"Hydro+" and chiral/spin hydrodynamics

HIC offer unique opportunities to explore the macroscopic quantum effects associated with *chirality* and/or *spin polarization*.

This in turn motivates extensive studies of hydro-like description of the dynamics of axial charge and spin density.

In parallel, a general framework, "hydro+", is developed to systemically describe hydro with additional slow modes.

Stephanov-YY, 1712.10305, PRD '18

I shall review this newly developed formalism, and illustrate its salient features in the context of chiral/spin hydrodynamics.

BEST COLLABORATION

Yi Yin

Chirality workshop,Tsinghua, Apr.9, 2019

“Hydro+”: general framework

Stephanov-YY, 1712.10305, PRD ’18

Hydro. as a low energy effective theory

Hydro. describes slow evolution of conserved densities, e.g, energy density e and momentum density.

$$\Gamma_{\mathrm{hydro}} \propto Q^2$$

Hydro. equation: conservation laws with constitutive relation obtained by gradient expansion.

$$\partial_\mu T^{\mu\nu} = 0\,.$$

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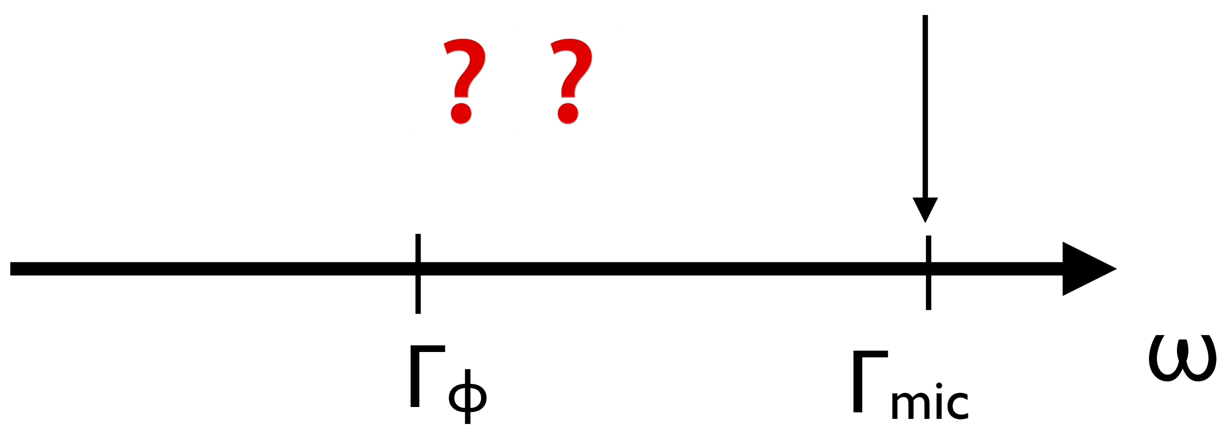


What happens if there is an additional slow mode ϕ? $\Gamma_\phi \ll \Gamma_{\mathrm{mic}}$

Parametrically slow mode(s)

Parametrically slow modes: smallness of Γ_ϕ is controlled by another small parameter $\delta_{(+)}$.

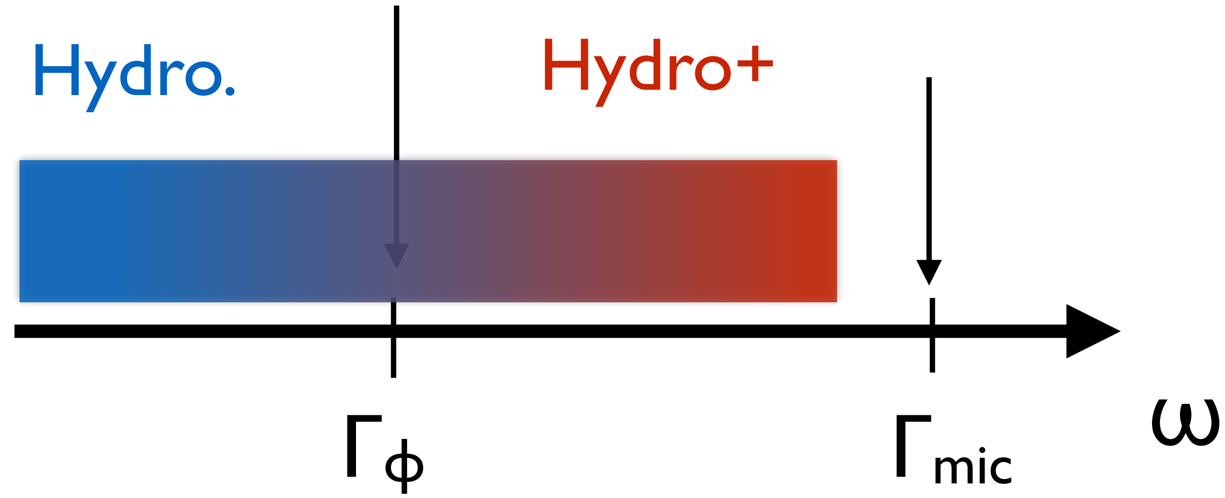
$$\lim_{\delta_{(+)}\to 0} \Gamma_\phi \to 0\,, \qquad \lim_{Q\to 0} \Gamma_\phi \neq 0\,, \qquad \Gamma_\phi \ll \Gamma_{\rm mic}\,.$$

