Acceleration as imaginary chemical potential

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Based on

- *G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, arXiv:1903.09697 [hep-th].*
- *G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, JHEP 1902, 146 (2019).*
- *G. Prokhorov, O. Teryaev and V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018).*

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Contents

- 1. Third-order corrections in acceleration and vorticity $\omega^2\omega_\mu$, $a^2\omega_\mu$ to the Chiral Vortical Effect (CVE).

  The concept of angular velocity as a real chemical potential.

- 2. Fourth order corrections a^4 in acceleration in the energy-momentum tensor.

  Unruh effect for fermions.

  The concept of acceleration as an imaginary chemical potential.

Methods

- Quantum-statistical density operator of Zubarev. *(D. N. Zubarev, A. V. Prozorkevich, S. A. Smolyanskii, Theoret. and Math. Phys., 40:3 (1979), 821-831)*.
- Covariant Wigner Function *(F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32)*.

Zubarev density operator

In the state of global thermodynamic equilibrium with thermal vorticity, the medium is described by the density operator of the form

D. N. Zubarev, A. V. Prozorkevich, S. A. Smolyanskii, Theoret. and Math. Phys., 40:3 (1979), 821-831.

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091.

$$\hat{\rho} = \frac{1}{Z} \exp\left\{ -\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}_x^{\mu\nu} + \zeta\hat{Q} \right\}$$

Effects associated with the vorticity and acceleration are described by the term with thermal vorticity

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Quantum statistical mean values can be found within the framework of perturbation theory. In particular, CVE was received

$$\langle j_\mu^5 \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega_\mu$$

Corrections to the chiral vortical effect

Parity allows the appearance of 3 types of terms in the third order of perturbation theory

$$\langle \hat{j}_5^\lambda(x)\rangle^{(3)} = A_1\omega^2\omega^\lambda + A_2 a^2\omega^\lambda + A_3(\omega a)a^\lambda$$

These coefficients were found based on the density operator for the case of massive fermions and in the chiral limit

G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, JHEP 1902, 146 (2019).

$$A_1 \to -\frac{1}{24\pi^2}\,, \quad A_2 \to -\frac{1}{8\pi^2}\,, \quad A_3 = 0$$

in the chiral limit

Thus, in the third order of perturbation theory, we have

$$\langle j_\mu^5\rangle = \Big(\frac{1}{6}\Big[T^2 - \frac{\omega^2}{4\pi^2}\Big] + \frac{\mu^2}{2\pi^2} - \frac{a^2}{8\pi^2}\Big)\omega_\mu + \mathcal{O}(\varpi^5)$$

Corrections to the chiral vortical effect

$$\langle j_\mu^5 \rangle = \Big(\frac{1}{6}\big[T^2 - \frac{\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2} - \frac{a^2}{8\pi^2}\Big)\omega_\mu + \mathcal{O}(\varpi^5)$$

Term, cubic in vorticity

A. Vilenkin, Phys. Rev. D 21 (1980) 2260.

Second order terms in the presence of axial chemical potential

M. Buzzegoli and F. Becattini, JHEP 1812, 002 (2018).

Since the third order term $A_3(\omega a)a^\lambda = 0$ the axial charge is conserved

$$\partial^\mu \langle j_\mu^5 \rangle = 0$$

Exact nonperturbative formula

From the Wigner function *(F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32)* it follows that in the case $a = 0$

G. Prokhorov, O. Teryaev and V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018).

$$\langle \mathbf{j}^5 \rangle_W = \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - \frac{\Omega}{2}) - n_F(E_p - \mu + \frac{\Omega}{2}) + $$
$$ + n_F(E_p + \mu - \frac{\Omega}{2}) - n_F(E_p + \mu + \frac{\Omega}{2}) \Big\} \mathbf{e}_\Omega$$

Look also: *M. Stone and J. Kim, Phys. Rev. D 98 (2018) 025012.*

A nonperturbative formula derived from the Wigner function coincides with the result obtained from the density operator up to the third order of the perturbation theory in the general case of massive fermions and if $a = 0$

$$\langle \boldsymbol{j}^5 \rangle_W^{(3)} = \langle \boldsymbol{j}^5 \rangle_\rho^{(3)} \qquad m \neq 0$$

Angular velocity as real chemical potential

$$\langle \mathbf{j}^5 \rangle_W = \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - \frac{\Omega}{2}) - n_F(E_p - \mu + \frac{\Omega}{2}) + $$
$$+ n_F(E_p + \mu - \frac{\Omega}{2}) - n_F(E_p + \mu + \frac{\Omega}{2}) \Big\} \mathbf{e}_\Omega$$

