{y^2}{\left(\frac{(2n+1)^2\pi^2}{M^2} + 1 + y^2\right)^3} \frac{2}{M} \right) =$$
$$\xrightarrow{M\to\infty} -i\frac{8e^2}{\pi^2}\epsilon^{\mu\nu\rho\sigma} q_{1\sigma} q_{2\rho} \left(\frac{3}{8}\cdot\frac{1}{3} - \frac{1}{16} \right) = -i\frac{e^2}{2\pi^2}\epsilon^{\mu\nu\rho\sigma} q_{1\sigma} q_{2\rho}$$

Limiting cases – absent current

$$=\underbrace{e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j\frac{\partial G^A(p)}{\partial p_k}\right\}(-2n'(p_0))}_{=:A}+\underbrace{e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0 G^A_M(p)\gamma^j\frac{\partial G^A_M(p)}{\partial p_k}\right\}g^A_M(p)(1-2n(p_0))}_{=:B}=0$$

$$2A=\frac{2e^2}{16\pi^4}4\pi\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}pp^2\mathrm{tr}\left\{\gamma^i\gamma^5\not{p}\gamma^j\left(\gamma^k-\frac{2p^k\not{p}}{(p_0-i0^+)^2-p^2}\right)\right\}\frac{-2n'(p_0)}{[(p_0-i0^+)^2-p^2]^2}=\frac{4ie^2}{\pi^3}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}pp^2\int_{-\infty}^{\infty}\mathrm{d}p_0\frac{p_0n'(p_0)}{[(p_0-i0^+)^2-p^2]^2}$$

$$=-\frac{2ie^2}{\pi^3}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}pp^2\int_{-\infty}^{\infty}\mathrm{d}p_0n'(p_0)\frac{\partial}{\partial p_0}\frac{1}{(p_0-i0^+)^2-p^2}=-\frac{2e^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}pp^2\int_{-\infty}^{\infty}\mathrm{d}p_0n''(p_0)\frac{\delta(p_0-p)-\delta(p_0+p)}{2p}=$$

$$=-\frac{e^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}ppn''(p)=\frac{e^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}pn'(p)=-\frac{e^2}{2\pi^2}\epsilon^{ijk},$$

$$2B=\frac{2e^2}{16\pi^4}4\pi\int_{-\infty}^{\infty}\mathrm{d}p_0(1-2n(p_0))\int_0^{\infty}\mathrm{d}p\mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0(\not{p}+M)\gamma^j\left(\gamma^k-\frac{2p^k(\not{p}+M)}{(p_0-i0^+)^2-p^2-M^2}\right)\right\}\frac{2M}{[(p_0-i0^+)^2-p^2-M^2]^3}=$$

$$=\frac{e^2}{\pi^3}\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}p\frac{M^2p^2(1-2n(p_0))}{[(p_0-i0^+)^2-p^2-M^2]^3}\mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0\gamma^j\gamma^k\right\}=-\frac{4ie^2}{\pi^3}\epsilon^{ijk}\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}p\frac{M^2p^2(1-2n(p_0))}{[(p_0-i0^+)^2-p^2-M^2]^3}=$$

$$=\frac{e^2}{2\pi^2}\epsilon^{ijk}=-2A$$

Limiting cases – constant ***B***

$$\frac{\partial \mathfrak{g}^{ij}}{\partial q_{1k}}\epsilon^{ljk}B_l = -\frac{e^2}{\pi^2}\int\limits_{-\infty}^{\infty}\mathrm{d}p_0\mathrm{sgn}(p_0)(1-2n(p_0))\int\limits_{0}^{\infty}\mathrm{d}p\delta\left(p_0^2-p^2-m^2\right)\int\limits_{-1}^{1}\mathrm{d}x$$

$$\left\{\frac{\partial}{\partial x}\frac{B_{\parallel}^i x(p_0q_{20}+q_2px+(x^2+1)p^2)+B_{\perp}^i x\left(p_0q_{20}+\left(\frac{1-x^2}{2}+1\right)p^2\right)}{(p_0+q_{20}+i0^+)^2-p^2-q_2^2-2q_2px-m^2}+\right.$$

$$\left.+B_{\perp}^i\frac{p^2}{(p_0+q_{20}+i0^+)^2-p^2-q_2^2-2q_2px-m^2}\right\}$$

$$J^i=\int_{q_2}e^{iq_2\cdot x}\widetilde{\mu_5}(q_{20},\mathbf{q}_2)\frac{1}{2}\left(\left.\frac{\partial\mathfrak{g}^{ij}}{\partial q_{1k}}\right|_{q_1=0,m\to 0}-\left.\frac{\partial\mathfrak{g}^{ij}}{\partial q_{1k}}\right|_{q_1=0,m=\infty}\right)\epsilon^{ljk}B_l$$