For example, fluctuations near a critical point equilibrates slowly due to the grow of correlation length ξ (critical slowing down).

$$\lim_{l_{\rm mic}/\xi\to 0} \Gamma_{\rm fluc} \to 0\,.$$

The emergence of parametrically slow mode(s) can be found in many interesting and relevant physical situations. (this talk: axial density and spin density).

Those modes would be of phenomenological important when $(\Gamma_\phi)^{-1}$ is comparable to the lifetime of the fireball.



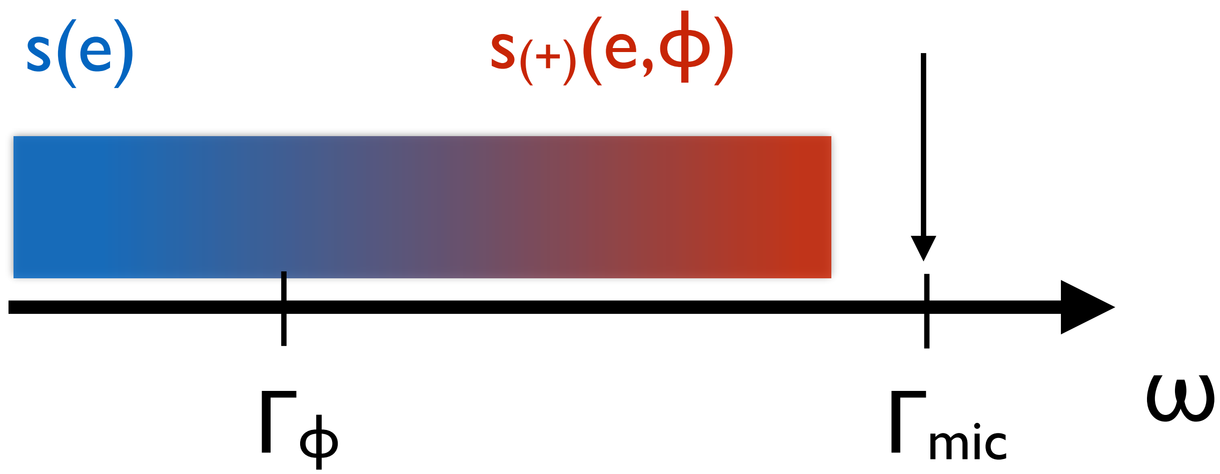
The presence of Γ_ϕ naturally divide the low frequency behavior of the system into two (qualitatively) different regimes.

Hydro regime: $\omega \ll \Gamma_\phi$, $\phi \Rightarrow$ its equilibrium value $\phi_{eq}(e)$.

"Hydro+" regime: $\omega \gg \Gamma_\phi$, ϕ is off-equilibrium and has to be treated as a mode independent of hydro modes.

"Hydro+" aims at formulating a hydro-like theory describing intertwined dynamics of hydro. d.o.f and ϕ.

Qualitative feature I: the generalization of thermal equilibrium



In "hydro+" regime, a macroscopic state is characterized by e, ϕ.

Generalized entropy $s_{(+)}$: log of the number microscopic states with given e, ϕ.

In principle, $s_{(+)}$ can be determined once ϕ is specified. E.g. : ϕ=n_A

From $s_{(+)}$, one could define other generalized thermodynamic functions such as $\beta_{(+)}$ and $p_{(+)}$.