The angular velocity enters the formula as a real chemical potential

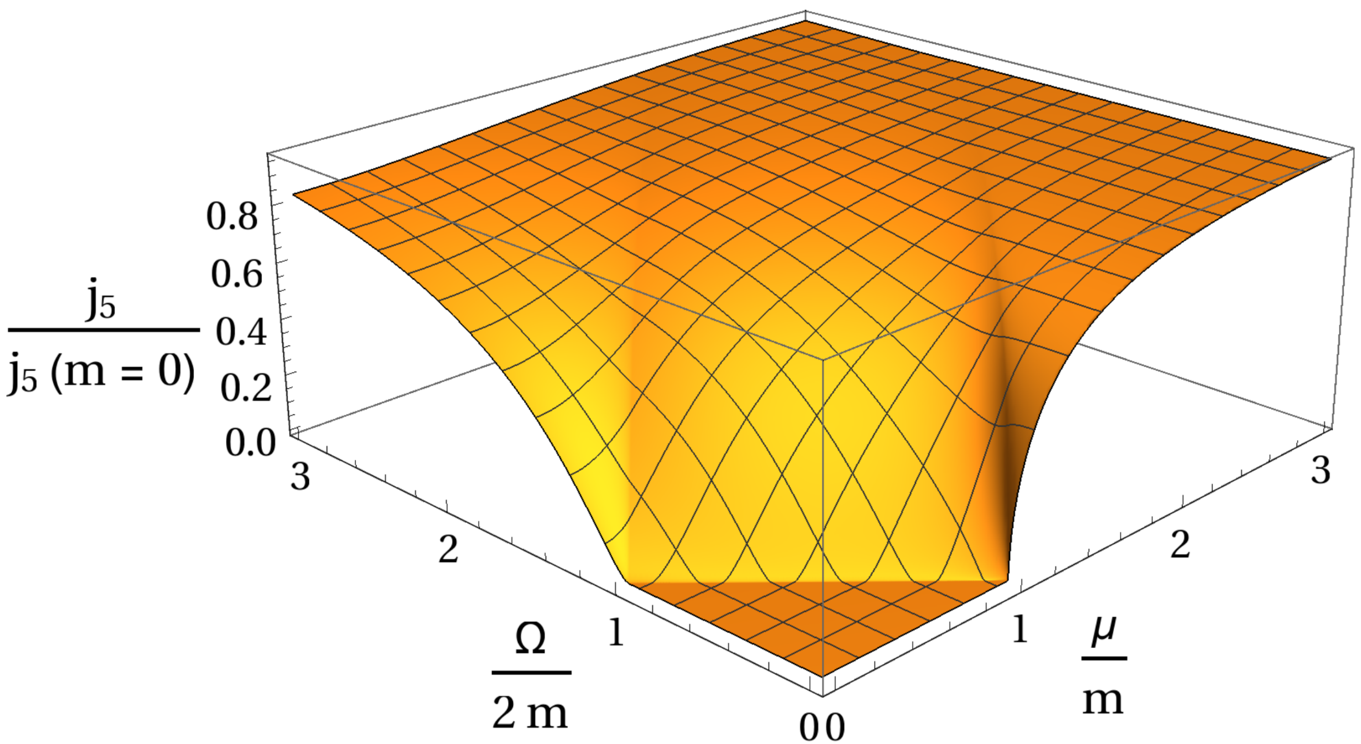
$$\mu \to \mu \pm \frac{\Omega}{2}$$

Analogy between angular velocity and chemical potential: *W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, Phys. Rev. C 97, no. 4, 041901 (2018)*.

Since the angular velocity behaves like a new chemical potential, it should be expected that there will be an effect similar to the vanishing of physical quantities at $|\mu| < m$ and $T \to 0$.

Angular velocity as real chemical potential

Such an effect really exists



Axial current at $T = 0$ is zero in the region $\Omega < 2(m - |\mu|)$.

Angular velocity as real chemical potential and acceleration as imaginary potential

The angular velocity enters the formula as a real chemical potential

$$\mu \to \mu \pm \frac{\Omega}{2}$$

If we consider the acceleration, then

$$\mu \to \mu \pm \frac{\Omega}{2} \pm \frac{i|\boldsymbol{a}|}{2}$$

acceleration appears as ***imaginary chemical potential***

G. Prokhorov, O. Teryaev and V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018).

Unruh effect from the point of view of quantum statistical mechanics

The meaning of the Unruh effect is that the accelerated observer sees the Minkowski vacuum as a medium filled with particles with a Unruh temperature proportional to the acceleration

$$T_U = \frac{a}{2\pi}$$

W. G. Unruh, Phys. Rev. D 14, 870 (1976).

Thus, the values of observables in the laboratory frame should be equal to zero when the proper temperature, measured by comoving observer equal to the Unruh temperature.

- **It has been shown for scalar particles by calculating quantum correlators**

F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018).