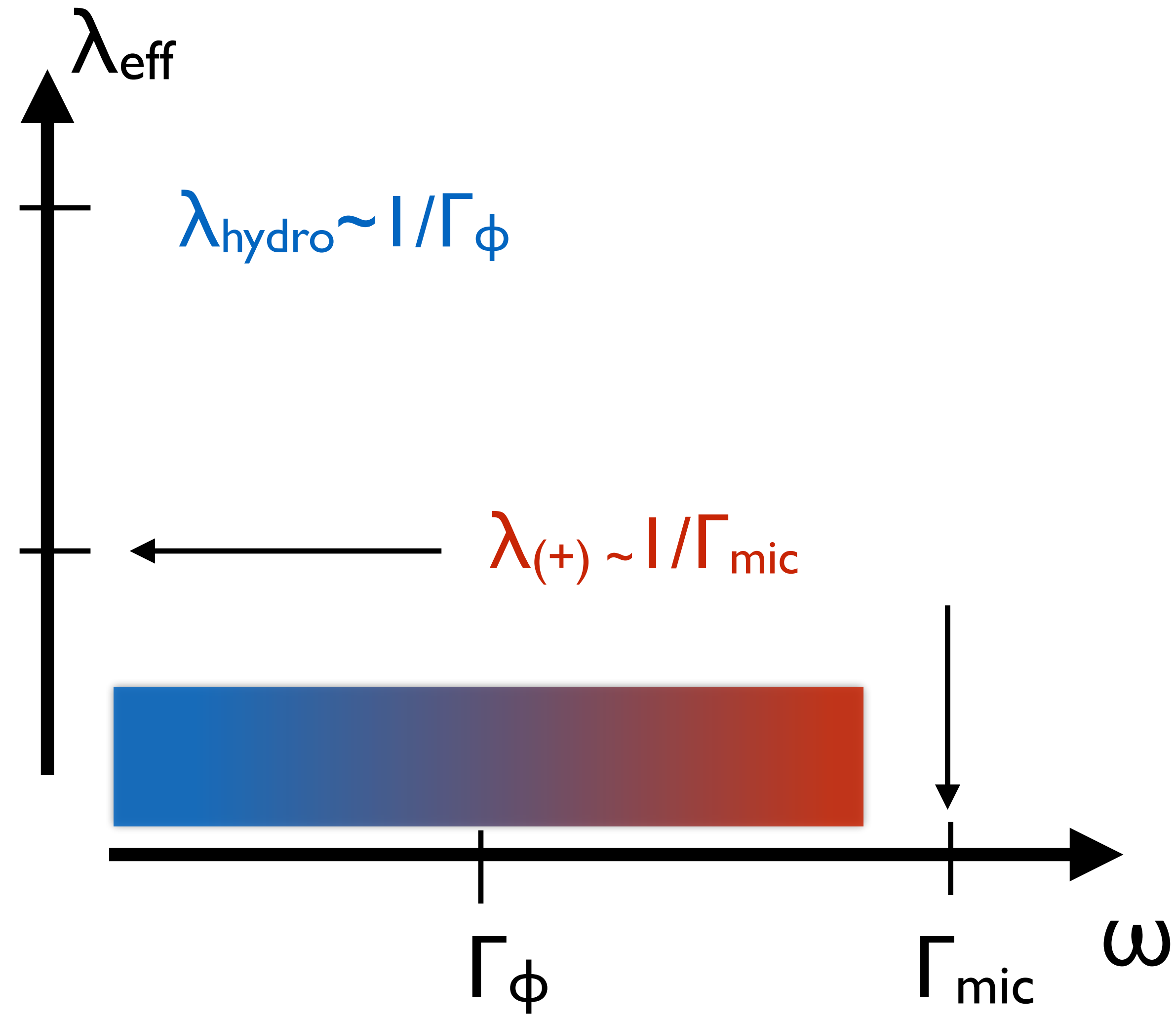
$$ds_{(+)} = \beta_{(+)} de + \dots$$

Qualitative feature II: transport coefficient

Slow equilibration of ϕ, large transport coefficient(s) λ in hydro regime.

$$\lambda_{\rm hydro} \propto \Gamma_\phi^{-1} + \mathcal{O}(\delta_{(+)})$$

For example, transport coefficients grow near the critical point.



However, "effective λ" would drop rapidly to a much smaller value in "hydro+" regime.

$$\lambda_{(+)} \propto \Gamma_{\rm mic}^{-1}$$

Construction of Hydro+

E.o.M for ϕ:

$$u^{\mu}\,\partial_{\mu}\phi = A_{\phi}(\partial\cdot u) + F_{\phi}(e,\phi)$$

$A_{\phi}(e, \phi)$ describes the response of ϕ to compression/expansion. E.g. for axial charge $\phi=n_A, A_{\phi} = n_A$.