For fermions, a similar effect was observed based on the Wigner function in the Boltzmann limit at a temperature $2T_U$

W. Florkowski, E. Speranza and F. Becattini, Acta Phys. Polon. B 49, 1409 (2018).

- This is due to the approximate nature of the Wigner function used.

Unruh effect for fermions: energy-momentum tensor

We will study the effects associated with acceleration based on the density operator. We will consider case $\mu = \mu_5 = 0$, $\omega^\mu = 0$ and $m = 0$.

Effects associated with acceleration are described by a term with a boost operator

$$\hat{\rho} = \frac{1}{Z} \exp\left\{ -\beta_\mu \hat{P}^\mu - \alpha_\mu \hat{K}_x^\mu \right\}$$

F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018).

In the fourth order of the perturbation theory, the energy-momentum tensor has the form

$$\begin{aligned} \langle \hat{T}^{\mu\nu} \rangle \; &= \; (\rho_0 + A_1 a^2 T^2 + A_2 a^4) u^\mu u^\nu - (p_0 + B_1 a^2 T^2 + B_2 a^4) \Delta^{\mu\nu} \\ &+ (A_3 T^2 + A_4 a^2) a^\mu a^\nu + \mathcal{O}(a^6) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \end{aligned}$$

The coefficients up to the second order for fermions $\rho_0,\ p_0,\ A_1,\ B_1,\ A_3$ are calculated in

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091.

Unruh effect for fermions: energy-momentum tensor

4-order terms were calculated in

G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, arXiv:1903.09697 [hep-th].

$$A_2 = -\frac{17}{960\pi^2}\,, \quad B_2 = \frac{A_2}{3} = -\frac{17}{2880\pi^2}\,, \quad A_4 = 0$$

in particular, the energy density takes the form

$$\begin{aligned} \rho_{Den} &= \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} + \mathcal{O}(a^6) \\ &= \frac{1}{240}\left(T^2 - \left(\frac{a}{2\pi}\right)^2\right)(17a^2 + 28\pi^2 T^2) + \mathcal{O}(a^6) \end{aligned}$$

The energy density is 0 at proper temperature $T = \frac{a}{2\pi}$, if we take into account the terms of 4 order. When substituting $T = \frac{a}{2\pi}$, we get $\frac{a^4}{960\pi^2}(7 + 10 - 17) = 0$.

Unruh effect for fermions: energy-momentum tensor

The same is true, in general, for the energy-momentum tensor

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

The remaining observables are 0 due to parity considerations and due to the conditions $\mu = \mu_5 = 0 \;, \quad \omega^\mu = 0 \,.$

Thus, the Minkowski vacuum in the laboratory system corresponds to the proper temperature measured by the comoving observer equal to the Unruh temperature.

- **Confirmation and generalization for the case of fermions of the result**

  F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018).

Integral representation, acceleration as imaginary chemical potential

It can be shown that the next mathematical equality is exactly fulfilled

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3 p}{(2\pi)^3}\Big(\frac{|\mathbf{p}| + ia}{1 + e^{\frac{|\mathbf{p}|}{T} + \frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1 + e^{\frac{|\mathbf{p}|}{T} - \frac{ia}{2T}}}\Big)$$

$$+4\int \frac{d^3 p}{(2\pi)^3} \frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}} - 1} \qquad (T > T_U)$$

Thus, at temperatures above T_U, the calculation result based on the density operator can be represented as integrals with Fermi distributions at temperature T and Bose at temperature T_U.

- In the first integral, the acceleration enters *as an imaginary chemical potential* $\pm\frac{ia}{2}$.

Motivation:

- Exact match with the fundamental result from the density operator at $T > T_U$
- True limit at $a \to 0$.
- Acceleration enters as an imaginary chemical potential into axial current.

Comparison with Wigner function

- **The additional motivation for introducing such an integral representation is the result for energy density, obtained using the Wigner function (beyond the Boltzmann approximation)**

$$\rho_{Wig} = 2\int \frac{d^3p}{(2\pi)^3}\varepsilon\Big(\frac{1}{1+e^{\frac{\varepsilon}{T}+\frac{ia}{2T}}} + \frac{1}{1+e^{\frac{\varepsilon}{T}-\frac{ia}{2T}}}\Big)$$

where acceleration also appears as imaginary chemical potential.