$F_{\phi}(e, \phi)$ is the "returning" force: $\lim_{Q\to 0} F_{\phi} \propto \Gamma_{\phi}\,(\phi - \phi_{\mathrm{eq}}(e))$

E.o.M for hydro. variables remain the same:

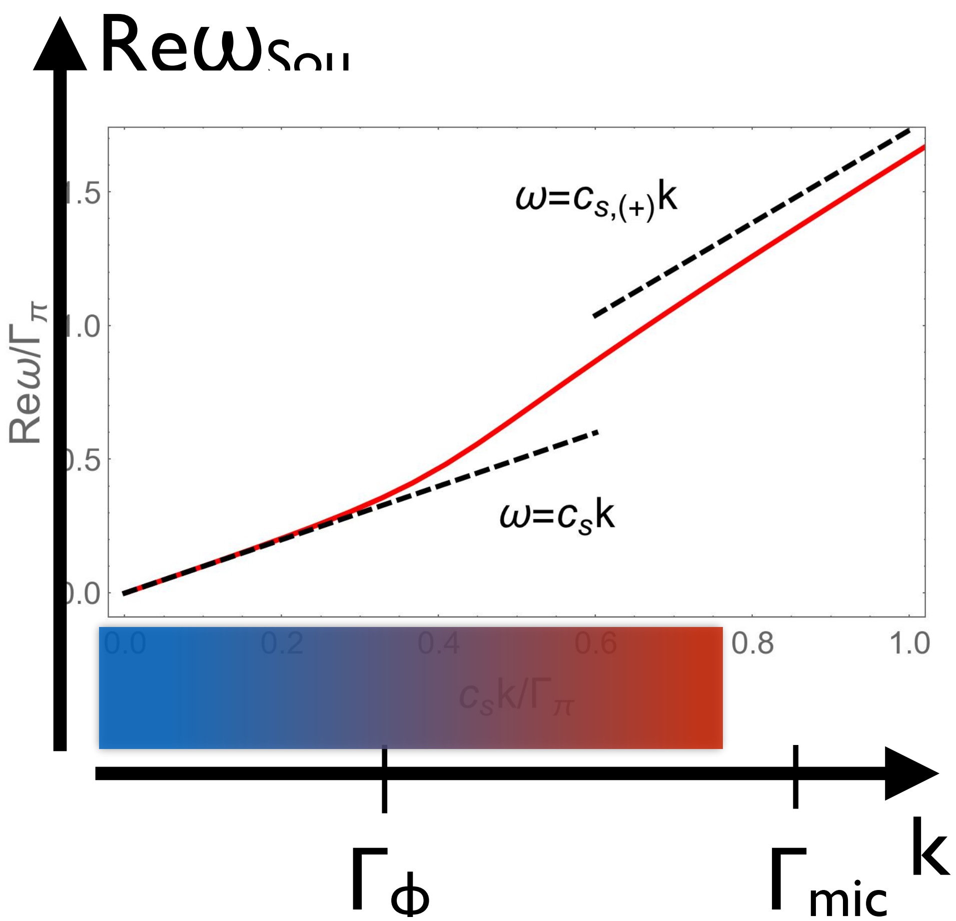
$$\partial_{\mu}\,T^{\mu\nu} = 0$$

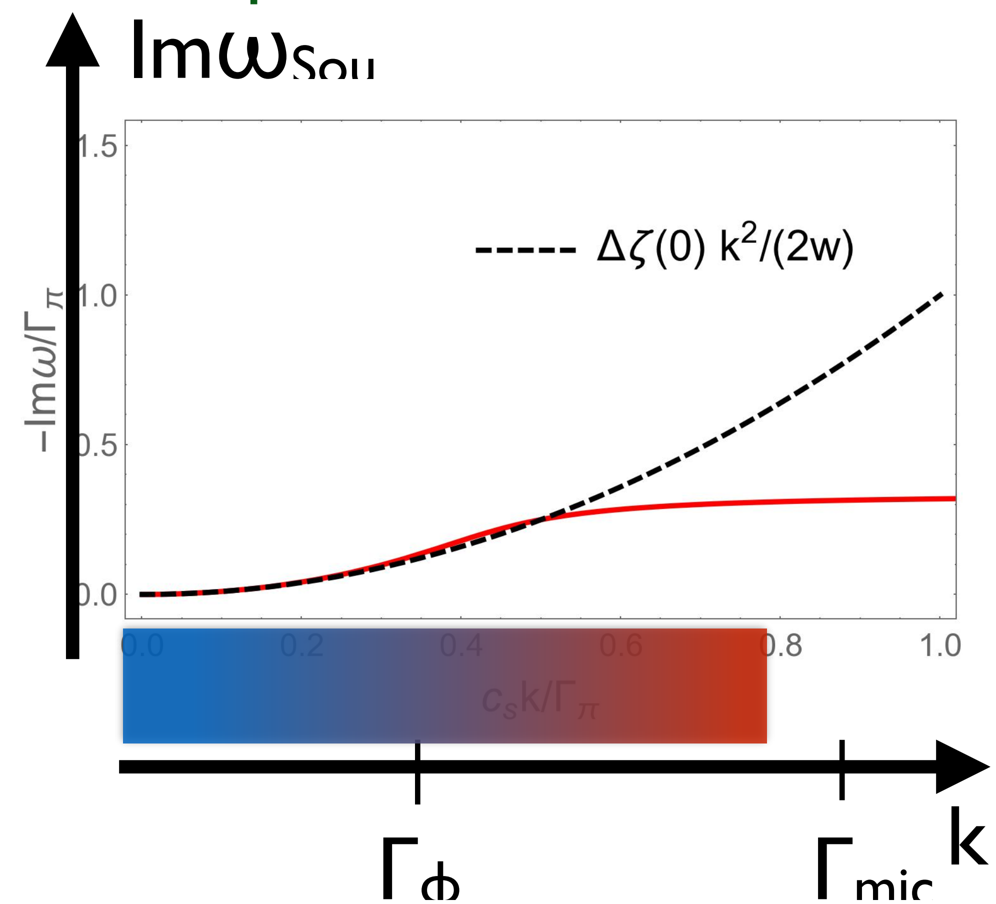
The constitutive relation for $T^{\mu\nu}$ and F_{ϕ} can be obtained by the double expansion in ($Q\, l_{mfp}$) and $\delta_{(+)}$. The generalized 2nd law imposes an important constraint:

$$\partial_{\mu}\,s^{\mu}_{(+)} \geq 0$$

Sound dispersion for "hydro+" with one scalar mode

Stephanov-YY, 1712.10305, PRD '18





Effective sound velocity becomes different.

$$c_s^2 = (\frac{\partial p}{\partial e}) \leq c_{s,(+)}^2 = (\frac{\partial p_{(+)}}{\partial e})_\phi .$$

Effective bulk viscosity also becomes different.

$$\zeta = (e+p)\frac{\Delta c_s^2}{\Gamma_\phi} \gg \zeta_{(+)}$$

So, generic parametrically slow mode(s). Now:

Applications to chiral/spin hydrodynamics

Axial charge density as a parametrically slow mode

The axial charge density n_A relaxes slowly in many situations.

Weyl/Dirac semimetal. (inter F.S collision rate is much smaller than intra F.S. collision rate)

Weakly coupled QGP.

$$\Gamma_A \sim \alpha_s^5 T \ll \Gamma_{\mathrm{mic}} \sim \alpha_s^2\, T$$

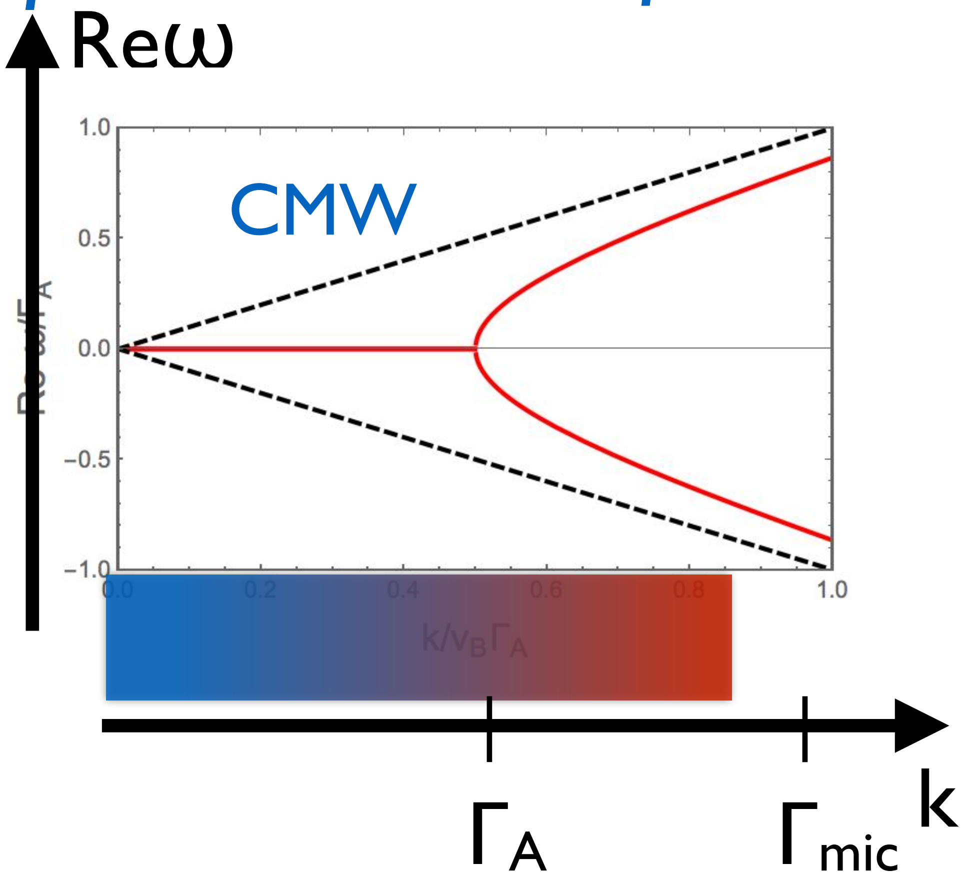
Maybe for sQGP. (Naively substituting α_s=0.2 into wQGP result gives $1/\Gamma_A$=10 fm)