The difference from the result obtained from the density operator

$$\rho = \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\Big(\frac{|\mathbf{p}|+ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}|-ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\Big)$$

$$+4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1} \qquad (T > T_U)$$

in red: modifications compared to the Wigner function

is related to the approximate nature of the Wigner function, which also leads to that $\rho_{Wig}(T_U) \neq 0$

W. Florkowski, E. Speranza and F. Becattini, Acta Phys. Polon. B 49, 1409 (2018).

Acceleration as imaginary chemical potential

The appearance of acceleration as an imaginary chemical potential leads to **additional terms** at $T < T_U$, with the integer part

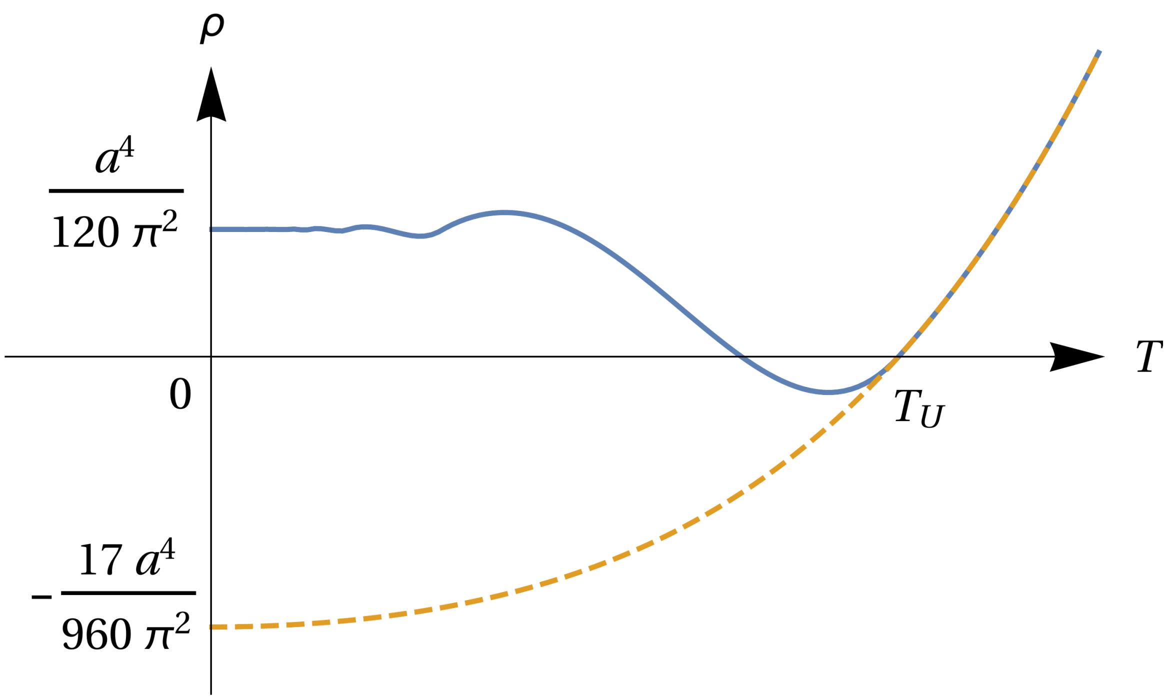
$$\rho = \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} + \left(\frac{\pi T^3 a}{3} + \frac{Ta^3}{4\pi}\right)\left\lfloor\frac{1}{2} + \frac{a}{4\pi T}\right\rfloor$$
$$-\left(\frac{T^2a^2}{2} + 2\pi^2T^4\right)\left\lfloor\frac{1}{2} + \frac{a}{4\pi T}\right\rfloor^2 - \frac{4\pi T^3 a}{3}\left\lfloor\frac{1}{2} + \frac{a}{4\pi T}\right\rfloor^3 + 4\pi^2T^4\left\lfloor\frac{1}{2} + \frac{a}{4\pi T}\right\rfloor^4$$

which lead to instabilities at $T = T_U/(2k+1)$, $k = 0, 1..$,

manifested in discontinuities in $\dfrac{\partial^2\rho}{\partial T^2}$.

- In particular, at $T = \dfrac{a}{2\pi}$ there is also a discontinuity of the function, which should be interpreted as another **manifestation of the Unruh effect.**

Comparison of perturbative formula and integral representation



The solid blue line is the energy density as a function of temperature, corresponding to the integral representation. *The dashed orange line* - the result of the fundamental calculation based on the density operator.

- **There is a difference in area $T < T_U$, which is forbidden according to**

F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018)

Conclusions

- Third-order corrections in acceleration and vorticity to CVE are calculated using the fundamental Zubarev density operator.
- This calculation confirms a non-perturbative formula derived from the Wigner function at zero acceleration, in which the angular velocity enters as a real chemical potential.
- The fourth order corrections are calculated in the energy-momentum tensor of massless fermions.
- It is shown that, taking into account these corrections, the energy-momentum tensor vanishes at a proper temperature measured by the comoving observer equal to the Unruh temperature. This is the essence of the Unruh effect and is a generalization of the result *(F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018))* for the case of fermions.
- An integral representation for the energy density is proposed, in which the acceleration enters as an imaginary chemical potential, which leads to instabilities at the Unruh temperature. *This formula can be considered as a modification of the result of the Wigner function.*

Thank you for your attention!