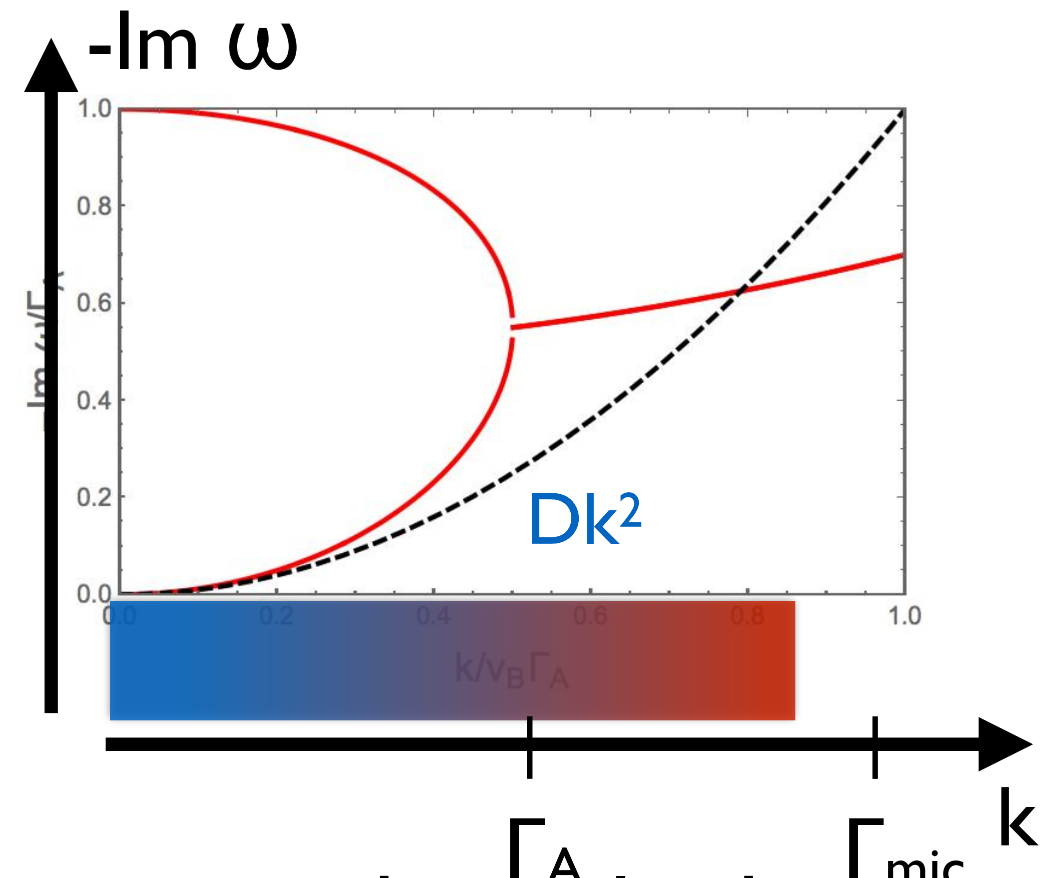
(Shu Lin and Yee, PRD 17; See Shu's talk)

If n_A is a parametrically slow mode, it will have interesting implications.

Dispersion relation for n_V+n_A

(Stephanov, Yee and YY PRD15, 1501.00222)





Consider vector charge conservation and axial charge non conservation.

$$\partial_\mu J^\mu_V = 0\,, \qquad \partial_\mu J^\mu_A = -\,\Gamma_A\, n_A$$

With external magnetic filed, n_A mixes with n_V due to CME and CSE.

(Large Diffusion in hydro. regime)

$$D = \frac{v^2_{CMW}}{\Gamma_A} \quad \Rightarrow \quad \sigma = \chi D = \frac{C_A^2 B^2}{\chi \Gamma_A}$$

Implications for HIC: signature of CMW is not only limited by the lifetime of B, but also that of n_A.

Magnetohydrodynamic (MHD) + n_A or "MHD+"

(Hattori, Hirono, Yee and YY, 1711.08450)

MHD variables (assuming $\sigma Q \ll 1$): e, u^μ, B^μ .

Magnetic field B is a hydro variable as its relaxation rate vanishes at small Q

$$\Gamma_B \sim Q^2/\sigma \, .$$

Electric field E and vector charge density n are not because their relaxation rates are proportional to conductivity σ.

"MHD+" variables: e, u^μ, B^μ, n_A .

E.o.M: conservation law, Bianchi identity + anomaly equation

$$\partial_\mu J_A^\mu = C_A \, B \cdot E \, .$$

To close "MHD+", we need to express E in term of "MHD+" variables (constitutive relation).

Constitutive relation of E in "MHD+"

$$\partial \cdot s_{(+)} = \left[-\beta \left(e + p_{(+)} \right) + s_{(+)} - H \cdot B + \mu_A n_A \right] (\partial \cdot u)$$
$$+ \left[\mu_A \left(C_A E \cdot B \right) + \mathcal{O}(\partial^2) \right] \geq 0$$

Let us assume gradient expansion is sufficient so that $E \sim \mathcal{O}(\partial)$.

Attempt 1: $\mu_A \sim \mathcal{O}(1)\,,$

Then $p_{(+)} = -\, e + \beta_{(+)}^{-1} s_{(+)} + H \cdot B + \mu_A n_A$ ✔

$E \propto C_A \mu_A B \sim \mathcal{O}(1)$ ✘

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$E \propto C_A \mu_A B \sim \mathcal{O}(1)$ ✗

Attempt 2: $\mu_A \sim \mathcal{O}(\partial)$

Then

$E \propto C_A \mu_A B \sim \mathcal{O}(\partial)$ ✓

$\mu_A n_A \sim \mathcal{O}(\partial)$ ✗

Let us try double expansion.

Double expansion

The gradient controls the "slowness" of hydro. modes. $\delta_{(+)}$ controls the slowness of "+" modes.

Natural extension: the double expansion in terms of $\delta_{(Q)}$ and $\delta_{(+)}$.

$$\mathcal{O}(\delta) : \delta_Q , \delta_{(+)} \qquad \mathcal{O}(\delta^2) : (\delta_Q^2) , (\delta_{(+)}^2) , (\delta_{(Q)}\delta_{(+)})$$

"Hydro+" variables: $\mathcal{O}(1)$; Others: $\mathcal{O}(\delta)$ or smaller.

For "MHD +", we identify $C_A=e^2/(2\pi^2) \ll 1$ as the small parameter $\delta_{(+)}$.

Does that work?

Results: the universality of CME with dynamical magnetic field

(Hattori, Hirono, Yee and YY, 1711.08450)

$$\partial \cdot s_{(+)} = (\ldots)(\partial \cdot u) + \left[\mu_A \left(C_A E \cdot B\right) + \mathcal{O}(\partial^2)\right] \geq 0$$

We found:

$$\overrightarrow{E} = \frac{1}{\sigma}\left(-C_A \mu_A \overrightarrow{B} + \nabla \times \overrightarrow{B}\right) \quad \Rightarrow \quad \vec{j} = C_A \mu_A \overrightarrow{B} + \sigma \overrightarrow{E}$$

Generalized 2nd law leads to the universal form for CME in "MHD+". Non-trivial since gauge field is dynamical. (CVE is not)

(Hou, Liu and Ren, PRD12)

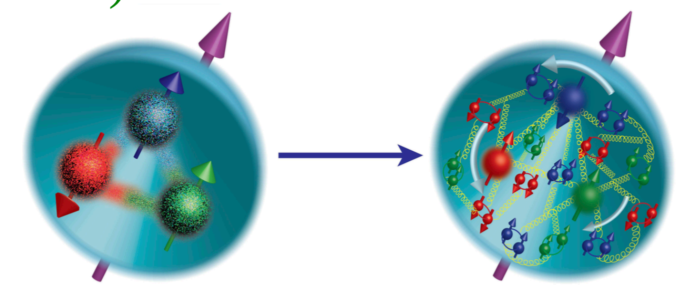
The relaxation rate:

$$\partial_\mu J^\mu_A = C_A B \cdot E. \quad \Rightarrow \quad \Gamma_A \propto C_A^2 \sim \delta^2_{(+)}$$

The double expansion is not only consistent, but also leads to interesting result.

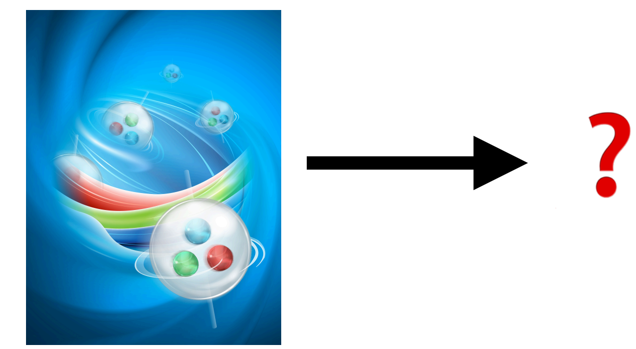
hydro+spin density

(Florkowski,Friman,Jaiswal, Speranza,PRC17;Hattori, Hongo, Huang, Matsuo and Taya, 1901.06615)
(See Becattini&Taya's talk)



Additional modes: spin density.

Gluon spin contributions can be significant in strongly interacting QGP.



Is spin density parametrically slow?

Yes if spin-orbital coupling is weak. How about sQGP?

Spin hydrodynamics: hydro+ (parametrically slow) tensor modes

Generalized entropy. Becattini, Florkowski, Speranza, PLB18

Prediction: slow equilibration of spin density would lead to a larger transport coefficient (not yet reported).

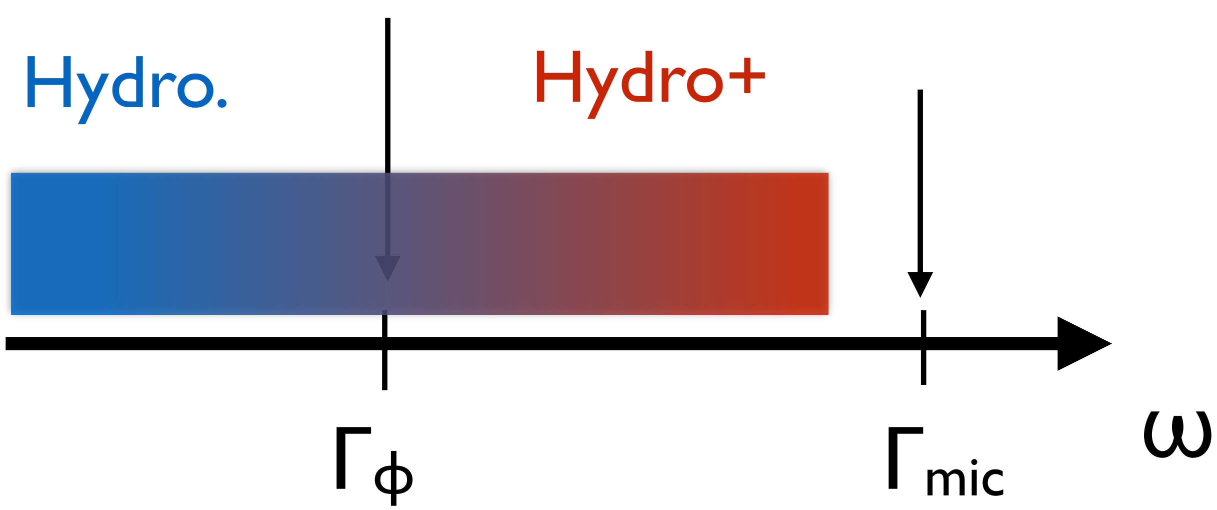
Conclusion and outlook

Conclusion and outlook I.

Understanding the criticality and chirality of QGP liquid motivates the extension of hydro. (or MHD) to include additional parametrically slow modes.

I present "Hydro+" as a general theory describing parametrically slow modes.

By demonstrating its features in the context of chiral/spin hydrodynamics, we see rich physics underlying fluid with parametrically slow modes



Conclusion and outlook II

If axial density/spin density are parametrically slow for QGP, it would lead to interesting effects for HIC.

Good progress is made on the numerical implication of "hydro+" for critical fluctuations based on different hydro codes (VH1+1, OSU hydro. and MUSIC).

Rajagopal-Ridgway-Weller-YY, in preparation; Lipei Du-Heinz; Chun.

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One can do the same for chiral/spin hydrodynamics in future.

Back-up

Outline

Review of "hydro+".

Application for chiral/spin hydrodynamics.

Conclusion.

Example: "hydro+" with one scalar mode

Stephanov-YY, 1712.10305, PRD '18

The constitutive relation at leading order.

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p_{(+)} \left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial)$$

$$p(\epsilon) \to p_{(+)}(\epsilon, \phi)$$

Similar for the viscous part

$$\zeta \to \zeta_{(+)}\,, \qquad \eta \to \eta_{(+)}$$

(The gradient of ϕ will only enter as $T^{\mu\nu}_{\mathcal{O}(\partial^2)} \sim \partial^\mu \phi \partial^\nu \phi$)